Lecture 2
Forward contracts

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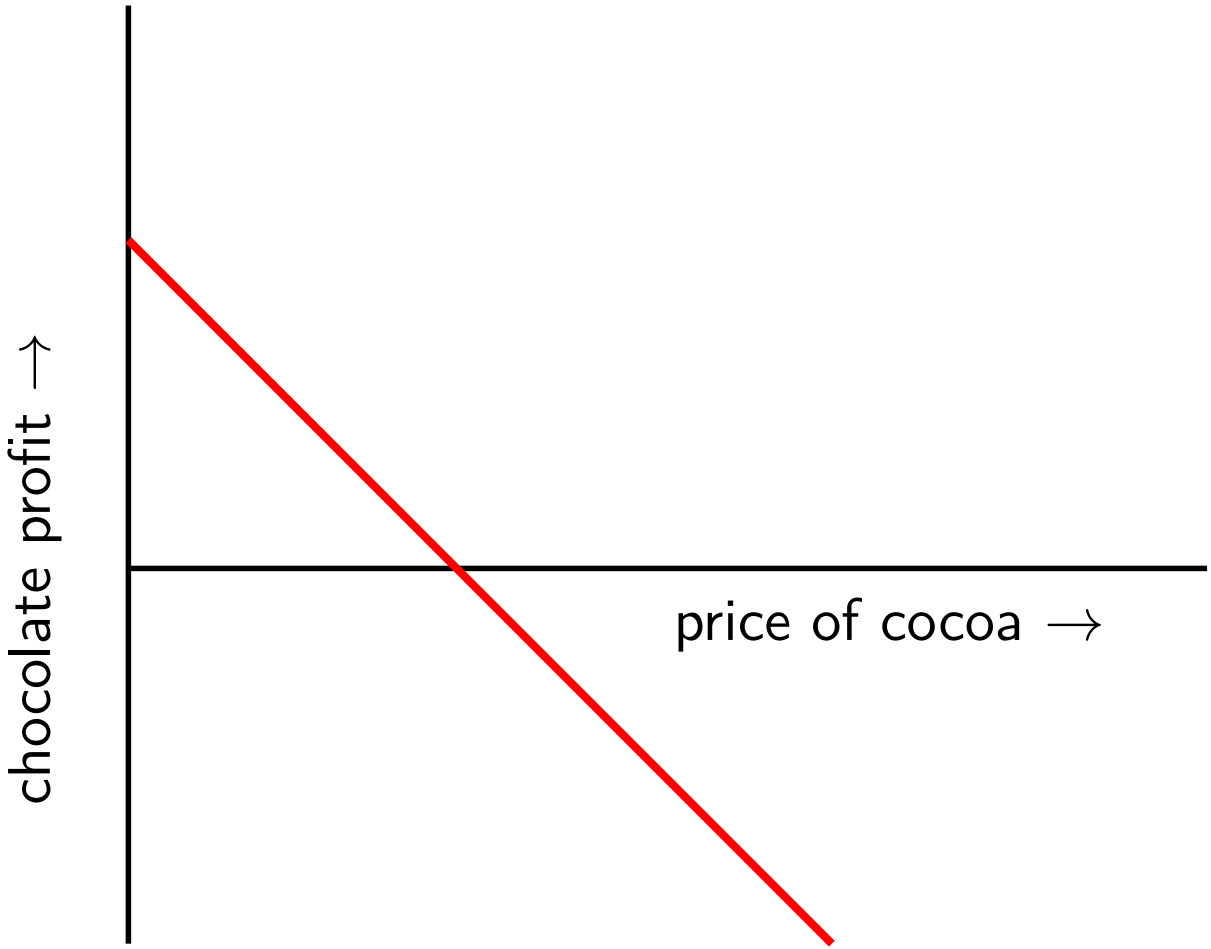
Chocolate and cocoa

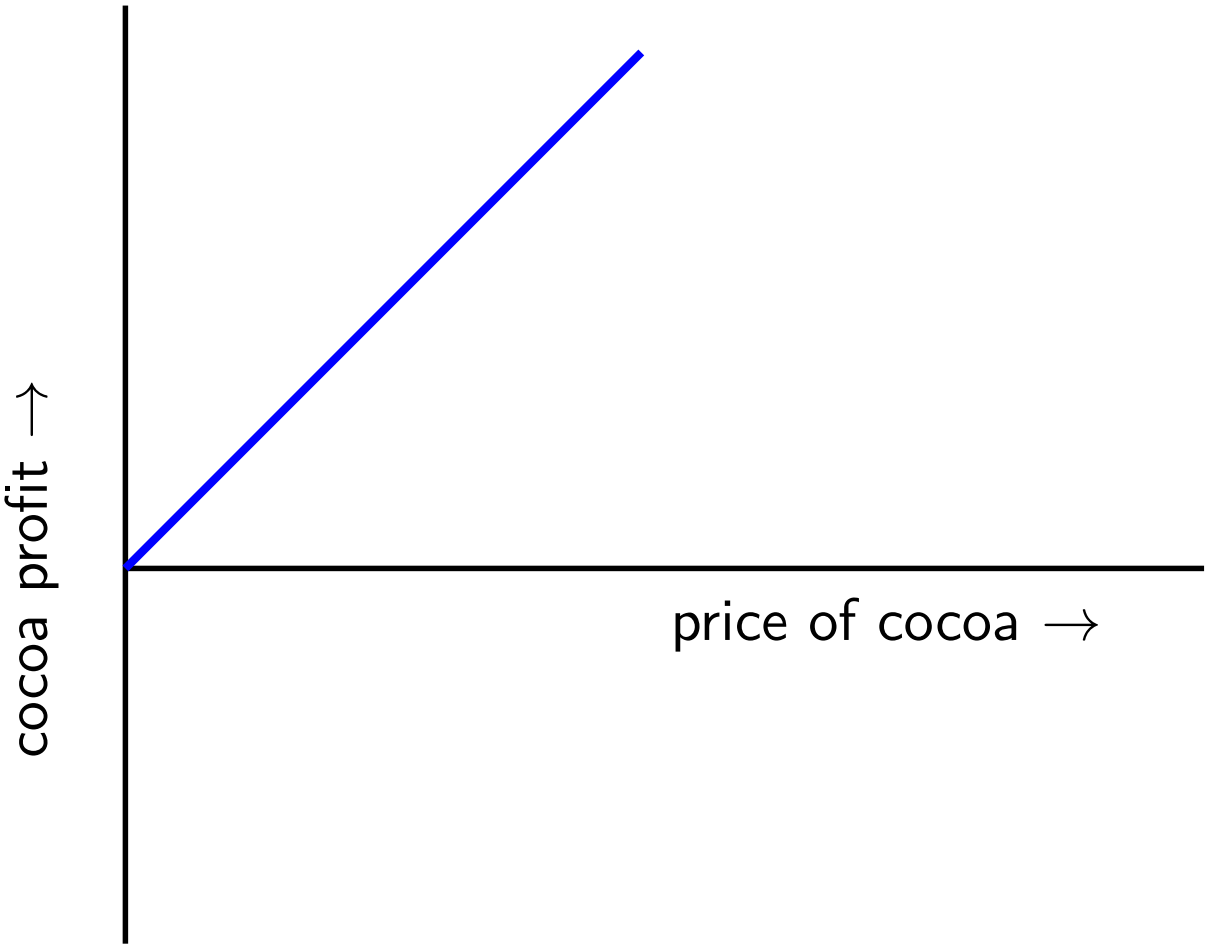
- The price of a Hershey chocolate bar is stable.
- But have you seen the price of cocoa?
- How does Hershey avoid passing on volatility to consumers?

Cocoa spot price

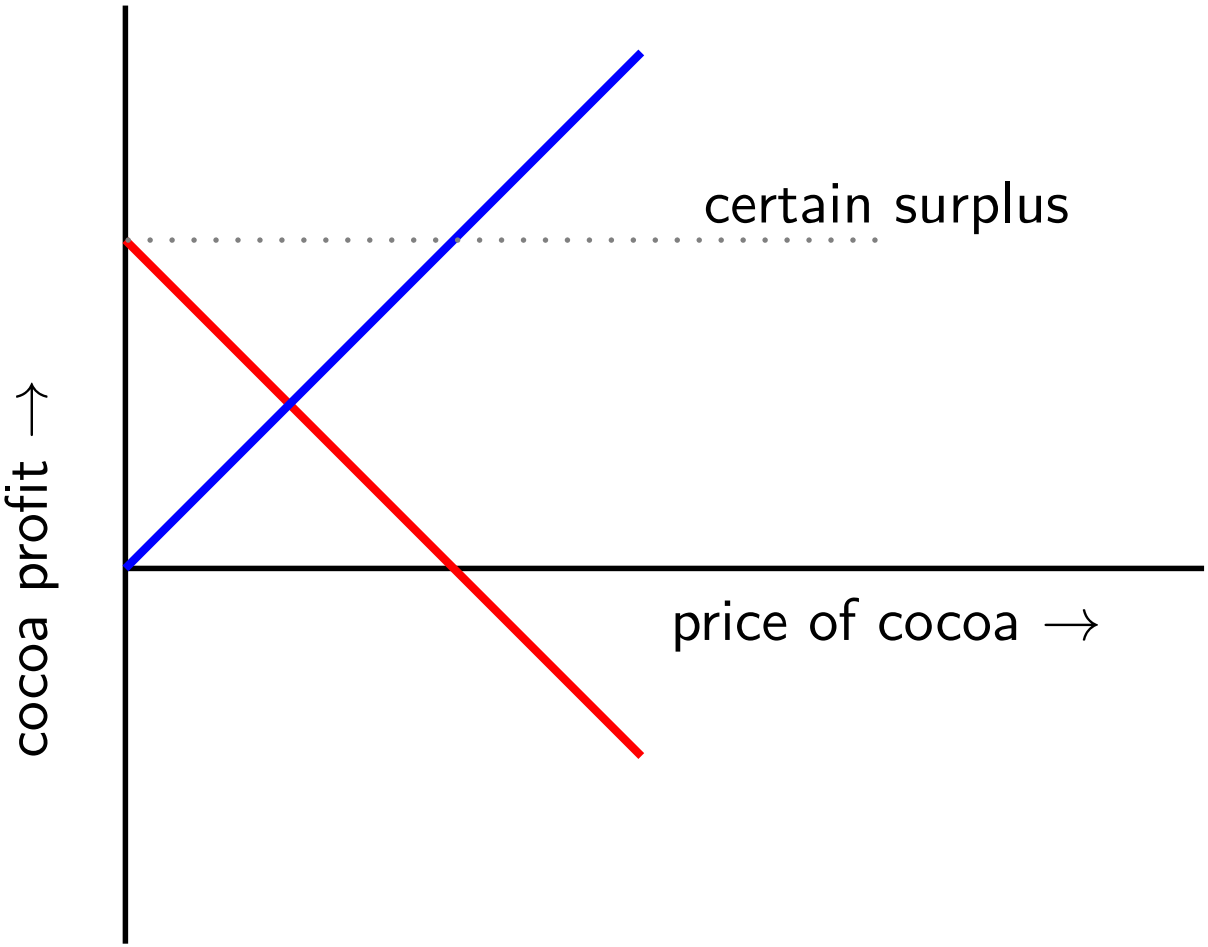
Hershey's unhedged exposure



Cocoa producer's unhedged exposure



Opposite exposures create demand for forward contracts



Roadmap: forward contracts

1. **Definitions and payoffs**
2. Application to risk management
3. Interest rates
4. Summary

Forward contracts

- A forward contract is an agreement to buy or sell an asset at a future date at a price specified today (called the forward price).
- A forward contract has two counterpaties:
  - The buyer (long) is obligated to pay the forward price.
  - The seller (short) is obligated to sell at the forward price.
- Typically, no money is exchanged when the contract is initiated. Contracts are usually cash-settled on the expiration date.

Forward contracts (sort of)

Agriculture

INDEX	UNITS	PRICE	CHANGE	%CHANGE	CONTRACT
C 1:COM **Corn (CBOT)**	USd/bu.	389.50	+3.75	+0.97%	Mar 2020
W 1:COM **Wheat (CBOT)**	USd/bu.	560.75	-3.75	-0.66%	Mar 2020
CC1:COM **Cocoa (ICE)**	USD/MT	2,583.00	-6.00	-0.23%	Mar 2020
CT1:COM **Cotton #2 (ICE)**	USd/lb.	71.40	+0.09	+0.13%	Mar 2020
LC1:COM **Live Cattle (CME)**	USd/lb.	127.53	-0.42	-0.33%	Apr 2020

Contract payoffs

- The payoff to a derivative security is the cash flow at expiration.

- The payoff to a long forward contract is:

$$X_T = S_T - F_{t,T}$$

where:

T = expiration date.

t = origination date (where $t = 0$ is today).

S_T = price of the underlying at date T.

$F_{t,T}$ = forward price agreed upon at date t for date T.

Timeline

- S_t spot.
- $F_{t,T}$ set.
- \$0 exchanged.

- S_T spot.
- $X_T = S_T - F_{t,T}$ to long.
- $-X_T = F_{t,T} - S_T$ to short.

Practice problem #1

The spot price of cocoa today is $S_0 = \$2,500$ per metric ton. The one-year forward price for cocoa is $F_{0,1} = \$2,750$. A buyer and seller agree to enter a forward contract for one ton of cocoa.

1. If the spot price in one year is $S_1 = \$2,600$, what is the payoff to the long party? The short party? How much money is exchanged at origination?

2. Plot the payoff to the long forward as a function of S_1. What is the minimum and maximum payoff?

3. Plot the payoff to the short forward as a function of S_1. What is the minimum and maximum payoff?

Practice problem #1 solutions

Practice problem #1 solutions

Practice problem #1 solutions

Roadmap: the basics of forward contracts

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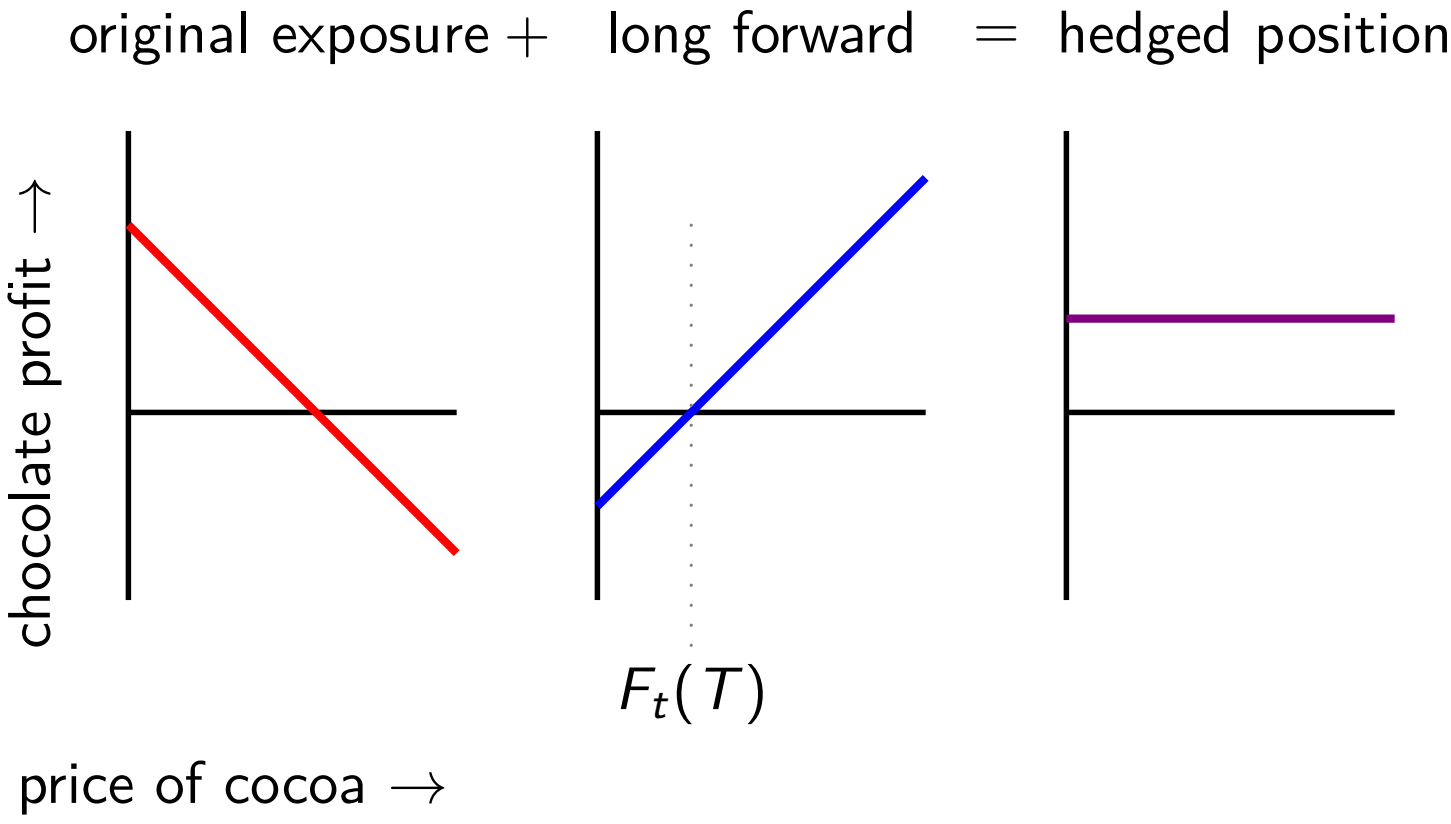
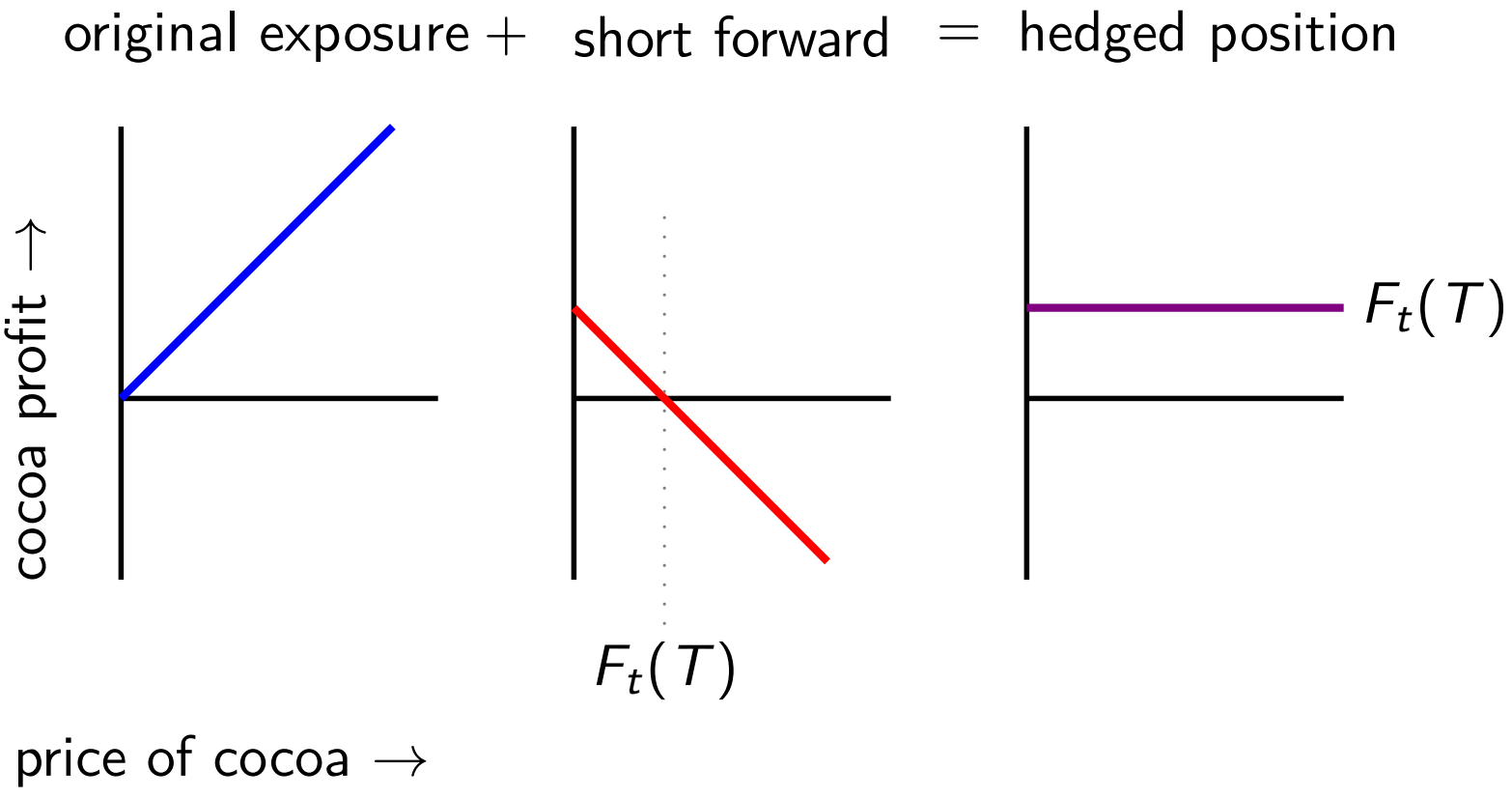
Forward contract insures both parties

- Hershey has a natural short position in cocoa.
- Cocoa producer has a natural long position in cocoa.
- Forward contract insures both parties against cocoa price risk.
- Risk sharing reduces profit volatility for both parties.

Hershey's hedged exposure



Cocoa producer's hedged exposure



Practice problem #2

A farmer is planning to grow 1 metric ton of cocoa. The farmer plans to sell the crop precisely one year from today. The cost of producing cocoa is $\$2,300$ per metric ton. Answer the following:

1. Is the farmer long or short cocoa? Graph the farmer's profit as a function of cocoa prices in one year.

2. How can the farmer hedge her exposure using forwards?

3. The one-year forward price is $\$2,600$. Graph the payoff of the forward as a function of cocoa prices in one year.

4. Suppose the farmer hedges using forwards. What is her profit if the spot price in one year is $\$2,000$? $\$2,600$?

Practice problem #2 solutions

Practice problem #2 solutions

Practice problem #2 solutions

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4. Summary

Technical note on interest rates

- Throughout the semester, we will assume there is a single risk-free interest rate of $r \geq 0$.
- Risk-free cash flows should be discounted at the risk-free rate.
- If $B_T \geq 0$ is a risk-free cash flow at time T, then the price of this cash flow at date $t < T$ is given by:

$$B_t = \begin{cases} \frac{B_T}{(1+r)^{T-t}} & \text{if } r \text{ is discretely compounded} \\ B_T e^{-r(T-t)} & \text{if } r \text{ is continuously compounded} \end{cases}$$

Practice problem #3

1. Suppose the discretely compounded risk-free rate is $r = 0.05$ and today is date $t = 0$. What is the price of a risk-free bond that pays \$100 at date $T = 1$?

2. Suppose today is date $t = 0$. The price of a risk-free bond that pays \$110 at date $T = 1$ is \$100. What is the discretely compounded risk-free rate?

3. Suppose the continuously compounded risk-free rate is $r = 0.10$ and today is date $t = 0$. What is the price of a risk-free bond that pays \$1 at date $T = 10$?

4. Suppose today is date $t = 0$. The price of a risk-free bond that pays \$100 at date $T = 3$ is \$95. What is the continuously compounded risk-free rate?

Practice problem #3 solutions

Practice problem #3 solutions

Roadmap: the basics of forward contracts

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Summary

- A forward contract is an agreement to buy or sell an asset at a future date at the forward price.
- Date T payoff of long forward originated at date 0:

$$X_T = S_T - F_{0,T}.$$

- Forwards can be used to hedge input and output price risk.
- Date 0 price of risk-free \$1 payoff at date T is $(1+r)^{-T}$ or e^{-rT}.

References

- Textbook chapters 2.1, 4.1, and 4.2.
- Hershey chocolate article is in the Wall Street Journal.
- Commodity prices from Bloomberg terminal and Bloomberg.
- Graphs are created using code on my Github.