This is not financial advice

Analysis of “free bet” gambling strategy

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1. Introduction

In May of 2021, the state of Colorado legalized online sports betting. There are now over two dozen legal sports betting platforms in the state of Colorado, with many platforms offering incentives via (‘promotions), to attract new users to these platforms. Promotions currently offered in the online sports betting world include ‘risk free bets’, ‘deposit match bonuses’, ‘wager match bonuses’ and ‘free bonus bets’. Due to the nature of the sports betting market and promotions, strategic betting can be used to guarantee a profit off these promotions. This paper will cover a few basic strategies/ideas for maximizing profit and minimizing risk of the current (Dec 2021) new customer promotion offered by the betting platform ‘WynnBet’.

2. TL;DR

If you can pass prob & stats 101 and maintain a set betting strategy, you can probably make money from “free bet” sports gambling promotions. However, 1: it might not be worth your time vs a normal job, 2: probably don't do it unless you understand the math and site-specific risks behind any strategies you want to execute, as errors could be very costly.

3. Definitions & Basic Intro to Sports Betting and Expected Value Statistics

If you have sports betting and expected value calculation knowledge, you can likely skip to Section 4.

The following terms will be referenced throughout this document. Basic understanding of these terms and definitions is essential to understanding the strategies discussed in Section 4. If you want to learn more about these terms - google/wikipedia can tell you everything you need to know. A lot of the language in this section is just straight copy/pasted from wikipedia and other sources.

Odds/American Sports Betting Odds - A notation for the amount of money paid out for winning a given bet. Negative american odds mean that the team/bet is favored to win, and positive american odds mean a team is an underdog/not favored to win. A favorite with odds of winning of -150 will return $150 for every $100 bet (for a $150-$100 = $50 profit). An underdog with odds of +150 will return $250 for every $100 bet (for a $250-$100 = $150 profit).

Vig - Refers to the percentage of money taken by a sports book from any event that is bet on in their platform. This is how betting platforms make money. The most common vig is 4.9%, but the vig varies between sportsbooks, and between differing types of bets/markets. The common 4.9% taken by the sportsbook is from the odds pairing of -110/-110. For an event with the odds of -110/-110, bettors on either side of the bet are paid out $90.91 for winning their bet. With $100 bet on each side of an event ($200 total), the betting platform will take in $200, and pay out $190.91 regardless of outcome. Therefore, the betting platform will profit $9.81 (regardless of outcome) per $200 dollars bet - $9.81/$200 = 4.9%.

Implied Probability -The probability of a bet winning, based on the odds that were given by a betting platform. If you bet on an outcome that has an odds of -150, your implied probability is = -(150)/-(150 + 100) = 0.6 (60% of the time, you are expected to win, 40% of the time, you are expected to lose).

It should be noted that use of this implied probability introduces some assumptions and margins of error/certainty into my models. Implied probability does not account for the vig, and also does not perfectly reflect the probability that a bet wins or loses. Instead of perfectly reflecting the probability that a team wins or loses, the odds given by a betting platform (and therefore implied probability) represent a profitable odds pairing for that betting platform. According to one small scale study that I found, implied odds vs actual outcomes vary as much as 5% on average. We need to keep this in mind when looking at the results of our models below.

Many sports betting strategies involve betting on low-probability events, where small changes to odds (and therefore probability of success) can have a large impact on expected value calculations. **Any and all comments/questions on improving probability for my models are welcome, as I would love to add to my understanding of sports gambling probability, and/or improve my accuracy.**

Free Bet - A promotional value given to a user by a betting platform, which can be used to make a bet, but cannot be withdrawn as cash until a bet (or many bets) is settled. Free bets do not operate exactly the same as a cash bet. With a free bet, users do not receive back the initial stake of the free bet, only the winnings of the bet. For example, a winning *cash* bet of $100 on a -150 odds bet will receive the winnings ($50) and the initial stake ($100) back, while a winning *free bet* of $100 on a -150 odds bet will only receive the winnings ($50) back.

Free bets often have many restrictions and/or conditions attached to them, at the discretion of the issuing betting platform. These conditions usually include, but are not limited to, time window and odds restrictions, playthrough requirements, and many more restrictions which are designed to keep you gambling and reduce your chances of

profiting from the free bet promotion. It is, therefore, pertinent to understand the restrictions of free bets on a platform-to-platform and promo-to-promo basis.

Playthrough Requirements -A stipulation that a betting platform puts on the cashing out of a 'free bet'. The platform/promotion under analysis in this paper (Wynnbet), requires a 1x playthrough for cashing out free bets. Which means, we must bet the free bet value one time before we are allowed to withdraw any cash. Other platforms and promotions have different requirements, although a 1x playthrough for 'free bets' is currently (Dec 2021) the most common.

Sports Betting Arbitrage - Betting on both sides of an event, (on different sports betting platforms), such that regardless of the outcome of the event, the bettor will profit. This is done by taking advantage of differing odds offered for the same event, across two or more betting platforms.

Sports Betting Hedging- Betting on both sides of an event, on different sports betting platforms, in order to reduce risk of a certain outcome. A complete hedge would therefore be a 0% arbitrage - where regardless of the outcome of an event, we will not make a profit, or lose any money.

Free Bet Conversion - Taking a guaranteed profit on a 'free bet' by finding a combination of odds that guarantee a profit by placing bets on both sides of the same event. Using an oddsjam free bet conversion tool (more on that later)- we can reliably expect this profit is ~70% of the free bet value. For example, given a 'free bet' of $100 and an event with an odds pairing of +420/-500, we can guarantee ourselves a profit of $70 off the free bet by betting $100 on the +420 side and $350 cash on the -500 side. We see that regardless of the outcome of the event, we guarantee ourselves that we will have $420 after the event takes place ((0+100*4.2 = $420 payout if our $100 free bet wins, and 350+350*.2= $420 payout if our $100 free bet loses), for a guaranteed profit of $70 off of our combination of a $100 free bet and a $350 hedge cash.

Probability Distribution- The mathematical function that gives the probabilities of occurrence of different possible outcomes for an event.

Histogram - A histogram is an approximate representation of the distribution of numerical data. A density histogram is a histogram that, instead of giving frequency of occurrence by outcome, gives probability of occurrence by outcome. A density histogram therefore gives a probability distribution of an experiment, and provides a great visualization of the distribution of potential outcomes and their probabilities. In our case, we will technically be looking at bar plots of potential profits and their probability. This is (pretty much) the same as looking at a density histogram and it'll have to do (because I couldn't get a nice looking density histogram working).

Expected Value- "EV" refers to the mathematical expected outcome of some event. The event requires inputs, action, and outputs. In our case, the input is the money bet on an event, the action is the event taking place, and output is the winnings (or loss) of the bet we placed. A simple example of expected value involves betting on a coin flip. Let's say a casino is running a special coin flip betting promotion. This promotion will pay you $2.00 for a result of "heads' - every time you wager $1.00. This promotion is wildly profitable (if we can do it multiple/many times), and we can see this by calculating the expected value of the promotion. Assuming the casino's offer is based on a fair coin, we have a 50% chance (p=.5) of getting a "heads", and profiting $2.00 (+ getting the $1.00 stake back), and a 50% chance of getting a "tails" and losing $1.00. Let's look at the expected value math. This is expressed as "E(some event)" = …

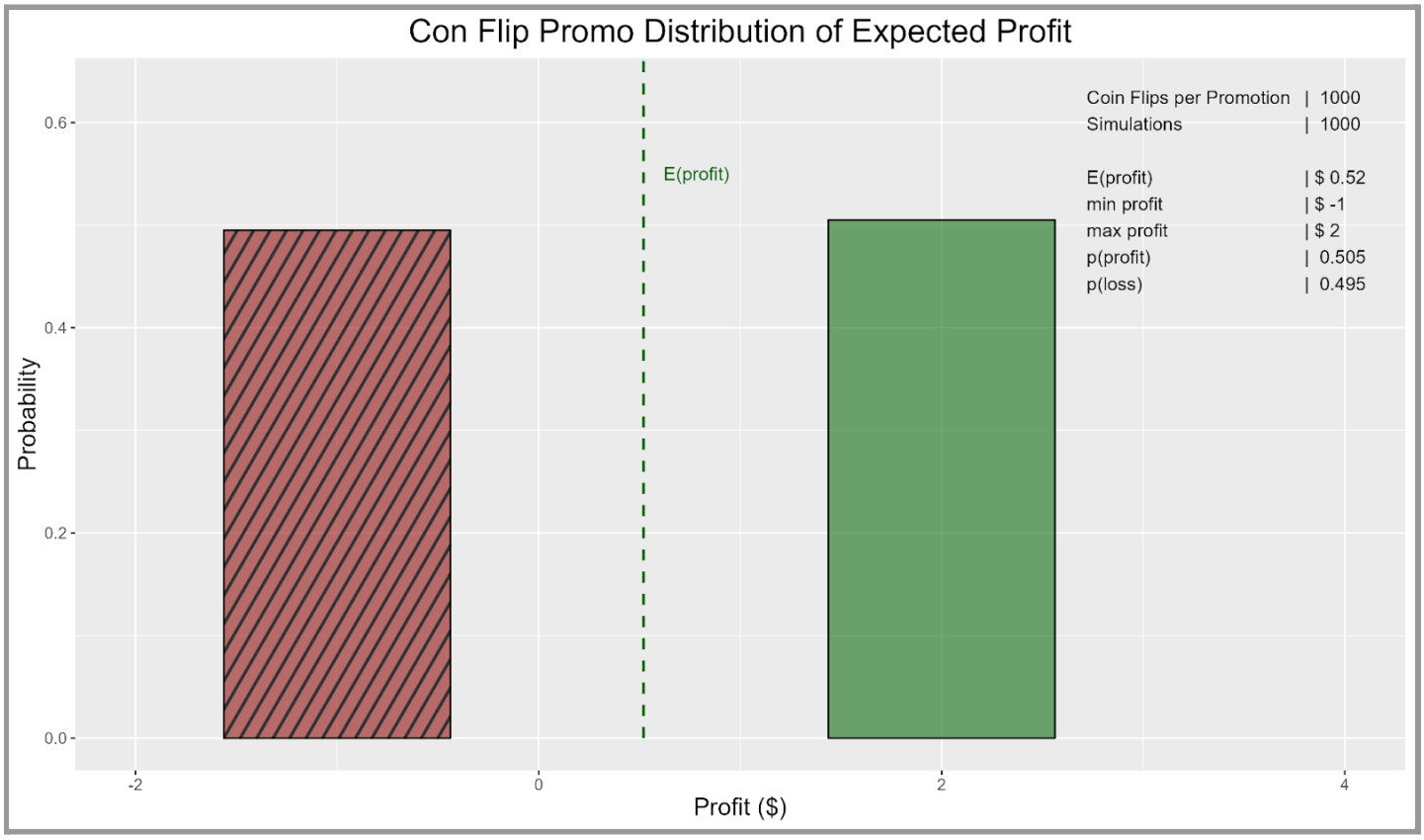
$$E(Coin\ flip\ promotion\ profit) = p(heads) * (heads\ profit) + p(tails) * (tails\ profit)$$

$$E(Coin\ flip\ promotion\ profit) = .5(\$2.00) + .5(-\$1.00) = \$1.00 - \$0.50 = \$0.50$$

We see that the coin flip casino promotion results in an expected profit of $0.50. For expected value calculations, a positive expected value means that profit is expected, and negative expected value means that a loss is expected. Of course, with only one coin flipped, we still have a very good chance (50%) of losing our $1.00 and ending up with a loss. However, if you are able to bet on 1000 individual coin flips, you are almost certainly guaranteed to make a profit, and you would expect that profit to be around $0.50*1000 = $500.

To illustrate these concepts, we will run two simulations of this promotion, and look at the expected profit distribution of each simulation. We will vary two variables - 1. The number of coin flips allowed per promotion, and 2. The number of times we attempt the promotion.

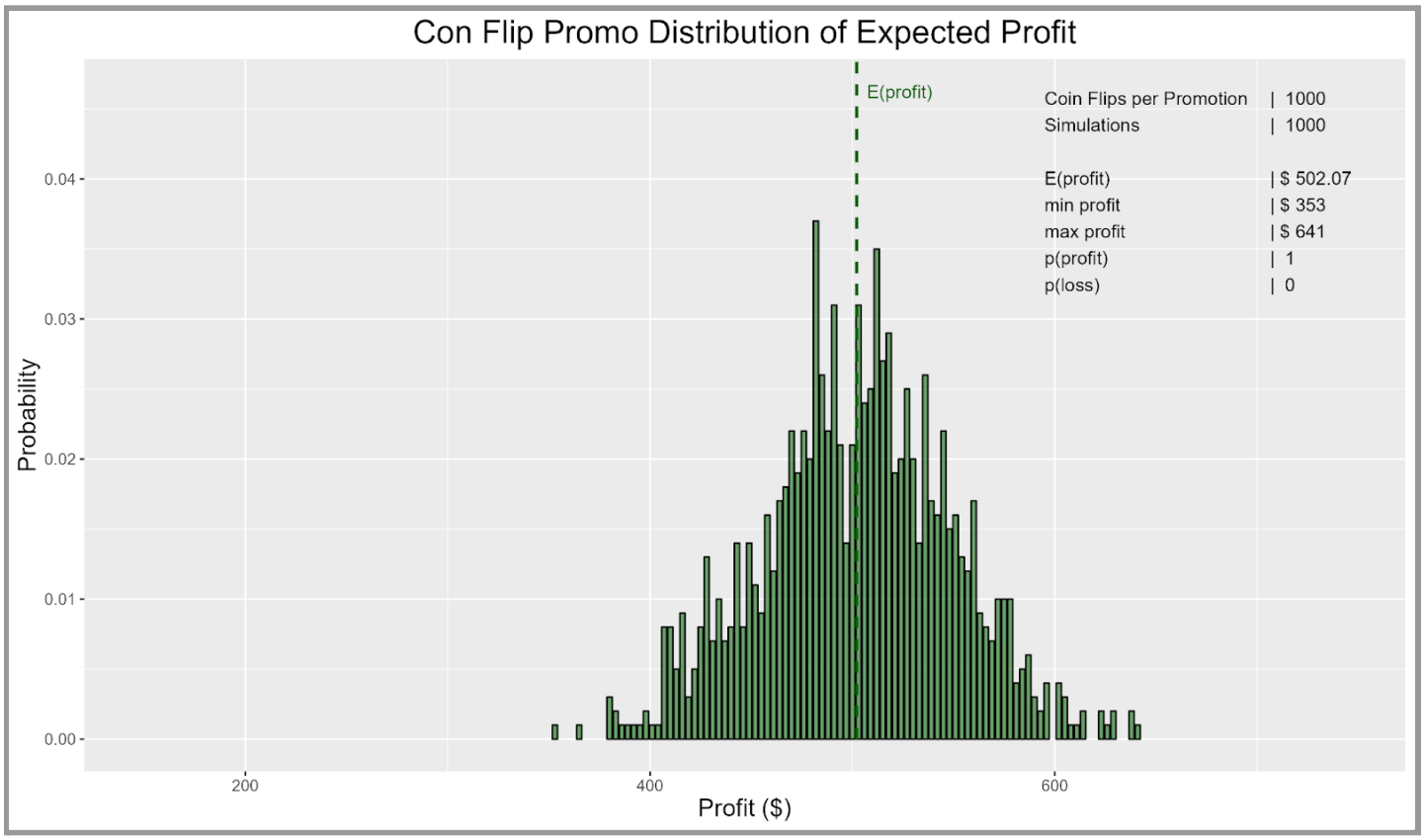
Simulation 1: Variable 1: The number of coin flips allowed per promotion: 1
Variable 2: The number of times we simulate a promotion: 1000



From this simulation 1 probability distribution, we can observe two important facts regarding this promotion: 1. Each simulation (of the 1000 simulations run) only has two possible outcomes - we either lose $1.00, or we profit $2.00. and 2. We have a (pretty much) equal chance of winning or losing money.

Let's now simulate this promotion again, but with 1000 coin flips made for each simulation (instead of just 1).

Simulation 2: Variable 1: The number of coin flips allowed per promotion: 1000
Variable 2: The number of times we simulate a promotion: 1000



From this simulation 2 probability distribution, we see that there is a range of profit outcomes between a minimum $353 and maximum $641 in profit. The key takeaways to apply to our gambling promotion analysis 1. The more bets we place, the more likely we are to land a profit around the mathematically expected profit and 2. We can calculate all the expected profits we want, but we only have one attempt per given promotion. Therefore, we need to do everything we can to find an expected distribution of profit where we would be happy landing in any location (a distribution that balances minimum return, expected profit, and potential maximum profit).

4. The WynnBet 200% Wager Match Promotion

One current promotion run by a sports betting platform in Colorado is the 200% deposit match bonus offered by Wynnbet. When a new user wagers (up to) $750 on their first bet on Wynnbet, Wynnbet will give 200% of that wager value to the user in the form of five $300 free bets (for a total value of $1500 in free bets). As with all betting platforms, this promotion includes many specific terms and conditions. For our analysis below, the only pertinent condition is that all bets (both the initial wager, and all bonus bets) may only be used for bets with odds of -150 or greater. (Other conditions include location/bonus expiration, bonuses being awarded on a particular schedule, etc)

5. Strategy Analysis

We will now apply the same expected value math and simulation as we did above (on the coin flip promotion) to the Wynnbet promotion. We will only cover static strategies (all of our bets are pre-determined before any bet outcomes, not adjusted off results of winning/losing bets).

Strategy: Static 0

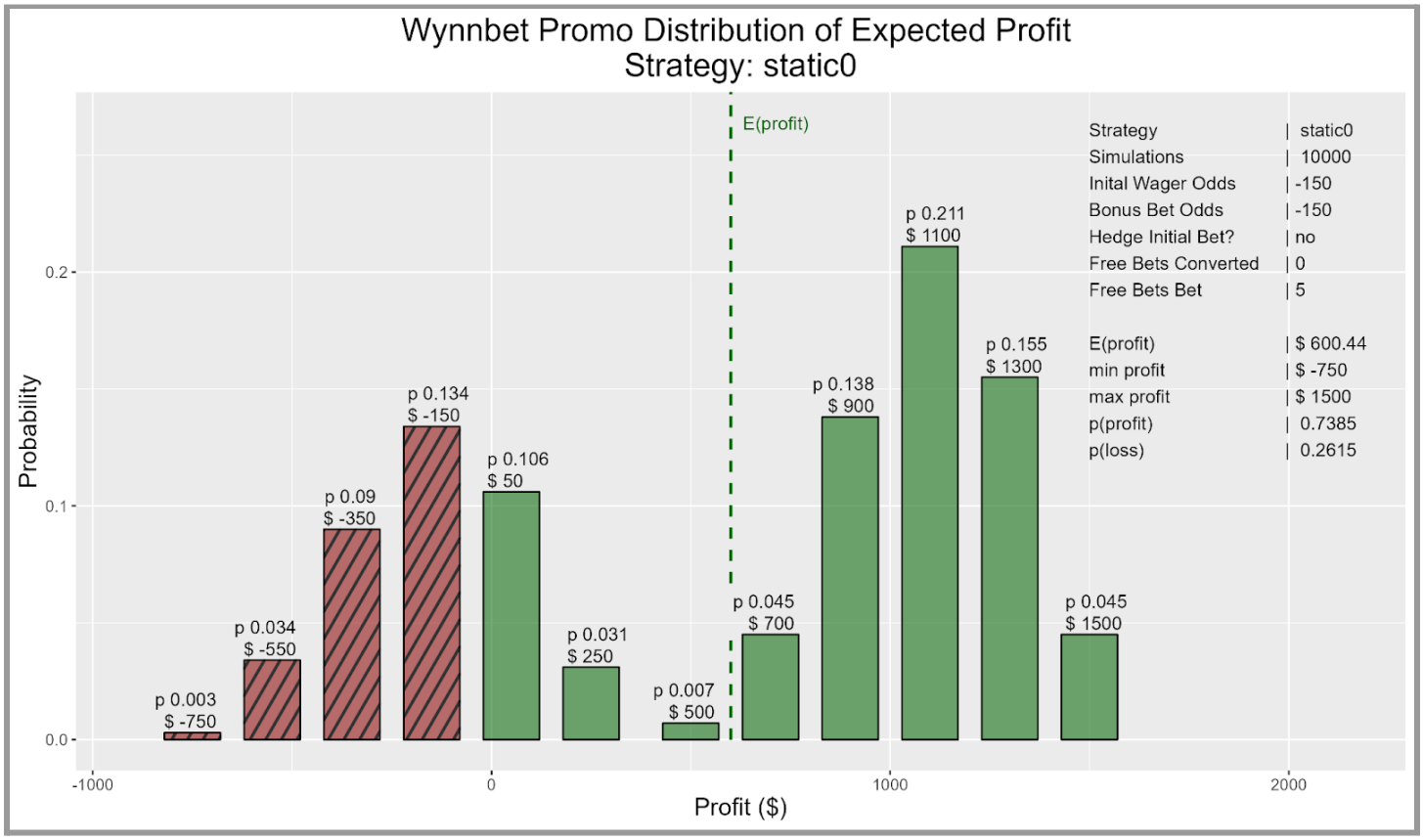
Strategy Static 0 represents our "base" strategy, which is the most basic version of playing through the Wynnbet promotion. We will wager $750 on a -150 bet in order to "earn" our free bets, and then wager each of the five free bets earned. For this base strategy, we are placing all bets on odds of -150, in order to give ourselves the "best chance of winning" each bet we place (as mentioned above, Wynnbet does not let us bet on outcomes with odds better than -150. ex: -200,-250, -20000…). Odds of -150 represent a slight favorite, and imply that the bet has a 60% chance of winning (and therefore a 40% chance of losing).

Static 0 Expected Profit Math:

$$E(Static0) = .6\left(750 \times \left(\frac{100}{150}\right)\right) + .4(-750) + 5\left(.6\left((300 \times \left(\frac{100}{150}\right)) + .4(-0)\right)\right)$$

$$E(Static0) = .6(500) + .4(-750) + 5(.6(200)) = 300 - 300 + 5(120) = \$600$$

Static 0 Distribution of Expected Profit:



Static 0 Conclusion:

Static Strategy 0 is expected to be profitable, with an expected profit of ~$600. However, as we can see from the probability distribution, we still have a p=.2615 chance of losing money, and a probability p=.003 chance of losing all of our money. We can improve on this in many ways. Fun observation: this distribution is bimodal, due to the difference between the initial cash wager and the free bets. The initial wager is a cash bet, and therefore we risk losing our initial $750 wager, while the free bets are “free” and therefore we do not lose any stake. This creates a bimodal distribution because we are either trying to claw our way back from a -$750 loss on the first via our bonus bets, or we won our first bet (+$500) and we are stacking bonus winnings on our initial wager profit.

Strategy: Static 1

Strategy Static 1 adds a hedge bet, in order to mitigate the risk of losing $750 on the first bet. Since we are betting $750 on a -150 odds outcome, we will then assume that the opposite of that bet will have an odds of +130, and use that for hedging. We can then calculate we need to hedge our initial bet with a stake of $543.48 on the +130 side (I used oddsjam arbitrage calculator for this). Now that we have hedged our initial bet, regardless of the outcome of the event we bet on, we will lose $43.43. This hedging severely limits our downside risk of losing money.

Static 1 Expected Profit Math:

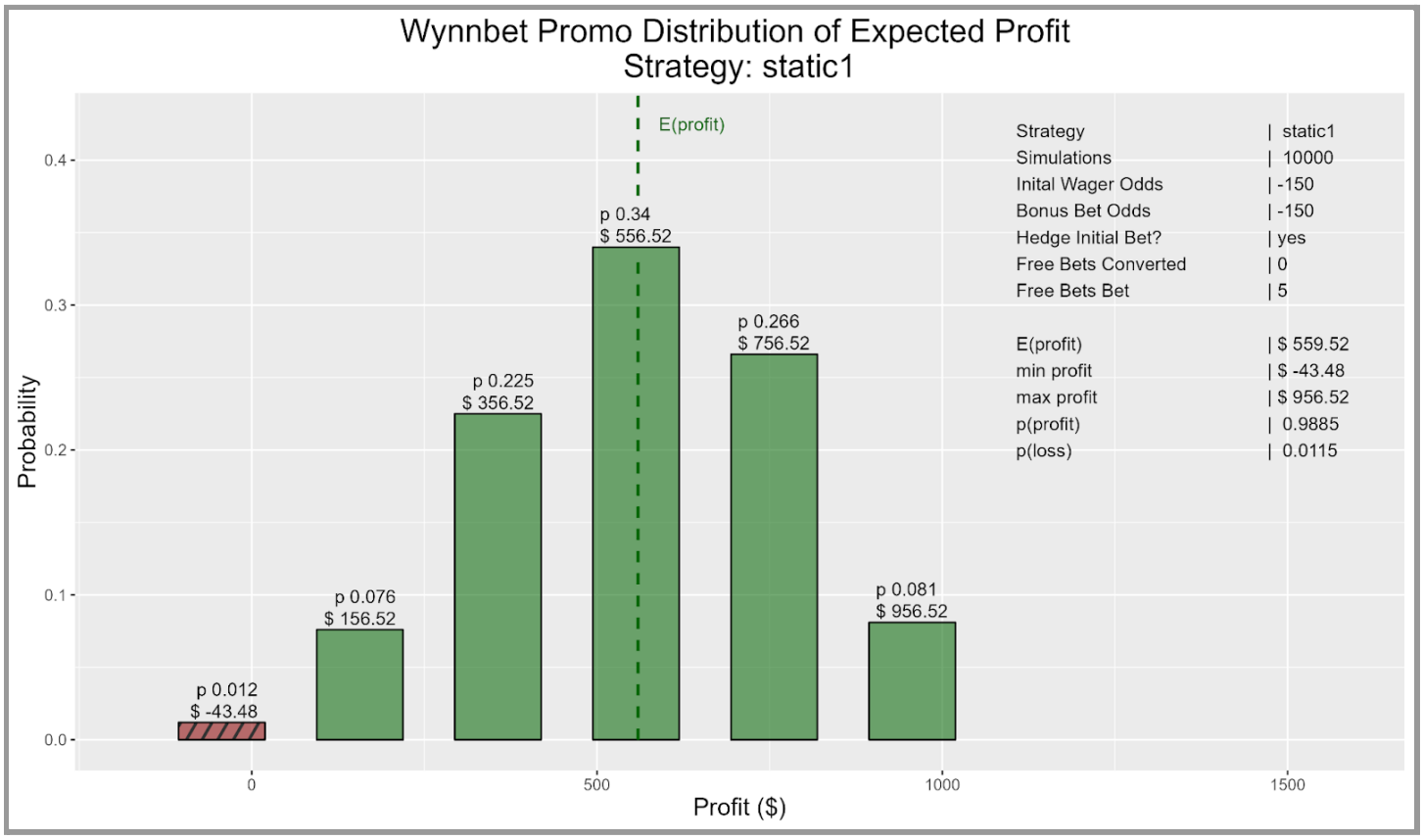
$$E(Static1) = .6\left(\left[750 \times \left(\frac{100}{150}\right)\right] - 543.48\right) + .4\left(\left[543.48 \times \left(\frac{130}{100}\right)\right] - 750\right)...$$

$$...+ 5\left(.6\left((300 \times \left(\frac{100}{150}\right)\right) + .4(-0)\right)\right)$$

$$E(Static1) = .6(500 - 543.48) + .4(706.5 - 750) + 5(.6(200))$$

$$E(Static1) = -26.088 - 17.4 + 5(120) = \$556.51$$

Static 1 Distribution of Expected Profit:



Static 1 Conclusion:

Strategy Static 1 is expected to be profitable, with an expected profit of ~$560. This expected value is less than our base case, and our maximum potential profit has gone down, but we now have only a p=.0115 chance of losing a maximum of $43.48, as opposed to our previous chance of p=.2672 chance of losing money, and a small chance of losing $750.

Strategy: Static 2

For strategy Static 2, we will now assume we can find a perfect 0% arbitrage bet to take as a hedging bet. This will eliminate the $43.48 loss of the previous strategy, and we now have no potential of losing any money.

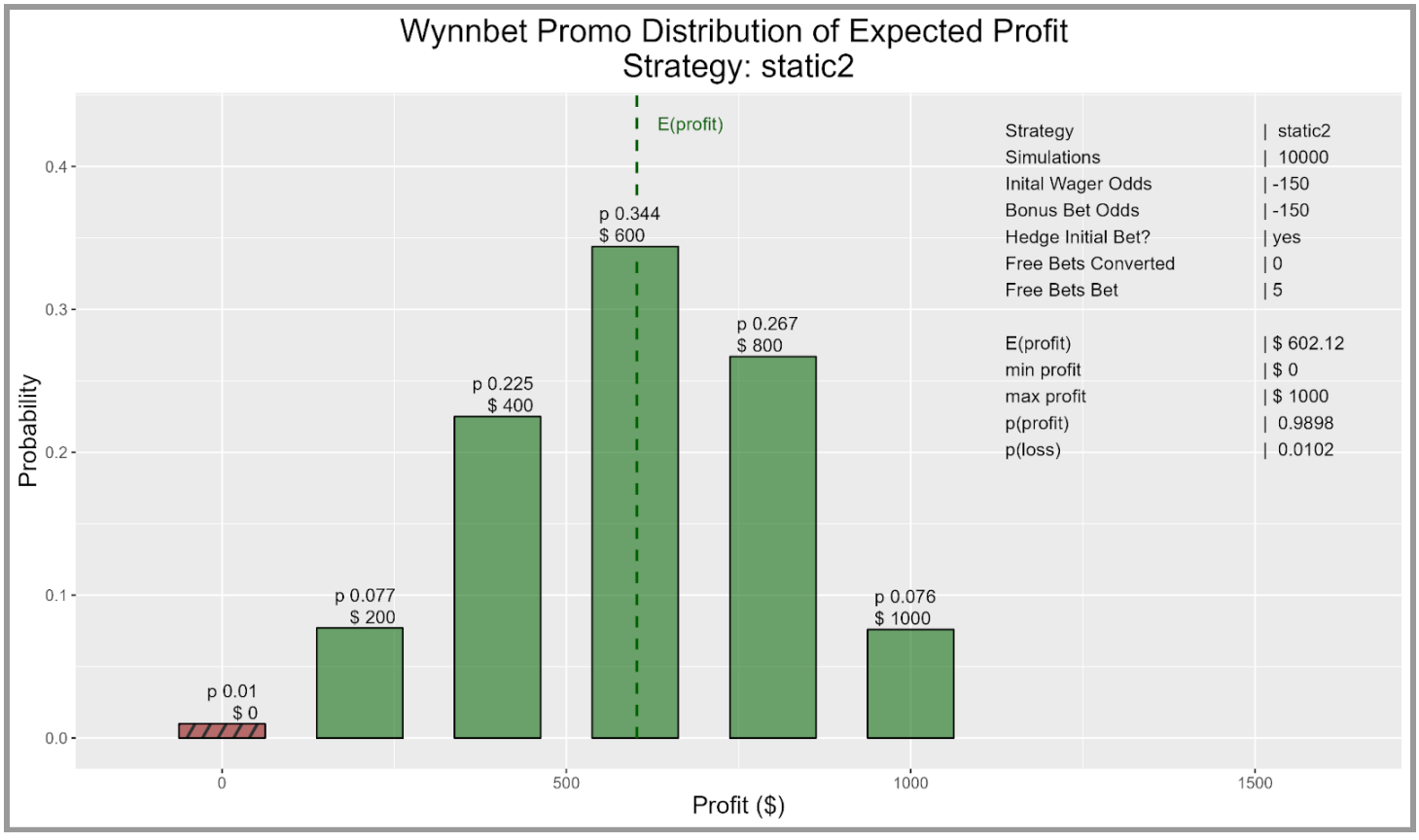
Static 2 Expected Profit Math:

$$E(Static2) = .6\left(\left[750 \times \left(\frac{100}{150}\right)\right] - 500\right) + .4\left(\left[500 \times \left(\frac{150}{100}\right)\right] - 750\right)...$$

$$...+ 5\left(.6\left((300 \times \left(\frac{100}{150}\right)) + .4(-0)\right)\right)$$

$$E(Static2) = .6(500 - 500) + .4(750 - 750) + 5(.6(200)) = \$600$$

Static 2 Distribution of Expected Profit:



Static 2 Conclusion:

Strategy Static 2 is expected to be profitable, with an expected profit of ~$600, zero risk of losing money, and a p=.01 chance of no winnings. We are back to the expected profit of the original strategy static 0, but we now have zero chance of losing money.

Strategy: Static 3

Strategy Static 2 does well, but we still have a (small) chance of not making any profit. This outcome is not ideal, since we're putting time and money on the line in search of a profit. In order to mitigate this with strategy Static 3, we will "convert" one of the free bets into a guaranteed profit (see "Free Bet Conversion" in Section 3) . We will assume that we are able to find an event to bet on which will convert 70% of the free bet value into cash.

Static 3 Expected Profit Math:

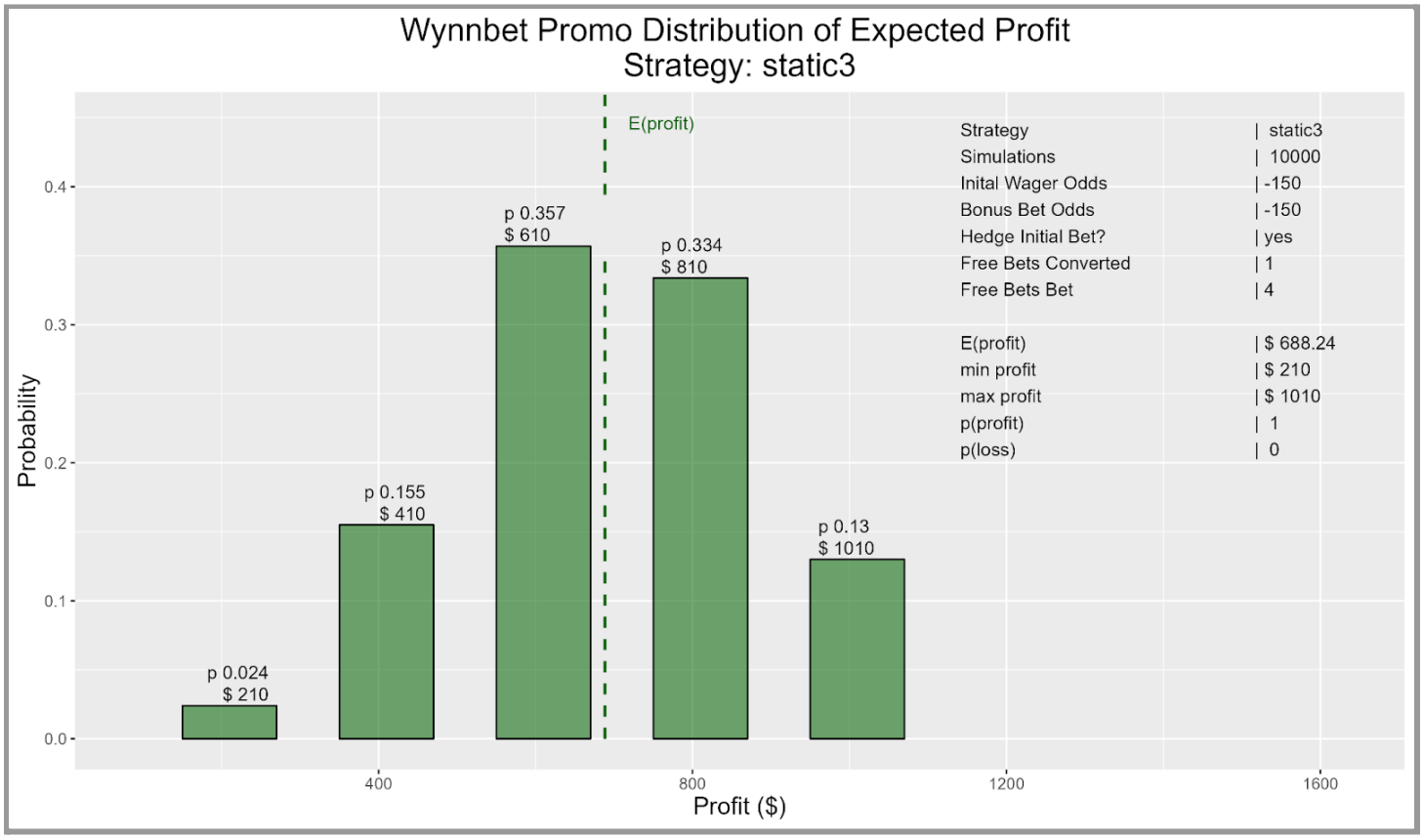
$$E(Static3) = .6\left(\left[750 \times \left(\frac{100}{150}\right)\right] - 500\right) + .4\left(\left[500 \times \left(\frac{150}{100}\right)\right] - 750\right)...$$

$$...+ 4\left(.6\left((300 \times \left(\frac{100}{150}\right)) + .4(-0)\right)\right) + 1(300 *.7)$$

$$E(Static3) = .6(500 - 500) + .4(750 - 750) + 4(.6(200)) + 210$$

$$E(Static3) = 4(120) + 210 = \$690$$

Static 3 Distribution of Expected Profit:



Static 3 Conclusion:

We now see that converting free bets is better than actually betting them (at -150), as our expected value went up with one free bet converted. Extending this to the extreme, we could convert all 5 bets, for guaranteed profit of 5*$210 = $1050. This would be an ideal strategy for some bettors, as you are therefore 100% guaranteed to make $1050 off of the Wynnbet promotion (aside from other risks - see Section 6). However, we can improve on this expected profit by changing the odds of our bonus bets.

Strategy: Static 4

We have now seen that we can make improvements via hedging our initial bet, and converting free bets. We also saw that converting free bets (at 70% conversion assumption) results in a higher return than betting the free bet at -150 odds. This is due to the fact that free bets only pay back the winnings of a bet, and do not return the stake of the free bet. If we calculate the expected value of a free bet placed on varying odds, we actually see that the "longer" odds we play with our free bet credit, the higher our expected value from that bet. In fact, as we increase our odds to +infinity, our expected profit approaches the value of the free bet stake:

$$E(\$100\ free\ bet, -\ 150\ odds) \ = \ .6\ *\ (100\ *\ (\tfrac{100}{150})\ )\ = \$40.00$$

$$E(\$100\ free\ bet, +\ 200\ odds) \ = \ .3333\ *\ (100\ *\ (\tfrac{200}{100})\ )\ = \$66.66$$

$$E(\$100\ free\ bet, +\ 400\ odds) \ = \ 2.*\ (100\ *\ (\tfrac{400}{100})\ )\ = \$80.00$$

$$E(\$100\ free\ bet, +\ 1,000\ odds) \ = \ .0909\ *\ (100\ *\ (\tfrac{1000}{100})\ )\ = \$90.09$$

$$E(\$100\ free\ bet, +\ 10,000\ odds) \ = \ .02\ *\ (100\ *\ (\tfrac{10000}{100})\ )\ = \$\,99.001$$

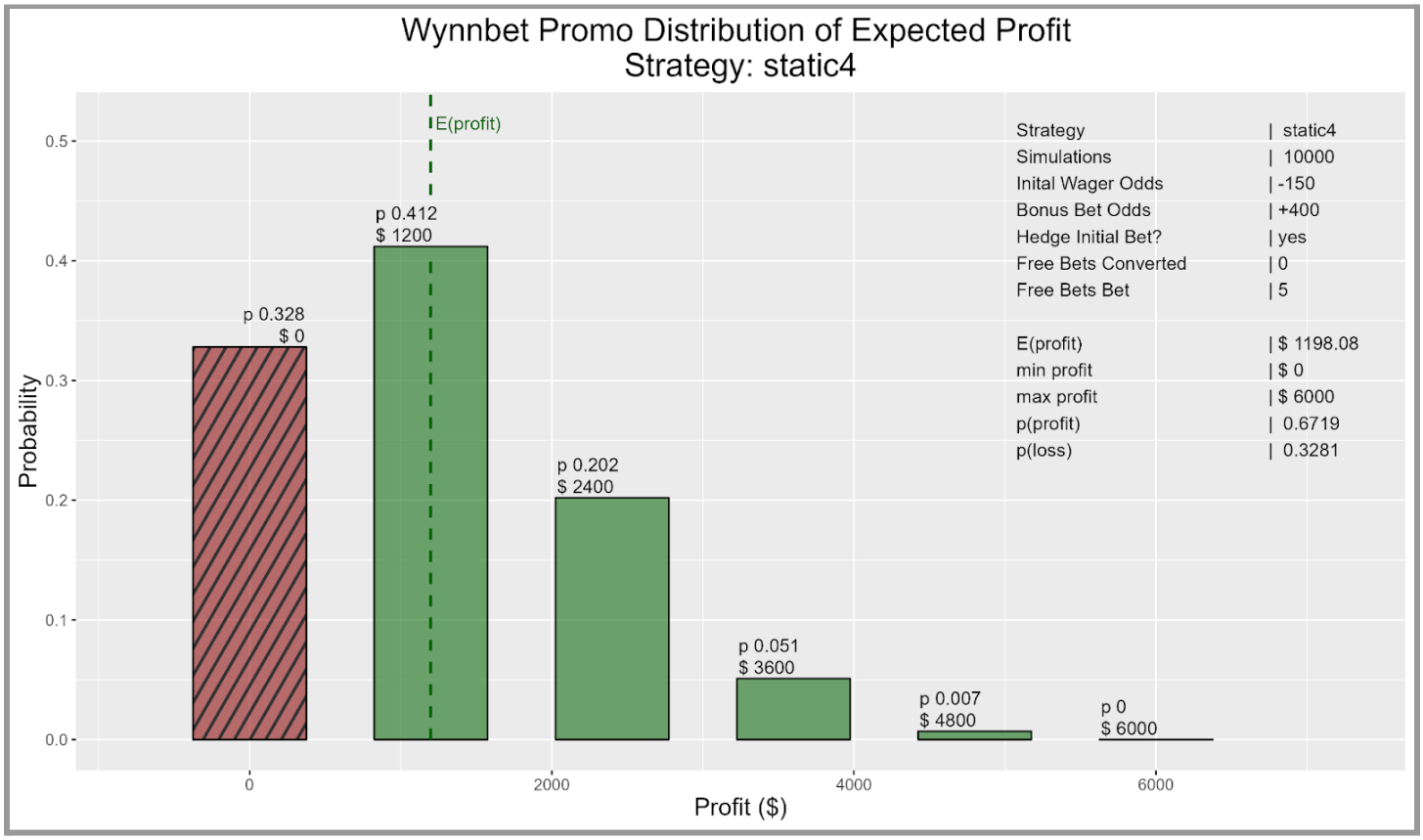
For strategy 4, we will 0% arbitrage our initial bet, and then place free bets on underdogs with odds of +400. Our odds of winning each free bet goes down, but when we win a bet, we get a much larger return, and therefore a higher expected profit.

Static 4 Expected Profit Math:

$$E(Static4)\ = 0\ +\ \ 5\Big(.2\Big((300\ \times\ (\tfrac{400}{100}))\ +\ .8\,(-\ 0)\Big)\Big)$$

$$E(Static4)\ = 5(.2(1200))\ =\ \$1200$$

Static 4 Distribution of Expected Profit:



Static 4 Conclusion:

We see that we have an almost 2x increase on our expected profit. However, with strategy Static 4, we have a p=.328 chance of not making any money (the outcome where all five of our +400 bets lose).

Strategy: Static 5

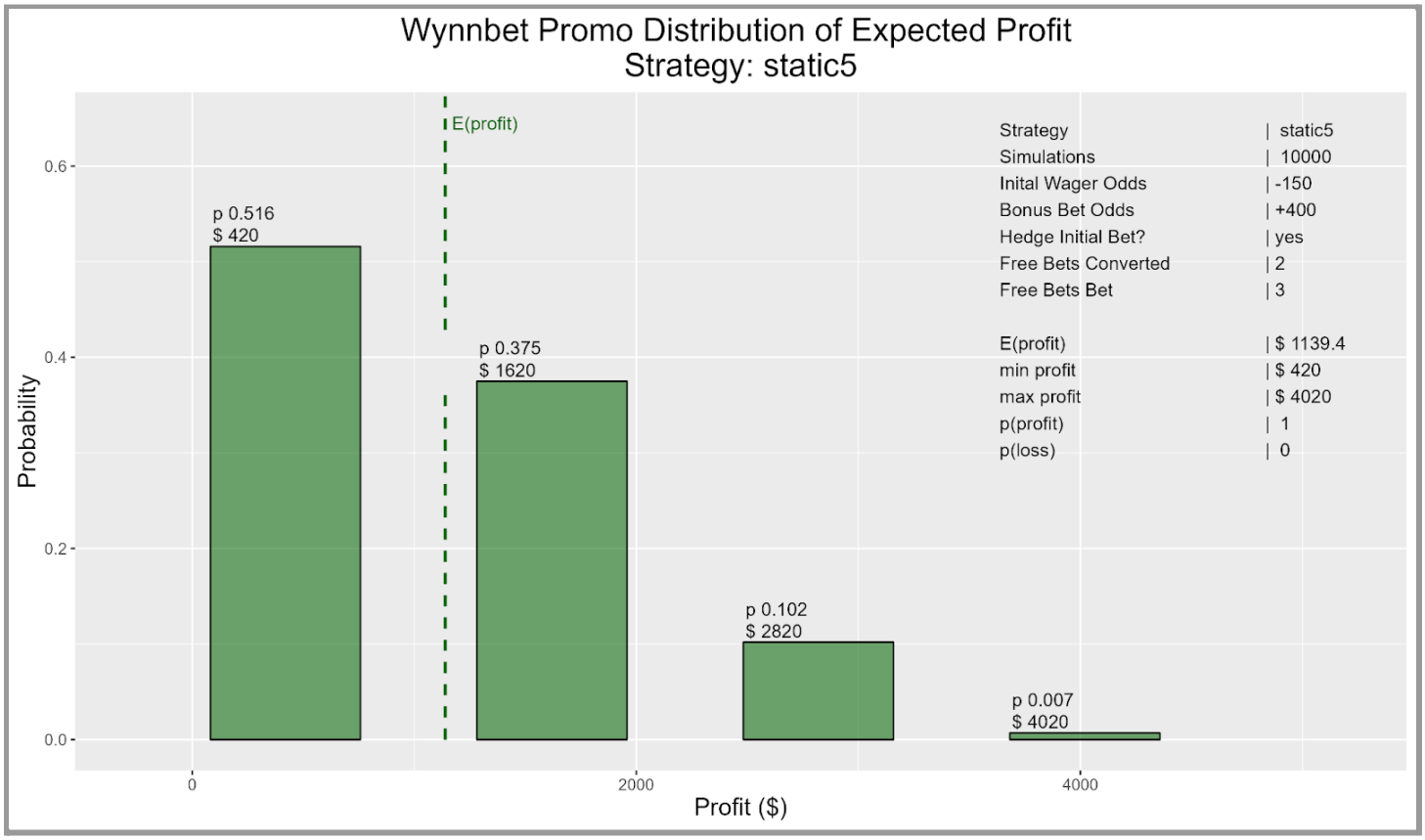
For strategy 5, we will hedge our initial bet, and combine the ideas of the Static 3 and Static 4 strategies by converting two free bets, and betting three free bets.

Static 5 Expected Profit Math:

$$E(Static5) = 0 + 3\Big(.2\Big((300 \times (\tfrac{400}{100})) + .8(-0)\Big)\Big) + 2(.7(300))$$

$$E(Static5) = 3(.2(1200)) + 420 = 720 + 420 = \$1140$$

Static 5 Distribution of Expected Profit:

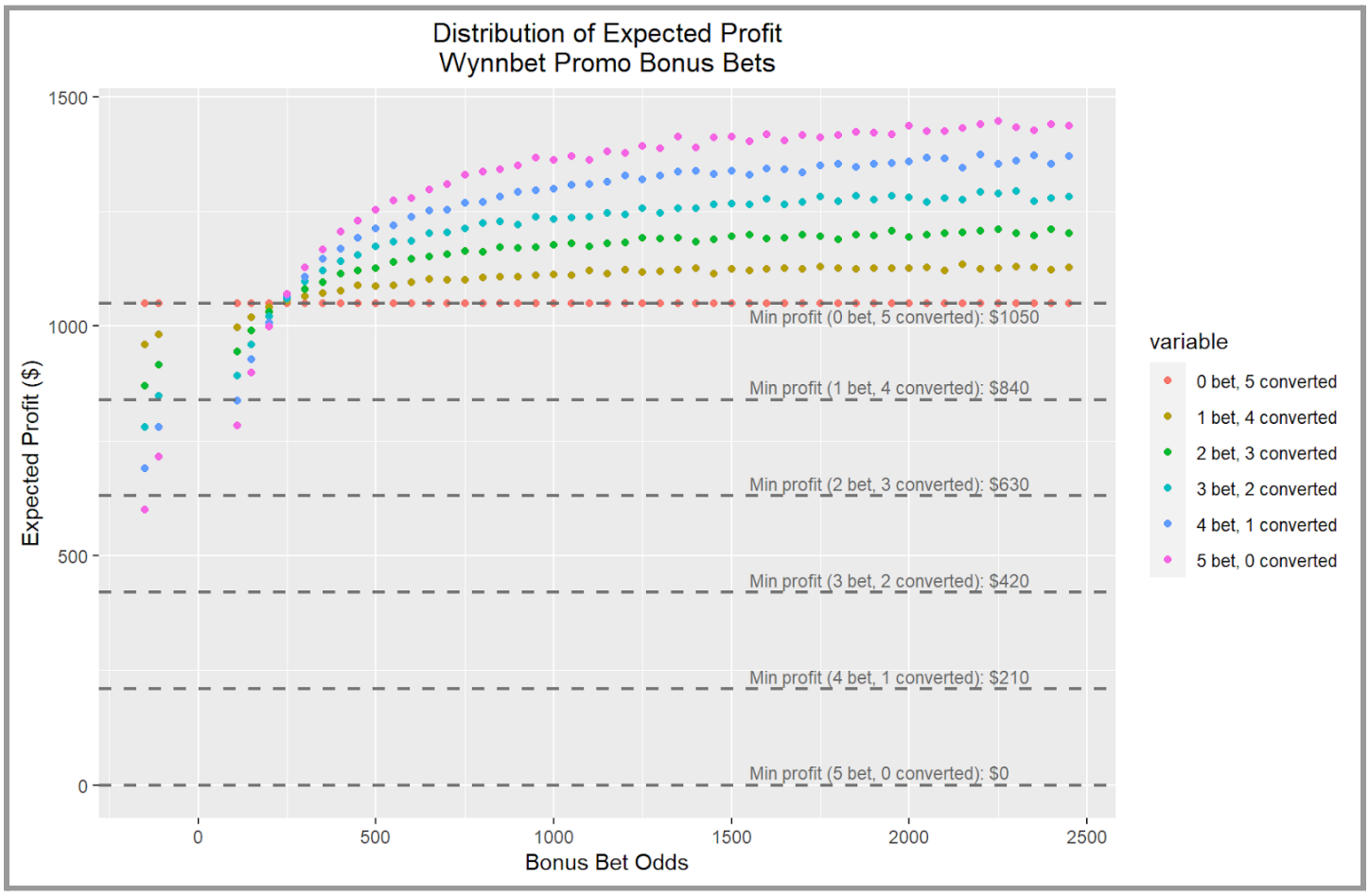


Static 5 Conclusion:

We see that combining our previous strategies into “Static5” gives us a nice balance of minimum profit vs expected profit, while also giving potential for a big payout (if we get lucky).

These strategies can be combined with each other in numerous ways, in order to adjust our minimum and potential profits as desired. The plot below shows the expected profit vs odds played for free “bonus bets”, across the six possible combinations of betting and converting the 5 Wynnbet free bets. Anyone implementing the strategies discussed should weigh the tradeoffs between minimum profit, expected profit, and maximum expected profit.

Note that the scatter plot does not follow perfectly smooth lines, as I was using monte carlo simulation for generating this graph. If I had done the pure math on the expected value, each scatter series would follow a nicer looking/smooth perfect math world curve.



Strategy: 6, 7, 8…

There are an infinite number of strategies that can be employed to profit from this promotion. The strategies discussed above, undoubtedly, do not cover the optimal strategy for maximizing the free bet expected value. Additionally, just because a strategy has the highest EV, does not mean it necessarily has a distribution of outcomes that we are happy with - ex making $0 in profit, any potential loss, etc. Future analysis will include dynamic strategies, strategies that include parlays, various levels of hedging free bets, etc - please let me know if you have high EV free bet strategy ideas for evaluation!

6. Various Risks, Disclaimers, Assumptions, etc

1. Gambling addiction
2. This list should not be taken as all-encompassing in any way
3. There is probably (definitely) at least one error somewhere in my math/calculations/assumptions/code
4. Betting platforms watch for 'bonus fraud', which is a term they may define in any way they see fit
5. Just like a casino, betting platforms can ban you at any time, for any reason they'd like
6. On-the-fly strategy changes and modification of strategy when bets do not go your way
7. Implied probability (and therefore anything we derive from it) is not a perfectly accurate prediction of outcome probability + random number generation from computers is a whole other thing
8. The author's code, stats knowledge, and sports betting knowledge is pretty amateur, and error prone
9. Taxes will be more work than you're probably used to

7. Shoutout to Oddsjam

Much of the strategy and information in this document came from the sports betting data/tool/resource site 'Oddsjam'. So much so, that they deserve their own entire section of this paper. This site is a fantastic and extremely valuable source of quantitative sports betting tools and education. None of the modeling I have done would be here without their site and the ideas I've gathered from their webpage and youtube channel. I am not affiliated with oddsjam - I've just found that their product has helped my profitability and has been worth a subscription while I work on free bet profit strategies.

https://oddsjam.com/
https://www.youtube.com/c/OddsJamInc

8. Contact & Free Beer Policy

All forms of questions, comments, and feedback are greatly appreciated - regardless of your familiarity with sports betting/math/stats/programming, or whether you view sports gambling positively or negatively.

Free Beer Policy - If you're around Denver, CO or any Ikon Pass ski resort in Colorado - I will buy you a beer (or water or coffee or whatever you want) for your feedback and thoughts in person!

Contact: david.sr.rose@gmail.com

9. Code

https://github.com/davidsrrose/gambling-functions

Any and all feedback is greatly appreciated - as I am currently brushing back up on my stochastic programming, learning R, and Git/Github use. I have never had another human review my code before, so even the most basic of comments would be extremely helpful to my learning process.