Lecture 4
Frictions

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What assumptions have we made so far?

- Zero cost of storage.
  - Financial asset: no dividends.
  - Commodity: no physical storage costs.
- Investor can short the underlying.
- Investment asset – i.e., no production.
- No transactions costs, can borrow at risk-free rate, ...

What happens if we introduce frictions?

Roadmap: the forward price with frictions

1. **Dividends.**
2. Commodity storage.
3. Convenience.
4. Limits to arbitrage.
5. Summary.

Dividends

- Financial assets (e.g., stocks) commonly pay dividends.
- Let $d > 0$ be the dividend paid on a stock at date T.
- Will the dividend increase or decrease the forward price?
- Think about the forward price decomposition:

$$F_{0,T} = S_0 + \text{cost of carry}.$$

Not-so replicating portfolio

- Dividends do not affect the payoff to a long forward:

$$X_T = S_T - F_{0,T}.$$

- Stock payoff now includes dividend (price is ex-dividend):

$$S_T + d.$$

- Leveraged stock portfolio no longer replicates forward:

$$(S_T + d) - F_{0,T} > S_T - F_{0,T}.$$

Downward adjusted replicating portfolio

- Form the following portfolio at date 0:
  1. Long one share of stock.
  2. Short a risk-free bond with face-value $F_{0,T} + d$.
- The payoff to this portfolio at date T is:

$$\begin{aligned} Y_T &= (S_T + d) - (F_{0,T} + d) \\ &= S_T - F_{0,T}. \end{aligned}$$

- This is the same payoff as the long forward!

The forward price with a dividend

- Apply LoP: value of forward today=value of replicating portfolio.
- Value of forward today is zero by construction:

$$X_0 = 0.$$

- Value of long stock, short bond portfolio today is:

$$Y_0 = S_0 - (F_{0,T} + d)\, e^{-rT}.$$

- Equating $X_0 = Y_0$ and solving for the forward price yields:

$$F_{0,T} = S_0 e^{rT} - d.$$

Practice problem

The price of AAPL stock is \$300 per share. Apple has comitted with certainty to make a \$5 divided payment per share in 6 months. The c.c. risk-free rate is one percent.

1. What is the six-month forward price of AAPL stock?

2. Is there an arbitrage if $F_{0,0.5} = \$310$? If so, construct a portfolio to capitalize upon it.

3. Is there an arbitrage if $F_{0,0.5} = \$290$? If so, construct a portfolio to capitalize upon it.

Practice problem scratch paper

Practice problem scratch paper

Practice problem scratch paper

Cost of carry with dividends

- The asset holder enjoys the dividend while the forward does not.
- Thus, forward must be price-compensated for deferred purchase.
- Another way to say this is dividends reduce the cost of carry:

$$\text{cost of carry} = S_0 \left(e^{rT} - 1 \right) - d.$$

- Forward curve is up (down) sloping if interest $>$ ($<$) dividends.

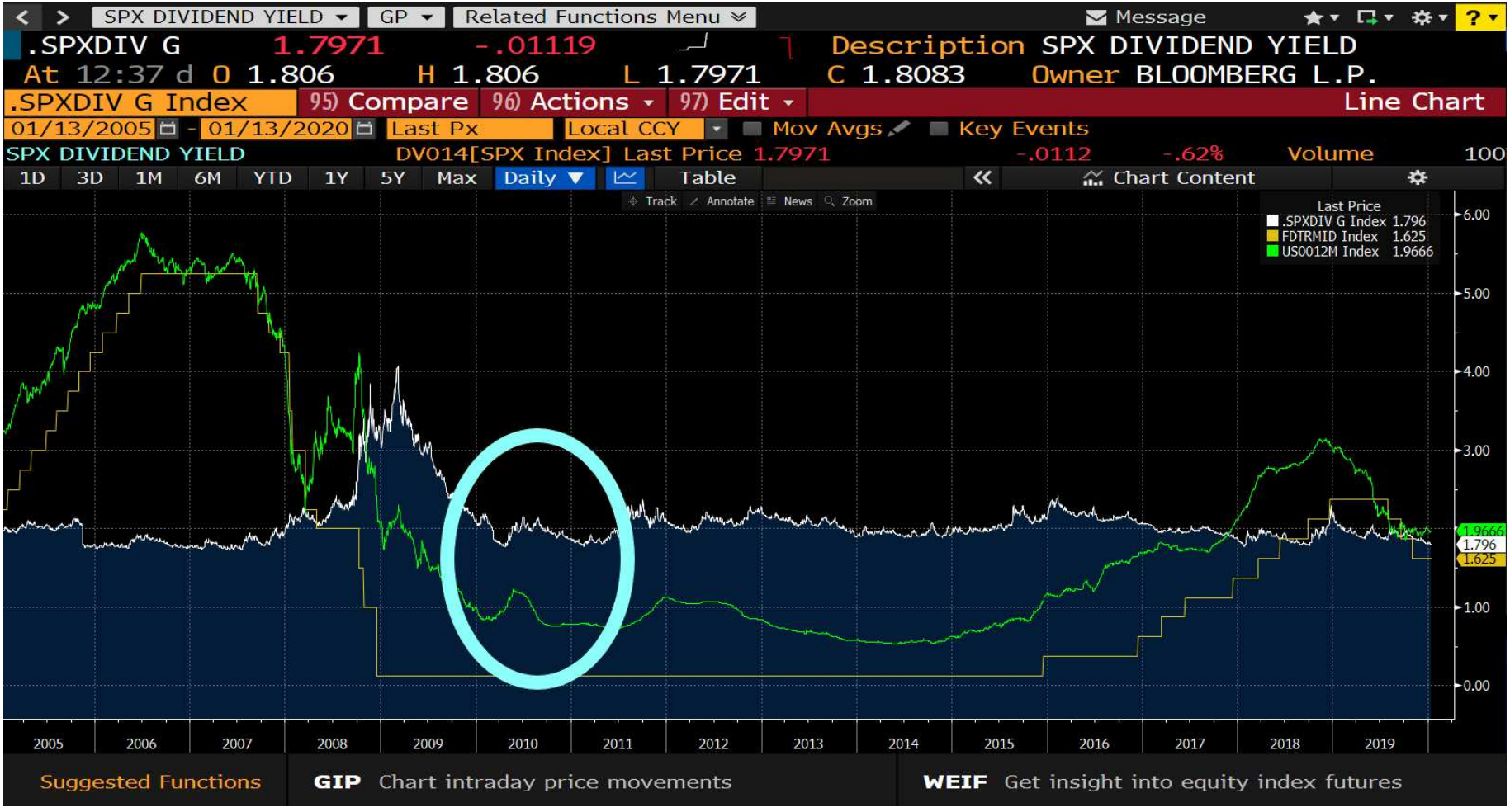
Upward sloping E-mini forward curve

S&P 500 dividend yield vs one year LIBOR

Downward sloping E-mini forward curve

Description	Last	Chg Settle	Time	Bid	Ask	Open Int	Volume	Yest Settle
21) Spot	s	--	Close	1135.60	1136.43			
22) Mar10	1132.25s	-13.00	Close	1131.75	1132.50	2454453	2031715	1145.25
23) Jun10	1127.50s	-12.75	Close	1120.00	1128.50	5936	1432	1140.25
24) Sep10	1122.75s	-13.00	Close	1075.00	1127.75	1394	19	1135.75
25) Dec10	1119.75s	-13.00	Close	1044.00	1154.75	11	3	1132.75
26) Mar11	1117.75s	-13.00	Close					1130.75

S&P 500 dividend yield vs one year LIBOR



General formulas for dividends

- For an asset with dividends $\{d_1, d_2, ..., d_n\}$ at dates $\{t_1, t_2, ..., t_n\}$ such that $0 \leq t_1 \leq ... \leq t_n \leq T$, the no-arbitrage forward price is:

$$F_{0,T} = S_0 e^{rT} - \sum_{i=1}^{n} d_i e^{r(T-t_i)}.$$

- For a financial asset with a continuous dividend yield $\delta > 0$:

$$F_{0,T} = S_0 e^{(r-\delta)T}.$$

- This latter formula is commonly used to price index forwards.

Practice problem

The price of the S&P 500 index is $\$3,300$ per share. The dividend yield on the S&P 500 index $\delta = 0.018$ per year with certainty. The c.c. risk-free rate is four percent. What is the ten year forward price for the S&P 500 index?

Practice problem scratch paper

Roadmap: the forward price with frictions

1. Dividends.
2. **Commodity storage.**
3. Convenience.
4. Limits to arbitrage.
5. Summary.

Commodity storage costs

- Commodities are costly to store (e.g., grain silos), if at all.
- Let $c > 0$ be the storage cost for date 0 to T, paid at T.
- Will the storage cost increase or decrease the forward price?
- Think about the forward price decomposition:

$$F_{0,T} = S_0 + \text{cost of carry}.$$

Not-so replicating portfolio

- Storage costs do not affect the payoff to a long forward:

$$X_T = S_T - F_{0,T}.$$

- Commodity payoff now includes storage cost:

$$S_T - c.$$

- Leveraged commodity portfolio no longer replicates forward:

$$(S_T - c) - F_{0,T} < S_T - F_{0,T}.$$

Upward adjusted replicating portfolio

- Form the following portfolio at date 0:
  1. Long one unit of the commodity.
  2. Short a risk-free bond with face-value $F_{0,T} - c$.
- The payoff to this portfolio at date T is:

$$\begin{aligned} Y_T &= (S_T - c) - (F_{0,T} - c) \\ &= S_T - F_{0,T}. \end{aligned}$$

- This is the same payoff as the long forward!

The forward price with storage costs

- Apply LoP: value of forward today=value of replicating portfolio.
- Value of forward today is zero by construction:

$$X_0 = 0.$$

- Value of long commodity, short bond portfolio today is:

$$Y_0 = S_0 - (F_{0,T} - c)\, e^{-rT}.$$

- Equating $X_0 = Y_0$ and solving for the forward price yields:

$$F_{0,T} = S_0 e^{rT} + c.$$

Practice problem

The price of gold is $\$1,500$ per ounce. For safety, gold must be stored in Fort Knox at a cost of \$45 per ounce per year, paid at the end of each year. The c.c. risk-free rate is five percent and the expected future spot price in one year is $\$1,525$.

1. What is the one-year forward price of gold?

2. What is the cost of carry for a long gold position?

2. Is there an arbitrage if $F_{0,1} = \$1,525$? If so, construct a portfolio to capitalize upon it.

3. Is there an arbitrage if $F_{0,1} = \$2,000$? If so, construct a portfolio to capitalize upon it.

Practice problem scratch paper

Practice problem scratch paper

Practice problem scratch paper

Cost of carry with storage costs

- The asset holder pays storage cost while the forward does not.
- Thus, forward must provide price compensation for deferring.
- Another way to say this is storage costs increase cost of carry:

$$\text{cost of carry} = S_0\left(e^{rT} - 1\right) + c.$$

- Forward curve is upward sloping based on this formula.

Upward sloping gold forward curve

Description	Last	Chg Settle	Time	Bid	Ask	Open Int	Volume	Yest Settle
21) Feb20	↓1550.10	-10.00	12:33	1550.00	1550.10	460509	256646	1560.10
22) Apr20	1556.40	-10.00	12:33	1556.30	1556.40	201731	54281	1566.40
23) Jun20	1562.00	-10.10	12:30	1561.90	1562.10	68517	4240	1572.10
24) Aug20	1568.00	-9.50	12:06	1567.30	1567.40	22226	1342	1577.50
25) Oct20	1575.70	-6.60	8:31	1572.10	1572.30	7513	29	1582.30
26) Dec20	1577.90	-9.10	12:19	1576.80	1576.90	18646	2433	1587.00
27) Feb21	1581.90	-9.60	10:07	1581.00	1581.20	4572	217	1591.50
28) Apr21		--	1/10	1584.60	1585.30	530	111	1595.40
29) Jun21		--	1/10	1588.40	1589.30	1757	21	1599.10
30) Aug21		--	1/10	1585.10	1600.90	1		1603.30
31) Oct21		--	1/10	1589.30	1605.60			1607.20
32) Dec21	1606.20	-5.00	8:52	1600.50	1601.10	806	1	1611.20
33) Jun22		--	1/10	1598.10	1623.10	4		1620.10
34) Dec22		--	1/10	1620.00	1627.50	47	1	1629.00
35) Jun23		--	1/10	1610.00	1644.10	5		1638.40

People actually perform storage arbitrage in real-life

The Wall Street Journal, "Oil Stockpile on Ship Shrinks"

THE WALL STREET JOURNAL.

Oil Stockpile on Ships Shrinks

Less oil is floating at sea on supertankers like the Maran Centaurus. AGENCE FRANCE-PRESSE/GETTY IMAGES

By Guy Chazan

Updated Feb. 1, 2010 12:01 am ET

General formulas for storage

- For storage costs $\{c_1, c_2, ..., c_n\}$ at dates $\{t_1, t_2, ..., t_n\}$ such that $0 \leq t_1 \leq ... \leq t_n \leq T$, the no-arbitrage forward price is:

$$F_{0,T} = S_0 e^{rT} + \sum_{i=1}^{n} c_i e^{r(T-t_i)}.$$

- For a commodity with a continuous storage cost $c > 0$:

$$F_{0,T} = S_0 e^{(r+c)T}.$$

Roadmap: the forward price with frictions

1. Dividends.
2. Commodity storage.
3. **Convenience.**
4. Limits to arbitrage.
5. Summary.

An arbitrage opportunity?

- It seems storable commodities should always be in contango.
  - Contango means an upward-sloping forward curve.
- But sometimes commodities are in backwardation.
  - Backwardation means a downward-sloping forward curve.
- Is this an obvious arbitrage?

Lean hogs in backwardation

The convenience yield

- Commodities may yield non-monetary benefits to owners.
- For example, Hershey needs cocoa to produce chocolate.
- Convenience yield is the non-monetary benefit of ownership.
- Convenience yield pushes down forward prices (like dividends).

Forward pricing with storage and convenience

- Let $y > 0$ be the c.c. convenience yield.
- The forward price with storage and convenience is:

$$F_{0,T} = S_0 e^{(r+c-y)T}.$$

- Forward curve will be in contango (backwardation) whenever storage costs and interest are $>$ ($<$) the convenience yield.

Practice problem

The spot price for cocoa is $\$2,000$ per ton. The c.c. risk-free rate is one percent and the c.c. storage cost is two percent. The six-month futures price of cocoa is $\$1,900$ per ton. What is the convenience yield for cocoa over this six month period?

Practice problem scratch paper

Roadmap: the forward price with frictions

1. Dividends.
2. Commodity storage.
3. Convenience.
4. **Limits to arbitrage.**
5. Summary.

Effects of limits to arbitrage

- *Transactions costs:* no-arbitrage "price bounds".
- *Short-sale restrictions:* no-arbitrage upper bound.
- *Barely-storable assets:* seasonality or weird stuff.
- *Constrained intermediaries:* persistent violations.

Roadmap: the forward price with frictions

1. Dividends.
2. Commodity storage.
3. Convenience.
4. Limits to arbitrage.
5. **Summary.**

Summary

- No-arbitrage forward price of an investment asset:
  - Non-dividend: $F_{0,T} = S_0 e^{rT}$.
  - Dividend: $F_{0,T} = S_0 e^{rT} - d$.
  - Storage cost: $F_{0,T} = S_0 e^{rT} + c$.
  - Convenience yield: $F_{0,T} = S_0 e^{(r+c-y)T}$.
- Limits to arbitrage create deviations from the model.

References

- Textbook chapters 5.2, 5.3, 6.1, 6.3, 6.4, and 6.6.
- Forward curves are from Bloomberg terminal.
- CDS basis graph is from Duffie (2010).
- Slides are created using code on my Github.

Transactions costs

- Setup for stock and forward:

  1. Bid (b) and ask (a) prices: $S^b < S^a$ and $F^b < F^a$.

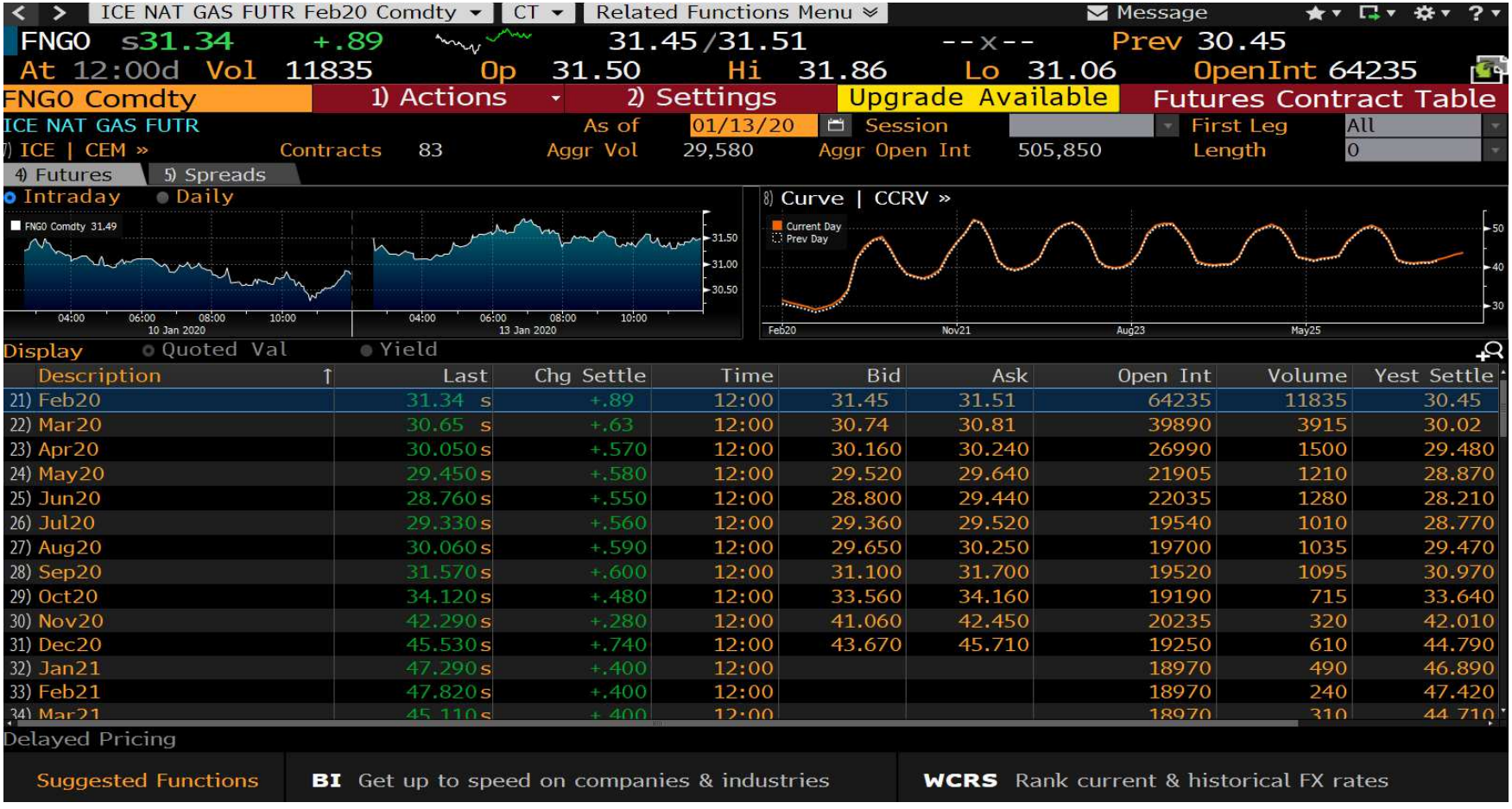
  2. Transaction cost of $k > 0$ (settlement is free).

  2. Borrow (b) and lending (ℓ) rates: $r^b > r^\ell$.

- The no-arbitrage forward price bounds are (Return):

$$\underbrace{\left(S_0^b - 2k\right) e^{r^\ell T}}_{< S_0 e^{rT}} \leq F_{0,T} \leq \underbrace{\left(S_0^a + 2k\right) e^{r^v T}}_{> S_0 e^{rT}}.$$

Natural gas seasonality



Return

Electricity intra-day seasonality

TABLE 6.7 Day-ahead price, by hour, for 1 megawatt-hour of electricity in New York City, March 21, 2011.

Time	Price	Time	Price	Time	Price	Time	Price
0000	$36.77	0600	$44.89	1200	$53.84	1800	$56.18
0100	$34.43	0700	$58.05	1300	$51.36	1900	$63.51
0200	$32.22	0800	$52.90	1400	$50.01	2000	$54.99
0300	$32.23	0900	$54.06	1500	$49.55	2100	$47.01
0400	$32.82	1000	$55.06	1600	$49.71	2200	$40.26
0500	$35.84	1100	$55.30	1700	$51.66	2300	$37.29

Data from Bloomberg

Return

Capital-constrained intermediaries

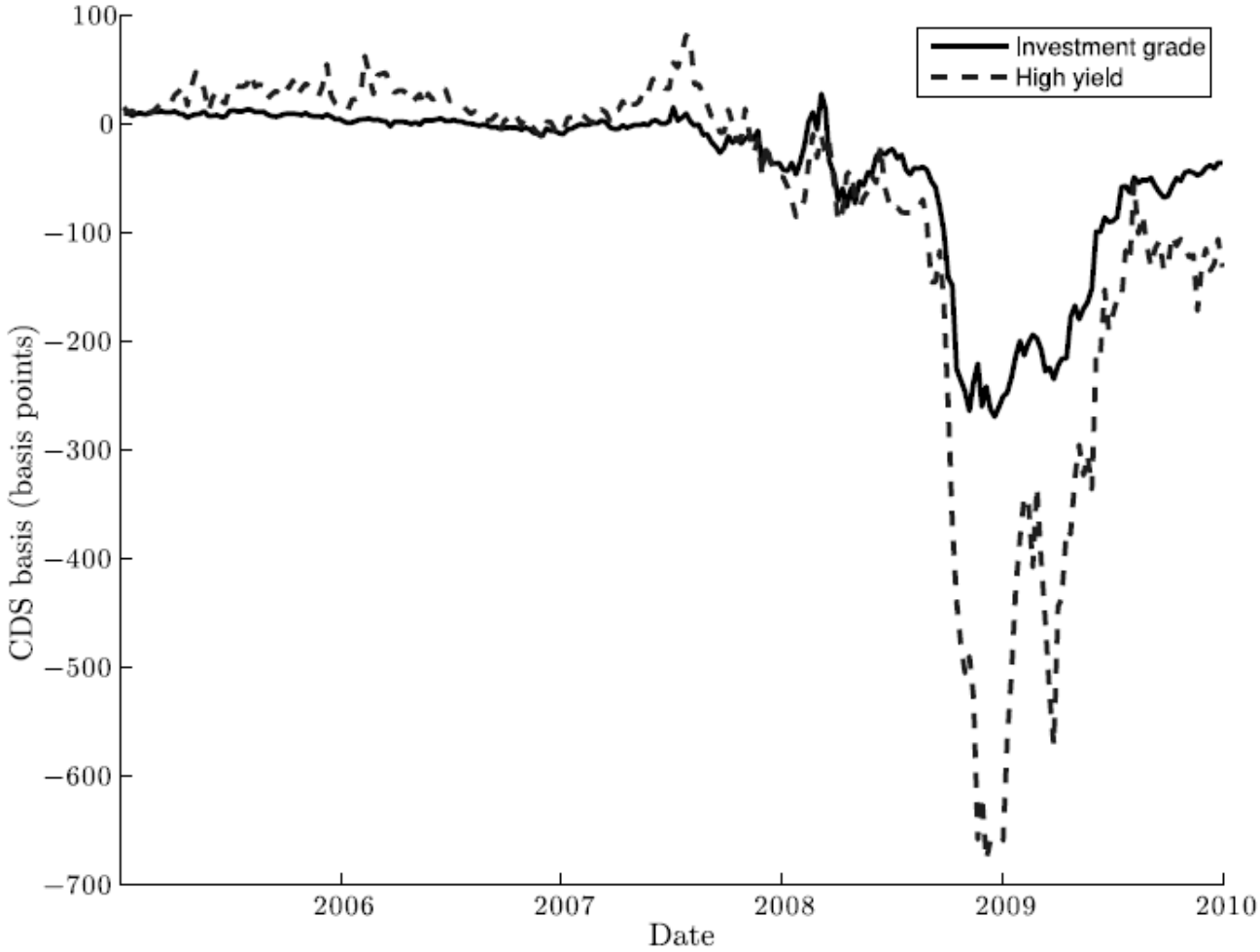


Figure 4. The corporate-bond CDS basis, the difference between the CDS rate and the associated par bond yield spread, is theoretically near zero in frictionless markets. As

Return