In[1]:=

```
(* :Title: DMDM-Z'Z'                                                          *)

(*
    This software is covered by the GNU General Public License 3.
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    Copyright (C) 1997-2020 Frederik Orellana
    Copyright (C) 2014-2020 Vladyslav Shtabovenko
*)

(* :Summary:  DMDM→Z'Z', Dark U1 model, matrix element squared, tree            *

(* ------------------------------------------------------------------------ *)
```

Annihilation of DM in two dark photons (U1D dark model)

Load FeynCalc and the necessary add-ons or other packages

In[2]:=

```
description="DM DM -> Z'Z', MDEOEWSB, matrix element squared, tree";
If[ $FrontEnd === Null,
    $FeynCalcStartupMessages = False;
    Print[description];
];
If[ $Notebooks === False,
    $FeynCalcStartupMessages = False
];
$LoadAddOns={"FeynArts"};
<<FeynCalc`
$FAVerbose = 0;

FCCheckVersion[9,3,1];
```

FeynCalc 9.3.1 (stable version). For help, use the documentation center, check out the wiki or visit the forum.

To save your and our time, please check our FAQ for answers to some common FeynCalc questions.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 256 (2020) 107478, arXiv:2001.04407.
- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 207 (2016) 432–444, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345–359.

FeynArts 3.11 (25 Mar 2022) patched for use with FeynCalc, for documentation see the manual or visit www.feynarts.de.

If you use FeynArts in your research, please cite

- T. Hahn, Comput. Phys. Commun., 140, 418–431, 2001, arXiv:hep-ph/0012260

Generate Feynman diagrams

Nicer typesetting

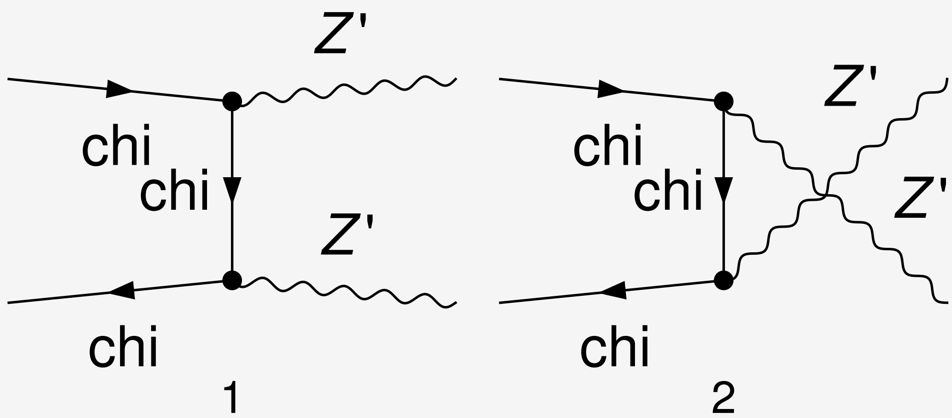
In[9]:=
```
MakeBoxes[p1,TraditionalForm]:="\!\(\*SubscriptBox[\(p\), \(1\)]\)";
MakeBoxes[p2,TraditionalForm]:="\!\(\*SubscriptBox[\(p\), \(2\)]\)";
MakeBoxes[k1,TraditionalForm]:="\!\(\*SubscriptBox[\(k\), \(1\)]\)";
MakeBoxes[k2,TraditionalForm]:="\!\(\*SubscriptBox[\(k\), \(2\)]\)";
```

In[13]:=
```
(*
diags = InsertFields[CreateTopologies[0, 2 -> 2], {F[2, {1}], -F[2, {1}]} ->
        {V[1], V[1]}, InsertionLevel -> {Classes},
        Restrictions->QEDOnly];

Paint[diags, ColumnsXRows -> {2, 1}, Numbering -> Simple,
    SheetHeader->None,ImageSize->{512,256}];
*)
```

In[14]:=
```
diags = InsertFields[CreateTopologies[0, 2 -> 2], {F[5], -F[5]} ->
        {V[10], V[10]}, InsertionLevel -> {Classes}, Model->"MDEOEWSB", ExcludePa

Paint[diags, ColumnsXRows -> {2, 1}, Numbering -> Simple,
    SheetHeader->None,ImageSize->{512,256}];
```



Obtain the amplitude

In[16]:=
```
amp[0] = FCFAConvert[CreateFeynAmp[diags], IncomingMomenta->{p1,p2},
    OutgoingMomenta->{k1,k2},UndoChiralSplittings->True,ChangeDimension->4,
    TransverseXPolarizationVectors->{k1,k2}, List->False, (*SMP->True,*)
    Contract->True]/.{CTWp->1,STWp->0,MassChi->Mx}
```

Out[16]=

$$(\varphi(-\overline{p_2}, \mathrm{Mx})).\left(-10\, i\, \mathrm{gBL}\left(\overline{\gamma}\cdot\overline{\varepsilon}^*(k_1)\right).\overline{\gamma}^6 - 9\, i\, \mathrm{gBL}\left(\overline{\gamma}\cdot\overline{\varepsilon}^*(k_1)\right).\overline{\gamma}^7\right).\left(\overline{\gamma}\cdot\left(\overline{k_1}-\overline{p_2}\right)+\mathrm{Mx}\right).\left(-10\, i\, \mathrm{gBL}\left(\overline{\gamma}\cdot\overline{\varepsilon}^*(k_2)\right).\overline{\gamma}^6 - 9\, i\, \mathrm{gBL}\left(\overline{\gamma}\cdot\overline{\varepsilon}^*(k_2)\right).\overline{\gamma}^7\right).(\varphi(\overline{p_1}, \mathrm{Mx}))\Big/\left((\overline{p_2}-\overline{k_1})^2-\mathrm{Mx}^2\right) + (\varphi(-\overline{p_2}, \mathrm{Mx})).\left(-10\, i\, \mathrm{gBL}\left(\overline{\gamma}\cdot\overline{\varepsilon}^*(k_2)\right).\overline{\gamma}^6 - 9\, i\, \mathrm{gBL}\left(\overline{\gamma}\cdot\overline{\varepsilon}^*(k_2)\right).\overline{\gamma}^7\right).\left(\overline{\gamma}\cdot\left(\overline{k_2}-\overline{p_2}\right)+\mathrm{Mx}\right).\left(-10\, i\, \mathrm{gBL}\left(\overline{\gamma}\cdot\overline{\varepsilon}^*(k_1)\right).\overline{\gamma}^6 - 9\, i\, \mathrm{gBL}\left(\overline{\gamma}\cdot\overline{\varepsilon}^*(k_1)\right).\overline{\gamma}^7\right).(\varphi(\overline{p_1}, \mathrm{Mx}))\Big/\left((\overline{p_2}-\overline{k_2})^2-\mathrm{Mx}^2\right)$$

Fix the kinematics

In[17]:=
```
FCClearScalarProducts[];
SetMandelstam[s, t, u, p1, p2, -k1, -k2, Mx, Mx, MZp, MZp];
```

Square the amplitude

In[19]:=
```
(*ampSquared[0] = (amp[0] (ComplexConjugate[amp[0]]))//
    FeynAmpDenominatorExplicit//
    DoPolarizationSums[#,k1,0]&//DoPolarizationSums[#,k2,0]&//
    FermionSpinSum[#, ExtraFactor -> 1/2^2]&//
    DiracSimplify//
    TrickMandelstam[#,{s,t,u,2SMP["m_e"]^2}]&//Simplify*)
```

We need to multiply by 1/2 to account for two identical particles in the final state

In[20]:=
```
ampSquared[0] = (amp[0] (ComplexConjugate[amp[0]]))//
    FeynAmpDenominatorExplicit//
    DoPolarizationSums[#,k1]&//DoPolarizationSums[#,k2]&//
    FermionSpinSum[#, ExtraFactor -> 1/(2*2^2)]&//
    DiracSimplify//
    TrickMandelstam[#,{s,t,u,2*MZp^2+2*Mx^2}]&//Simplify
```

Out[20]=

$$-\frac{1}{4\,\mathrm{MZp}^4\left(\mathrm{Mx}^2-t\right)^2\left(\mathrm{Mx}^2-u\right)^2}\mathrm{gBL}^4\Big(2\,\mathrm{Mx}^{12}-3\,\mathrm{Mx}^{10}\left(482\,\mathrm{MZp}^2+t+u\right)+\mathrm{Mx}^8\left(195\,844\,\mathrm{MZp}^4+2892\,\mathrm{MZp}^2\,(t+u)+(t+u)^2\right)+$$
$$2\,\mathrm{Mx}^6\left(2166\,\mathrm{MZp}^6-1444\,\mathrm{MZp}^4\,(t+u)-2\,\mathrm{MZp}^2\left(452\,t^2+1265\,t\,u+452\,u^2\right)+t\,u\,(t+u)\right)-$$
$$\mathrm{Mx}^4\Big(198\,732\,\mathrm{MZp}^8-128\,156\,\mathrm{MZp}^6\,(t+u)+\mathrm{MZp}^4\left(95\,753\,t^2+450\,718\,t\,u+95\,753\,u^2\right)-$$
$$2\,\mathrm{MZp}^2\left(181\,t^3+1265\,t^2\,u+1265\,t\,u^2+181\,u^3\right)+t\,u\left(t^2+4\,t\,u+u^2\right)\Big)+$$
$$\mathrm{Mx}^2\Big(198\,732\,\mathrm{MZp}^8\,(t+u)-10\,\mathrm{MZp}^6\left(253\,t^2+52\,056\,t\,u+253\,u^2\right)+$$
$$32\,400\,\mathrm{MZp}^4\left(t^3+7\,t^2\,u+7\,t\,u^2+u^3\right)-2\,\mathrm{MZp}^2\,t\,u\left(181\,t^2+361\,t\,u+181\,u^2\right)+t^2\,u^2\,(t+u)\Big)+$$
$$33\,122\,\mathrm{MZp}^4\left(\mathrm{MZp}^4\left(t^2-8\,t\,u+u^2\right)+4\,\mathrm{MZp}^2\,t\,u\,(t+u)-t\,u\left(t^2+u^2\right)\right)\Big)$$

DoPolarizationSums[exp,k]
sums over the three polarizations of an external massive vector boson with momentum k and mass k^2.

Expand the ampSquared

In[21]:=
```
AMP2 = -(1/(4 MZp^4 (Mx^2-t)^2 (Mx^2-u)^2)) gBL^4 (2 Mx^12-3 Mx^10 (482 MZp^2+t+u
```

Out[21]=

$$-\frac{1}{4\,\mathrm{MZp}^4\left(\mathrm{Mx}^2-t\right)^2\left(\mathrm{Mx}^2-u\right)^2}\mathrm{gBL}^4\Big(2\,\mathrm{Mx}^{12}-3\,\mathrm{Mx}^{10}\left(482\,\mathrm{MZp}^2+t+u\right)+\mathrm{Mx}^8\left(195\,844\,\mathrm{MZp}^4+2892\,\mathrm{MZp}^2\,(t+u)+(t+u)^2\right)+$$
$$2\,\mathrm{Mx}^6\left(2166\,\mathrm{MZp}^6-1444\,\mathrm{MZp}^4\,(t+u)-2\,\mathrm{MZp}^2\left(452\,t^2+1265\,t\,u+452\,u^2\right)+t\,u\,(t+u)\right)-$$
$$\mathrm{Mx}^4\Big(198\,732\,\mathrm{MZp}^8-128\,156\,\mathrm{MZp}^6\,(t+u)+\mathrm{MZp}^4\left(95\,753\,t^2+450\,718\,t\,u+95\,753\,u^2\right)-$$
$$2\,\mathrm{MZp}^2\left(181\,t^3+1265\,t^2\,u+1265\,t\,u^2+181\,u^3\right)+t\,u\left(t^2+4\,t\,u+u^2\right)\Big)+$$
$$\mathrm{Mx}^2\Big(198\,732\,\mathrm{MZp}^8\,(t+u)-10\,\mathrm{MZp}^6\left(253\,t^2+52\,056\,t\,u+253\,u^2\right)+$$
$$32\,400\,\mathrm{MZp}^4\left(t^3+7\,t^2\,u+7\,t\,u^2+u^3\right)-2\,\mathrm{MZp}^2\,t\,u\left(181\,t^2+361\,t\,u+181\,u^2\right)+t^2\,u^2\,(t+u)\Big)+$$
$$33\,122\,\mathrm{MZp}^4\left(\mathrm{MZp}^4\left(t^2-8\,t\,u+u^2\right)+4\,\mathrm{MZp}^2\,t\,u\,(t+u)-t\,u\left(t^2+u^2\right)\right)\Big)$$

In[22]:=
```
k1={E1,p1,0,0}; (*Halzen and Martin book*)
k2={E1,-p1,0,0};
k3={E1,p3*ct,p3*st,0};
k4={E1,-p3*ct,-p3*st,0};

guv={{1,0,0,0},{0,-1,0,0},{0,0,-1,0},{0,0,0,-1}};
MV = Simplify[{S->(k1 + k2).guv.(k1 + k2),   T->(k1 - k3).guv.(k1 - k3),   U ->(k
MV // MatrixForm
```

Out[27]//MatrixForm=

$$\begin{pmatrix} S\to 4\,\mathrm{E1}^2 \\ T\to -(p_1-\mathrm{ct}\,\mathrm{p3})^2-\mathrm{p3}^2\,\mathrm{st}^2 \\ U\to -(\mathrm{ct}\,\mathrm{p3}+p_1)^2-\mathrm{p3}^2\,\mathrm{st}^2 \end{pmatrix}$$

In[28]:=
```
Phase space factor
cinematic={E1->Sqrt[(p1)^2+Mx^2],p3->Sqrt[(p1)^2+Mx^2-MZp^2],p1->Mx*v*(1+0*v^2/2)
F1=Simplify[(1/(64*π^2*S)*Sqrt[((S-(MZp+MZp)^2)(S-(MZp-MZp)^2))/((S-(Mx+Mx)^2)(S-
F2=Normal[Series[F1,{v,0,2}]]
Sqrt[1-MZp^2/Mx^2]/(256 Mx^2 π^2 v)+((-2 Mx^2+3 MZp^2) v)/(512 Mx^4 Sqrt[(Mx^2-MZ
```

Out[28]= factor Phase space

Out[30]=

$$\frac{\sqrt{1-\frac{\mathrm{MZp}^2}{\mathrm{Mx}^2}}}{256\,\pi^2\,\mathrm{Mx}^2\,v}+\frac{v\left(2\,\mathrm{MZp}^2-\mathrm{Mx}^2\right)}{512\,\pi^2\,\mathrm{Mx}^4\sqrt{\frac{\mathrm{Mx}^2-\mathrm{MZp}^2}{\mathrm{Mx}^2}}}$$

Out[31]=

$$\frac{\sqrt{1-\frac{\mathrm{MZp}^2}{\mathrm{Mx}^2}}}{256\,\pi^2\,\mathrm{Mx}^2\,v}+\frac{v\left(3\,\mathrm{MZp}^2-2\,\mathrm{Mx}^2\right)}{512\,\pi^2\,\mathrm{Mx}^4\sqrt{\frac{\mathrm{Mx}^2-\mathrm{MZp}^2}{\mathrm{Mx}^2}}}$$

In[32]:=
```
F3 = Simplify[AMP2/.{u->U,t->T,s->S}/.MV//.cinematic/.st^2-> 1-ct^2]
```

Out[32]=

$$\Big(\mathrm{gBL}^4\,\mathrm{Mx}^2\Big(16\left(\mathrm{ct}^2-1\right)\mathrm{Mx}^{10}\,v^2\left(v^2+1\right)^3\left(\left(\mathrm{ct}^2-1\right)v^2-1\right)+2\,\mathrm{MZp}^{10}\left(33\,122\left(\mathrm{ct}^2+1\right)v^2-32\,039\right)-16\,\mathrm{Mx}^8\,\mathrm{MZp}^2\left(v^2+1\right)^2\left(\left(358\,\mathrm{ct}^2-720\right)v^2+\left(3\,\mathrm{ct}^4+356\,\mathrm{ct}^2-359\right)v^4-361\right)-8\,\mathrm{Mx}^6\,\mathrm{MZp}^4\left(v^2+1\right)\left(33\,122\left(\mathrm{ct}^4-1\right)v^6+2\left(31\,682\,\mathrm{ct}^2-79\,377\right)v^2+\left(33\,116\,\mathrm{ct}^4+63\,365\,\mathrm{ct}^2-161\,281\right)v^4-30\,595\right)+4\,\mathrm{Mx}^4\,\mathrm{MZp}^6\left(132\,488\,\mathrm{ct}^2\left(\mathrm{ct}^2-1\right)v^6-\left(2515\,\mathrm{ct}^2+381\,582\right)v^2+\left(132\,484\,\mathrm{ct}^4-135\,005\,\mathrm{ct}^2-255\,231\right)v^4-126\,351\right)+\mathrm{Mx}^2\,\mathrm{MZp}^8\left(\left(456\,472\,\mathrm{ct}^2+249\,093\right)v^2-66\,244\left(4\,\mathrm{ct}^4-7\,\mathrm{ct}^2+3\right)v^4+318\,946\right)\Big)\Big)\Big/\Big(\mathrm{MZp}^4\left(-4\,\mathrm{Mx}^4\left(v^2+1\right)\left(\left(\mathrm{ct}^2-1\right)v^2-1\right)+4\,\mathrm{Mx}^2\,\mathrm{MZp}^2\left(\left(\mathrm{ct}^2-1\right)v^2-1\right)+\mathrm{MZp}^4\right)^2\Big)$$

In[33]:=
```
F4 = Collect[Normal[Series[F3,{v,0,2}]],{v,v^2}]
```

Out[33]=

$$\left(2\, gBL^4\, Mx^2 \left(722\, Mx^4 + 31\,317\, Mx^2\, MZp^2 - 32\,039\, MZp^4\right)\right) \Big/ \left(MZp^2 \left(MZp^2 - 2\, Mx^2\right)^2\right) - \left(gBL^4\, Mx^2\, v^2 \left(16\, ct^2\, Mx^{10} - 5824\, ct^2\, Mx^8\, MZp^2 + 17\,392\, ct^2\, Mx^6\, MZp^4 + 1\,023\,756\, ct^2\, Mx^4\, MZp^6 - 969\,096\, ct^2\, Mx^2\, MZp^8 - 66\,244\, ct^2\, MZp^{10} - 16\, Mx^{10} + 32\, Mx^8\, MZp^2 - 524\,200\, Mx^6\, MZp^4 + 8\, Mx^4\, MZp^6 + 263\,531\, Mx^2\, MZp^8 - 66\,244\, MZp^{10}\right)\right) \Big/ \left(MZp^4 \left(2\, Mx^2 - MZp^2\right)^4\right)$$

In[42]:=
```
F5=Simplify[F4/.v->0]
```

Out[42]=

$$\left(2\, gBL^4\, Mx^2 \left(722\, Mx^4 + 31\,317\, Mx^2\, MZp^2 - 32\,039\, MZp^4\right)\right) \Big/ \left(MZp^3 - 2\, Mx^2\, MZp\right)^2$$

dσ/dΩ

In[43]:=
```
F7=Simplify[F2*F4]
```

Out[43]=

$$\left(1 \Big/ \left(512\, \pi^2\, Mx^2\, v \sqrt{1 - \frac{MZp^2}{Mx^2}} \left(MZp^3 - 2\, Mx^2\, MZp\right)^4\right)\right) gBL^4 \left(Mx^2 \left(v^2 - 2\right) - 2\, MZp^2 \left(v^2 - 1\right)\right) \left(16 \left(ct^2 - 1\right) Mx^{10}\, v^2 - 16\, Mx^8\, MZp^2 \left(\left(364\, ct^2 - 2\right) v^2 + 361\right) + 8\, Mx^6\, MZp^4 \left(\left(2174\, ct^2 - 65\,525\right) v^2 - 30\,595\right) + 4\, Mx^4\, MZp^6 \left(\left(255\,939\, ct^2 + 2\right) v^2 + 126\,351\right) + Mx^2\, MZp^8 \left(\left(263\,531 - 969\,096\, ct^2\right) v^2 - 318\,946\right) - 2\, MZp^{10} \left(33\,122 \left(ct^2 + 1\right) v^2 - 32\,039\right)\right)$$

σ(v)=∫ (dσ/dΩ)dΩ

In[44]:=
```
Integrate[((F7*2Pi*Sin[t])//.{st->Sin[t],ct->Cos[t]}),{t,0,Pi}];
sigma=Normal[Series[%,{v,0,2}]]
```

Out[45]=

$$\left(gBL^4 \left(Mx^2 - MZp^2\right)^2 \left(722\, Mx^2 + 32\,039\, MZp^2\right)\right) \Big/ \left(32\, \pi\, Mx^2\, MZp^2\, v \sqrt{\frac{Mx^2 - MZp^2}{Mx^2}} \left(2\, Mx^2 - MZp^2\right)^2\right) + \left(gBL^4\, v \left(16\, Mx^{12} - 1484\, Mx^{10}\, MZp^2 + 599\,834\, Mx^8\, MZp^4 - 543\,301\, Mx^6\, MZp^6 - 396\,174\, Mx^4\, MZp^8 + 569\,714\, Mx^2\, MZp^{10} - 228\,605\, MZp^{12}\right)\right) \Big/ \left(96\, \pi\, Mx^2\, MZp^4 \sqrt{\frac{Mx^2 - MZp^2}{Mx^2}} \left(2\, Mx^2 - MZp^2\right)^4\right)$$

σ .vr

In[46]:=
```
sigmavr=Simplify[(sigma*v)/.v->(vr/2)]
```

Out[46]=
$$\left(gBL^4\left(Mx^2-MZp^2\right)\left(16\,Mx^{10}\,vr^2-4\,Mx^8\,MZp^2\left(367\,vr^2-8664\right)+2\,Mx^6\,MZp^4\left(299\,183\,vr^2+734\,280\right)+3\,Mx^4\,MZp^6\left(18\,355\,vr^2-1\,010\,808\right)+3\,Mx^2\,MZp^8\left(637\,892-113\,703\,vr^2\right)+MZp^{10}\left(228\,605\,vr^2-384\,468\right)\right)\right)\Big/\left(384\,\pi\,Mx^2\sqrt{1-\frac{MZp^2}{Mx^2}}\left(MZp^3-2\,Mx^2\,MZp\right)^4\right)$$

S - wave part

In[58]:=
```
F8= Simplify[sigmavr/.vr->0/.MZp->r*Mx]
```

Out[58]=
$$\frac{gBL^4\left(1-r^2\right)^{3/2}\left(32\,039\,r^2+722\right)}{32\,\pi\,Mx^2\,r^2\left(r^2-2\right)^2}$$

In[59]:=
```
722/(32*4)
```

In[60]:=
```
N[722/(2*4)]
```

Out[60]= 90.25

P - wave part

In[55]:=
```
F9=Simplify[Simplify[D[D[sigmavr,vr],vr]/2/.vr->0/.MZp->r*Mx]*vr^2/.(228605 r^10-34
```

Out[55]=
$$-\frac{gBL^4\sqrt{1-r^2}\left(367\,r^2-4\right)vr^2}{96\,\pi\,Mx^2\,r^4\left(r^2-2\right)^4}$$

In[57]:=
```
4/96
```

Out[57]=
$$\frac{1}{24}$$