Computer Vision, Assignment 2

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February 2025

1 Calibrated vs. Uncalibrated Reconstruction

1.1 Exercise 1

The camera equation states

$$\lambda x \sim HX$$

Assume in 3D space we have

$$x \sim PX$$

A projective transformation in 3D space is represented by a 4×4 invertible matrix T:

$$X' = TX$$

where X' is a transformed version of X. Adding $I = T^{-1}T$ into the camera equation gives us:

$$x \sim PX = PT^{-1}TX = (PT^{-1})X'$$

We see that we get a projection for X':

$$P' = PT^{-1}$$

and we obtain:

$$x \sim P'X'$$

showing that we can always obtain a new solution from TX for any projective trans- formation T of 3D space.

1.2 Computer Exercise 1

1.2.1 # COMPUTER EXERCISE 1.1

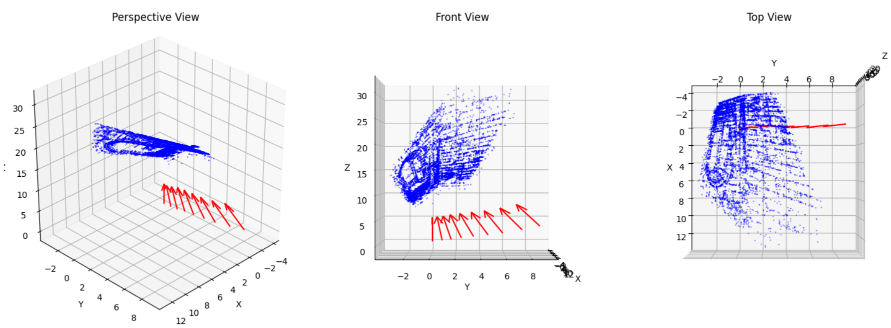

Figure 1: Perspective, Front, and Top views of the reconstructed point cloud.

It looks like a reasonable reconstruction where you can see the resemblance of a wall meeting another wall like in the pictures. However, it does not seem to be close to 90° and more like 20° when viewing it from the front and from the top.

1.2.2 # COMPUTER EXERCISE 1.2

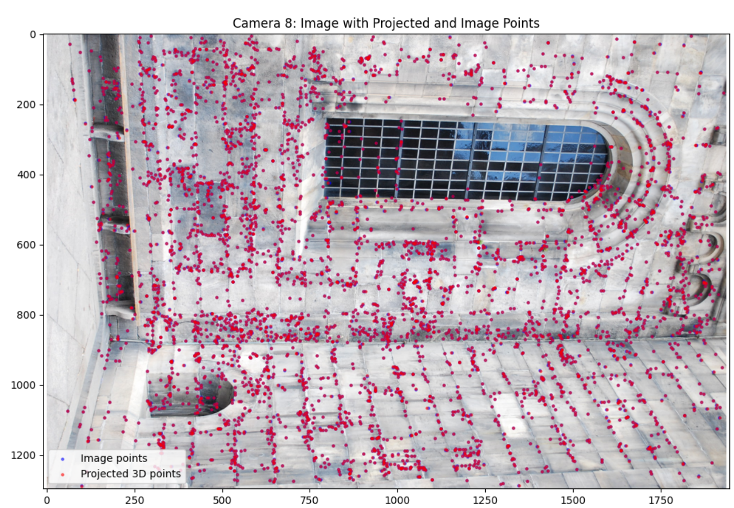

Figure 2: Projected points and image points onto image.

It looks like the projected points and the image points are very close in the picture, almost like they completely overlap. Investigating this further I discovered there is a small difference with the mean difference being 0.24 pixels for camera 7, and around 0.2 for the other cameras too.

1.2.3 # COMPUTER EXERCISE 1.3

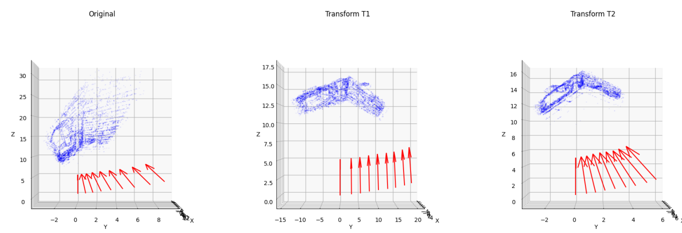

Figure 3: Projected points and image points using T1 and T2 viewed from a Front view.

We clearly see an improvement in the angle between the walls displaying a higher angular change closer to 90°. I looked around with from some other views/perspectives since it's hard to tell the 3D plot from only a 2D picture and I think both of them gives a clear improvement and seem compared to without the first projection. The alleged 90° angel looks to be around 60°, so not all the way there but definitely reasonable.

1.2.4 # COMPUTER EXERCISE 1.4

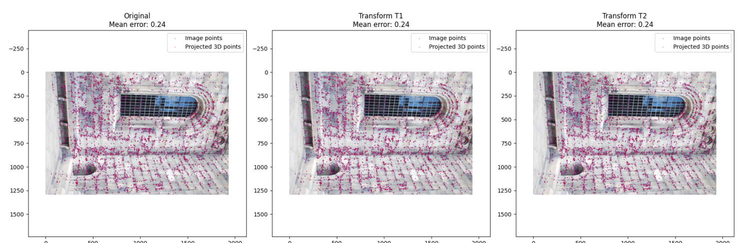

Figure 4: Projected points and image points in Image using T1 and T2.

We see that just as the theory says

$$x \sim PX \iff$$
$$x \sim (PT^{-1})(TX) \iff$$
$$x \sim P'X'$$

the projections stay the same.

1.3 Exercise 2

With a calibrated camera we reduce the degrees of freedom fixating the intrinsic parameters such as focal length, principal point, and skew reducing the possible projective transformations.

The corresponding statement for calibrated cameras is that instead of allowing any projective transformations, they only allow transformations that preserve the camera's intrinsic parameters.

We can allow

- **Euclidean transformations**: Rotation and translation.
- **Similarity transformations**: Euclidean transformations plus uniform scaling.
- **Affine transformations** (in some cases): Preserving parallelism.

2 Camera Calibration

2.1 Exercise 3

2.1.1 Exercise 3.1

Verifying that K^{-1} is the inverse we check if $KK^{-1} = I$.

$$KK^{-1} = \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{bmatrix} \iff$$

$$\begin{bmatrix} f \cdot \frac{1}{f} & 0 & f \cdot \left(-\frac{x_0}{f}\right) + x_0 \cdot 1 \\ 0 & f \cdot \frac{1}{f} & f \cdot \left(-\frac{y_0}{f}\right) + y_0 \cdot 1 \\ 0 & 0 & 1 \cdot 1 \end{bmatrix}$$

$$\iff \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Similiar we can verify that K^{-1} can be factorized as AB by checking if $AB = K^{-1}$

$$AB = \begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \iff$$

$$\begin{bmatrix} 1/f & 0 & (1/f \cdot -x_0) \\ 0 & 1/f & (1/f \cdot -y_0) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\iff \begin{bmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{bmatrix} = K^{-1}$$

Interpretations of A and B:

- A scales the coordinates to normalize for focal length, effectively mapping pixel coordinates to normalized camera coordinates.
- B shifts the image coordinates so that the optical center becomes the new origin. A translation matrix.

2.1.2 Exercise 3.2

- Applying K^{-1} removes the effects of intrinsic parameters like focal length and translation.
- The principal point (x_0, y_0) moves to the origin $(0, 0)$ in normalized coordinates.

- A point at distance f from the principal point maps to the unit circle, with distance 1.

2.1.3 Exercise 3.3

$$K = \begin{bmatrix} 320 & 0 & 320 \\ 0 & 320 & 240 \\ 0 & 0 & 1 \end{bmatrix}$$

which gives $f = 320, x_0 = 320, y_0 = 240$.

$$K^{-1} = \begin{bmatrix} 1/320 & 0 & -320/320 \\ 0 & 1/320 & -240/320 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/320 & 0 & -1 \\ 0 & 1/320 & -0.75 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying K^{-1} to the given points gives us the normalization:

For $(0, 240)$ we get:

$$K^{-1} \cdot \begin{bmatrix} 0 \\ 240 \\ 1 \end{bmatrix} = \begin{bmatrix} (-1) \cdot 1 \\ (1/320) \cdot 240 + (-0.75) \cdot 1 \\ 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

For $(640, 240)$**:**

$$K^{-1} \cdot \begin{bmatrix} 640 \\ 240 \\ 1 \end{bmatrix} = \begin{bmatrix} (1/320) \cdot 640 + (-1) \cdot 1 \\ (1/320) \cdot 240 + (-0.75) \cdot 1 \\ 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

The angle between the viewing rays projecting to these points, lines in 3D, is given by the scalar product of the lines, which we immediately realize is $-1 + 1 = 0$. Therefore we conclude that the angle between the lines is $90°$.

2.1.4 Exercise 3.4

- The **camera center** is the 3D point where all viewing rays intersect, given by $C = -R^T t$, alternatively $RC + t = 0$. As an algebraical note, this is the same as $(R + t)C = 0$ since the third row in C is 0.

$$K(R + t)C = 0 \iff /]/[KRC + Kt = 0 \iff$$
$$KRC = -Kt \iff$$
$$(KR)^{-1}KRC = -(KR)^{-1}Kt \iff$$
$$C = -\underbrace{R^{-1}}_{R^T}K^{-1}Kt \iff$$
$$C = -R^T t$$

Therefore, $\begin{bmatrix} R & t \end{bmatrix}$ and $K \begin{bmatrix} R & t \end{bmatrix}$ give the same camera center C.

- The **principal axis** is the direction the camera is pointing, given by the third column of R, i.e., $R_3 = R^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

  The camera matrix with K is:

  $$K[R\ t]$$

  which gives the the third row of KR as (translation doesn't affect principal axis):

  $$\underbrace{k_3^T}_{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T} R = R_3$$

2.2 Exercise 4

2.2.1 Exercise 4.1

The camera's intrinsic parameters are:

- Focal length:$f = 1000$
- Principal point: $(500, 500)$
- Skew: 0
- Aspect ratio: 1

The intrinsic matrix K is then:

$$K = \begin{bmatrix} 1000 & 0 & 500 \\ 0 & 1000 & 500 \\ 0 & 0 & 1 \end{bmatrix}$$

Which gives K^{-1}:

$$K^{-1} = \begin{bmatrix} \frac{1}{1000} & 0 & -\frac{500}{1000} \\ 0 & \frac{1}{1000} & -\frac{500}{1000} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{1000} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{1000} & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

To normalize the camera, we compute $K^{-1}P$:

$$K^{-1}P = \begin{bmatrix} \frac{1}{1000} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{1000} & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 & -250 & 250\sqrt{3} & 500 \\ 0 & 500\left(\sqrt{3} - \frac{1}{2}\right) & 500\left(1 + \frac{\sqrt{3}}{2}\right) & 500 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1000}{1000} & \frac{-250}{1000} + \frac{1}{4} & \frac{250\sqrt{3}}{1000} - \frac{\sqrt{3}}{4} & \frac{500}{1000} - \frac{1}{2} \\ 0 & \frac{500(\sqrt{3}-\frac{1}{2})}{1000} + \frac{1}{4} & \frac{500(1+\frac{\sqrt{3}}{2})}{1000} - \frac{\sqrt{3}}{4} & \frac{500}{1000} - \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$$

2.2.2 Exercise 4.2

For an image size of 1000×1000 pixels, we normalize the corner points and center using:

$$K^{-1}\begin{bmatrix}x\\y\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{1000} & 0 & -\frac{1}{2}\\ 0 & \frac{1}{1000} & -\frac{1}{2}\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}$$

For the corner points we get:

$$(0,0) \to K^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\1\end{bmatrix} = \begin{bmatrix}-0.5\\-0.5\\1\end{bmatrix}$$

$$(0,1000) \to K^{-1}\begin{bmatrix}0\\1000\\1\end{bmatrix} = \begin{bmatrix}-\frac{1}{2}\\1-\frac{1}{2}\\1\end{bmatrix} = \begin{bmatrix}-0.5\\0.5\\1\end{bmatrix}$$

$$(1000,0) \to K^{-1}\begin{bmatrix}1000\\0\\1\end{bmatrix} = \begin{bmatrix}1-\frac{1}{2}\\-\frac{1}{2}\\1\end{bmatrix} = \begin{bmatrix}0.5\\-0.5\\1\end{bmatrix}$$

$$(1000,1000) \to K^{-1}\begin{bmatrix}1000\\1000\\1\end{bmatrix} = \begin{bmatrix}1-\frac{1}{2}\\1-\frac{1}{2}\\1\end{bmatrix} = \begin{bmatrix}0.5\\0.5\\1\end{bmatrix}$$

For the image center we get:

$$(500,500) \to K^{-1}\begin{bmatrix}500\\500\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{2}-\frac{1}{2}\\\frac{1}{2}-\frac{1}{2}\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

Exercise 5, RQ Factorization and Computation of

Exercise 5.1

We verify KR by:

$$KR = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} \begin{bmatrix} R_1^T \\ R_2^T \\ R_3^T \end{bmatrix} = \begin{bmatrix} aR_1^T + bR_2^T + cR_3^T \\ dR_2^T + eR_3^T \\ fR_3^T \end{bmatrix}$$

since R_1^T, R_2^T, R_3^T are row-vectors naturally it becomes the same with the value in the first column always getting multiplied by the value in the first row, and similar with the second and third row-column pair.

We have the camera matrix

$$P \;=\; \begin{pmatrix} 800\sqrt{2} & 0 & 2400\sqrt{2} & 4000 \\ -700\sqrt{2} & 1400 & 700\sqrt{2} & 4900 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 3 \end{pmatrix}$$

which we can divide into

$$P = \begin{bmatrix} M & \mathbf{p} \end{bmatrix},$$

where M is the 3×3 block on the left and $\mathbf{p}$ is the last column denoting translation. We want to decompose

$$M \;=\; K\,R$$

with

- K upper triangular and with positive diagonal entries,
- R an orthogonal rotation matrix ($R^\top R = I$).

Exercise 5.2

First we want to identify R_3^T and f.

From the product

$$K\,R \;=\; \begin{pmatrix} a\,R_1^T \;+\; b\,R_2^T \;+\; c\,R_3^T \\ d\,R_2^T \;+\; e\,R_3^T \\ f\,R_3^T \end{pmatrix},$$

the third row of M is $f\,R_3^T$. In our case, the third row of M (from P) is:

$$(-\tfrac{1}{\sqrt{2}},\ 0,\ \tfrac{1}{\sqrt{2}}).$$

We want R_3^T to be a unit vector, so we compute its length:

$$\sqrt{\left(-\tfrac{1}{\sqrt{2}}\right)^2 + 0^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2} \;=\; \sqrt{\tfrac{1}{2}+\tfrac{1}{2}} \;=\; 1.$$

The row is already unit length, therefore

$$f \;=\; 1, \quad R_3^T \;=\; \left(-\tfrac{1}{\sqrt{2}},\, 0,\, \tfrac{1}{\sqrt{2}}\right).$$

Exercise 5.2

Thereafter we want to find R_2^T, d, e.

From the second row of M and our expression for KR we get:

$$\begin{bmatrix}-700\sqrt{2} & 1400 & 700\sqrt{2})\end{bmatrix} = dR_2^T + e\,R_3^T.$$

To find e, we take the dot product with R_3 (which is a unit vector):

$$\begin{bmatrix}-700\sqrt{2} & 1400 & 700\sqrt{2})\end{bmatrix} \cdot R_3 = (dR_2^T + e\,R_3^T) \cdot R_3$$

$$\iff \begin{bmatrix}-700\sqrt{2} & 1400 & 700\sqrt{2})\end{bmatrix} \cdot R_3 = e$$

$$= (-700\sqrt{2})\left(-\tfrac{1}{\sqrt{2}}\right) + (700\sqrt{2})\left(\tfrac{1}{\sqrt{2}}\right) = 700 + 700 = 1400.$$

Rewriting our equation so we get d on one side gives us:

$$d\,R_2^T \;=\; M_2^T \;-\; e\,R_3^T$$

$$=\; \begin{bmatrix}-700\sqrt{2} & 1400 & 700\sqrt{2}\end{bmatrix} - 2\cdot 700 \begin{bmatrix}-\tfrac{1}{\sqrt{2}} & 0 & \tfrac{1}{\sqrt{2}}\end{bmatrix}$$

$$= \begin{bmatrix}0 & 1400 & 0\end{bmatrix}.$$

We know R_2^T is of length one, therefore we quickly realize:

$$d = 1400, \quad R_2^T = (0,\, 1,\, 0).$$

Exercise 5.2

Lastly, we use the first row to find R_1^T, a, b, c

The first row of M to KR is:

$$\begin{bmatrix}800\sqrt{2} & 0 & 2400\sqrt{2}\end{bmatrix} = a\,R_1^T + b\,R_2^T + c\,R_3^T$$

Using the orthogonality between the rows meaning the dot product is either 0 if they are different or 1 if they are the same we get from $R_2^T = \begin{bmatrix}0 & 1 & 0\end{bmatrix}$ that:

$$b = M_1^T \cdot R_2 \;=\; 0.$$

Similarly, from $R_3^T = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$ we get:

$$c \;=\; M_1^T \cdot R_3 \;=\; \begin{bmatrix} 800\sqrt{2} & 0 & 2400\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \;=\; -800 + 2400 \;=\; 1600.$$

From knowing c and b we get that:

$$M_1^T = aR_1^T + 1600\, R_3^T, \iff a\, R_1^T \;=\; M_1^T - 1600\, R_3^T.$$

$$1600\, R_3^T = 1600 \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{1600}{\sqrt{2}} & 0 & \frac{1600}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -800\sqrt{2} & 0 & 800\sqrt{2} \end{bmatrix}$$

$$M_1^T - 1600\, R_3^T = \begin{bmatrix} 800\sqrt{2} & 0 & 2400\sqrt{2} \end{bmatrix} - \begin{bmatrix} -800\sqrt{2} & 0 & 800\sqrt{2} \end{bmatrix} \;=\; \begin{bmatrix} 1600\sqrt{2} & 0 & 1600\sqrt{2} \end{bmatrix}.$$

The length of $\begin{bmatrix} 1600\sqrt{2} & 0 & 1600\sqrt{2} \end{bmatrix}$ is

$$1600\sqrt{2}\,\sqrt{1^2 + 1^2} \;=\; 1600\sqrt{2}\,\sqrt{2} \;=\; 1600 \times 2 \;=\; 3200.$$

Since R_1^T must be a unit vector,

$$a = 3200, \quad R_1^T \;=\; \frac{1}{3200} \begin{bmatrix} 1600\sqrt{2} & 0 & 1600\sqrt{2} \end{bmatrix} \;=\; \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \;=\; \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

We collect these results and get:

$$K \;=\; \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \;=\; \begin{pmatrix} 3200 & 0 & 1600 \\ 0 & 1400 & 1400 \\ 0 & 0 & 1 \end{pmatrix},$$

and

$$R \;=\; \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

From K, we interpret it in the usual camera-calibration form:

$$K \;=\; \begin{pmatrix} \alpha_x & \gamma & u_0 \\ 0 & \alpha_y & v_0 \\ 0 & 0 & 1 \end{pmatrix} \;=\; \begin{pmatrix} 3200 & 0 & 1600 \\ 0 & 1400 & 1400 \\ 0 & 0 & 1 \end{pmatrix}.$$

- $\alpha_x = 3200$ (focal length in x),
- $\alpha_y = 1400$ (focal length in y),
- $\gamma = 0$ (no skew),
- Principal point $(u_0, v_0) = (1600,\ 1400)$,
- Aspect ratio $= \alpha/\beta = 3200/1400 = 16/7$.

Computer Exercise 2

K matrix from P_2 with T_1 (normalized and rounded)

$$K_1 = \begin{bmatrix} -2391.5 & 225.7 & 942.7 \\ 0 & 623.9 & 813.6 \\ 0 & 0 & 1 \end{bmatrix}$$

K matrix from P_2 with T_2 (normalized and rounded)

$$K_2 = \begin{bmatrix} -2394.0 & 0 & 932.4 \\ 0 & 2398.1 & 628.3 \\ 0 & 0 & 1 \end{bmatrix}$$

Since K_1 and K_2 are not proportional by a scalar factor, they do not represent the same transformation.

Exersice 6, Direct Linear Transformation DLT

Exercise 6.1

The given problem is to solve the linear least squares system:

$$\min_{v} \|Mv\|^2.$$

Since choosing $v = 0$ leads to $\|Mv\|^2 = 0$, the minimum value is trivially 0.

Exercise 6.2

To remove the trivial solution, we instead solve:

$$\min_{\|v\|^2=1} \|Mv\|^2.$$

This constraint forces v to be a nonzero vector on the unit sphere.

We use the singular value decomposition of M:

$$M = U\Sigma V^T,$$

where:

- U is an $m \times m$ orthogonal matrix,
- Σ is an $m \times n$ diagonal matrix with singular values $\sigma_1 \geq \sigma_2 \geq \cdots \geq 0$,
- V is an $n \times n$ orthogonal matrix.

Applying M to a vector v is then the same as:

$$Mv = U\Sigma V^T v.$$

For a vector w and an orthogonal matrix we get:

$$\|Uw\|^2 = w^T \underbrace{U^T U}_{I} w = w^T w = \|w\|^2.$$

since the inverse of an orthogonal matrix is it's transpose.

If we choose $w = \Sigma V^T v$, then taking the squared norm gives us:

$$\|Mv\|^2 = \|U \underbrace{\Sigma V^T v}_{w} \|^2 = \| \underbrace{\Sigma V^T v}_{w} \|^2,$$

.

Since V is an orthogonal matrix, the transformation $v \mapsto \tilde{v} = V^T v$ preserves the norm similarly like U:

$$\|V^T v\|^2 = v^T \underbrace{V^T V}_{I} v = \|v\|^2 = 1.$$

Exercise 6.3

Define $\tilde{v} = V^T v$. The problem then becomes:

$$\min_{\|\tilde{v}\|^2=1} \|\Sigma\tilde{v}\|^2.$$

This reformulation maintains the same minimum value because U and V^T are orthogonal transformations.

Once a minimum $\tilde{v}_\star$ is found, we obtain $v_\star$ via:

$$v_\star = V\tilde{v}_\star.$$

Since V is orthogonal and $\|\tilde{v}_\star\| = 1$, we know that the solution remains valid and $\|v_\star\| = 1$.

If $v_\star$ is a solution, then $-v_\star$ is also a solution, since:

$$\|M(-v_\star)\|^2 = \| - Mv_\star\|^2 = \|Mv_\star\|^2.$$

Therefore there are at least two solutions: $\pm v_\star$.

Exercise 6.4

If $\mathrm{rank}(M) < n$, then the smallest singular value $\sigma_n = 0$. In this case, the minimum value of $\|Mv\|^2$ is zero, and any unit vector in the null space of M solves the problem (for instance the vector). A solution can be chosen as:

$$\tilde{v}_\star = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

Then the corresponding solution in the original space is:

$$v_\star = V\tilde{v}_\star = V \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} V_{1n} \\ V_{2n} \\ \vdots \\ V_{nn} \end{bmatrix}$$

that will say last column of V. If more singular values are 0 there will be more solution, but assuming $\sigma_n = 0$ last column of V will always be a solution.

Exercise 7

$$P = N^{-1}\widetilde{P}.$$

Computer Exercise 3

COMPUTER EXERCISE 3.1

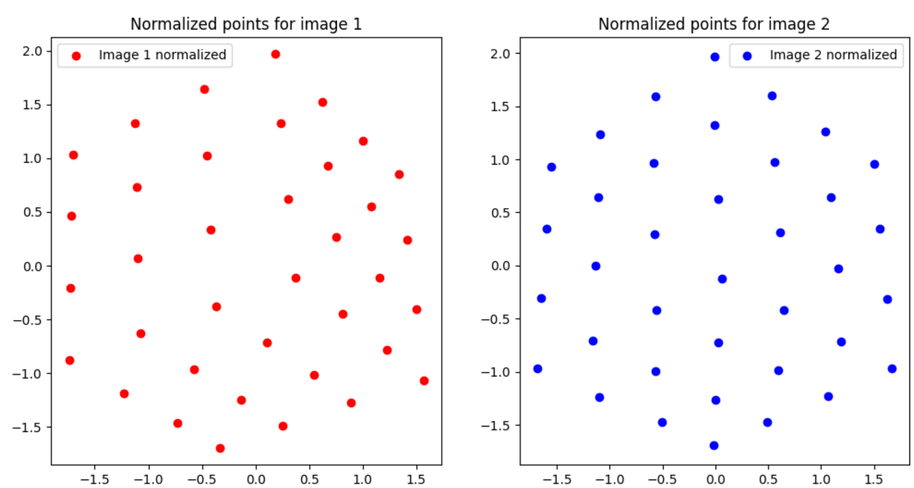

Figure 5: Normalized points for image 1 and image 2

Yes, the points look like they are centered around (0, 0), and that the distance vary almost uniformely from 0 to 2 with a mean distance of 1.

COMPUTER EXERCISE 3.2

$$\text{Smallest singular value (Image 1): } 0.0510121207472845$$

$$\|M_1 v\| = 0.05101212074728396$$

$$\text{Smallest singular value (Image 2): } 0.04380505110696644$$

$$\|M_2 v\| = 0.043805051106966406$$

We see that they are very close to 0 and that the minimal value of $\|M_2 v\|$ is the singular value using the solution of last row in V^T.

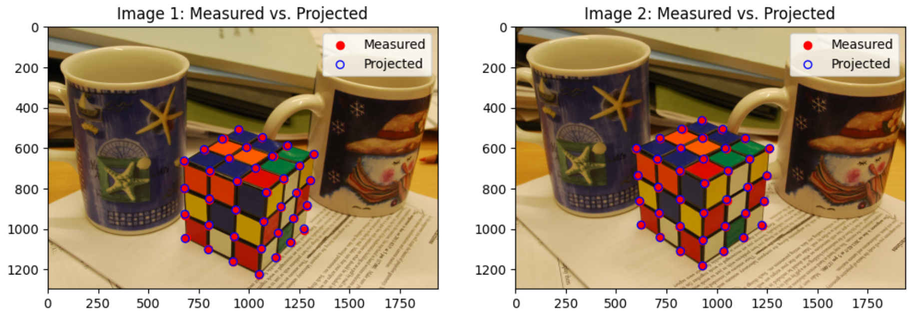

Figure 6: Measured vs Projected points

COMPUTER EXERCISE 3.3

They are very close to each other, in most cases even on top of each other.

COMPUTER EXERCISE 3.4

Camera 1

$$K_1 = \begin{bmatrix} 37.0338 & -0.1042 & 14.9809 \\ 0 & 37.0063 & 10.6103 \\ 0 & 0 & 0.0152893 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 0.55497 & -0.04814 & -0.83048 \\ -0.37927 & 0.87388 & -0.30410 \\ 0.74038 & 0.48375 & 0.46672 \end{bmatrix}$$

$$t_1 = \begin{bmatrix} 0.8623 \\ 6.2303 \\ 28.4334 \end{bmatrix}$$

Camera 2

$$K_2 = \begin{bmatrix} 37.0375 & -0.1685 & 13.1785 \\ 0 & 37.2621 & 11.9302 \\ 0 & 0 & 0.0154914 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0.71185 & 0.00228 & -0.70233 \\ -0.36828 & 0.85270 & -0.37051 \\ 0.59803 & 0.52240 & 0.60783 \end{bmatrix}$$

$$t_2 = \begin{bmatrix} 0.95596 \\ 4.8276 \\ 28.1724 \end{bmatrix}$$

Working on motivating about truly atm.

 # COMPUTER EXERCISE 3.4

I get the RMS error to

$$e_{\mathrm{RMS},1} = 3.5700 \text{ pixels}$$

$$e_{\mathrm{RMS},2} = 3.1647 \text{ pixels}$$

Have not had time to do everything again.

Computer Exercise 4

Working on writing it in python atm

Computer Exercise 5

Working on getting CE4 done first