Lecture 2
Forward contracts

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Chocolate and cocoa

- The price of a Hershey chocolate bar is stable.
- But have you seen the price of cocoa?
- How does Hershey avoid passing on volatility to consumers?

Cocoa price

- INSERT

Roadmap: the basics of forward contracts

1. **Definitions**
2. Payoffs
3. Application of forwards
4. Interest rates

Forward contracts

- A forward contract is an agreement to buy or sell an asset at a future date at a price specified today and called the forward price.
- A forward contract has two counterpaties:
  - The buyer (long) is obligated to pay the forward price.
  - The seller (short) is obligated to sell at the forward price.
- Typically, no money is exchanged when the contract is initiated. Contracts are usually cash-settled on the expiration date.

Forward contract timeline

- INSERT FIGURE. Communicate definitions of origination date (= today), expiration date (= tomorrow), price date agreed upon, etc.

Roadmap: the basics of forward contracts

1. Definitions
2. **Payoffs**
3. Application of forwards
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Contract payoffs

- The payoff to a derivative security is the cash flow at expiration.

- The payoff X_T to a long forward contract is:

$$X_T = S_T - F_{0,T}$$

where:

T = expiration date in years.

0 = origination date (i.e., today).

S_T = price of the underlying at date T.

$F_{0,T}$ = forward price agreed upon at date 0 for date T.

Forward contract timeline

- INSERT FIGURE. ADD IN LABELS WITH THE NOTATION.

Practice problem #1

The spot price of cocoa today is $S_0 = \$2,500$ per metric ton. The one-year forward price for cocoa is $F_{0,1} = \$2,750$. A buyer and seller agree to enter a forward for one ton of cocoa.

1. If the spot price of cocoa in one year is $S_1 = \$2,600$, then what is the payoff to the long party? The short party? How much money is exchanged on the origination date?

2. Plot the payoff to the long forward as a function of S_1. What is the minimum and maximum payoff?

3. Plot the payoff to the short forward as a function of S_1. What is the minimum and maximum payoff?

Practice problem #1 solutions

1. The payoff to the long party is:

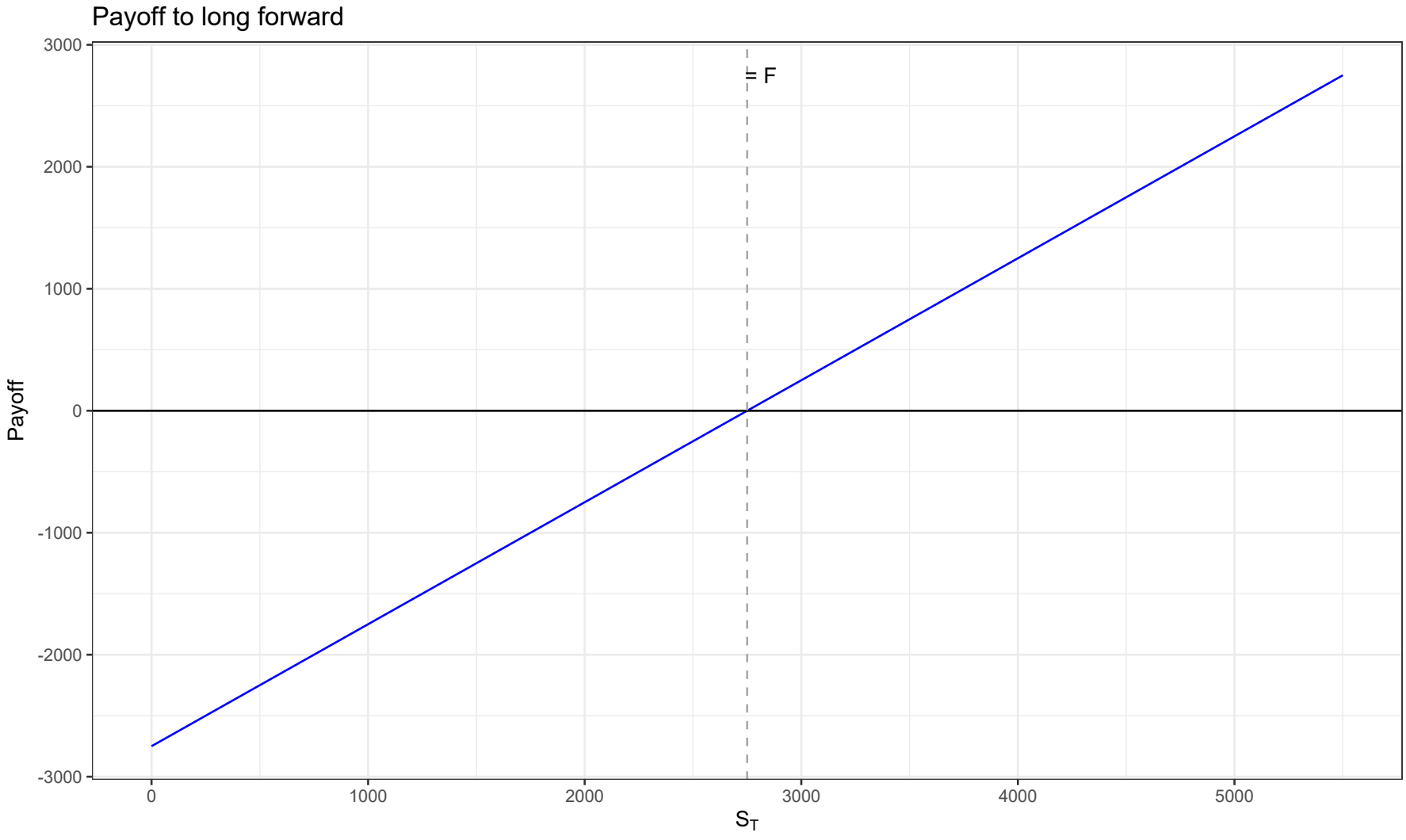
$$\begin{aligned} X_1 &= S_1 - F_{0,1} \\ &= 2,600 - 2,750 \\ &= -150 \end{aligned}$$

The payoff to the short party is:

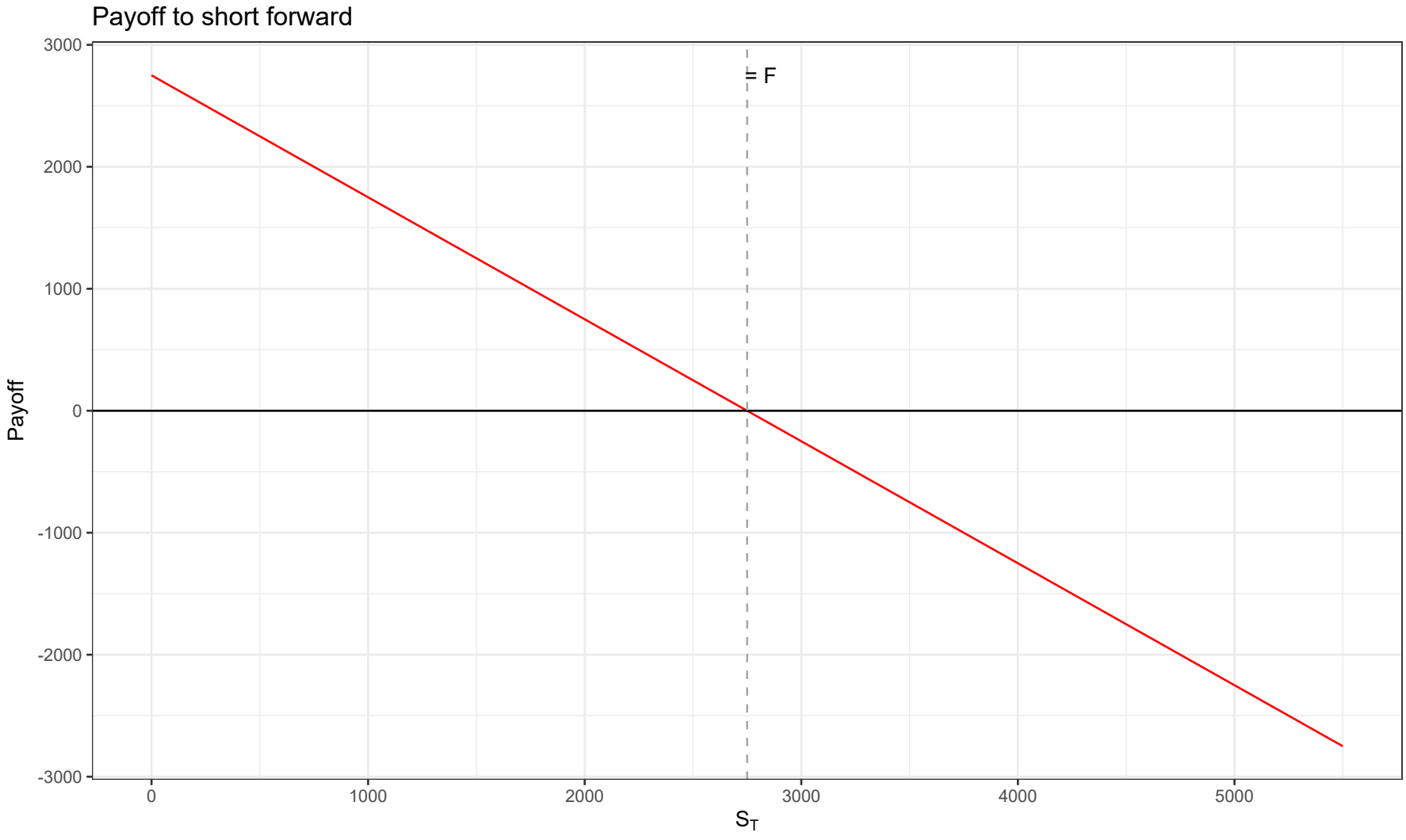
$$\begin{aligned} -X_1 &= F_{0,1} - S_1 \\ &= 150 \end{aligned}$$

No money is exchanged hands on the origination date.

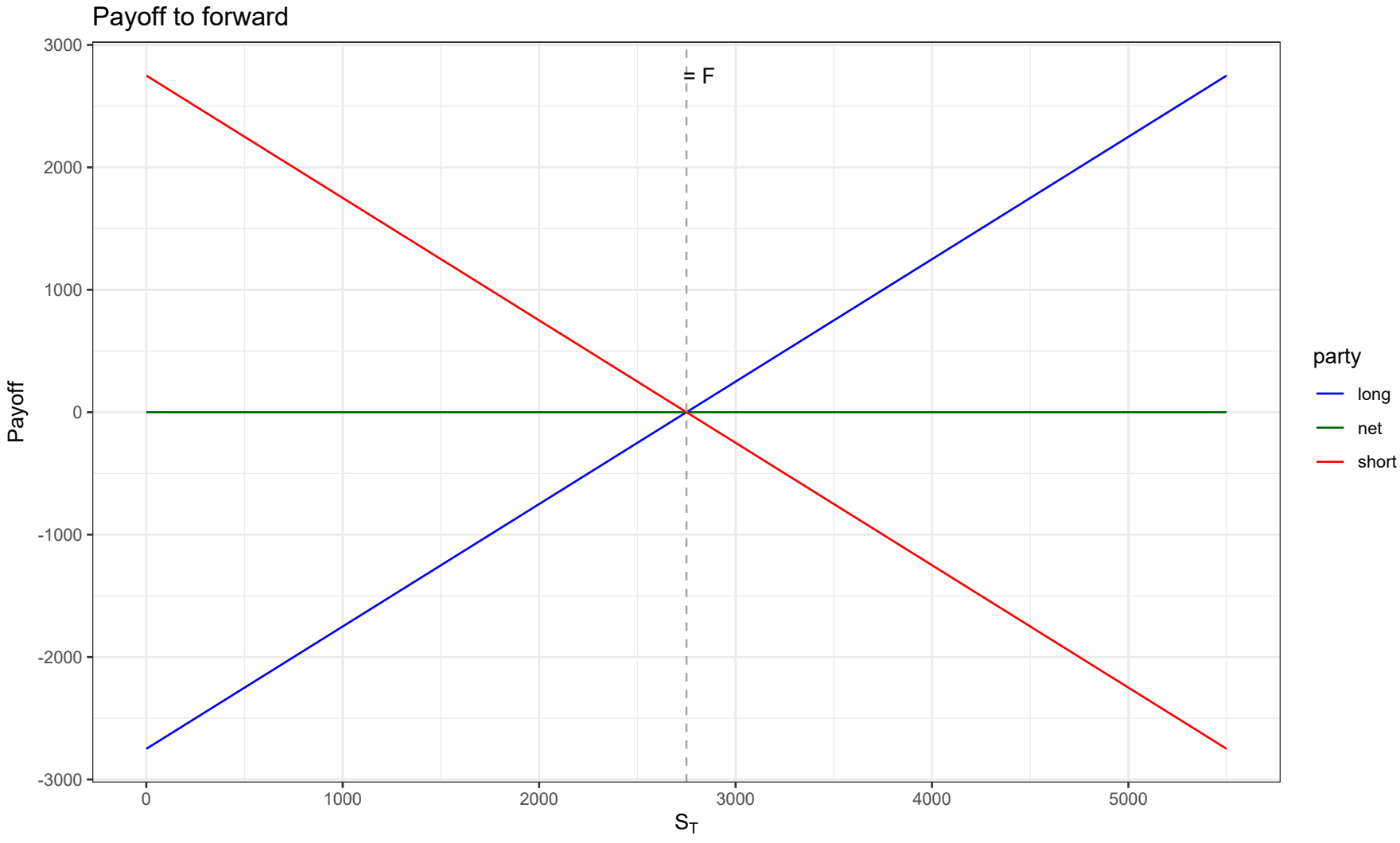
Practice problem #1 solutions



Practice problem #1 solutions



Practice problem #1 solutions



Roadmap: the basics of forward contracts

1. Definitions
2. Payoffs
3. **Applications of forwards**
4. Interest rates

Application of forwards

- Two applications: Risk management and speculation.
- Risk management give the example.
- Speculation is because of the lvereage.
- In our previous example, Hershey uses cocoa as an input in production. Hence, it has a natural short position in cocoa.

- On the other hand, farmers output cocoa and have natural long positions.

- A forward contract can be used to transfer risks between Hershey and cocoa farmers and capture certain surplus.

Roadmap: the basics of forward contracts

1. Definitions
2. Payoffs
3. Application of forwards
4. **Interest rates**

An aside on risk-free interest rates

- Throughout the semester, we will assume there is a single risk-free interest rate of $r \geq 0$.
- Risk-free cash flows should be discounted at the risk-free rate.
- The date 0 price of a risk-free payment of $B_T \geq 0$ at date T is:
  - Discretely compounded interest rate $= \frac{B_T}{(1+r)^{(T-t)}}$.
  - Continuously compounded interest rate $= B_T \cdot e^{-r(T-t)}$.

Practice problem #3

Suppose $r = 0.05$ and today is date $t = 0$. Compute the prices of the following securities assuming discrete compounding:

1. A risk-free security that pays \$1 at date $T = 1$.
2. A risk-free zero-coupon bond with a face value of \$100 and maturity of $T = 5$.
3. A $T = 3$ year risk-free coupon bond with annual coupons of \$5 and face value of \$100.

Practice problem #3 solutions

1. The price is $\$1 \cdot (1+0.05)^{-1} = \$0.952$.

2. The price is $\$100 \cdot (1+0.05)^{-5} = \$78.353$.

3. The price is $\sum_{i=1}^{3} 5 \cdot (1+0.05)^{-i} + \$100 \cdot (1+0.05)^{-3} = \$100$.

Practice problem #4

Suppose $r = 0.05$ and today is date $t = 0$. Compute the prices of the following securities assuming continuous compounding:

1. A risk-free security that pays \$1 at date $T = 1$.

2. A risk-free zero-coupon bond with a face value of \$100 and maturity of $T = 5$.

3. A $T = 3$ year risk-free coupon bond with annual coupons of \$5 and face value of \$100.

Practice problem #3 solutions

1. The price is $\$1 \cdot e^{-0.05} = \$0.951$.

2. The price is $\$100 \cdot e^{-0.05 \cdot 5} = \$77.88$.

3. The price is $\sum_{i=1}^{3} 5 \cdot e^{-0.05 \cdot i} + \$100 \cdot e^{-0.05 \cdot 3} = \$99.43$.

Summary

- A forward contract is an agreement to buy or sell an asset at a future date at the forward price.
- Date T payoff of long forward originated at date 0:

$$X_T = S_T - F_{0,T}.$$

- Forwards can be used to hedge input and output price risk.
- Date 0 price of date T risk-free \$1 payoff is $(1+r)^{-T}$ or e^{-rT}.

References

- Textbook chapters XXX, XXX, and XXX.
- Hershey chocolate article is in the Wall Street Journal.
- Cocoa prices are from Bloomberg screen XXX.
- Graphs are created using code on my Github.