Lecture 2
Forward contracts

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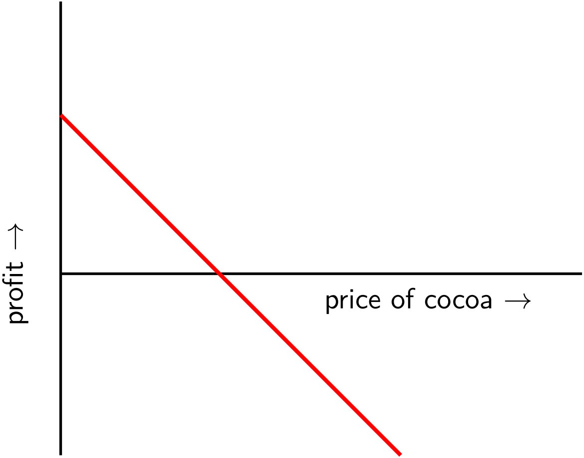
Spring 2020

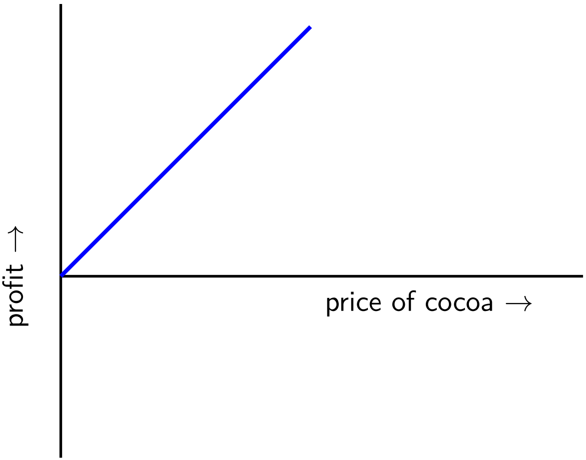
Chocolate and cocoa

- The price of a Hershey chocolate bar is stable.
- But have you seen the price of cocoa?
- How does Hershey avoid passing on volatility to consumers?

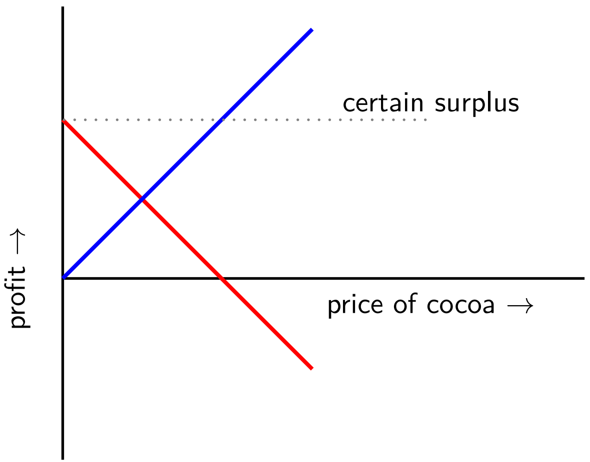
Hershey's unhedged exposure



Cocoa producer's unhedged exposure



Opportunity arises to share risks



Basic idea behind forward contract

- Hershey and the producer can lock-in a price for cocoa.
- This will fully insure both of them against cocoa price risk.
- Moreover, Hershey can keep chocolate bar prices stable.

Roadmap: forward contracts

1. **Definitions and payoffs**
2. Basic risk management
3. Interest rates
4. Summary

Forward contracts

- A forward contract is an agreement to buy or sell an asset at a future date at a price specified today (called the forward price).

Agriculture

INDEX	UNITS	PRICE	CHANGE	%CHANGE	CONTRACT	TIME (EST)
C 1:COM **Corn (CBOT)**	USd/bu.	389.25	+13.75	+3.66%	Mar 2020	1/17/2020
W 1:COM **Wheat (CBOT)**	USd/bu.	570.50	+5.25	+0.93%	Mar 2020	1/17/2020
CC1:COM **Cocoa (ICE)**	USD/MT	2,797.00	+85.00	+3.13%	Mar 2020	1/17/2020

Forward contract details

- Every forward contract has a buyer and a seller:
  - Buyer (long) is obligated to pay the forward price.
  - Seller (short) is obligated to sell at the forward price.
- Typically, no money is exchanged when the contract is initiated.
- Contracts are usually cash-settled on the expiration date.

Contract payoffs

- The payoff to a derivative is its cash flow or value at expiration.

- The payoff to a long forward contract is:

$$X_T = S_T - F_{t,T}$$

where:

T = expiration date in years (timeline).

t = origination date (usually $t = 0$ or "today").

S_T = price of the underlying asset at date T.

$F_{t,T}$ = forward price agreed upon at date t for date T.

Practice problem

Today's spot price of cocoa is $S_0 = \$2,500$ per ton. The one-year forward price for cocoa is $F_{0,1} = \$2,750$. A buyer and seller agree to enter a forward contract for one ton of cocoa.

1. Assume the future spot price is $S_1 = \$2,600$. Calculate the payoff to the long and short parties.
2. Plot a payoff diagram for the long forward. What is the minimum and maximum payoff?
3. Plot a payoff diagram for the short forward. What is the minimum and maximum payoff?

Practice problem scratch paper

Practice problem scratch paper

Practice problem scratch paper

Roadmap: the basics of forward contracts

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2. **Basic risk management**
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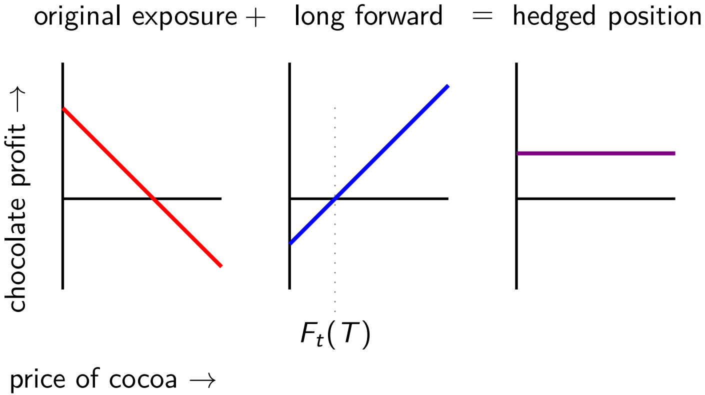
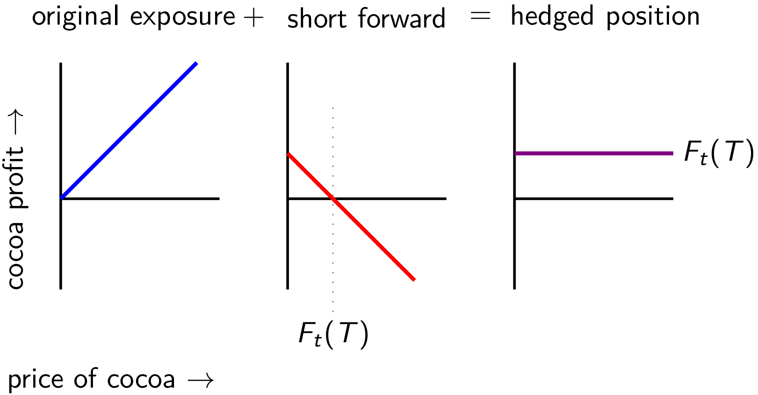
Hershey's hedged exposure



Cocoa producer's hedged exposure



Practice problem

A farmer is planning to grow one ton of cocoa. She will sell her crop one year from today. The cost of producing cocoa is $\$2,300$ per ton, payable in one year.

1. Is the farmer long or short cocoa? Plot the farmer's unhedged profit as a function of S_1.
2. The one-year forward price is $F_{0,1} = \$2,600$. Plot the payoff diagram of a long forward contract.
3. How should the farmer hedge using forwards? What is her hedged profit if $S_1 = \$2,000$? $S_1 = \$2,600$? $S_1 = \$0$?

Practice problem scratch paper

Practice problem scratch paper

Practice problem scratch paper

Roadmap: the basics of forward contracts

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Technical note on interest rates

- Throughout the semester, we will assume there is a single continuously compounded (c.c.) risk-free interest rate of r.

- Discounting risk-free cash flows using c.c:

$$\mathrm{PV}_{t,T}(\$1) = e^{-r(T-t)}$$

- Converting n-times per annum compounded rate r^* to c.c. rate:

$$r = n \cdot \log\left(1 + \frac{r^*}{n}\right)$$

Practice problem

The c.c. risk-free interest rate is $r = 0.05$. What is the price of a risk-free zero-coupon bond that pays \$100 in five years?

Practice problem scratch paper

Practice problem

The price of a risk-free zero-coupon bond that pays \$500 in ten years is \$400. What is the c.c. risk-free interest rate?

Practice problem scratch paper

Practice problem

The semi-annually compounded risk-free rate is 0.10. What is the equivalent continuously compounded risk-free rate?

Practice problem scratch paper

Roadmap: the basics of forward contracts

1. Definitions and payoffs
2. Basic risk management
3. Interest rates
4. **Summary**

Summary

- A forward contract is an agreement to buy or sell an asset at a future date at the forward price.

- Date T payoff of long forward originated at date 0:

$$X_T = S_T - F_{0,T}.$$

- Forwards can be used to hedge input and output price risk.

- Date 0 price of risk-free \$1 payoff at date T is e^{-rT}.

References

- Textbook chapters 2.1, 4.1, 4.2, and appendix B.2.
- Hershey chocolate article is in the Wall Street Journal.
- Commodity prices from Bloomberg terminal and Bloomberg.
- Slides are created using code on my Github.

Cocoa spot price

Return

Timeline

Return

origination

expiration

t T

- S_t spot.
- $F_{t,T}$ set.
- \$0 exchanged.

- S_T spot.
- $X_T = S_T - F_{t,T}$ to long.
- $-X_T = F_{t,T} - S_T$ to short.