3d grafika

(3) Ray - object intersection

- ray Sphere intersection

ray $p(t) = e + td$ and an implicit surface $f(p) = 0$
intersection points occurs when points on the ray satisfy the implicit ~~[illegible]~~ equation

$$f(p(t)) = 0 \quad \text{or} \quad f(e+td) = 0$$

sphere with center $c\ (x_c, y_c, z_c)$ and radius R can be represented by the implicit equation

$$(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 - R^2 = 0$$

in vector form

$$(p-c)(p-c) - R^2 = 0$$

any point p that satisfies this equation is on the sphere. If we plug points on the ray $p(t) = e+td$ into this equation we get $(e+td-c)\cdot(e+td-c) - R^2 = 0$

rearranging terms $(d \cdot d)t^2 + 2d\cdot(e-c)t + (e-c) - R^2 = 0$

$$\Rightarrow At^2 + Bt + C$$

discriminant $B^2 - 4AC$ tells us how many solutions there are

$D<0$ dont't intersect

$D>0$ 1. ray enters sphere ~~and leaves~~ 2. it leaves

$D=0$ ray grazes the sphere

$$t = \frac{-d\cdot(e-c) \pm \sqrt{(d\cdot(e-c))^2 - (d\cdot d)\cdot((e-c)(e-c) - R^2)}}{(d\cdot d)}$$

in actual implementation you first check the value of discriminant before computing other terms

the normal vector at point p is given by the gradient $n = 2(p-c)$ unit normal $= (p-c)/R$

Ray triangle intersection
to intersect a ray with a parametric surface we set up a system of equations where Cartesian coordinates all match

$$x_e + t x_d = f(u,v)$$
$$y_e + t y_d = g(u,v) \quad \text{or} \quad e + td = f(u,v)$$
$$z_e + t z_d = h(u,v)$$

Parametric equations can be written in vector form vertices of a triangle are a, b, c and then the intersection occurs when $e + td = a + \beta(b-a) + \gamma(c-a)$

for some t, β and γ the intersection p will be at $e + td$ ~~[illegible]~~

we know the intersection is inside the triangle if $\beta > 0$, $\gamma > 0$ $\beta + \gamma < 1$

to solve for t, β and γ in equation we expend it form its vector form

$$x_e + t x_d = x_a + \beta(x_b - x_a) + \gamma(x_c - x_a)$$
$$y_e + t y_d = y_a + \beta(y_b - y_a) + \gamma(y_c - y_a)$$
$$z_e + t z_d = z_a + \beta(z_b - z_a) + \gamma(z_c - z_a)$$

fastest classic method to solve is Cramer's rule

$$\beta = \frac{\begin{vmatrix} x_a - x_e & x_a - x_c & x_d \\ y_a - y_e & y_a - y_c & y_d \\ z_a - z_e & z_a - z_c & z_d \end{vmatrix}}{|A|} \qquad \gamma = \frac{\begin{vmatrix} x_a - x_b & x_a - x_e & x_d \\ y_a - y_b & y_a - y_e & y_d \\ z_a - z_b & z_a - z_e & z_d \end{vmatrix}}{|A|}$$

$$t = \frac{\begin{vmatrix} x_a - x_b & x_a - x_c & x_a - x_e \\ y_a - y_b & y_a - y_c & y_a - y_e \\ z_a - z_b & z_a - z_c & z_a - z_e \end{vmatrix}}{|A|}$$

where matrix $A = \begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix}$ and $|A|$ denotes the determinat of A

linear systems with dummy variables

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} j \\ k \\ l \end{bmatrix}$$

...imabales gives us

$$p = \frac{j(ei - hf) + k(gf - di) + l(dh - eg)}{M} \qquad t = \frac{f(ak - jb) + e(jc - al) + d(bl - kc)}{M}$$

$$j = \frac{i(ak - jb) + h(jc - al) + g(bl - kc)}{(M)}$$

$$M = a(ei - hf) + b(gf - di) + c(dh - eg)$$