Find extrema on displayed graph.

3. referenced graph pg. 286

   min: r

   maxmax: s

   loc min: r, b

   loc max: c

   neither: d, a

5. referenced graph pg. 286

   min: undefined

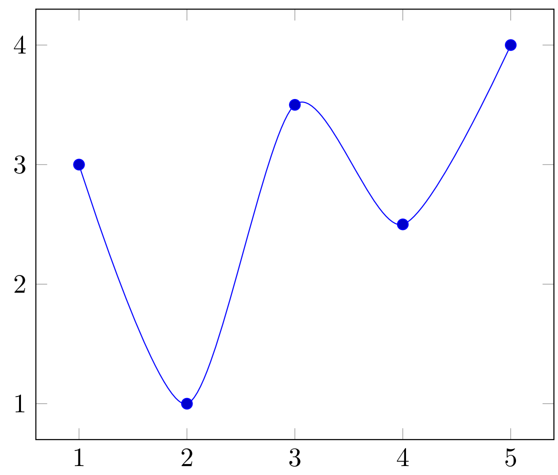
   maxmax: 5

   loc min: 2, 3

   loc max: 5

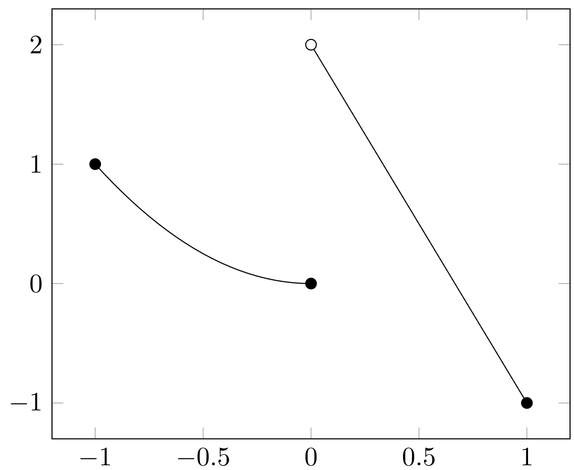
Sketch a graph with the given constraints.

7. Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4.



Create graph from function and find extrema.

7. $f(x) = \begin{cases} x^2 & \text{if } -1 \leq x \leq 0, \\ 2-3x & \text{if } 0 < x \leq 1 \end{cases}$



Absolute min = -1, Local min = 0. No abs or local max.

Find the critical numbers of the function.

31. $3x^4 + 8x^3 - 48x^2$ Polynomials are differentiable for all $x \in \mathbb{R}$.

$$\begin{aligned} f'(x) &= 4x^3 + 24x^2 - 96x && \text{find derivative} \\ 0 &= x^3 + 2x^2 - 8x && \text{find root(s)} \\ 0 &= x(x^2 + 2x - 8) \\ 0 &= x(x+4)(x-2) \end{aligned}$$

$\boxed{x = 0,\ 4, \text{ and } 2}$

35. $g(y) = \frac{y-1}{y^2-y+1}$

$$g'(y) = \frac{(y^2 - y + 1)(1) - (y-1)(2y-1)}{(y^2 - y + 1)^2} \qquad \text{quotient rule}$$

Set denominator and numerator to 0 to find undefined inputs and roots respectively.

$$\begin{aligned} 0 &= y^2 - y + 1 && \text{denominator factor} \\ y &= 0 \end{aligned}$$

$$\begin{aligned} 0 &= y^2 - y + 1 - 2y^2 + 3y - 1 && \text{numerator} \\ 0 &= -y^2 + 2y \\ y &= 2 \end{aligned}$$

$\boxed{x = 0 \text{ and } 2}$

39. $h(t) = t^{3/4} - 2t^{1/4}$

$$h'(t) = \frac{3}{4}t^{-1/4} - \frac{1}{2}t^{-3/4} \quad \text{find derivative}$$

$$h'(t) = \frac{3}{4t^{1/4}} - \frac{1}{2t^{3/4}} \quad \text{rewrite exponents}$$

$$h'(t) = \frac{3}{4t^{1/4}} \cdot \frac{t^{2/4}}{t^{2/4}} - \frac{1}{2t^{3/4}} \cdot \frac{2}{2} \quad \text{get common denominators}$$

$$h'(t) = \frac{3t^{1/2} - 2}{4t^{3/4}}$$

$0 = 4t^{3/4}$ set denominator to 0 to evaluate where function is undefined

$t = 0$

$0 = 3t^{1/2} - 2$ set numerator to 0 to find root(s)

$$t^{1/2} = \frac{2}{3} \implies t = \frac{4}{9}$$

$\boxed{t = 0 \text{ and } \frac{4}{9}}$

43. $f(x) = x^{1/3}(4 - x)^{2/3}$

$$f'(x) = x^{1/3}\frac{2}{3}(4 - x)^{-1/3}(-1) + \frac{1}{3}x^{-2/3}(4 - x)^{2/3} \quad \text{product rule}$$

$$f'(x) = \frac{-2x^{1/3}}{3(4 - x)^{1/3}} + \frac{(4 - x)^{2/3}}{3x^{2/3}} \quad \text{rewrite as rational expression}$$

$$f'(x) = \frac{-2x^{1/3}}{3(4 - x)^{1/3}} * \frac{x^{2/3}}{x^{2/3}} + \frac{(4 - x)^{2/3}}{3x^{2/3}} * \frac{(4 - x)^{1/3}}{(4 - x)^{1/3}} \quad \text{get common denominators}$$

$$f'(x) = \frac{-2x + (4 - x)}{3x^{2/3}(4 - x)^{1/3}}$$

set both factors in denominator to 0 to evaluate where function is undefined, set numerator to 0 to find root(s).

$0 = 3x^{2/3}$ denominator factor #1

$x = 0$

$0 = (4 - x)^{1/3}$ denominator factor #2

$x = 4$

$0 = -2x - x + 4$ numerator

$0 = -2x - x + 4$

$4 = 2x + x$

$4 = x(2 + 1)$

$4 = 3x$

$$x = \frac{4}{3}$$

$\boxed{x = 0,\ 4 \text{ and } \frac{4}{3}}$

47. $g(x) = x^2 \ln x$

$$\begin{aligned}
g'(x) &= x^2\left(\frac{1}{x}\right) + 2x\ln x && \text{product rule} \\
g'(x) &= x + 2x\ln x && \\
0 &= x + 2x\ln x && \text{find root(s)} \\
-x &= 2x\ln x && \\
\frac{-x}{2x} &= lnx && x\text{'s on left side of expression cancel} \\
-\frac{1}{2} &= lnx && \\
e^{-1/2} &= e^{lnx} = x && \text{use e to isolate remaining x}
\end{aligned}$$

$\boxed{x = \frac{1}{\sqrt{e}}}$

Find the value of $f(x)$ at absolute min and max of the function on the given interval.

51. $f(x) = 12 + 4x - x^2$, [0, 5]

$$\begin{aligned}
f'(x) &= -2x + 4 && \text{find derivative} \\
0 &= -2x + 4 && \text{find root(s)} \\
-4 &= -2x && \text{root is 2}
\end{aligned}$$

Original function is a polynomial so it is continuous for all real numbers. Plug in roots and endpoints to original function.

$$\begin{aligned}
f(2) &= 12 + 4(2) - (2)^2 = 12 + 8 - 4 = 16 \\
f(0) &= 12 + 4(0) - (0)^2 = 12 + 0 - 0 = 12 \\
f(5) &= 12 + 4(5) - (5)^2 = 12 + 20 - 25 = 7
\end{aligned}$$

$\boxed{\text{max: } f(2) = 16, \text{ min: } f(5) = 7}$

55. $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$, [-2, 3]

$$\begin{aligned}
f'(x) &= 12x^3 - 12x^2 - 24x && \text{find derivative} \\
0 &= 12x^3 - 12x^2 - 24x && \text{find roots} \\
0 &= (x^2 - x - 2)(12x) && \text{find factors} \\
0 &= (x-2)(x+1)(12x) && \text{roots are 2, -1, and 0}
\end{aligned}$$

Original function is polynomial so it is continuous for all real numbers. Plug in roots and endpoints into original function.

$$f(-2) = 3(-2)^4 - 4(-2)^3 - 12(-2)^2 + 1 \qquad \text{critical val \#1}$$
$$f(-2) = (3 * 16) - (4 * -8) - (12 * 4) + 1$$
$$f(-2) = 48 - (-32) - 48 + 1$$
$$f(-2) = 33$$

$$f(3) = 3(3)^4 - 4(3)^3 - 12(3)^2 + 1 \qquad \text{critical val \#2}$$
$$f(3) = (3 * 81) - (4 * 27) - (12 * 9) + 1$$
$$f(3) = 243 - 108 - 108 + 1$$
$$f(3) = 28$$

$$f(2) = 3(2)^4 - 4(2)^3 - 12(2)^2 + 1 \qquad \text{critical val \#3}$$
$$f(2) = (3 * 16) - (4 * 8) - (12 * 4) + 1$$
$$f(2) = 48 - 32 - 48 + 1$$
$$f(2) = -31$$

$$f(0) = 3(0)^4 - 4(0)^3 - 12(0)^2 + 1 \qquad \text{critical val \#4}$$
$$f(0) = 1$$

$$f(-1) = 3(-1)^4 - 4(-1)^3 - 12 - (1)^2 + 1 \qquad \text{critical val \#5}$$
$$f(-1) = 3 - 4 + 12 + 1$$
$$f(-1) = 12$$

$\boxed{\text{max: } f(-2) = 33, \text{ min: } f(2) = -31}$

59. $f(x) = t - \sqrt[3]{t}$, [-1, 4]

$$f(x) = t - t^{1/3} \qquad \text{rewrite function with exponents}$$

$$f'(x) = 1 - \frac{1}{3}t^{-2/3} \qquad \text{find derivative}$$

$$f'(x) = 1 - \frac{1}{3t^{2/3}} \qquad \text{simplify exponents}$$

Radical is undefined when denominator is 0, so $f(0)$ is a critical value. Set denominator to 1 to find the root, where the function evaluates to 0.

$$3t^{2/3} = 1 \quad \Longrightarrow \quad t^{2/3} = \frac{1}{3} \quad \Longrightarrow \quad t = \frac{\sqrt{1^3}}{\sqrt{3^3}} = \frac{\sqrt{3}}{9}$$

Plug in critical values into original equation.

$$f(-1) = -1 - \sqrt[3]{-1} \qquad \text{critical val \#1}$$
$$f(-1) = -1 + 1$$
$$f(-1) = 0$$

$$f(4) = 4 - \sqrt[3]{4} \qquad \text{critical val \#2}$$
$$f(4) = 4 - 1.58 = 2.41$$

$$f(0) = 0 - \sqrt[3]{0} = 0 \qquad \text{critical val \#3}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - \sqrt[3]{\frac{\sqrt{3}}{9}} \qquad \text{critical val \#4}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - \left(3^{-2} * 3^{1/2}\right)^{1/3}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - \left(3^{-2/3} * 3^{1/6}\right)$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - 3^{-3/6}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - \frac{1}{\sqrt{3}}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = -0.38$$

$$\boxed{\text{max: } f(4) = 2.41, \ \text{min: } f\left(\tfrac{\sqrt{3}}{9}\right) = -0.38}$$

63. $f(x) = x^{-2}\ln x, \ [\frac{1}{2}, 4]$

$$f'(x) = x^{-2}\left(\frac{1}{x}\right) + -2x^{-3}\ln x \qquad \text{find derivative}$$
$$f'(x) = x^{-3} - 2x^{-3}\ln x$$
$$f'(x) = x^{-3}(1 - 2\ln x)$$
$$0 = x^{-3}(1 - 2\ln x) \qquad \text{find root(s)}$$
$$0 = 1 - 2\ln x \qquad \text{remove factor, } \ x = 0$$
$$\frac{1}{2} = lnx$$
$$e^{1/2} = x$$

Critical numbers are $e^{1/2}$, $\frac{1}{2}$, and 4. 0 is not included because it is not in domain of original function. Plug critical numbers into original function and evaluate.

$$f(e^{1/2}) = (e^{1/2})^{-2} \ln e^{1/2} \qquad \text{critical val \#2}$$
$$f(e^{1/2}) = (e^{-1}) \left(\frac{1}{2}\right)$$
$$f(e^{1/2}) = \left(\frac{1}{2e}\right)$$
$$f(e^{1/2}) = 0.18$$

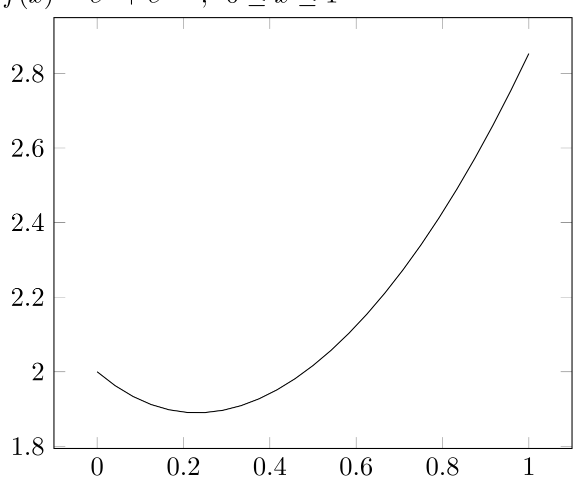
$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{-2} \ln \frac{1}{2} \qquad \text{critical val \#3}$$
$$f\left(\frac{1}{2}\right) = 4 \ln \frac{1}{2}$$
$$f\left(\frac{1}{2}\right) = -2.77$$

$$f(4) = 4^{-2} \ln 4 \qquad \text{critical val \#4}$$
$$f(4) = \frac{\ln 4}{16}$$
$$f(4) = 0.09$$

$\boxed{\text{max: } f(e^{1/2}) = 0.18, \ \text{min: } f\left(\frac{1}{2}\right) = -5.77}$

(a) Use a graph to estimate extrema to 2 decimal places (b) Find extrema using calculus.

70. $f(x) = e^{x} + e^{-2x}, \ \ 0 \leq x \leq 1$



(a) $\boxed{\text{max: } f(1) = 2.85, \ \text{min: } f(0.23) = 0.89}$

$$
\begin{aligned}
f'(x) &= e^x + e^{-2x}(-2) && \text{find derivative} \\
f'(x) &= e^x - \frac{2}{e^{2x}} \\
f'(x) &= \frac{e^x}{1} * \frac{e^{2x}}{e^{2x}} - \frac{2}{e^{2x}} \\
f'(x) &= \frac{e^{3x} - 2}{e^{2x}}
\end{aligned}
$$

Denominator can't be 0. Set numerator to 0 to find roots

$$
\begin{aligned}
0 &= e^{3x} - 2 \\
2 &= e^{3x} \\
\ln 2 &= 3x \\
x &= \frac{\ln 2}{3}
\end{aligned}
$$

Critical values are 0, 1 and $\frac{\ln 2}{3}$. Plug into original equation.

$$
\begin{aligned}
f(0) &= e^0 + e^{-2*0} \\
f(0) &= 2
\end{aligned}
$$

$$
\begin{aligned}
f(1) &= e^1 + e^{-2*1} \\
f(1) &= e + \frac{1}{e^2} \\
f(1) &= \frac{1}{e} \\
f(1) &= 0.37
\end{aligned}
$$

$$
\begin{aligned}
f\left(\frac{\ln 2}{3}\right) &= e^{\left(\frac{\ln 2}{3}\right)} + e^{-2\left(\frac{\ln 2}{3}\right)} \\
f\left(\frac{\ln 2}{3}\right) &= \frac{e^{\left(\frac{\ln 2}{3}\right)}}{1} + \frac{1}{e^{\left(\frac{2\ln 2}{3}\right)}} \\
f\left(\frac{\ln 2}{3}\right) &= \frac{e^{\left(\frac{\ln 2}{3}\right)}}{1} \cdot \frac{e^{\left(\frac{2\ln 2}{3}\right)}}{e^{\left(\frac{2\ln 2}{3}\right)}} + \frac{1}{e^{\left(\frac{2\ln 2}{3}\right)}} \\
f\left(\frac{\ln 2}{3}\right) &= \frac{e^{\left(\frac{\ln 2 + 2\ln 2}{3}\right)} + 1}{e^{\left(\frac{2\ln 2}{3}\right)}} && \text{how to simplify further?}
\end{aligned}
$$