Exercises for lecture 2

1. For the system $\dot{x}(t) = Ax(t)$ with

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

determine all initial conditions $x(0)$ such that any of the elements of $x(t) \to \infty$ as $t \to \infty$.

2. For the system $x(k+1) = Ax(k) + Bu(k)$ with

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

determine all initial conditions $x(0)$ such that there exists an input sequence $u(k)$ so that the state remains at $x(0)$, i. e. $x(k) = x(0)$ for all $k \geq 0$.

3. Consider continuous-time system $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t)$ with

$$A = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = [1 \quad 0].$$

Choose α and $x(0)$ such that $u(t) = \mathrm{e}^{\alpha t}$ produces $y(t) = \mathrm{e}^{\alpha t}$, i. e. the system behaves as a unity gain.

4. For a continuous-time system described by $\dot{x}(t) = Ax(t)$ with

$$A = \begin{bmatrix} -1 & \alpha & 2 \\ 0 & -2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

determine all the values of $\alpha \in \Re$ such that the system contains as many modes as possible.

5. A certain man put a pair of rabbits in a place surrounded on all sides by a wall. What will be the number of pairs of rabbits in a month k if it is supposed that every month each pair begs a new pair which from the second month on becomes productive? Suppose that the rabbits do not die. Write corresponding state-space equations and solve them in explicit form (i. e. as a scalar function of k).