Find extrema on displayed graph.

3. referenced graph pg. 286

   min: r

   maxmax: s

   loc min: r, b

   loc max: c

   neither: d, a

5. referenced graph pg. 286

   min: undefined

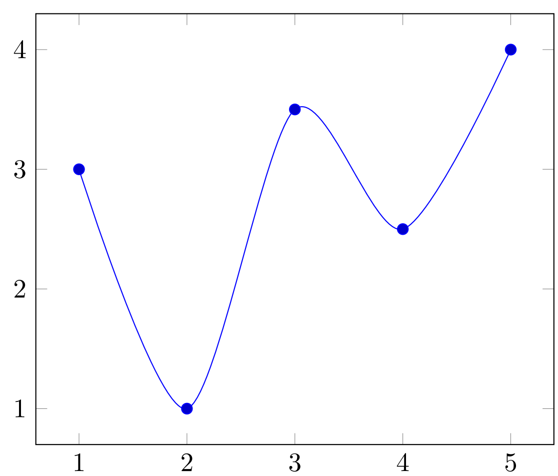
   maxmax: 5

   loc min: 2, 3

   loc max: 5

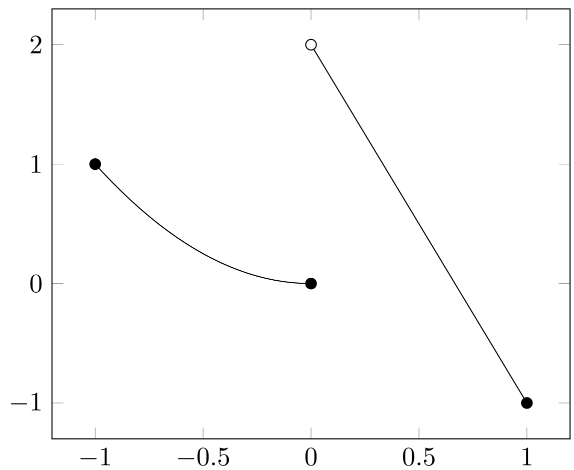
Sketch a graph with the given constraints.

7. Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4.



Create graph from function and find extrema.

7. $f(x) = \begin{cases} x^2 & \text{if } -1 \le x \le 0, \\ 2 - 3x & \text{if } 0 < x \le 1 \end{cases}$



Absolute min = -1, Local min = 0. No abs or local max.

Find the critical numbers of the function.

31. $3x^4 + 8x^3 - 48x^2$ Polynomials are differentiable for all $x \in \mathbb{R}$.

$$\begin{aligned} f'(x) &= 4x^3 + 24x^2 - 96x && \text{find derivative} \\ 0 &= x^3 + 2x^2 - 8x && \text{find root(s)} \\ 0 &= x(x^2 + 2x - 8) \\ 0 &= x(x+4)(x-2) \end{aligned}$$

$\boxed{x = 0,\ 4,\ \text{and } 2}$

35. $g(y) = \frac{y-1}{y^2-y+1}$

$$\begin{aligned} g'(y) &= \frac{(y^2 - y + 1)(1) - (y-1)(2y-1)}{(y^2 - y + 1)^2} && \text{quotient rule} \\ g'(y) &= && \text{how to simplify from here?} \end{aligned}$$

39. $h(t) = t^{3/4} - 2t^{1/4}$

$$\begin{aligned} h'(t) &= \frac{3}{4}t^{-1/4} - \frac{1}{2}t^{-3/4} && \text{find derivative} \\ h'(t) &= \frac{3}{4t^{1/4}} - \frac{1}{2t^{3/4}} && \text{rewrite exponents} \\ h'(t) &= \frac{3}{4t^{1/4}} \cdot \frac{t^{2/4}}{t^{2/4}} - \frac{1}{2t^{3/4}} \cdot \frac{2}{2} && \text{get common denominators} \\ h'(t) &= \frac{3t^{1/2} - 2}{4t^{3/4}} \end{aligned}$$

$$\begin{aligned} 0 &= 4t^{3/4} && \text{set denominator to 0 to evaluate where function is undefined} \\ t &= 0 \end{aligned}$$

$$0 = 3t^{1/2} - 2 \qquad \text{set numerator to 0 to find root(s)}$$

$$t^{1/2} = \frac{2}{3} \implies t = \frac{4}{9}$$

$\boxed{t = 0 \text{ and } \frac{4}{9}}$

43. $f(x) = x^{1/3}(4-x)^{2/3}$

$$f'(x) = x^{1/3}(-4+x)(-1) + (4-x)^{2/3}\frac{1}{3}x^{2/3} \qquad \text{product rule}$$

$$f'(x) = \frac{x^{1/3}(-4+x)}{3} + \frac{(4-x)^{2/3}}{3x^{2/3}} \qquad \text{rewrite as rational expression}$$

$$f'(x) = \frac{x^{1/3}(-4+x)}{3} \cdot \frac{x^{2/3}}{x^{2/3}} + \frac{(4-x)^{2/3}}{3x^{2/3}} \qquad \text{get common denominators}$$

$$f'(x) = \frac{x(-4+x) + (4-x)^{2/3}}{3x^{2/3}}$$

$$0 = 3x^{2/3} \qquad \text{set denominator to 0 to evaluate where function is undefined}$$
$$x = 0$$

$$0 = x(-4+x) + (4-x)^{2/3} \qquad \text{set numerator to 0 to find root(s)}$$
$$0 = (4-x)(-x + (4-x)^{1/3}) \qquad \text{find factors, } x = 4$$
$$0 = -x + (4-x)^{1/3} \qquad \text{isolate factor by removing } (4-x)$$
$$x = \qquad \text{how to simplify from here? missing } x = \frac{4}{3}$$

$\boxed{x = 0 \text{ and } 4}$

47. $g(x) = x^2 \ln x$

$$g'(x) = x^2\left(\frac{1}{x}\right) + 2x\ln x \qquad \text{product rule}$$
$$g'(x) = x + 2x\ln x$$
$$0 = x + 2x\ln x \qquad \text{find root(s)}$$
$$-x = 2x\ln x$$
$$\frac{-x}{2x} = lnx \qquad x\text{'s on left side of expression cancel}$$
$$-\frac{1}{2} = lnx$$
$$e^{-1/2} = e^{lnx} = x \qquad \text{use e to isolate remaining x}$$

$\boxed{x = \frac{1}{\sqrt{e}}}$

Find the value of $f(x)$ at absolute min and max of the function on the given interval.

51. $f(x) = 12 + 4x - x^2$, [0, 5]

$$\begin{aligned} f'(x) &= -2x + 4 && \text{find derivative} \\ 0 &= -2x + 4 && \text{find root(s)} \\ -4 &= -2x && \text{root is 2} \end{aligned}$$

Original function is a polynomial so it is continuous for all real numbers. Plug in roots and endpoints to original function.

$$\begin{aligned} f(2) &= 12 + 4(2) - (2)^2 = 12 + 8 - 4 = 16 \\ f(0) &= 12 + 4(0) - (0)^2 = 12 + 0 - 0 = 12 \\ f(5) &= 12 + 4(5) - (5)^2 = 12 + 20 - 25 = 7 \end{aligned}$$

$\boxed{\text{max: } f(2) = 16,\ \text{min: } f(5) = 7}$

55. $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$, [-2, 3]

$$\begin{aligned} f'(x) &= 12x^3 - 12x^2 - 24x && \text{find derivative} \\ 0 &= 12x^3 - 12x^2 - 24x && \text{find roots} \\ 0 &= (x^2 - x - 2)(12x) && \text{find factors} \\ 0 &= (x-2)(x+1)(12x) && \text{roots are 2, -1, and 0} \end{aligned}$$

Original function is polynomial so it is continuous for all real numbers. Plug in roots and endpoints into original function.

$$\begin{aligned} f(-2) &= 3(-2)^4 - 4(-2)^3 - 12(-2)^2 + 1 && \text{critical val \#1} \\ f(-2) &= (3 * 16) - (4 * -8) - (12 * 4) + 1 \\ f(-2) &= 48 - (-32) - 48 + 1 \\ f(-2) &= 33 \end{aligned}$$

$$\begin{aligned} f(3) &= 3(3)^4 - 4(3)^3 - 12(3)^2 + 1 && \text{critical val \#2} \\ f(3) &= (3 * 81) - (4 * 27) - (12 * 9) + 1 \\ f(3) &= 243 - 108 - 108 + 1 \\ f(3) &= 28 \end{aligned}$$

$$\begin{aligned} f(2) &= 3(2)^4 - 4(2)^3 - 12(2)^2 + 1 && \text{critical val \#3} \\ f(2) &= (3 * 16) - (4 * 8) - (12 * 4) + 1 \\ f(2) &= 48 - 32 - 48 + 1 \\ f(2) &= -31 \end{aligned}$$

$$\begin{aligned} f(0) &= 3(0)^4 - 4(0)^3 - 12(0)^2 + 1 && \text{critical val \#4} \\ f(0) &= 1 \end{aligned}$$

$$\begin{aligned} f(-1) &= 3(-1)^4 - 4(-1)^3 - 12 - (1)^2 + 1 && \text{critical val \#5} \\ f(-1) &= 3 - 4 + 12 + 1 \\ f(-1) &= 12 \end{aligned}$$

$\boxed{\text{max: } f(-2) = 33,\ \text{min: } f(2) = -31}$

59. $f(x) = t - \sqrt[3]{t}$, [-1, 4]

$$f(x) = t - t^{1/3} \qquad \text{rewrite function with exponents}$$

$$f'(x) = 1 - \frac{1}{3}t^{-2/3} \qquad \text{find derivative}$$

$$f'(x) = 1 - \frac{1}{3t^{2/3}} \qquad \text{simplify exponents}$$

Radical is undefined when denominator is 0, so $f(0)$ is a critical value. Set denominator to 1 to find the root, where the function evaluates to 0.

$$3t^{2/3} = 1 \implies t^{2/3} = \frac{1}{3} \implies t = \frac{\sqrt{1^3}}{\sqrt{3^3}} = \frac{\sqrt{3}}{9}$$

Plug in critical values into original equation.

$$\begin{aligned} f(-1) &= -1 - \sqrt[3]{-1} \\ f(-1) &= -1 + 1 \\ f(-1) &= 0 \end{aligned} \qquad \text{critical val \#1}$$

$$\begin{aligned} f(4) &= 4 - \sqrt[3]{4} \\ f(4) &= 4 - 1.58 = 2.41 \end{aligned} \qquad \text{critical val \#2}$$

$$f(0) = 0 - \sqrt[3]{0} = 0 \qquad \text{critical val \#3}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - \sqrt[3]{\frac{\sqrt{3}}{9}} \qquad \text{critical val \#4}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - \left(3^{-2} * 3^{1/2}\right)^{1/3}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - \left(3^{-2/3} * 3^{1/6}\right)$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - 3^{-3/6}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = \frac{\sqrt{3}}{9} - \frac{1}{\sqrt{3}}$$

$$f\left(\frac{\sqrt{3}}{9}\right) = -0.38$$

$$\boxed{\text{max: } f(4) = 2.41, \ \text{min: } f\left(\tfrac{\sqrt{3}}{9}\right) = -0.38}$$

63. $f(x) = x^{-2} \ln x, \ [\frac{1}{2}, 4]$

$$\begin{aligned}
f'(x) &= x^{-2}\left(\frac{1}{x}\right) + -2x^{-3}\ln x && \text{find derivative}\\
f'(x) &= x^{-3} - 2x^{-3}\ln x\\
f'(x) &= x^{-3}(1 - 2\ln x)\\
0 &= x^{-3}(1 - 2\ln x) && \text{find root(s)}\\
0 &= 1 - 2\ln x && \text{remove factor,} \quad x = 0\\
\frac{1}{2} &= lnx\\
e^{1/2} &= x
\end{aligned}$$

Critical numbers are $e^{1/2}$, $\frac{1}{2}$, and 4. 0 is not included because it is not in domain of original function. Plug critical numbers into original function and evaluate.

$$\begin{aligned}
f(e^{1/2}) &= (e^{1/2})^{-2}\ln e^{1/2} && \text{critical val \#2}\\
f(e^{1/2}) &= (e^{-1})\left(\frac{1}{2}\right)\\
f(e^{1/2}) &= \left(\frac{1}{2e}\right)\\
f(e^{1/2}) &= 0.18
\end{aligned}$$

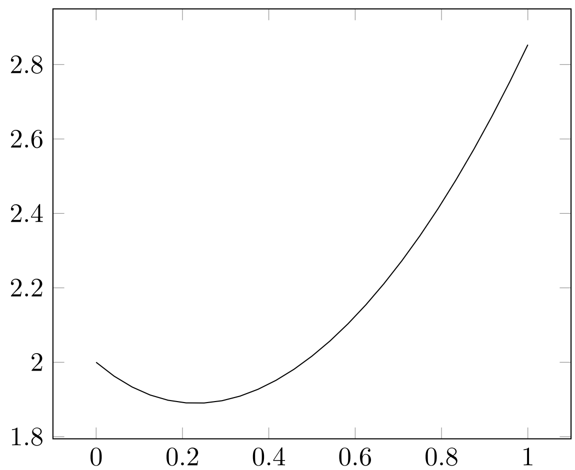
$$\begin{aligned}
f\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right)^{-2}\ln\frac{1}{2} && \text{critical val \#3}\\
f\left(\frac{1}{2}\right) &= 4\ln\frac{1}{2}\\
f\left(\frac{1}{2}\right) &= -2.77
\end{aligned}$$

$$\begin{aligned}
f(4) &= 4^{-2}\ln 4 && \text{critical val \#4}\\
f(4) &= \frac{\ln 4}{16}\\
f(4) &= 0.09
\end{aligned}$$

$$\boxed{\text{max: } f(e^{1/2}) = 0.18, \ \text{ min: } f\left(\tfrac{1}{2}\right) = -5.77}$$

(a) Use a graph to estimate extrema to 2 decimal places (b) Find extrema using calculus.

70. $f(x) = e^x + e^{-2x}, \ 0 \le x \le 1$



(a) $\boxed{\text{max: } f(1) = 2.85, \ \text{min: } f(0.23) = 0.89}$

$$\begin{aligned} f'(x) &= e^x + e^{-2x}(-2) && \text{find derivative} \\ f'(x) &= && \text{How to simplify without taking log of negative number?} \end{aligned}$$