Flattening Polygonal Linkages via Uniform Angular Motion

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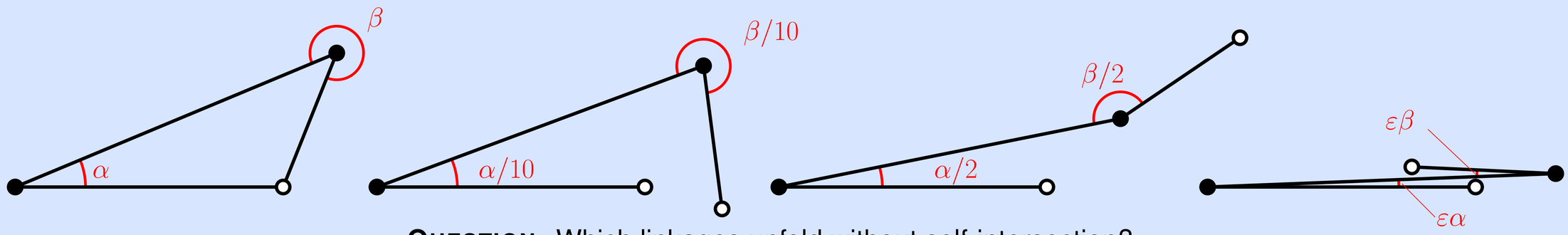
MODEL: Fixed edge lengths, flexible joints, and each angle changes at a uniform rate to 0, π, or 2π.

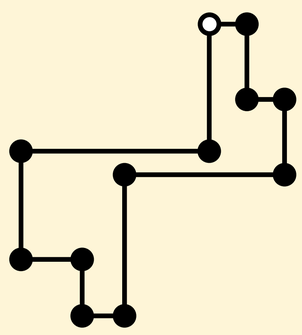


QUESTION: Which linkages unfold without self-intersection?

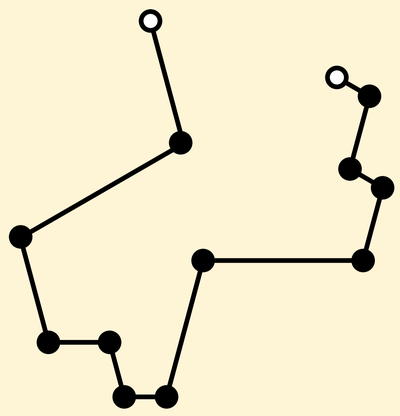
LINKAGES THAT UNFOLD INTO A STRAIGHT LINE

- **Convex Orthogonal Polygons**

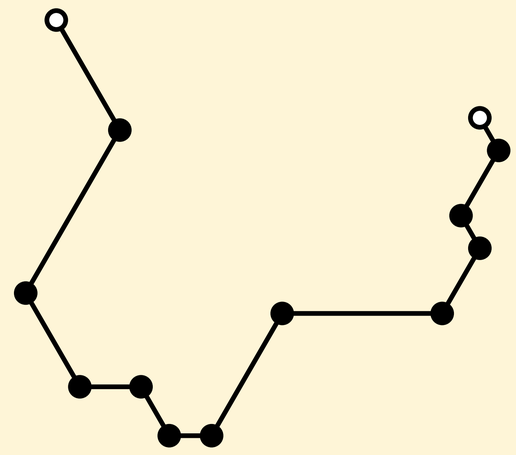
$t = 0$



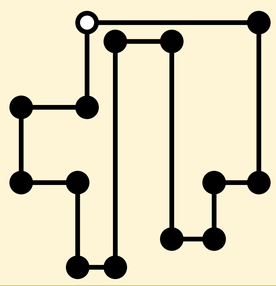
$t = \frac{1}{6}$



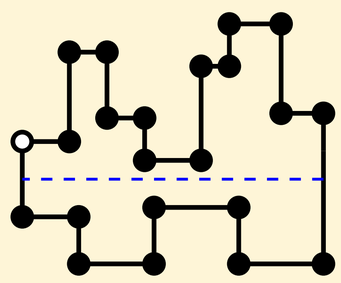
$t = \frac{1}{3}$



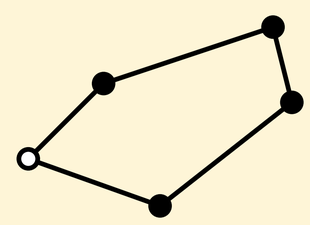
- **Orthogonal Polygons with one concavity**



- **Two terrains**



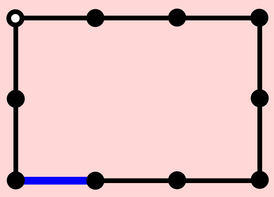
- **Convex Polygons**



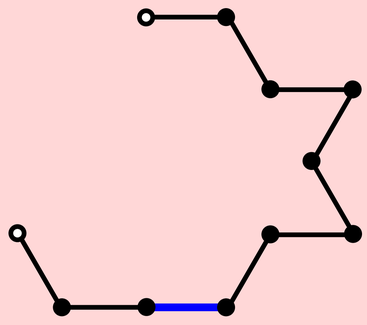
LINKAGES THAT RECONFIGURE INTO A ZIGZAG

- **Rectangle formed by unit length segments.**

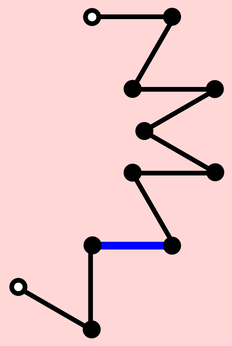
$t = 0$



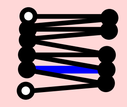
$t = \frac{1}{3}$



$t = \frac{2}{3}$

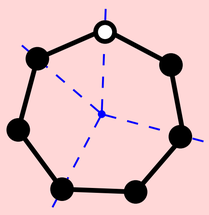


$t = 1 - \varepsilon$

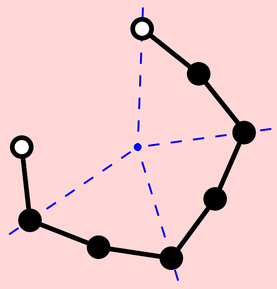


- **Regular Polygons, Triangles, Quadrilaterals and Convex Pentagons**

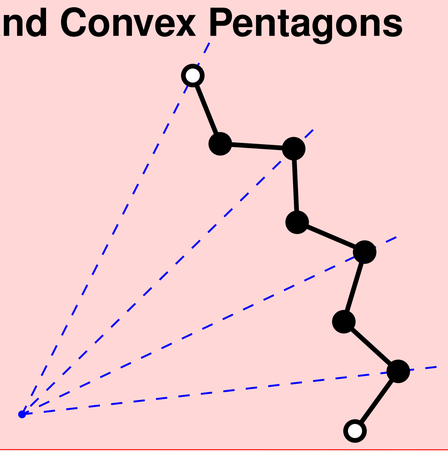
$t = 0$



$t = \frac{1}{6}$

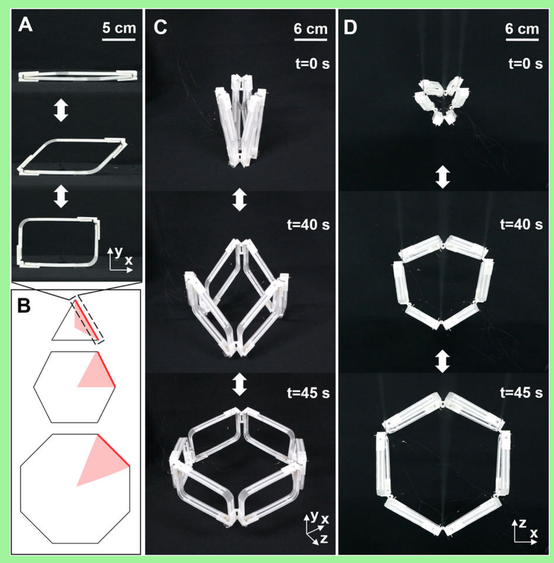


$t = \frac{1}{2}$



$t = 1 - \varepsilon$

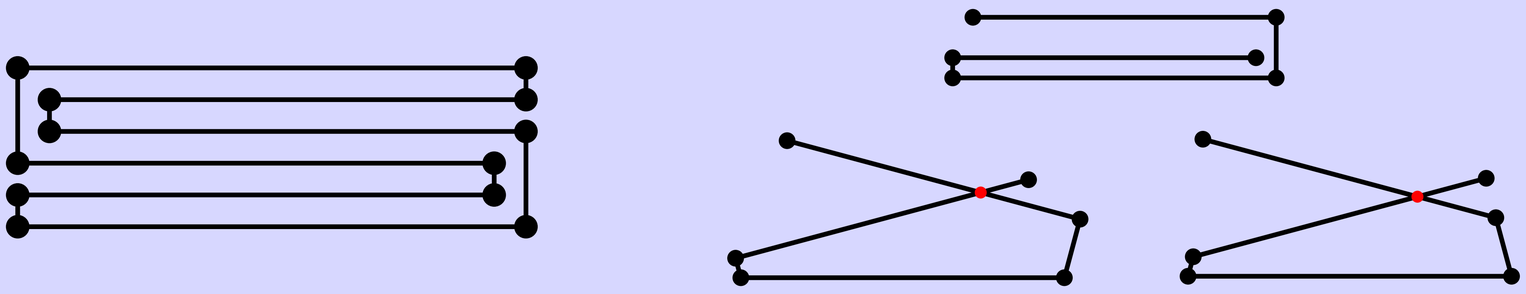
MOTIVATION



Wei Wang, Hugo Rodrigue, Sung-Hoon Ahn,
Deployable Soft Composite Structures.
DOI:10.1038/srep20869, 2016.

LINKAGES THAT UNFOLD INTO NEITHER A STRAIGHT LINE NOR A ZIG-ZAG

- **Orthogonal Polygons with two concavities/x-Monotone Polygons**



References

[1] Robert Connelly and Erik D. Demaine, Geometry and topology of polygonal linkages, in *Handbook of Discrete and Computational Geometry*, third edition, 2017, chapter 9, pages 233–256.

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