State indeterminate form or evaluate limit.

Indeterminate forms:

$$\frac{0}{0} \qquad \frac{\infty}{\infty} \qquad 0\cdot\infty \qquad \infty-\infty \qquad 0^0 \qquad 1^\infty \qquad \infty^0$$

1.

a. $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}$

b. $\lim_{x\to a}\frac{f(x)}{p(x)}=\frac{0}{\infty}=0$

c. $\lim_{x\to a}\frac{h(x)}{p(x)}=\frac{1}{\infty}=0$

d. $\lim_{x\to a}\frac{p(x)}{f(x)}=\frac{\infty}{0}=$ false indeterminate

e. $\lim_{x\to a}\frac{p(x)}{q(x)}=\frac{\infty}{\infty}$

2.

a. $\lim_{x\to a}[f(x)p(x)]=0\cdot\infty$

b. $\lim_{x\to a}[h(x)p(x)]=1\cdot\infty=1$?

c. $\lim_{x\to a}[p(x)q(x)]=\infty\cdot\infty$ false indeterminate

3.

a. $\lim_{x\to a}[f(x)-p(x)]=0+\infty=\infty$

b. $\lim_{x\to a}[p(x)-q(x)]=\infty-\infty$

c. $\lim_{x\to a}[p(x)+q(x)]=\infty+\infty$ false indeterminate

4.

a. $\lim_{x\to a}[f(x)]^{g(x)}=0^0$

b. $\lim_{x\to a}[f(x)]^{p(x)}=0^\infty$ false indeterminate

c. $\lim_{x\to a}[h(x)]^{p(x)}=1^\infty$

d. $\lim_{x\to a}[p(x)]^{f(x)}=\infty^0$

e. $\lim_{x\to a}[p(x)]^{q(x)}=\infty^\infty$ false indeterminate

f. $\lim_{x\to a}\sqrt[q(x)]{p(x)}=\sqrt[\infty]{\infty}=\infty^{\frac{1}{\infty}}=\infty^0$?

Find the limit. Use L'Hospital's Rule if possible.

13. $\lim_{x\to\pi/4}\frac{\sin x-\cos x}{\tan x-1}$

$$=\frac{\sin\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)-1}$$

$$=\frac{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}}{1-1}$$

L'Hospital's Rule is applicable

$$\lim_{x\to\pi/4} \frac{\frac{dx}{dy}(\sin x - \cos x)}{\frac{dx}{dy}(\tan x - 1)}$$

$$= \lim_{x\to\pi/4} \frac{\cos x + \sin x}{\sec^2 x}$$

$$= \frac{\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)}{\sec^2\left(\frac{\pi}{4}\right)}$$

$$= \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{\sqrt{2}^2}$$

$$= \frac{\frac{2\sqrt{2}}{2}}{2}$$

$$= \boxed{\frac{\sqrt{2}}{2}}$$

17. $\lim_{x\to1} \dfrac{\sin(x-1)}{x^3+x-2}$

$$= \frac{\sin(1-1)}{1^3+1-2}$$
$$= \frac{0}{0}$$

L'Hospital's Rule is applicable

$$\lim_{x\to1} \frac{\frac{dx}{dy}(\sin(x-1))}{\frac{dx}{dy}(x^3+x-2)}$$

$$= \lim_{x\to1} \frac{\cos(x-1)(1)}{(3x)^2+1}$$

$$= \frac{\cos(1-1)}{3(1)^2+1}$$

$$= \frac{\cos(0)}{4}$$

$$= \boxed{\frac{1}{4}}$$

21. $\lim_{x\to0^+} \dfrac{\ln x}{x}$

$$= \frac{\ln 0}{0}$$

$$= \frac{-\infty}{1}$$

$$= \boxed{-\infty}$$

25. $\lim_{x\to 0} \dfrac{\sqrt{1+2x}-\sqrt{1-4x}}{x}$

$$= \frac{\sqrt{1+2(0)}-\sqrt{1-4(0)}}{0}$$

L'Hospital's Rule is applicable

$$\lim_{x\to 0} \frac{\frac{dx}{dy}(\sqrt{1+2x}-\sqrt{1-4x})}{\frac{dx}{dy}(x)}$$

$$= \lim_{x\to 0} \frac{\frac{dx}{dy}((1+2x)^{1/2}-(1-4x)^{1/2})}{\frac{dx}{dy}(x)}$$

$$= \lim_{x\to 0} \frac{(1+2x)^{-1/2}-(-2)(1-4x)^{-1/2}}{1}$$

$$= \lim_{x\to 0} \frac{\frac{1}{(1+2x)^{1/2}}+\frac{2}{(1-4x)^{1/2}}}{1}$$

$$= \frac{\frac{1}{(1+2(0))^{1/2}}+\frac{2}{(1-4(0))^{1/2}}}{1}$$

$$= \frac{1+2}{1}$$

$$= \boxed{3}$$

29. $\lim_{x\to 0} \dfrac{\tanh x}{tan x}$

$$\frac{\tanh 0}{\tan 0} \qquad \text{Using} \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{\tanh 0}{\tan 0}$$

$$\frac{0}{0}$$

L'Hospital's Rule is applicable

$$\frac{\frac{dx}{dy}(\tanh x)}{\frac{dx}{dy}(\tan x)}$$

$$= \frac{\operatorname{sech}^2 0}{\sec^2 0} \qquad \text{Using} \operatorname{sech} x = \frac{1}{e^x - e^{-x}}$$

$$= \frac{1}{1}$$

$$= \boxed{1}$$

33. $\lim_{x\to 0} \dfrac{x3^x}{3^x - 1}$

$$= \frac{(0)3^0}{3^(0) - 1}$$

$$= \frac{0}{0}$$

L'Hospital's Rule is applicable

$$\lim_{x\to 0} \frac{\frac{dx}{dy}(x3^x)}{\frac{dx}{dy}(3^x - 1)}$$

$$= \lim_{x\to 0} \frac{x(3^x)(\ln 3) + (1)(3^x)}{(3^x)(\ln 3)}$$

$$= \frac{0(3^0)(\ln 3) + (1)(3^0)}{(3^0)(\ln 3)}$$

$$= \frac{0+1}{\ln 3}$$

$$= \boxed{\frac{1}{\ln 3}}$$

37. $\lim_{x\to 0^+} \dfrac{\arctan(2x)}{\ln x}$

$$= \frac{\arctan(2(0))}{\ln 0}$$

$$= \frac{0}{-\infty}$$

L'Hospital's Rule is applicable

$$\lim_{x\to 0^+} \frac{\frac{dx}{dy}(\arctan(2x))}{\frac{dx}{dy}(\ln x)}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{1+x^2}}{\frac{1}{x}}$$

$$= \frac{\frac{1}{1+(0)^2}}{\frac{1}{0}}$$

$$= \frac{\frac{1}{1+0}}{\infty} \qquad \infty \text{ because } 0^+$$

$$= \boxed{0}$$

41. $\lim_{x\to 0} \dfrac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$

$$= \frac{\cos 0 - 1 + \frac{1}{2}0^2}{0^4}$$

$$= \frac{1 - 1 + \frac{1}{2}}{0}$$

L'Hospital's Rule is applicable (substitution not shown)

$$\lim_{x\to 0} \frac{-\sin x + x}{4x^3}$$

L'Hospital's Rule is applicable (substitution not shown)

$$\lim_{x\to 0} \frac{-\cos x + 1}{12x^2}$$

L'Hospital's Rule is applicable (substitution not shown)

$$\lim_{x\to 0} \frac{\sin x}{24x}$$

L'Hospital's Rule is applicable

$$\lim_{x\to 0} \frac{\cos x}{24}$$

$$\frac{\cos 0}{24}$$

$$= \boxed{\frac{1}{24}}$$

45. $\lim_{x\to 0} \sin(5x)\csc(3x)$

$$= \sin 0 \csc 0$$

$$= \frac{\sin 5x}{\tan 3x}$$

$$= \frac{\sin 0}{\tan 0}$$

$$= \frac{0}{0}$$

L'Hospital's Rule is applicable

$$= \lim_{x\to 0} \frac{\frac{dx}{dy}(\sin 5x)}{\frac{dx}{dy}(\tan 3x)}$$

$$= \lim_{x\to 0} \frac{\cos 5x(5)}{\sec^2 3x(3)}$$

$$\frac{\cos 0(5)}{\sec^2 0(3)}$$

$$= \frac{1(5)}{1(3)}$$

$$= \boxed{\frac{5}{3}}$$

49. $\lim_{x\to 1^+} \ln x \tan(\pi x/2)$

$$= \lim_{x\to 1^+} \frac{\ln(\pi x/2)\sin(\pi x/2)}{\cos(\pi x/2)}$$

$$= \frac{\ln(\pi/2)\sin(\pi/2)}{\cos(\pi/2)}$$

$$= \frac{\ln(\pi/2)}{0}$$

L'Hospital's Rule is applicable.

$$= \lim_{x\to 1^+} \frac{\frac{dx}{dy}(\ln(\pi x/2)\sin(\pi x/2))}{\frac{dx}{dy}(\cos(\pi x/2))}$$

$$= \lim_{x\to 1^+} \frac{\ln(\pi x/2)\cos(\pi x/2)(\pi/2) + \frac{1}{(\pi x/2)}(\pi/2)\sin(\pi x/2)}{-\sin(\pi x/2)(\pi/2)}$$

$$= \frac{\ln(\pi/2)\cos(\pi/2)(\pi/2) + \frac{1}{(\pi/2)}(\pi/2)\sin(\pi/2)}{-\sin(\pi/2)(\pi/2)}$$

$$= \frac{\ln(\pi/2)(0)(\pi/2) + 1(1)}{-1(\pi/2)}$$

$$= -\frac{1}{\pi/2}$$

$$= \boxed{-\frac{2}{\pi}}$$

53. $\lim_{x\to 0^+} \dfrac{1}{x} - \dfrac{1}{e^x - 1}$

$$= \lim_{x\to 0^+} \frac{1}{x} - \frac{1}{e^x - 1}$$

$$= \lim_{x\to 0^+} \frac{1}{x} \cdot \frac{e^x - 1}{e^x - 1} - \frac{1}{e^x - 1} \cdot \frac{x}{x}$$

$$= \lim_{x\to 0^+} \frac{(e^x - 1) - 1}{x(e^x - 1)}$$

$$= \lim_{x\to 0^+} \frac{e^x - 2}{xe^x - x}$$

$$= \frac{-1}{0}$$

L'Hospital's Rule is applicable.

$$\lim_{x\to 0^+} \frac{\frac{dx}{dy}(e^x - 2)}{\frac{dx}{dy}(xe^x - x)}$$

$$= \lim_{x\to 0^+} \frac{e^x}{xe^x + 1e^x - 1}$$

$$= \frac{e^0}{0(e^0) + 1(e^0) - 1}$$

$$= \frac{1}{1 - 1}$$

L'Hospital's Rule is applicable.

$$\lim_{x\to 0^+} \frac{\frac{dx}{dy}(e^x)}{\frac{dx}{dy}(xe^x + e^x - 1)}$$

$$= \lim_{x\to 0^+} \frac{e^x}{xe^x + 1e^x + e^x}$$

$$= \frac{e^0}{0e^0 + 1e^0 + e^0}$$

$$= \frac{1}{0 + 1 + 1}$$

$$= \boxed{\frac{1}{2}}$$

57. $\lim_{x\to 0^+} x^{\sqrt{x}}$

$$= \lim_{x\to 0^+} x^{\sqrt{x}}$$

$$= \lim_{x\to 0^+} e^{\sqrt{x}\ln x}$$

$$= \lim_{x\to 0^+} e^{\sqrt{0}\ln 0}$$

$$= \boxed{1} \qquad \text{because } 0^+$$

61. $\lim_{x\to 1^+} x^{1/(1-x)}$

$$= \lim_{x\to 1^+} e^{\ln(x^{1/(1-x)})}$$

$$= e^{\lim_{x\to 1^+} \ln(x^{1/(1-x)})}$$

Isolate the exponent.

$$\lim_{x\to 1^+} \ln\left(x^{1/(1-x)}\right)$$

$$= \lim_{x\to 1^+} \frac{1}{1-x}\ln x$$

$$= \frac{1}{1-1}\ln 1$$

$$= \frac{0}{0}$$

L'Hospital's Rule is applicable.

$$\lim_{x\to 1^+} \frac{\frac{dx}{dy}(\ln x)}{\frac{dx}{dy}(1-x)}$$

$$= \lim_{x\to 1^+} \frac{\frac{1}{x}}{-1}$$

$$= \lim_{x\to 1^+} -\frac{1}{x}$$

$$= -\frac{1}{1}$$

$$= -1$$

Substitute exponent into modified original function.

$$e^{(-1)}$$

$$= \boxed{\frac{1}{e}}$$

65. $\lim_{x\to0^+}(4x+1)^{\cot x}$

$$= \lim_{x\to0^+} e^{\ln\left((4x+1)^{\cot x}\right)}$$

$$= e^{\lim_{x\to0^+}\left(\ln\left((4x+1)^{\cot x}\right)\right)}$$

Isolate the exponent.

$$\lim_{x\to0^+} \ln\left((4x+1)^{\cot x}\right)$$

$$= \lim_{x\to0^+} \cot x \ln(4x+1)$$

$$= \lim_{x\to0^+} \frac{\ln(4x+1)\cos x}{\sin x}$$

$$= \frac{\ln(4(0)+1)\cos 0}{\sin 0}$$

$$= \frac{0\cdot 1}{0}$$

L'Hospital's Rule is applicable.

$$= \lim_{x\to0^+} \frac{\frac{dx}{dy}(\ln(4x+1)\cos x)}{\frac{dx}{dy}(\sin x)}$$

$$= \lim_{x\to0^+} \frac{\ln(4x+1)(-\sin x) + \frac{1}{(4x+1)}(4)(\cos x)}{\cos x}$$

$$= \frac{\ln(4(0)+1)(-\sin 0) + \frac{1}{(4(0)+1)}(4)(\cos 0)}{\cos 0}$$

$$= \frac{(0)(0) + \frac{1}{1}(4)(1)}{1}$$

$$= 4$$

Substitute exponent into modified original function.

$$\boxed{e^4}$$