Lecture 2
Forward contracts

David Sovich

University of Kentucky

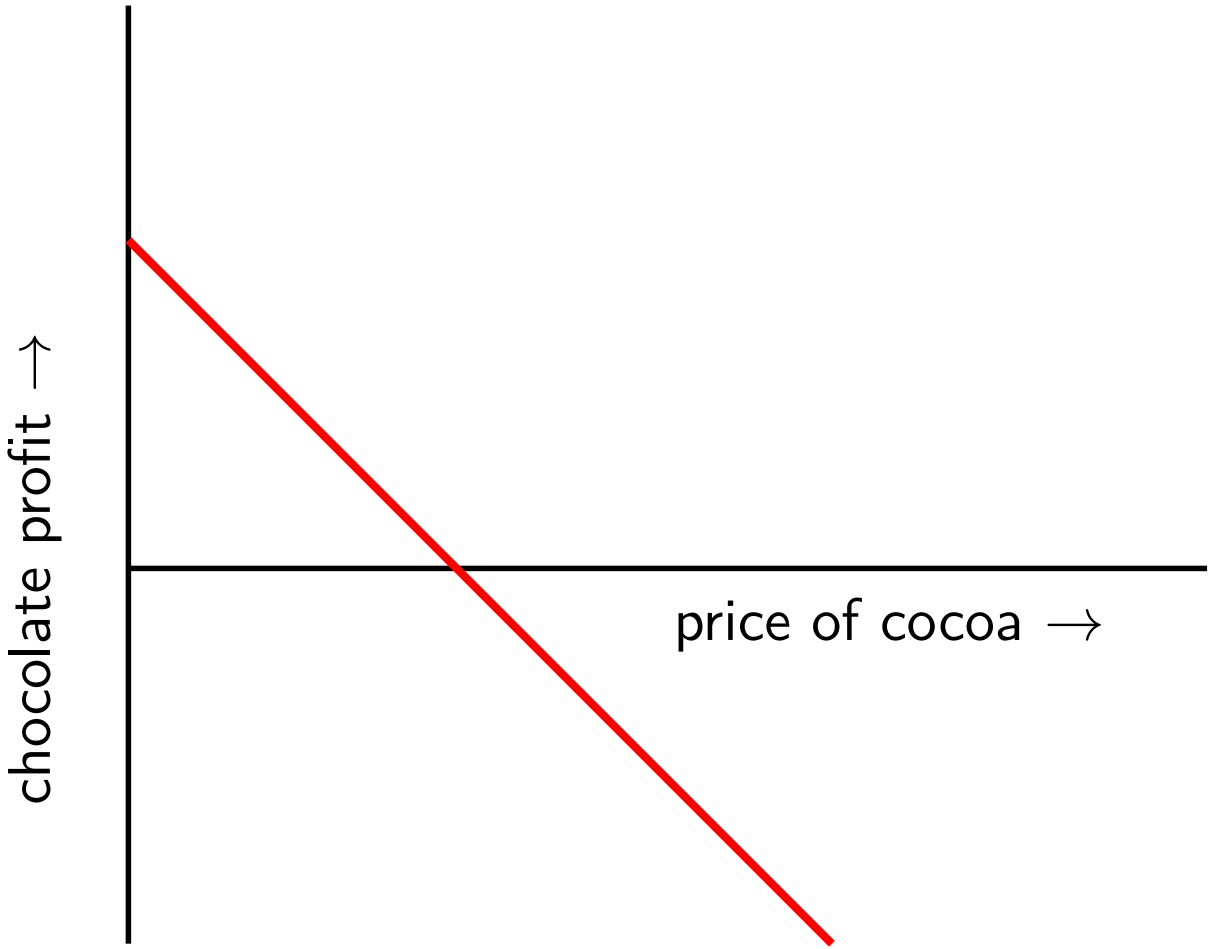
January 12, 2020

Chocolate and cocoa

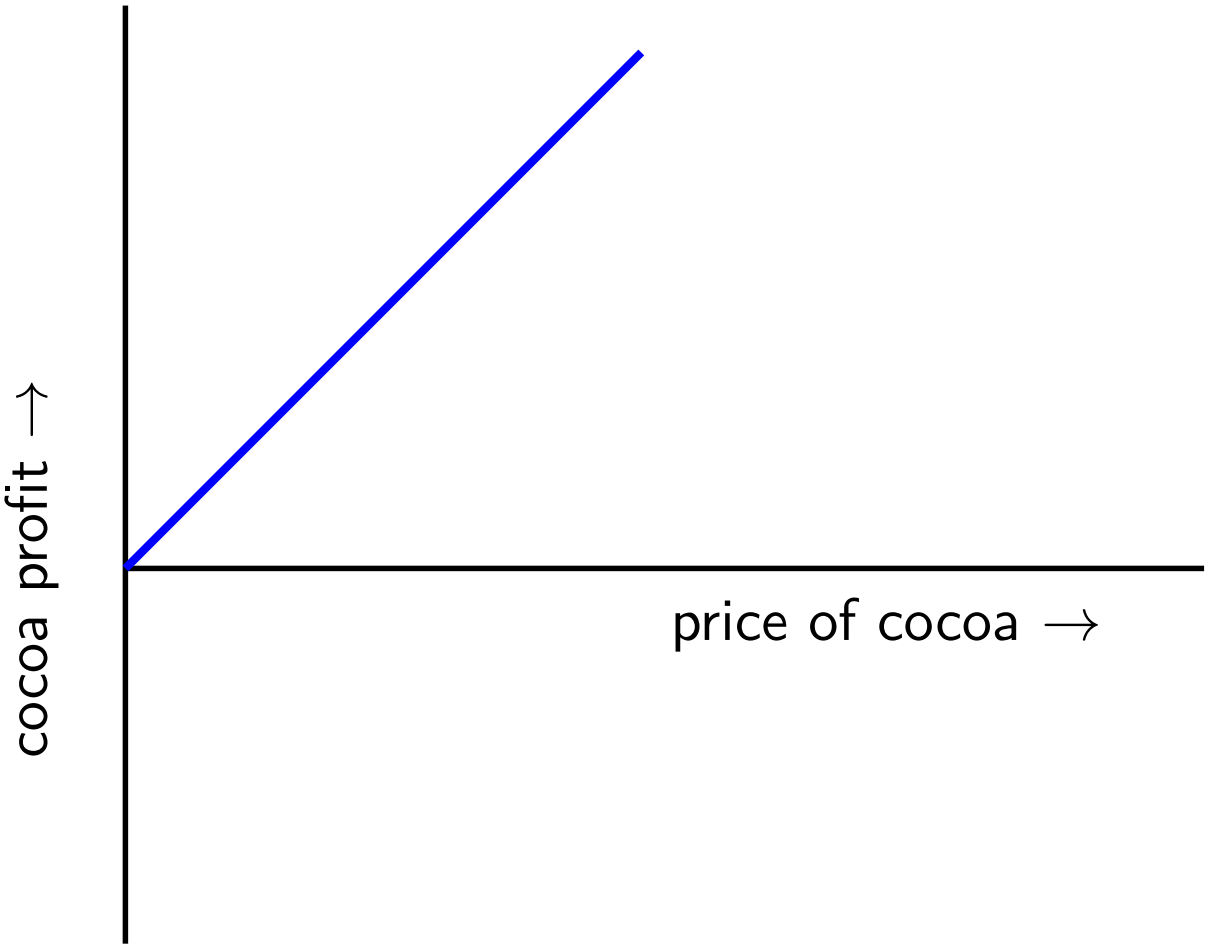
- The price of a Hershey chocolate bar is stable.
- But have you seen the price of cocoa?
- How does Hershey avoid passing on volatility to consumers?

Cocoa spot price

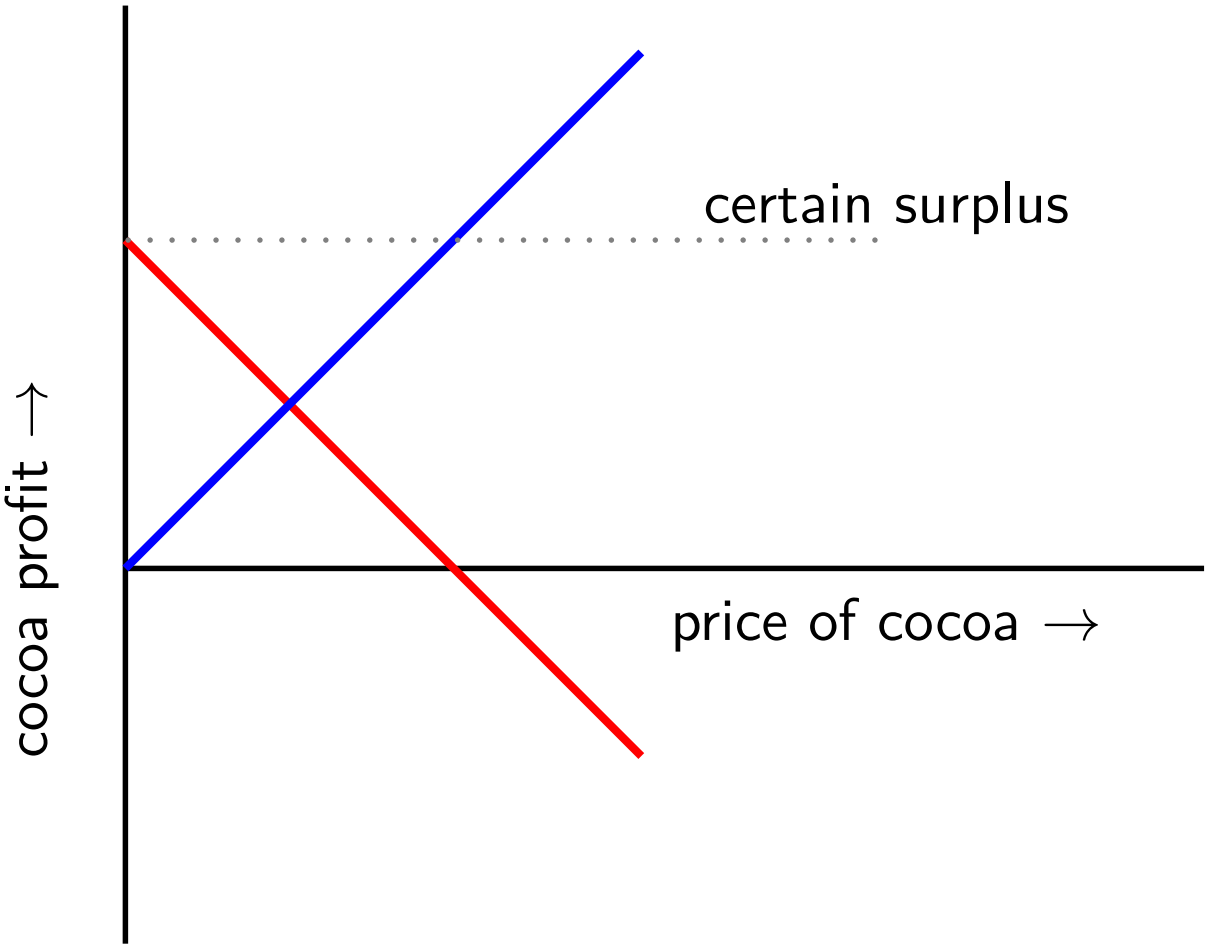
Hershey's unhedged exposure



Cocoa producer's unhedged exposure



Opposite exposures create demand for forward contracts



Roadmap: forward contracts

1. **Definitions and payoffs**
2. Application to risk management
3. Interest rates
4. Summary

Forward contracts

- A forward contract is an agreement to buy or sell an asset at a future date at a price specified today (called the forward price).
- A forward contract has two counterpaties:
  - The buyer (long) is obligated to pay the forward price.
  - The seller (short) is obligated to sell at the forward price.
- Typically, no money is exchanged when the contract is initiated. Contracts are usually cash-settled on the expiration date.

Forward contracts (sort of)

Agriculture

INDEX	UNITS	PRICE	CHANGE	%CHANGE	CONTRACT
C 1:COM **Corn (CBOT)**	USd/bu.	389.50	+3.75	+0.97%	Mar 2020
W 1:COM **Wheat (CBOT)**	USd/bu.	560.75	-3.75	-0.66%	Mar 2020
CC1:COM **Cocoa (ICE)**	USD/MT	2,583.00	-6.00	-0.23%	Mar 2020
CT1:COM **Cotton #2 (ICE)**	USd/lb.	71.40	+0.09	+0.13%	Mar 2020
LC1:COM **Live Cattle (CME)**	USd/lb.	127.53	-0.42	-0.33%	Apr 2020

Contract payoffs

- The payoff to a derivative security is the cash flow at expiration.

- The payoff, X_T, to a long forward contract is:

$$X_T = S_T - F_{t,T}$$

where:

T = expiration date.

t = origination date (where $t = 0$ is today).

S_T = price of the underlying at date T.

$F_{t,T}$ = forward price agreed upon at date t for date T.

Timeline

- S_t spot.
- $F_{t,T}$ set.
- \$0 exchanged.

- S_T spot.
- $X_T = S_T - F_{t,T}$ to long.
- $-X_T = F_{t,T} - S_T$ to short.

Practice problem #1

The spot price of cocoa today is $S_0 = \$2,500$ per metric ton. The one-year forward price for cocoa is $F_{0,1} = \$2,750$. A buyer and seller agree to enter a forward for one ton of cocoa.

1. If the spot price of cocoa in one year is $S_1 = \$2,600$, then what is the payoff to the long party? The short party? How much money is exchanged on the origination date?

2. Plot the payoff to the long forward as a function of S_1. What is the minimum and maximum payoff?

3. Plot the payoff to the short forward as a function of S_1. What is the minimum and maximum payoff?

Practice problem #1 solutions

Practice problem #1 solutions

Practice problem #1 solutions

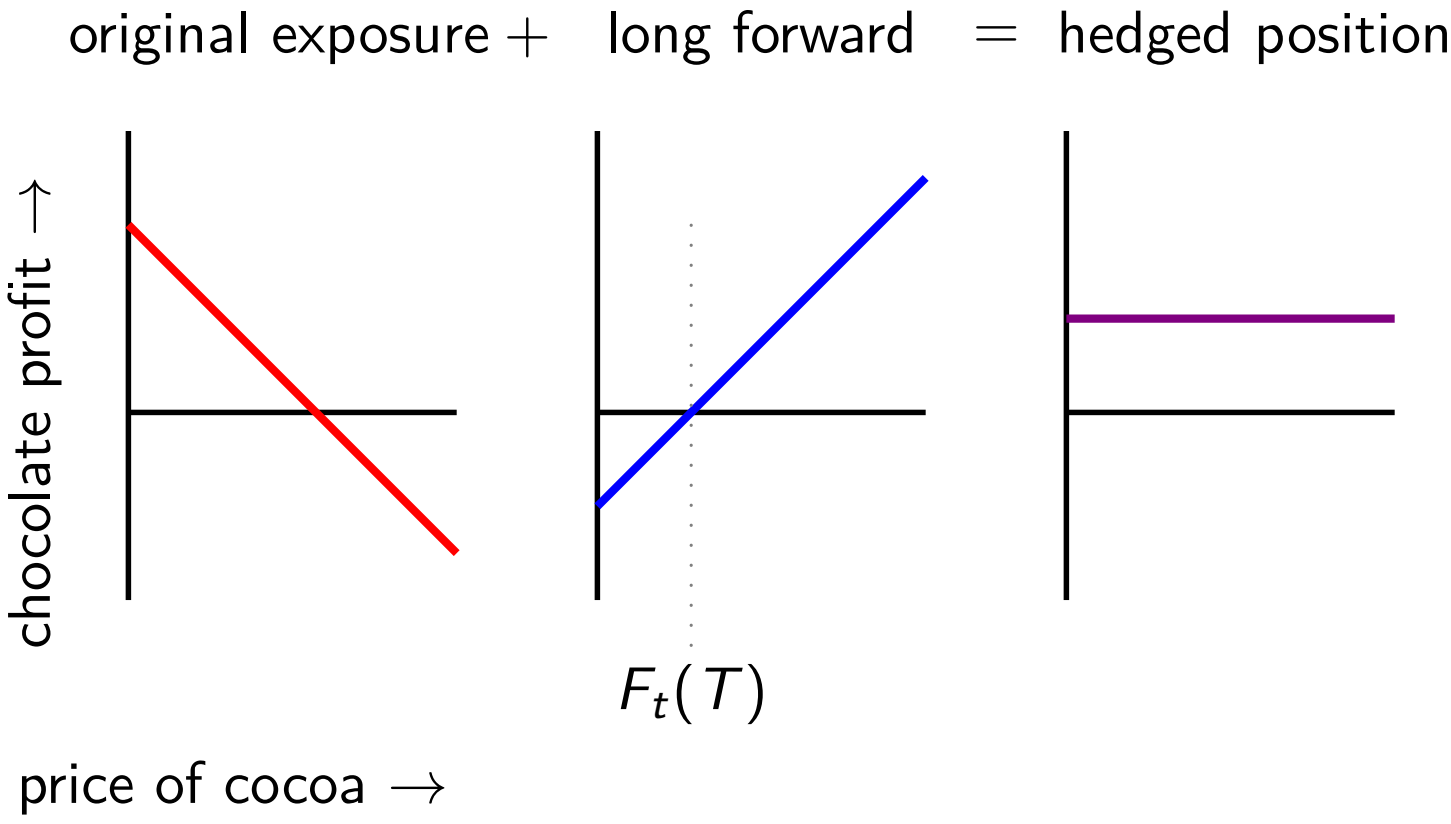
Roadmap: the basics of forward contracts

1. Definitions and payoffs
2. **Application to risk management**
3. Interest rates
4. Summary

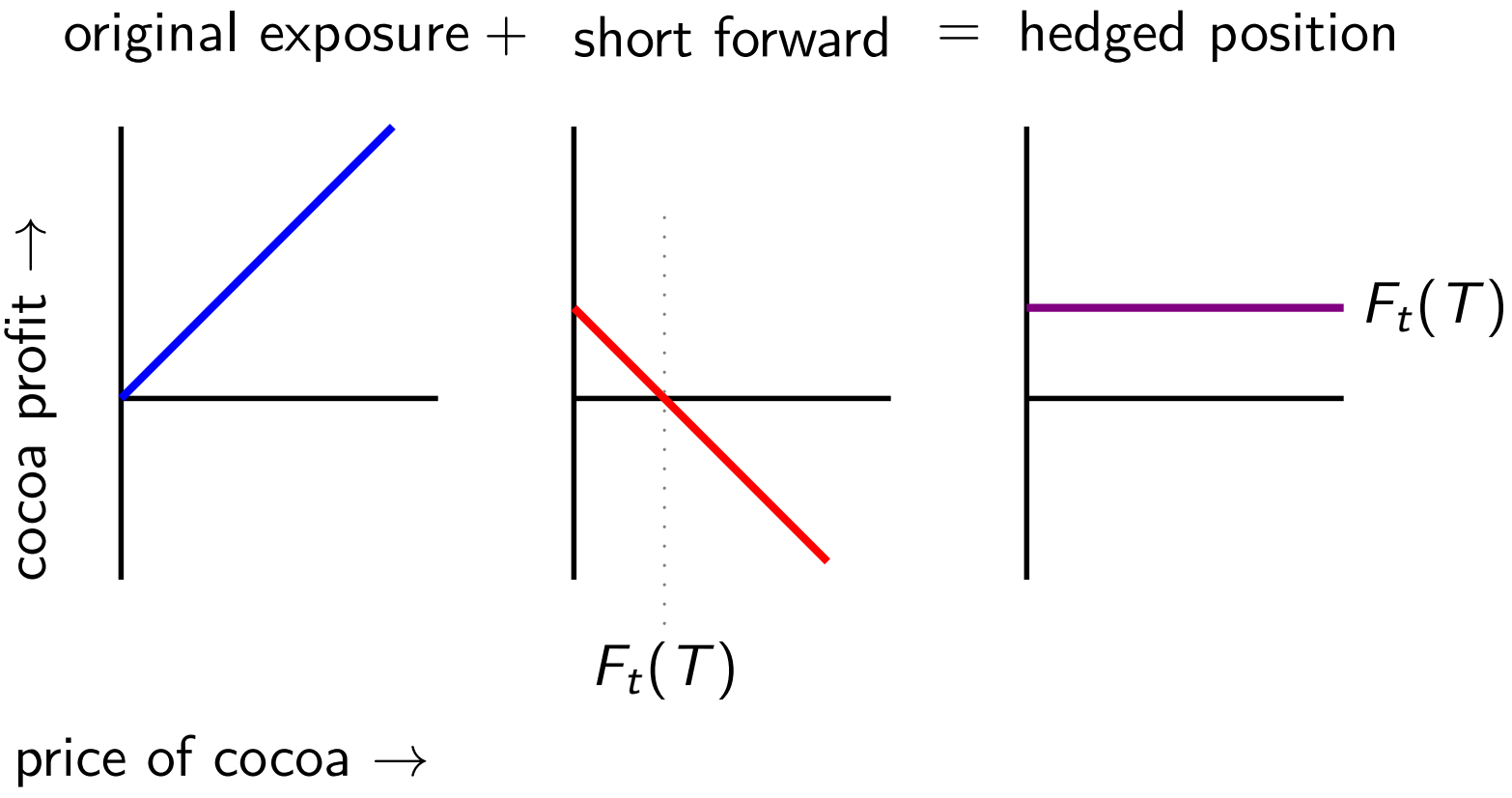
Forward contract insures both parties

- Hershey has a natural short position in cocoa.
- Cocoa producer has a natural long position in cocoa.
- Forward contract insures both parties against cocoa price risk.
- Risk sharing reduces profit volatility for both parties.

Hershey's hedged exposure



Cocoa producer's hedged exposure



Practice problem #2

A farmer is planning to grow 1,000 metric tons of cocoa over the next year. The farmer plans to sell the crop precisely one year from today. The cost of producing cocoa is \$2,300 per metric ton payable one year from today. Answer the following questions:

1. Is the farmer long or short cocoa? Graph the farmer's profit per ton as a function of cocoa prices in one year.

2. The one-year forward price for cocoa is \$2,600 per metric ton. How can the farmer hedge her exposure using forward contracts? Graph the payoff per metric ton of the forward contract as a function of cocoa prices in one year.

3. Suppose the farmer hedges her entire production using forward contracts. What is her profit per ton if the spot price in one year is \$2,000? \$2,600? \$2,900?

Practice problem #2 solutions

Practice problem #2 solutions

Practice problem #2 solutions

Roadmap: the basics of forward contracts

1. Definitions and payoffs
2. Application to risk management
3. **Interest rates**
4. Summary

Technical note on interest rates

- Throughout the semester, we will assume there is a single risk-free interest rate of $r \geq 0$.
- Risk-free cash flows should be discounted at the risk-free rate.
- If $B_T \geq 0$ is a risk-free cash flow at time T, then the price of this cash flow at date $t < T$ is given by:

$$B_t = \begin{cases} \frac{B_T}{(1+r)^{T-t}} & \text{if } r \text{ is discretely compounded} \\ B_T e^{-r(T-t)} & \text{if } r \text{ is continuously compounded} \end{cases}$$

Practice problem #3

Suppose the discretely compounded risk-free rate is $r = 0.05$ and today is date $t = 0$. Compute the prices of the following:

1. A risk-free security that pays \$1 at date $T = 1$.

2. A risk-free zero-coupon bond with a face value of \$100 and maturity of $T = 5$.

3. A $T = 3$ year risk-free coupon bond with annual coupons of \$5 and face value of \$100.

Practice problem #3 solutions

Practice problem #4

Suppose the continuously compounded risk-free rate is $r = 0.05$ and today is date $t = 0$. Compute the prices of the following:

1. A risk-free security that pays \$1 at date $T = 1$.

2. A risk-free zero-coupon bond with a face value of \$100 and maturity of $T = 5$.

3. A $T = 3$ year risk-free coupon bond with annual coupons of \$5 and face value of \$100.

Practice problem #4 solutions

Roadmap: the basics of forward contracts

1. Definitions and payoffs
2. Application to risk management
3. Interest rates
4. **Summary**

Summary

- A forward contract is an agreement to buy or sell an asset at a future date at the forward price.
- Date T payoff of long forward originated at date 0:

$$X_T = S_T - F_{0,T}.$$

- Forwards can be used to hedge input and output price risk.
- Date 0 price of risk-free \$1 payoff at date T is $(1+r)^{-T}$ or e^{-rT}.

References

- Textbook chapters XXX, XXX, and XXX.
- Hershey chocolate article is in the Wall Street Journal.
- Commodity prices from Bloomberg terminal and Bloomberg.
- Graphs are created using code on my Github.