

# Chapter 6

## Design for Testability and Built-In Self-Test

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# Outline

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- Basics
- Design-for-Testability (DFT) Techniques
  - Ad Hoc DFT
  - Structural Methods
    - Scan
    - Partial Scan
    - BIST
    - Boundary Scan
    - Syndrome-Testable Design
    - C-Testable Design
- Built-In Self-Test (BIST) Techniques
  - Signature Analysis
  - Pseudorandom Pattern Generator (PRPG)
  - Built-In Logic Block Observer (BILBO)
- Summary

# Definitions

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## □ Definition

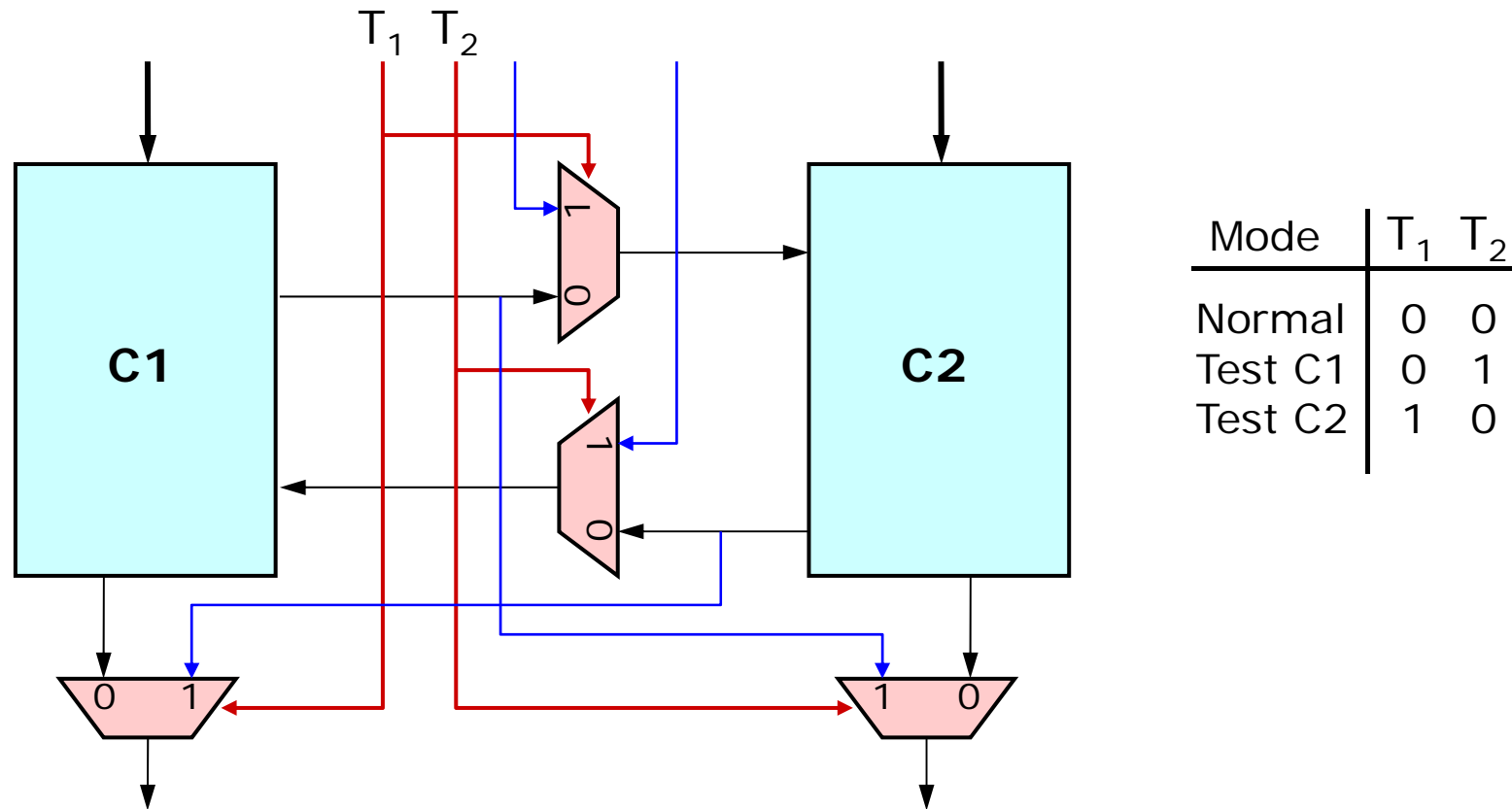
- A fault is ***testable*** if there exists a well-specified procedure to expose it, which is implementable with a reasonable cost using current technologies. A circuit is ***testable with respect to a fault set*** when each and every fault in this set is testable

## □ Definition

- ***Design for testability (DFT)*** refers to those design techniques that make test generation and test application cost-effective
- Electronic systems contain three types of components: (a) digital logic, (b) memory blocks, and (c) analog or mixed-signal circuits
- In this chapter, we discuss DFT techniques for digital logic

# Ad Hoc DFT Guidelines

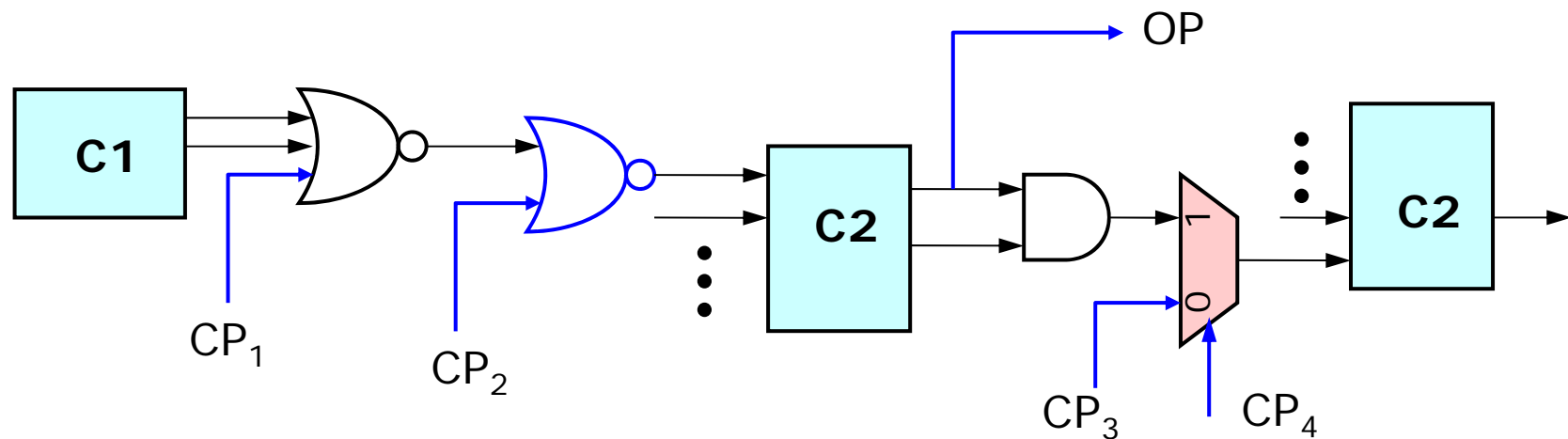
- Partition large circuits into smaller subcircuits to reduce test generation cost (using MUXed and/or scan chains)



# Ad Hoc DFT Guidelines

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- Insert test points to enhance controllability & observability
  - Test points: control points & observation points



# Ad Hoc DFT Guidelines

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- ❑ Design circuits to be easily initializable
- ❑ Provide logic to break global feedback paths
- ❑ Partition large counter into smaller ones
- ❑ Avoid the use of redundant logic
- ❑ Keep analog and digital circuits physically apart
- ❑ Avoid the use of asynchronous logic
- ❑ Consider tester requirements (pin limitation, etc)
- ❑ Etc

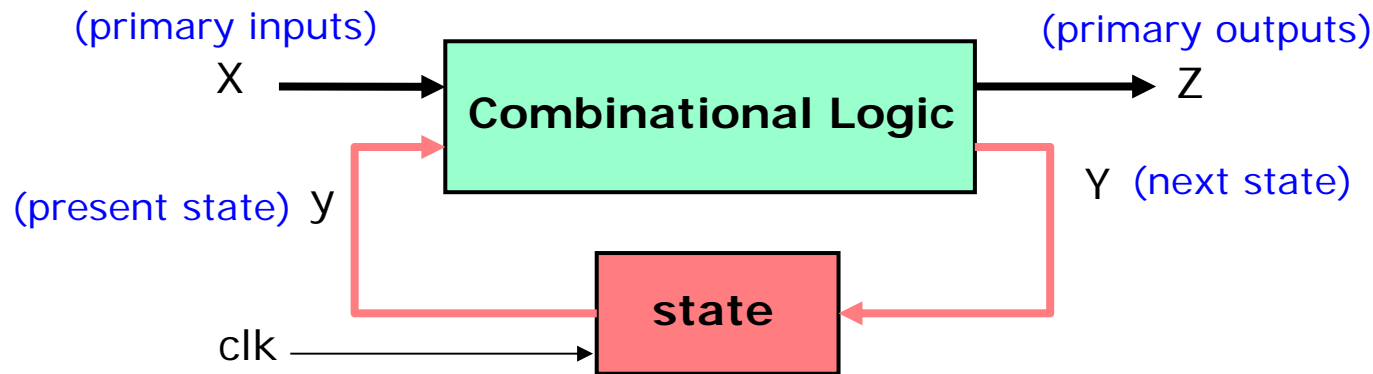
# Scan Design Approaches

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- They are effective for circuit partitioning
- They provide controllability and observability of internal state variables for testing
- They turn the sequential test problem into a combinational one
- Four major approaches
  - Shift-register modification
  - Scan path
  - Level-sensitive scan design (LSSD)
  - Random access
- Circuit is designed using pre-specified design rules.

# Scan Design Approaches

- Consider a representation of sequential circuits

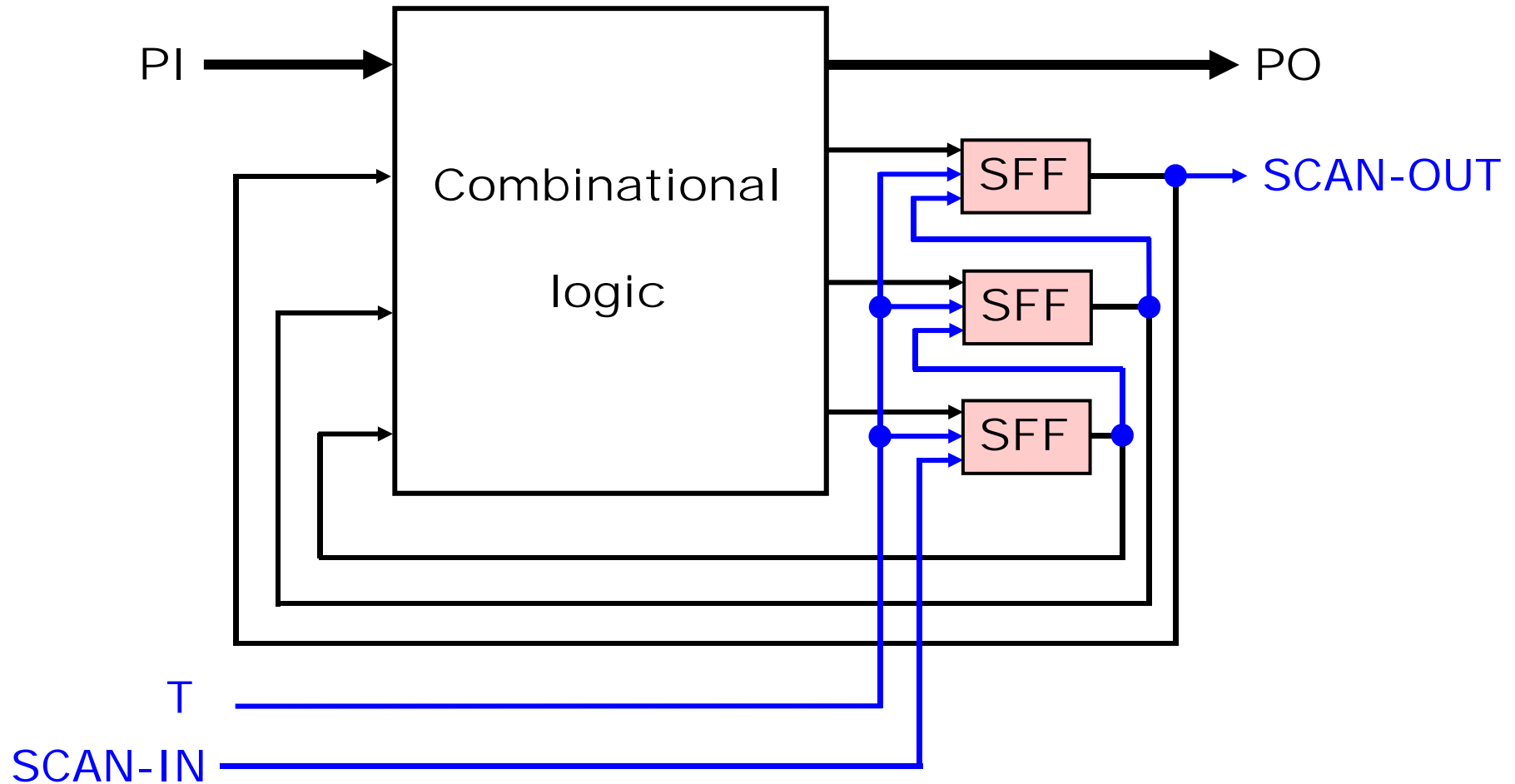


- To make elements of state vector controllable and observable, we add
  - A TEST mode pin (T)
  - A SCAN-IN pin (SI)
  - A SCAN-OUT pin (SO)
  - A MUX (switch) in front of each FF (M)



# Adding Scan Structure

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# Scan Test Generation & Design Rules

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## □ Test pattern generation

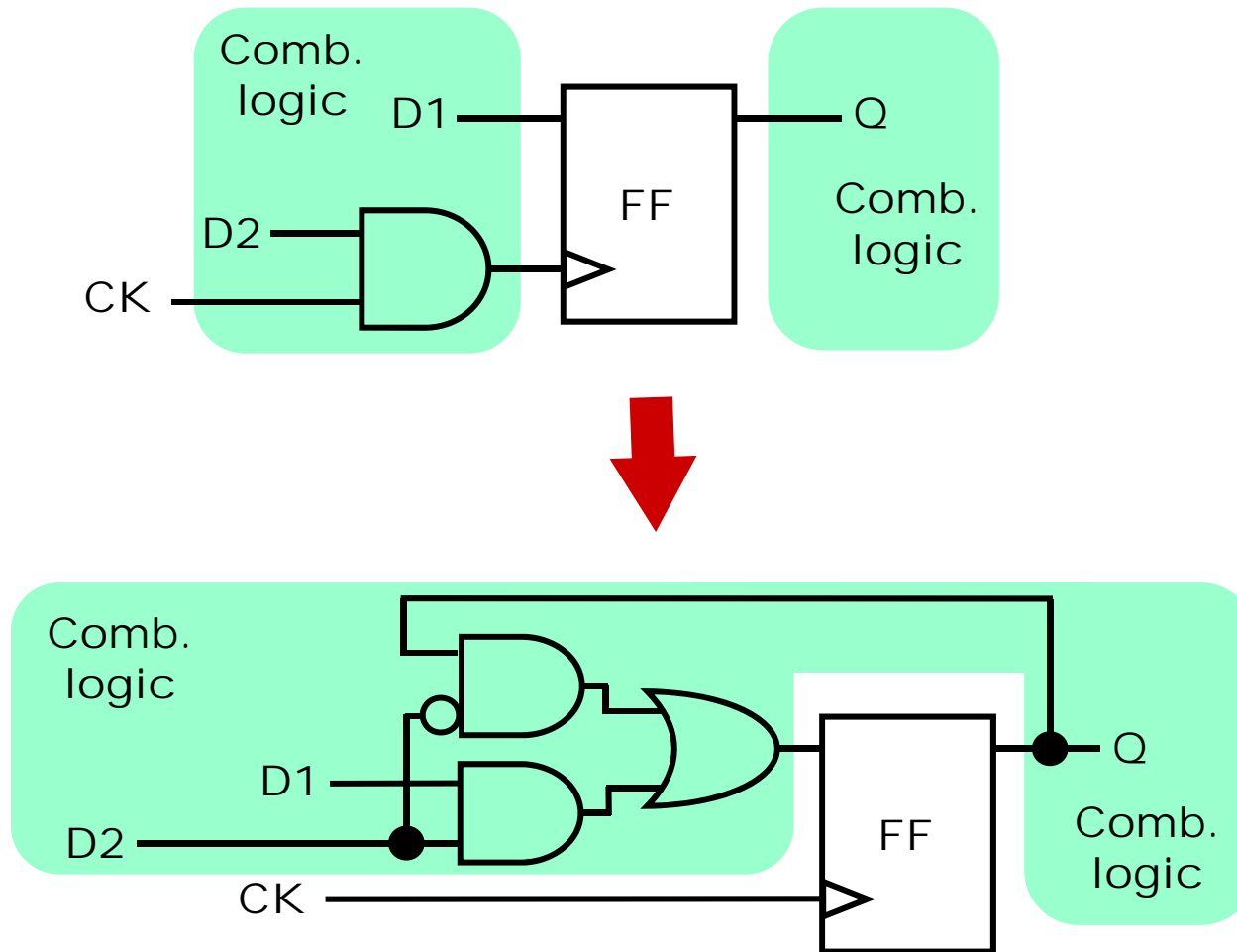
- Use combinational ATPG to obtain tests for all testable faults in the combinational logic
- Add shift register tests and convert ATPG tests into scan sequences for use in manufacturing test

## □ Scan design rules

- Use only clocked D-type of flip-flops for all state variables
- At least one PI pin must be available for test; more pins, if available, can be used
- All clocks must be controlled from PIs
- Clocks must not feed data inputs of flip-flops

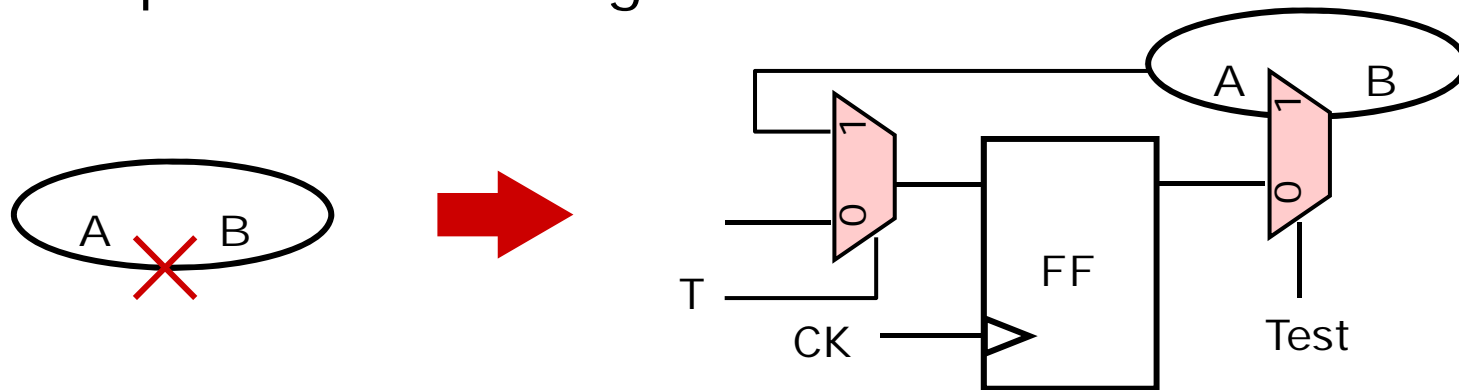
# Correcting a Rule Violation

- All clocks must be controlled from PIs

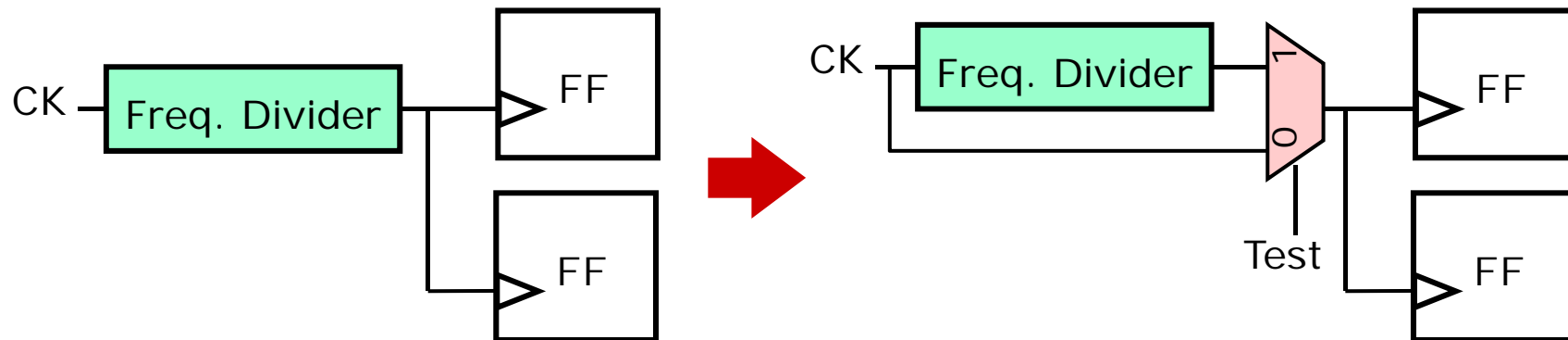


# Correcting a Rule Violation

- Adding a scan FF and a mux allows a feedback loop to be opened for testing

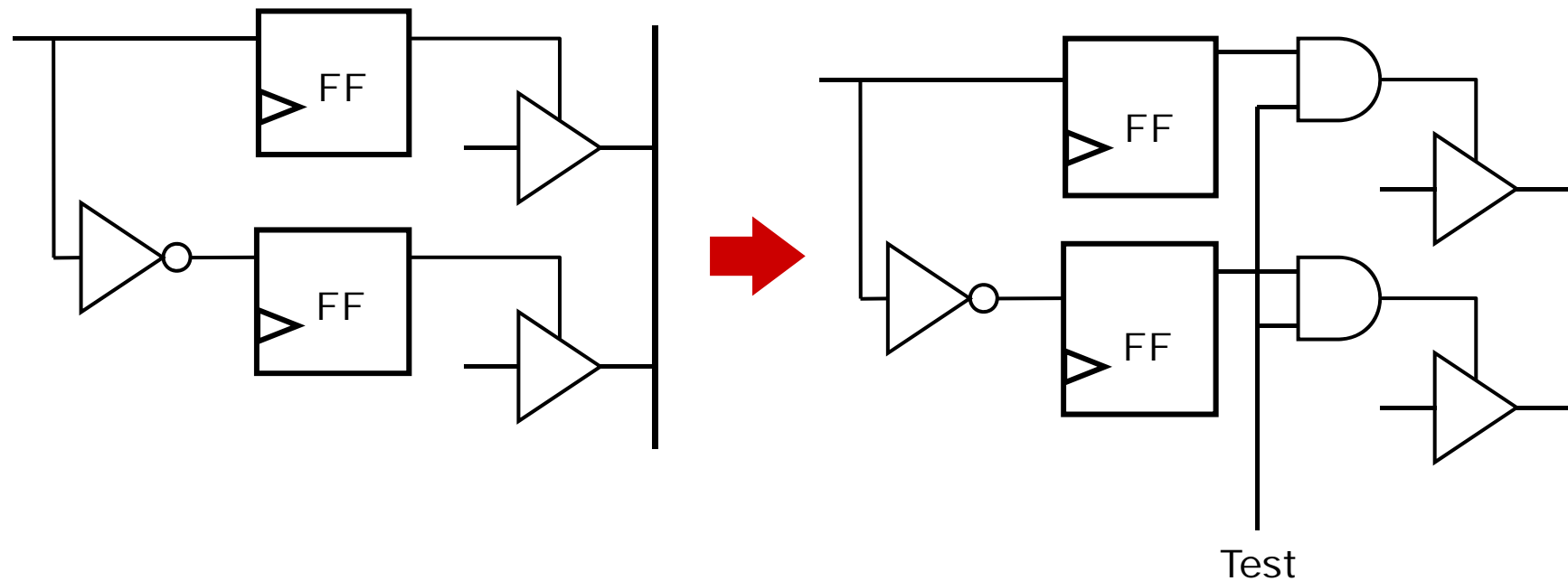


- Testing derived clocks requires the use of a mux to bypass the division stages



# Correcting a Rule Violation

- ❑ The AND gates keep the bus drivers from being activated by the normal logic during testing



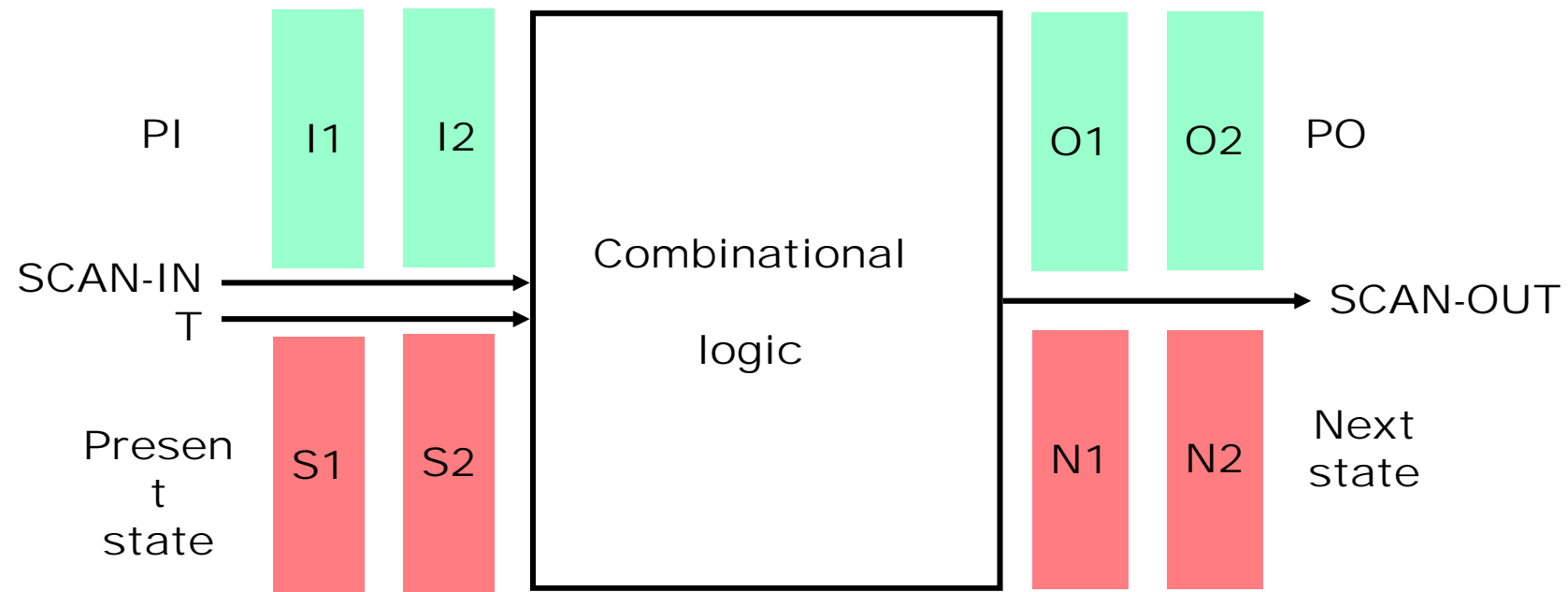
# Scan Test Procedure

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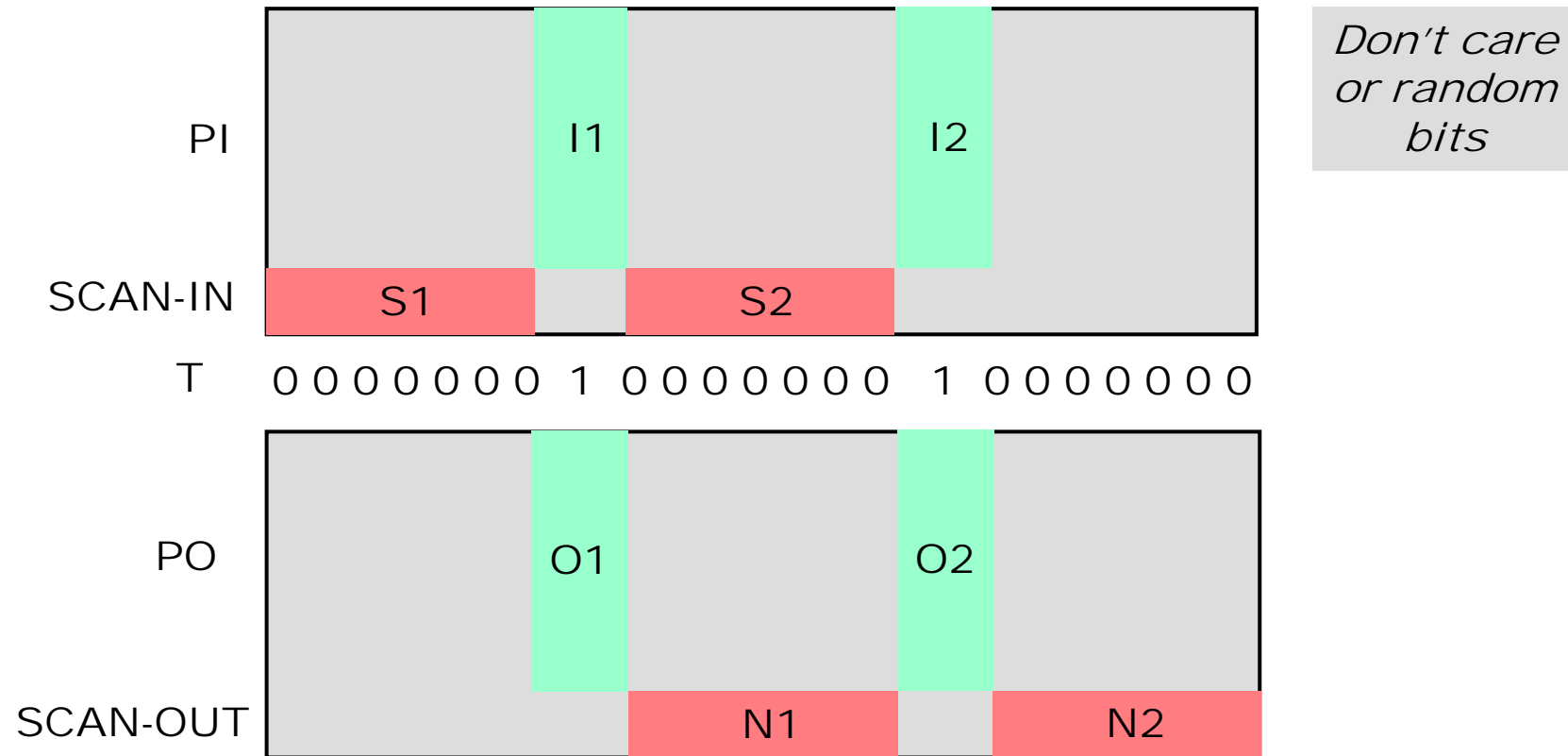
- ❑ **Step 1:** Switch to the shift-register mode and check the SR operation by shifting in an alternating sequence of 1s and 0s, e.g., 00110 (functional test)
- ❑ **Step 2:** Initialize the SR---load the first pattern
- ❑ **Step 3:** Return to the normal mode and apply the test pattern
- ❑ **Step 4:** Switch to the SR mode and shift out the final state while setting the starting state for the next test. Go to **Step 3**

# Combining Test Vectors

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# Combining Test Vectors



*Sequence length =  $(n_{\text{comb}} + 1) n_{\text{sff}} + n_{\text{comb}}$  clock periods*

$n_{\text{comb}}$  = number of combinational vectors

$n_{\text{sff}}$  = number of scan flip-flops



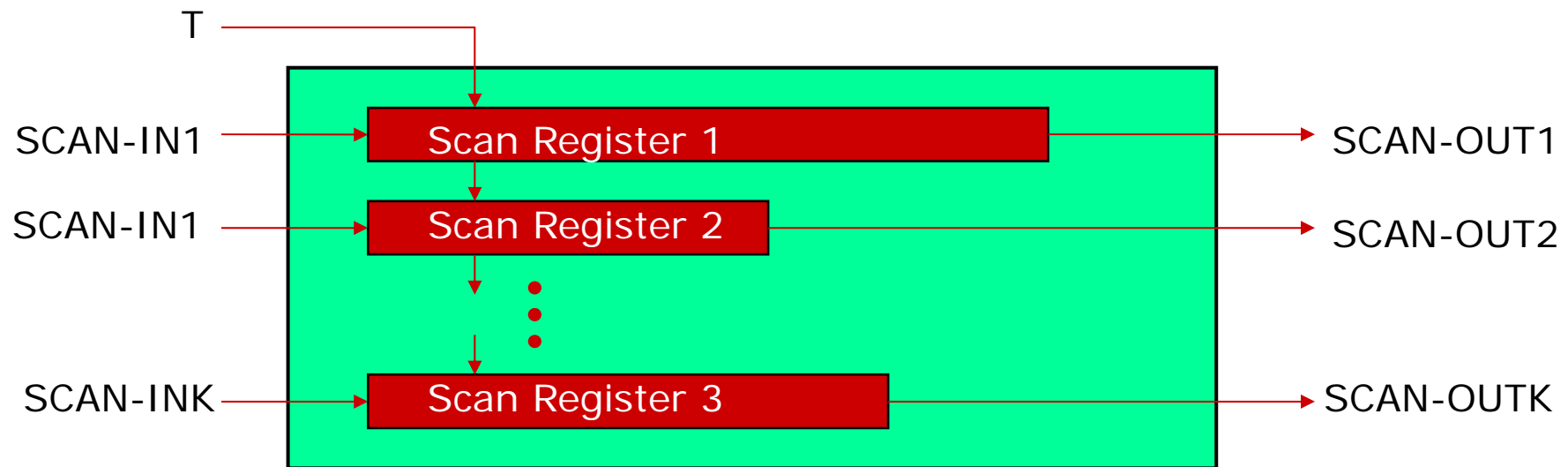
# Testing Scan Register

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- ❑ Scan register must be tested prior to application of scan test sequences
- ❑ A shift sequence 00110011 . . . of length  $n_{\text{sff}} + 4$  in scan mode (TC=0) produces 00, 01, 11 and 10 transitions in all flip-flops and observes the result at SCAN-OUT output
- ❑ Total scan test length:  
 $(n_{\text{comb}} + 2)n_{\text{sff}} + n_{\text{comb}} + 4$  *clock periods*
- ❑ Example: 2,000 scan flip-flops, 500 comb. vectors, total scan test length  $\sim 10^6$  clocks
- ❑ Multiple scan registers reduce test length

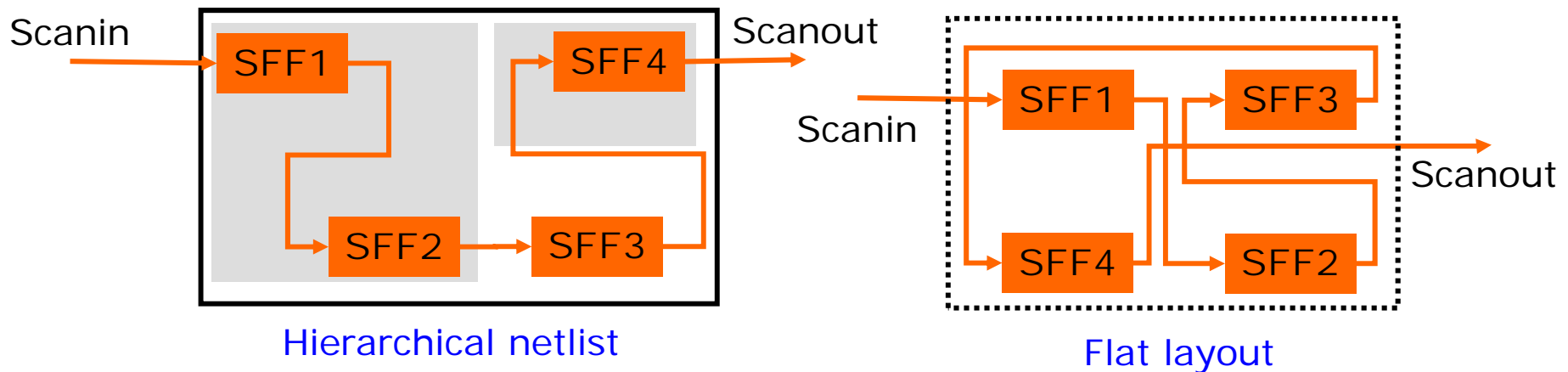
# Multiple Scan Registers

- ❑ Scan flip-flops can be distributed among any number of shift registers, each having a separate *SCAN-IN* and *SCAN-OUT* pin
- ❑ Test sequence length is determined by the longest scan shift register
- ❑ Just one *test control* (TC) pin is essential

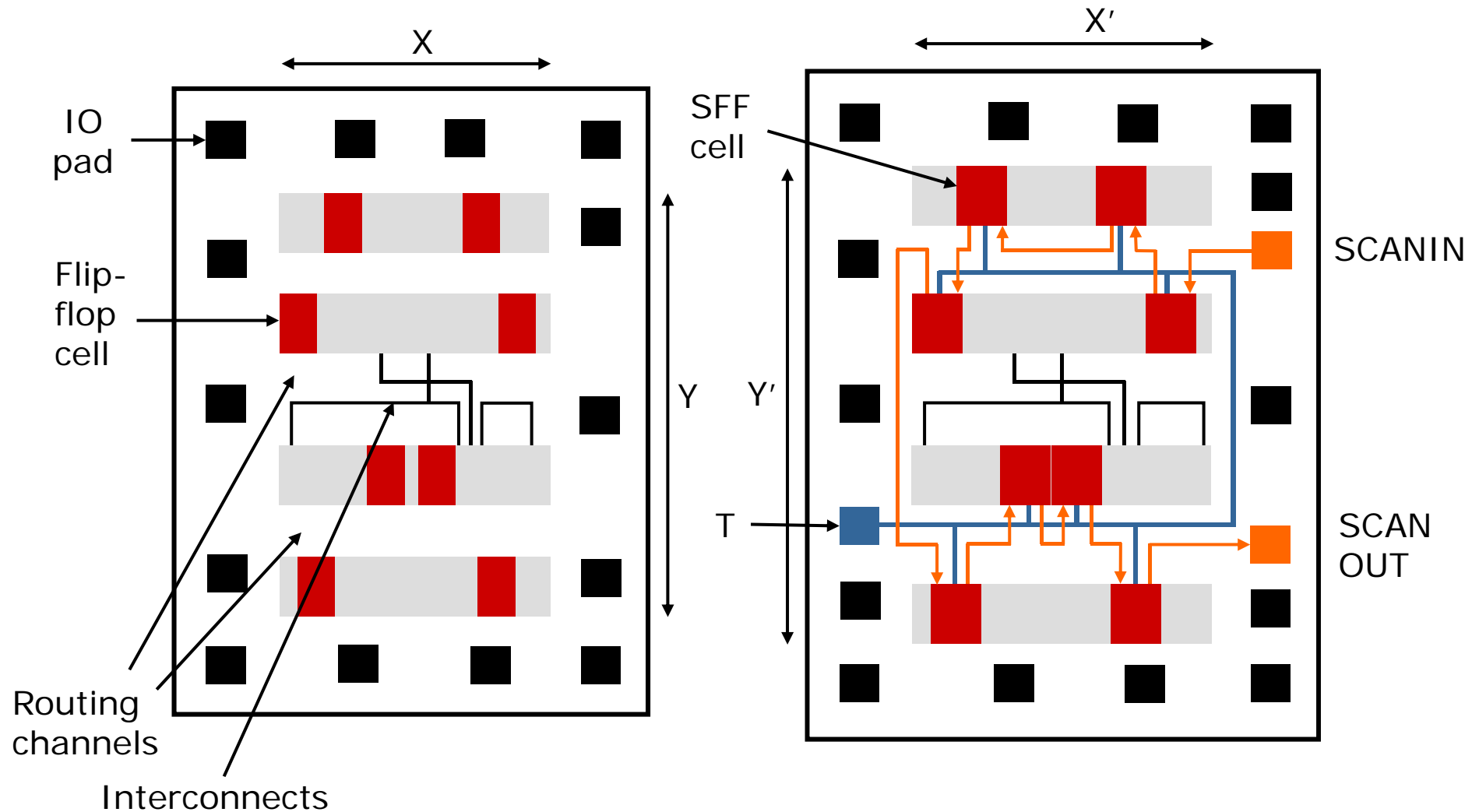


# Hierarchical Scan

- ❑ Scan flip-flops are chained within subnetworks before chaining subnetworks
- ❑ Advantages:
  - Automatic scan insertion in netlist
  - Circuit hierarchy preserved – helps in debugging and design changes
- ❑ Disadvantage: Non-optimum chip layout

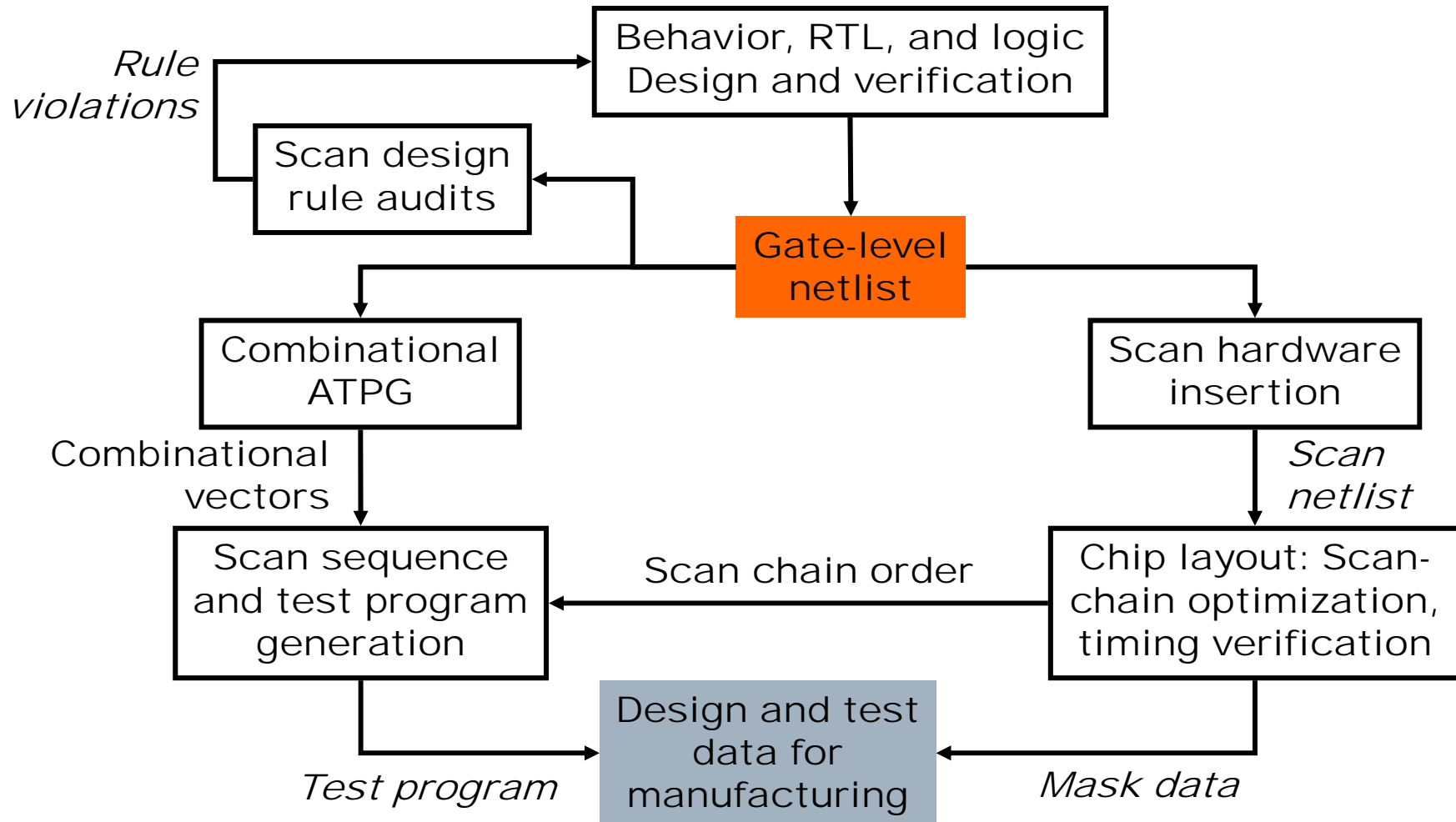


# Optimum Scan Layout

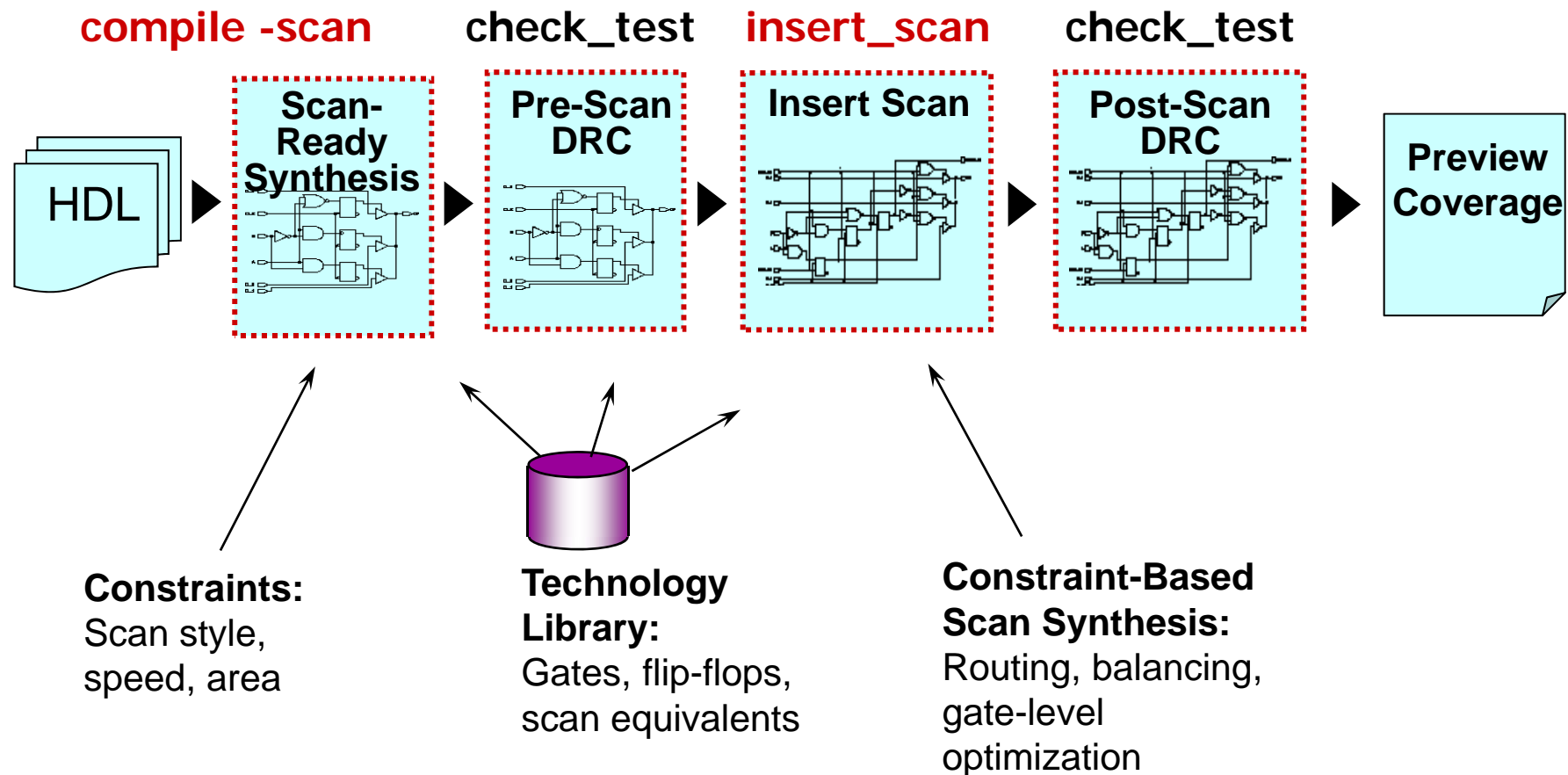


Active areas:  $XY$  and  $X'Y'$

# Automated Scan Design



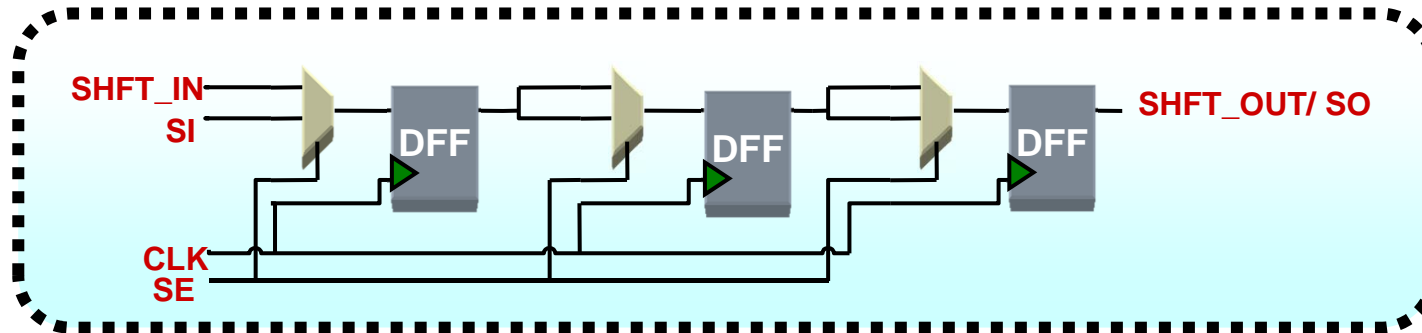
# An Example of DFT Compiler Flow



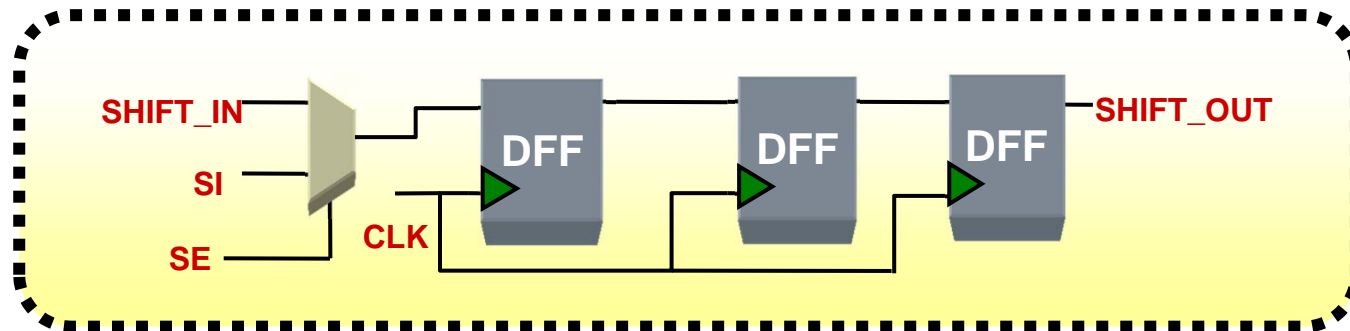
Source: H.-J. Huang, CIC

# Shift Registers

Scan added:

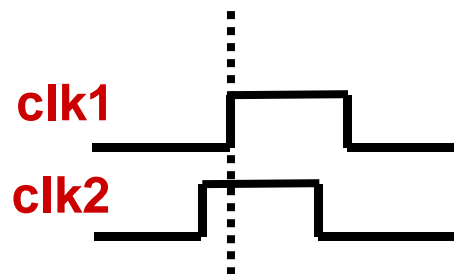
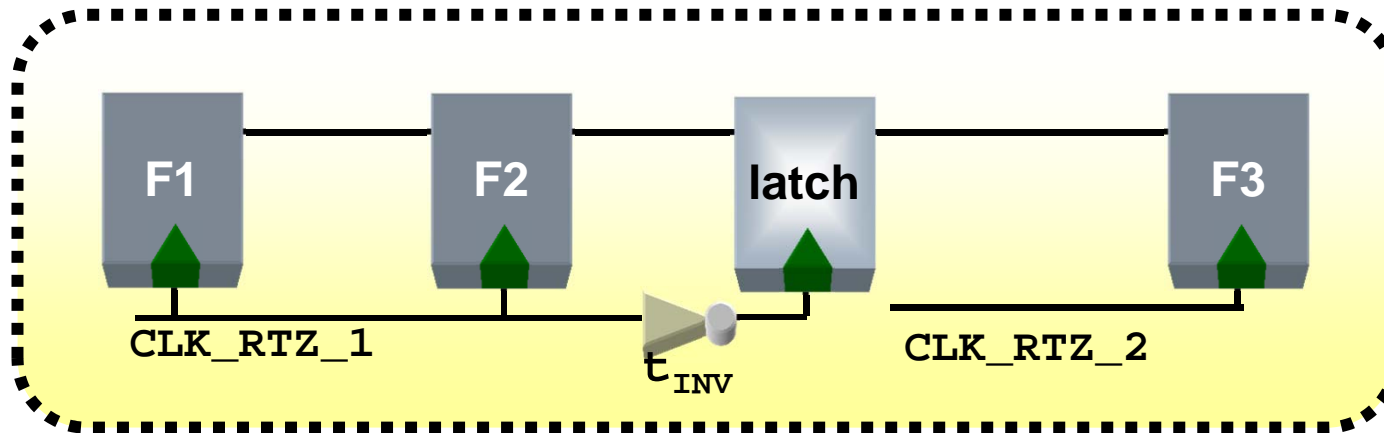


Revised:

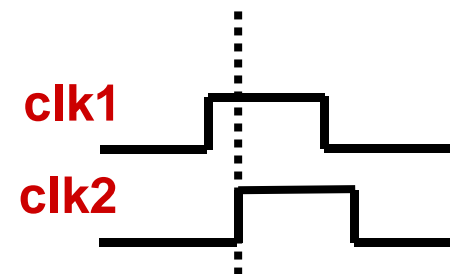


Source: H.-J. Huang, CIC

# Lockup Latch Insertion



OK!



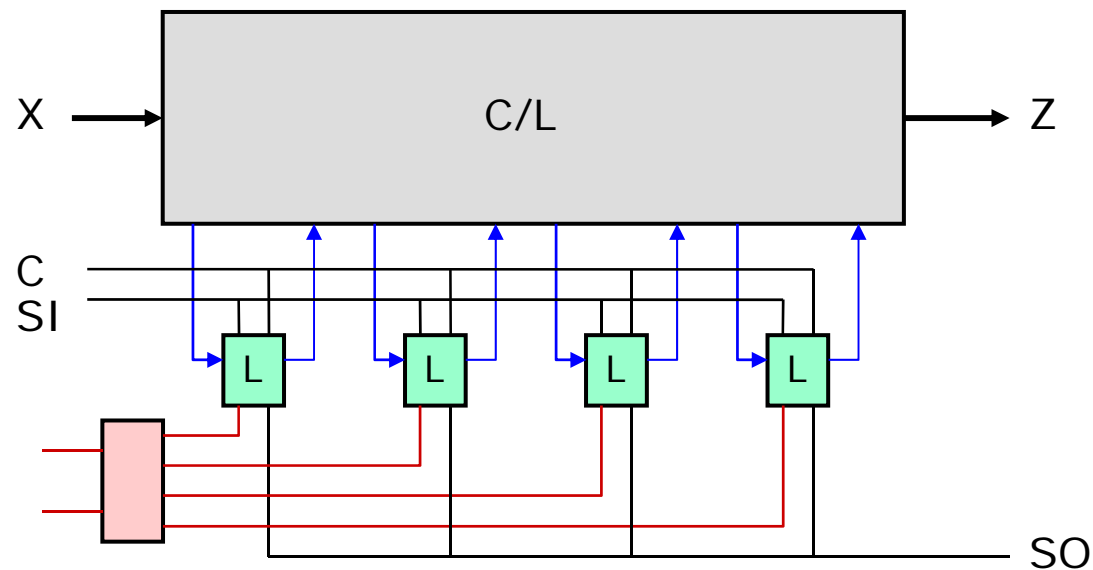
Big Problem !!  
Rearrange clock domain or  
insert lockup latch

Source: H.-J. Huang, CIC



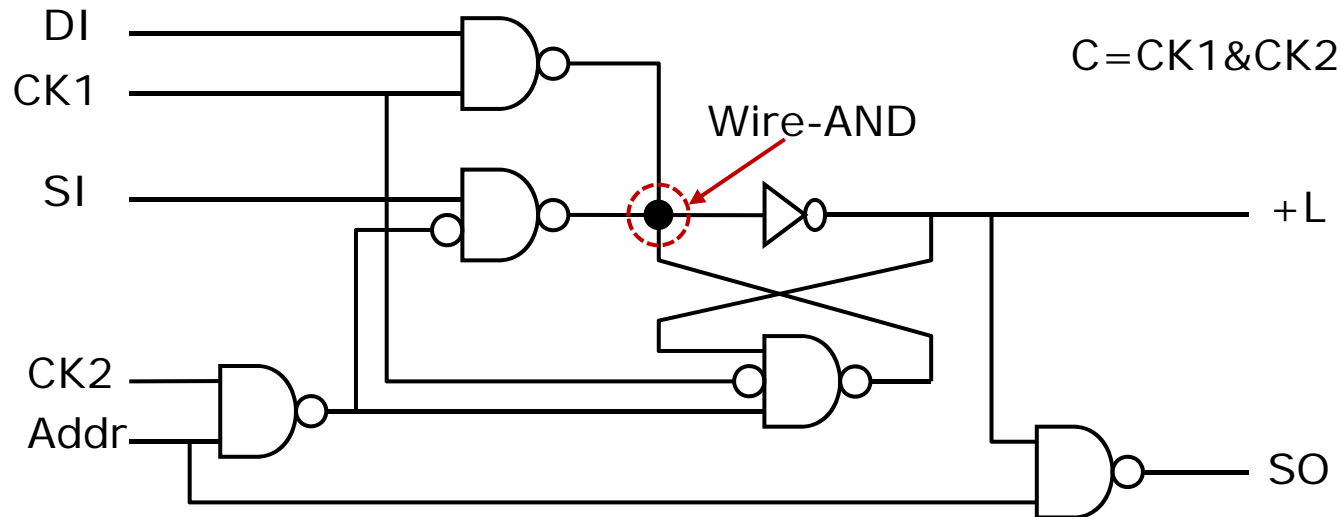
# Random Access Scan

- ❑ Uses addressable latches
- ❑ Provides random access to FFs via multiplexing—address selection



# Random Access Scan

□ Random access scan cell



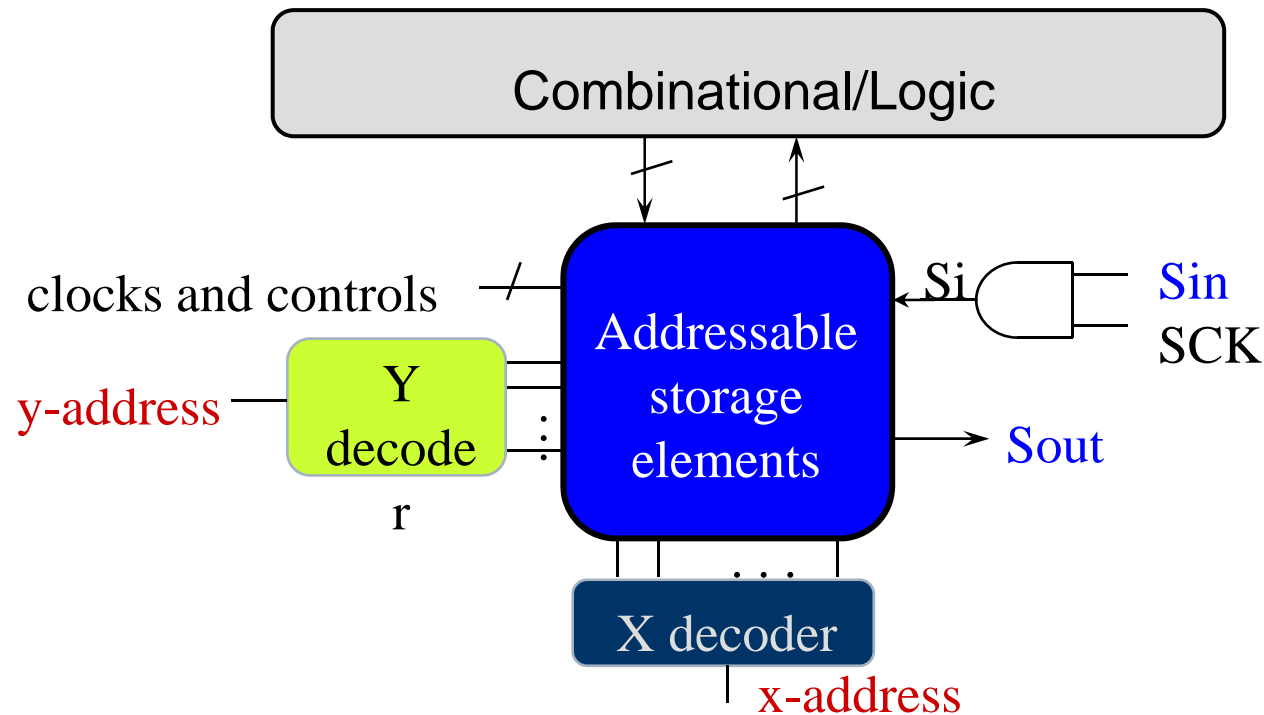
## Advantages

- Fast; minimal impact on normal path
- Fast for testing—random access
- Ability to ‘watch’ a node in normal operation mode

## ❑ Disadvantages

- Hardware cost is large; more pins added

# Random Access Architecture



- During normal operation the storage cells operate in their parallel-load mode
- To scan in a bit, the appropriate cell is addressed, the data are applied to  $s_{in}$

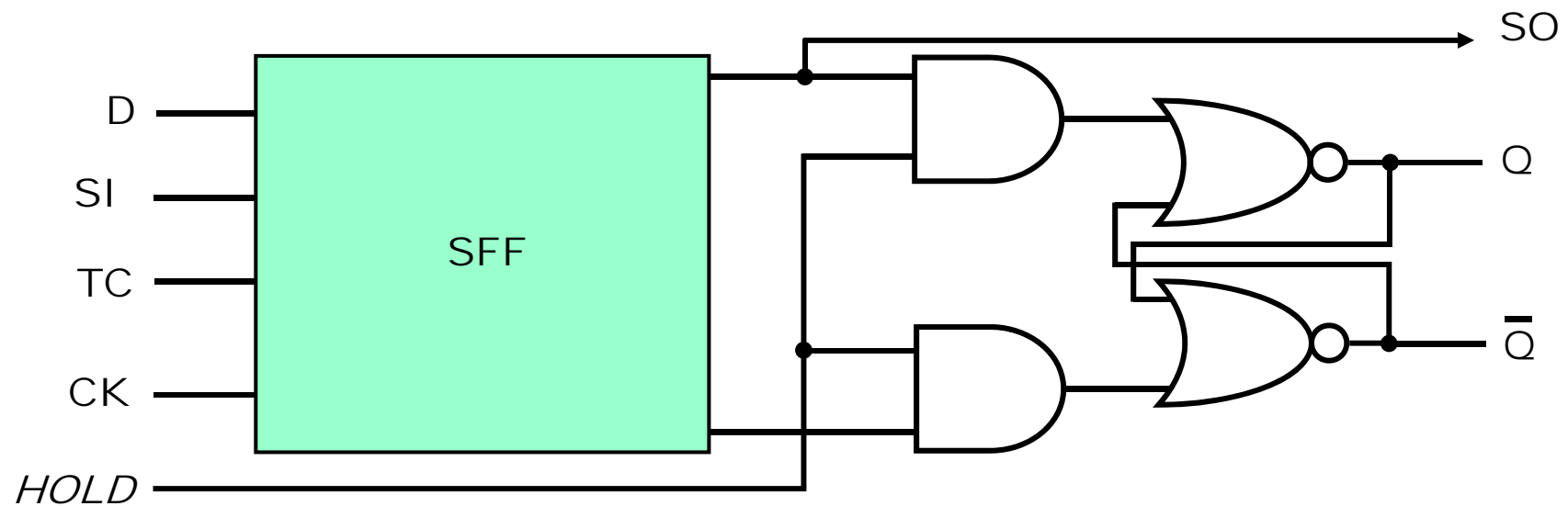
# Test Procedure

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1. Set test input to all test points
2. Apply the master reset signal to initialize all memory elements
3. Set scan-in address and data, and then apply the scan clock
4. Repeat step 3 until all internal test inputs are scanned in
5. Clock once for normal operation
6. Check states of the output points
7. Read the scan-out states of all memory elements by applying appropriate X-Y signals

# Scan-Hold FFs (SHFFs)

- $HOLD=0 \rightarrow Q \text{ \& } Q'$  are fixed

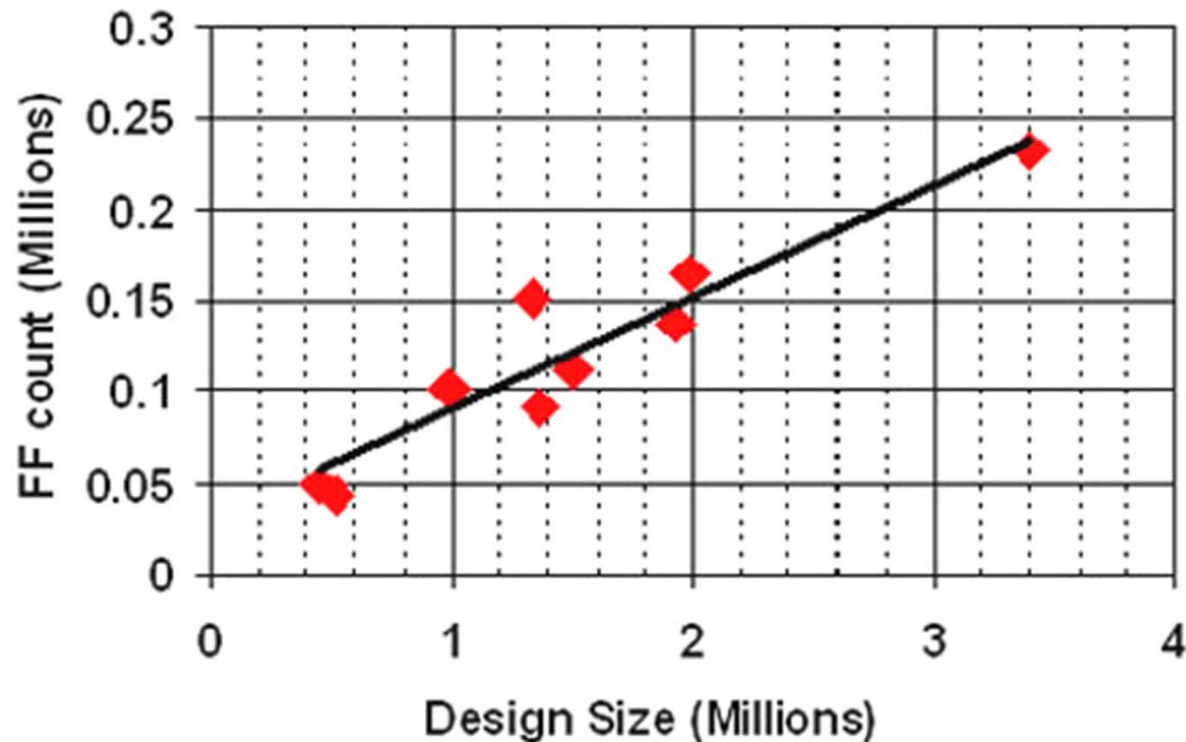


- The control input *HOLD* keeps the output steady at previous state of flip-flop
- Applications
  - Reduce power dissipation during scan, etc.

# Scan Enters the Nanometer Era

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- Trend in flip flop count with design size

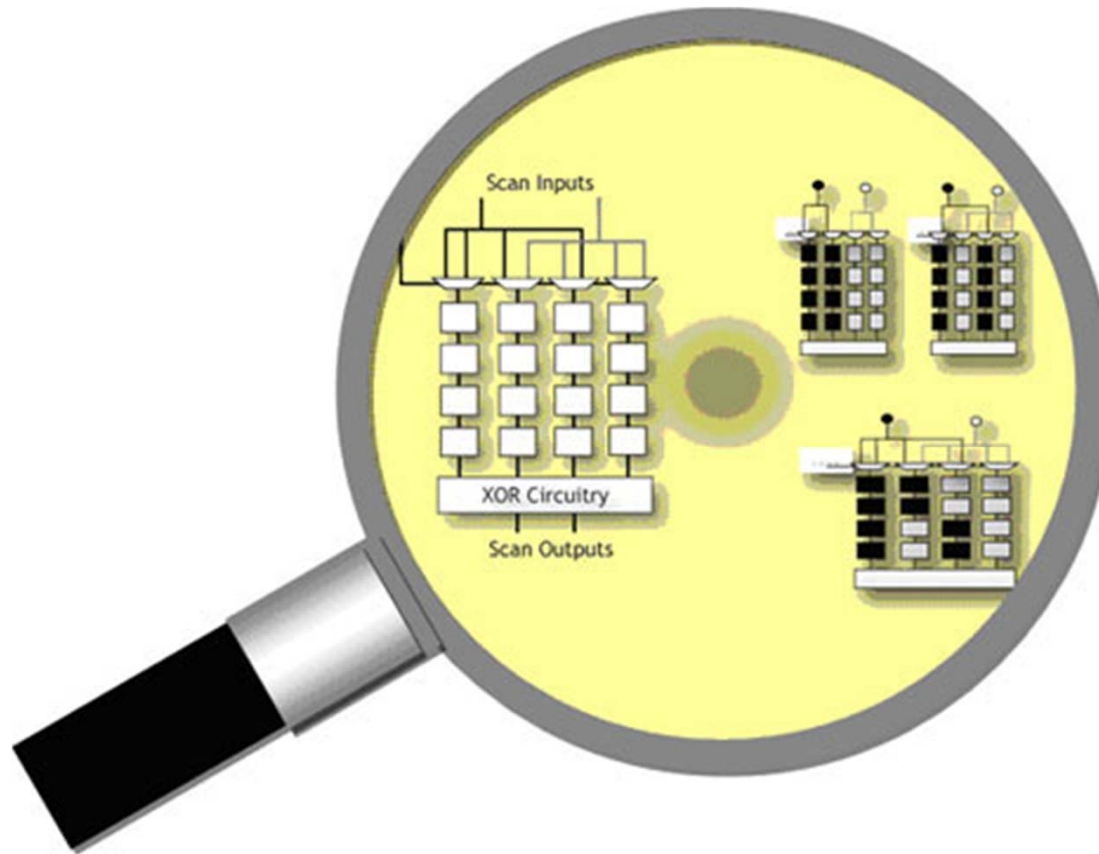


[Source: EE Times]

# Scan Enters the Nanometer Era

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## □ Adaptive scan architecture

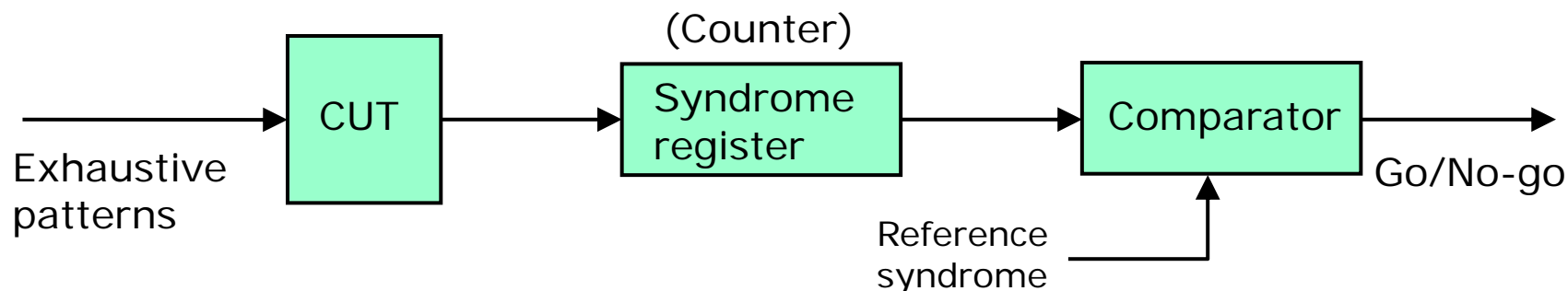


[Source: EE Times]

# Syndrome-Testable Design

## □ Definition

- The syndrome of a Boolean function  $f$  is  $S(f) \equiv \frac{k(f)}{2^n}$ , where  $k$  is the number of 1s (minterms) in  $f$  and  $n$  is the number of independent input variables
- A typical syndrome testing set-up



- $0 \leq S(f) \leq 1$
- A circuit is syndrome testable iff  $\forall$  fault  $\alpha, S(f) \neq S(f_\alpha)$
- Syndromes of logic gates

Gate	$AND_n$	$OR_n$	$XOR_n$	$NOT$
$S$	$1 / 2^n$	$1 - (1 / 2^n)$	$1 / 2$	$1 / 2$



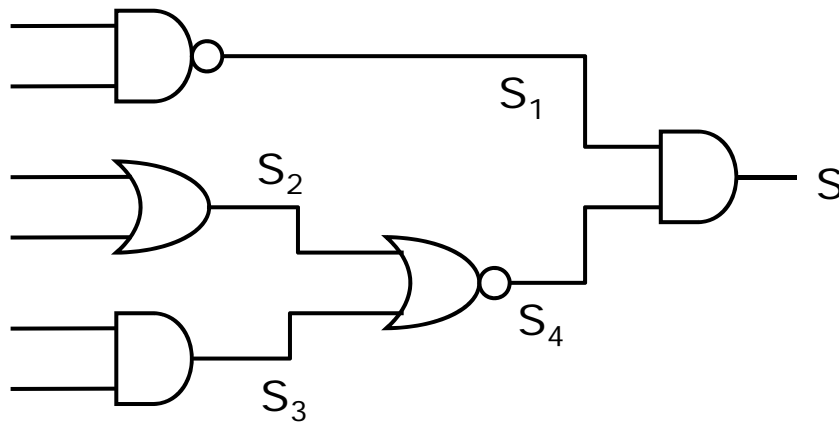
# Syndrome Computation

- Consider a circuit having 2 blocks,  $f$  and  $g$ , with unshared inputs

O/P Gate	<i>OR</i>	<i>AND</i>	<i>XOR</i>	<i>NAND</i>	<i>NOR</i>
$S$	$S_f + S_g - S_f S_g$	$S_f S_g$	$S_f + S_g - 2S_f S_g$	$1 - S_f S_g$	$1 - S_f - S_g + S_f S_g$

- Example

- Calculate the syndrome of the following circuit



$$S_1 = 1 - 1/4 = 3/4$$

$$S_2 = 1 - 1/4 = 3/4$$

$$S_3 = 1/8$$

$$S_4 = 1 - S_2 - S_3 + S_2 S_3 = 7/32$$

$$S = S_1 S_4 = 21/128$$

# Syndrome-Testable Design

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- Consider the function  $f = xz + y\bar{z}$ . The circuit is syndrome untestable
  - $S_f = 1/2$
  - If the circuit has a fault  $\alpha \equiv z/0$ , then the corresponding syndrome of the faulty circuit is  $S'_f = 1/2$
  - Thus the circuit is syndrome untestable
- A realization C of a function  $f$  is said to be syndrome-testable if no single stuck-at fault causes the circuit to have the same syndrome as the fault-free circuit
- Syndrome is a property of ***function***, not of ***implementation***

# Syndrome-Testable Design

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## □ Definition

- A logic function is **unate** in a variable  $x_i$  if it can be represented as an SOP or POS expression in which the variable  $x_i$  appears either only in an uncomplemented form or only in a complemented form
- For example:
  - $f(x_1, x_2) = \overline{x_1} \overline{x_2} + \overline{x_1} x_2$  no unate
  - $f(x_1, x_2, x_3) = x_1 x_2 + x_2 x_3 + x_1 x_3$  unate in  $x_2, x_3$ , not unate in  $x_1$

## □ Theorem

- A 2-level irredundant circuit realizing a unate function in all its variables is syndrome testable

# Syndrome-Testable Design

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## □ Theorem

- Any 2-level irredundant circuit can be made syndrome-testable by adding control inputs to the AND gates

## □ For example

- The function  $f = xz + y\bar{z}$  is syndrome untestable
- Now add a control input  $c \ni f' = cxz + y\bar{z}$ , where
$$C \equiv \begin{cases} 1 & \text{when in normal operation mode} \\ \text{normal i/p} & \text{when in test mode} \end{cases}$$
- $S' = 3/8, f'_\alpha = y$ , and  $S'_\alpha = 1/2 \neq S' \rightarrow$  Syndrome testable

## □ Drawbacks

- Only for combinational logic
- Exhaustive; modification doubles test set size

# Introduction to Built-In Self-Test

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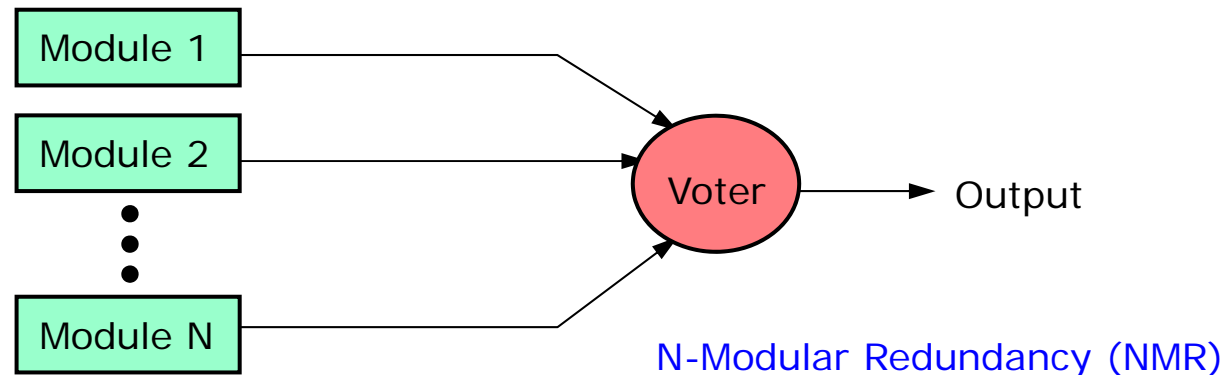
- Built-in self-test (BIST):
  - The capability of a circuit (chip/board/system) to test itself
- Advantages of BIST
  - Test patterns generated on-chip → controllability increased
  - (Compressed) response evaluated on-chip → observability increased
  - Test can be on-line (concurrent) or off-line
  - Test can run at circuit speed → more realistic; shorter test time; easier delay testing
  - External test equipment greatly simplified, or even totally eliminated
  - Easily adopting to engineering changes

# Introduction to Built-In Self-Test

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## □ On-line BIST

- Concurrent (EDAC, NMR, totally self-checking checkers, etc.):
  - Coding or modular redundancy techniques (fault tolerance)

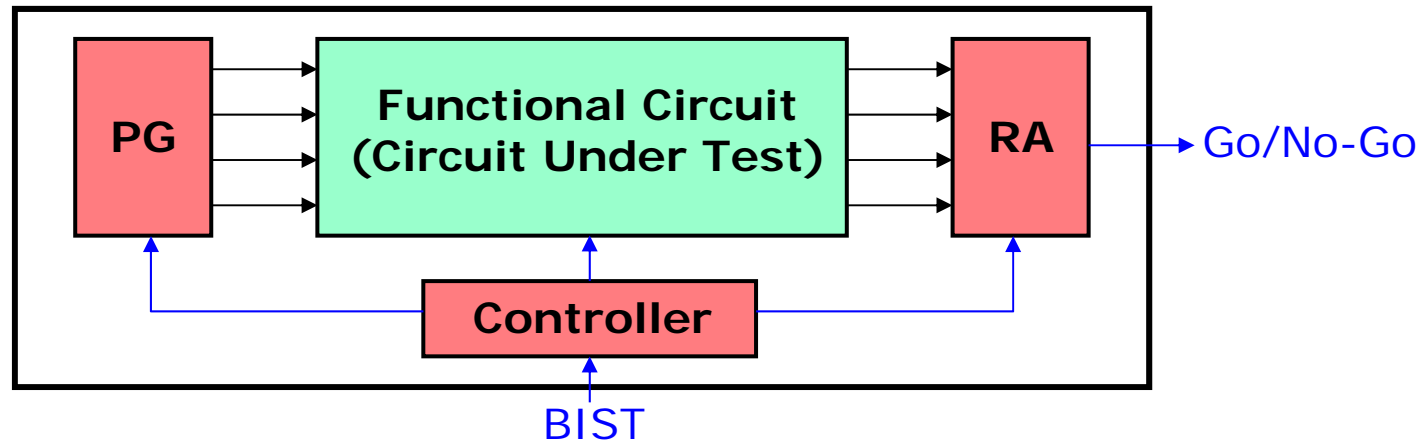


- Instantaneous correction of errors caused by temporary or permanent faults
- Nonconcurrent (diagnostic routines):
  - Carried out while a system is in an idle state

# Introduction to Built-In Self-Test

## □ Off-line BIST

- A typical BIST architecture



- Test generation

- Prestored TPG, e.g., ROM or shift register
- Exhaustive TPG, e.g., binary counter
- Pseudo-exhaustive TPG, e.g., constant-weight counter, combined LFSR and SR
- Pseudo-random pattern generator, e.g., LFSR

# Introduction to Built-In Self-Test

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- Response analysis
  - Check-sum
  - Ones counting
  - Transition counting
  - Parity checking
  - Syndrome analysis
  - Etc.
- Linear feedback shift register (LFSR) can be both the test generator and response analyzer
- We need a gold unit to generate the good signature or a simulator



# Signature Analysis

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- A compression technique based on the concept of cyclic redundancy checking (CRC) and realized in hardware using linear feedback shift registers
- Definition
  - A function  $f(x_1, x_2, \dots, x_n)$  is said to be linear if it can be expressed in the form
$$f = a_0 \oplus a_1 x_1 \oplus a_2 x_2 \oplus \dots \oplus a_n x_n$$
where  $a_i \in \{0,1\} \forall i = 0,1,\dots,n$ 
    - There are  $2^{n+1}$  linear functions of  $n$  variables
    - Linear operations: modulo addition, module scalar multiplication, & delay
    - Nonlinear operations: AND, OR, NAND, NOR, etc.

# Linear Feedback Shift Register

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## □ Definition

- A linear feedback shift register is a shift register with feedback paths which consist only of unit delays and XOR operators

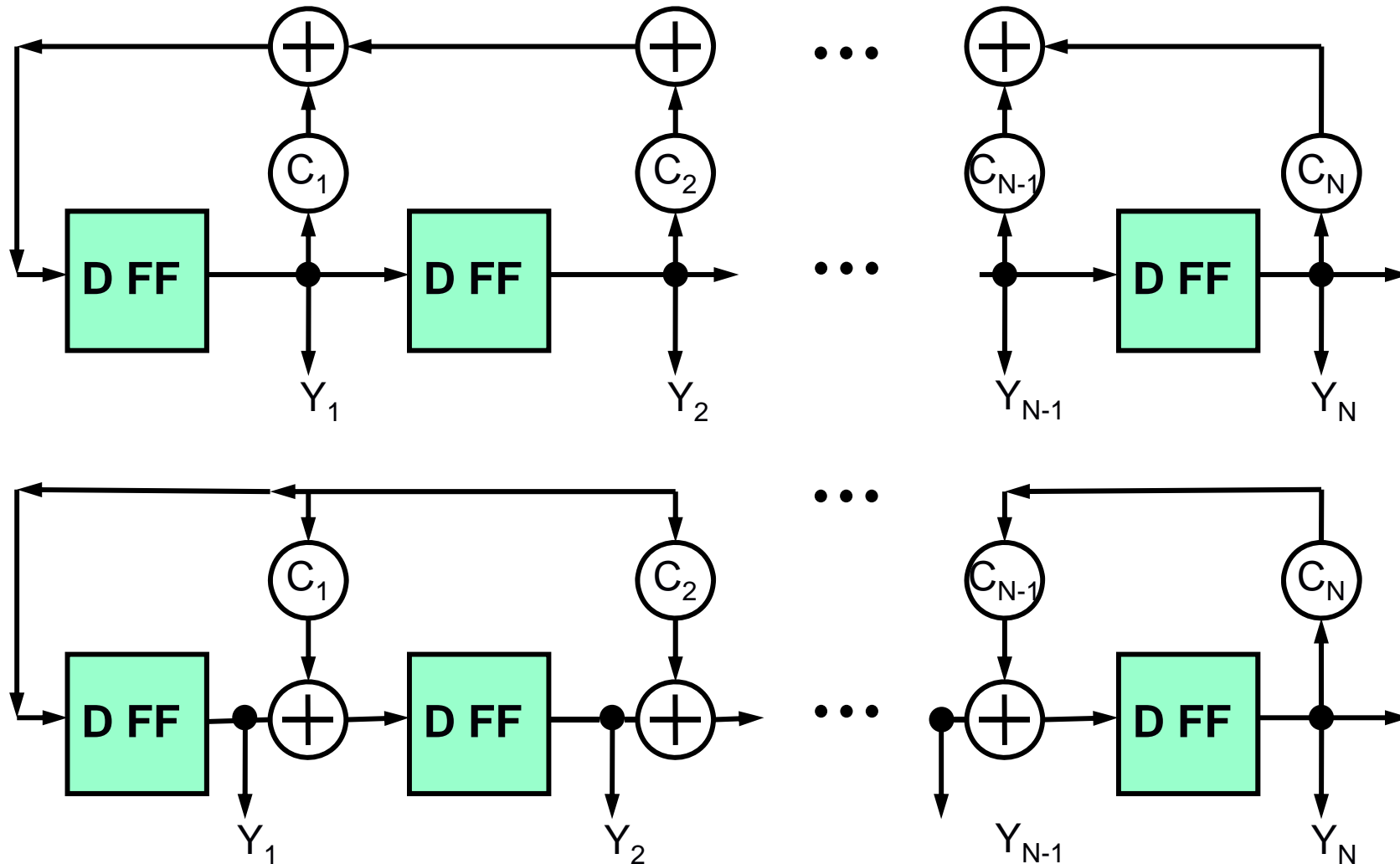
## □ Let $M$ =fault-free circuit response, $B$ =faulty circuit response, and $E$ =error syndrome (Hamming), where $E=M \oplus B \rightarrow$ thus $M=B \oplus E$ and $B=M \oplus E$

- We need a circuit to take  $B$  as input and compact it but still be able to tell if  $M \neq B$

## □ LFSR is considered as a popular approach for test response compaction

# Structures of LFSR

- Two types of generic standard LFSRs



# Mathematical Foundation of LFSR

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- As a function of time,  $Y_j$  can be expressed as
  - $Y_j(t) = Y_{j-1}(t-1)$  for  $j \neq 0$
  - Hence  $Y_j(t) = Y_0(t-j)$
- If we denote the translation operator as  $X^k$ , where  $k$  is the time translation unit
  - $Y_j(t) = Y_0(t) X^j$
- On the other hand,  $Y_0(t)$  can be expressed as
  - $Y_0(t) = \sum_{j=1}^N C_j Y_j(t)$
- Then
  - $Y_0(t) = \sum_{j=1}^N C_j Y_0(t) X^j$  for  $1 \leq j \leq N$

# Mathematical Foundation of LFSR

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□ We can rewrite the  $Y_0(t)$  as

■ 
$$Y_0(t) = Y_0(t) \sum_{j=1}^N C_j X^j$$

■ Also, 
$$Y_0(t) \left( \sum_{j=1}^N C_j X^j + 1 \right) = 0$$

□ We can rewrite this expression as  $Y_0(t)P_N(X) = 0$

■ For nontrivial solution,  $Y_0(t) \neq 0$ , we must have

$$P_N(X) = 0$$

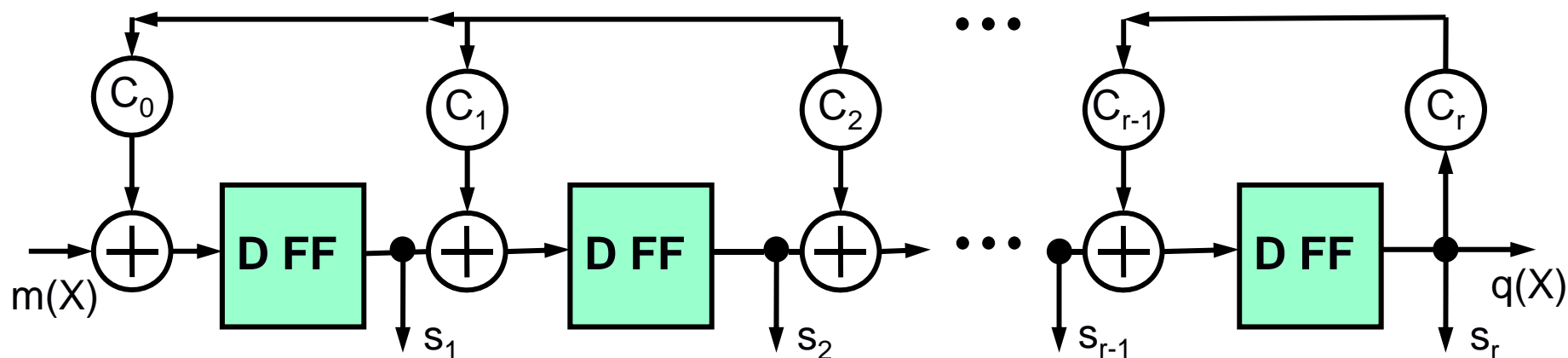
where 
$$P_N(X) = 1 + \sum_{j=1}^N C_j X^j$$

■  $P_N(X)$  is called the *characteristic polynomial* of the LFSR

# LFSR for Signature Analysis

- A serial input stream  $m_n, m_{n-1}, \dots, m_1, m_0$  entering the LFSR can be considered as the coefficients of a polynomial

- $m(X) = m_n X^n + m_{n-1} X^{n-1} + \dots + m_1 X + m_0$



- The LFSR is said to have a characteristic polynomial defined as follows

- $c(X) = c_r X^r + c_{r-1} X^{r-1} + \dots + c_1 X + c_0$

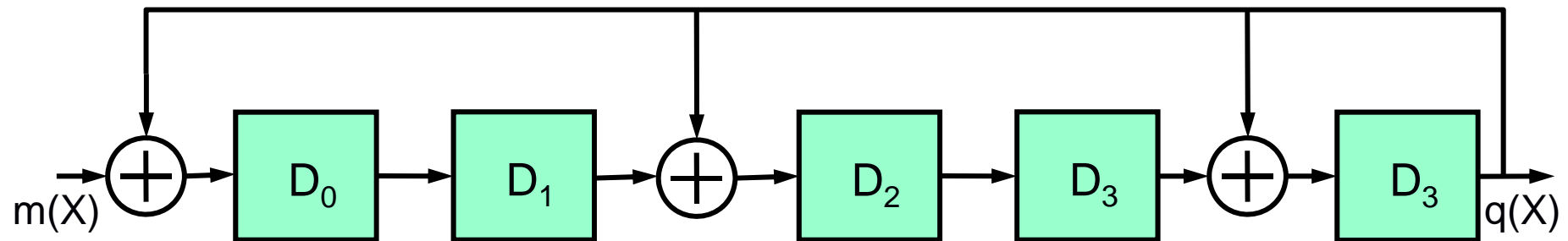
# LFSR for Signature Analysis

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- Assume that the initial state of the LFSR is  $D_i=0, i=0, \dots, r-1$ , then the LFSR effectively divides any  $m(X)$  by  $c(X)$ , i.e.,
  - $m(X) = q(X) \bullet C(X) + s(X)$
- The quotient  $q(X)$  appears serially at the output of the SR. The remainder  $s(X)$  is in the SR after  $n+1$  shifts:
  - $s(X) = s_r X^r + s_{r-1} X^{r-1} + \dots + s_1 X + s_0$

# An Example

- The following LFSR divides any  $m(X)$  by  $c(X) = X^5 + X^4 + X^2 + 1$



- Suppose  $m(x) = X^7 + X^6 + X^5 + X^4 + X^2 + 1$ , then  $q(X) = X^2 + 1$ , and  $s(X) = X^4 + X^2$

I/P	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	O/P
10101111	0	0	0	0	0	-
101	0	1	1	1	1	-
10	0	0	0	1	0	1
1	0	0	0	0	1	01
-	0	0	1	0	1	101



# Signature Analysis

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- Let  $m(X)$  be the input polynomial of degree  $k-1$ ,  $q(X)$  the quotient, and  $s(X)$  the signature (remainder).  
Then
  - $m(X) = q(X)c(X) + s(X)$
- The error syndrome can be represented as a polynomial  $e(X)$ 
  - E.g., let  $m(X) = X^4 + X^3 + 1$  (11001), and an erroneous input  $b(X) = X^3 + X + 1$  (01011), then the error syndrome is  $11001 \oplus 01011 = 10010$ , and is represented by  $e(X) = X^4 + X$
- In general, an erroneous input polynomial can be represented by
  - $B(X) = m(X) + e(X)$

# Signature Analysis

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- Theorem1: *Input streams  $m(X)$  and  $b(X)$  have the same signature iff  $e(X)$  is a multiple of  $c(X)$* 
  - Proof: an error is not detected when  $m(X)$  and  $b(X)$  have the same signature, i.e.,  $b(x) = q'(X)c(X) + s(X)$ . Since  $m(X) = q(X)c(X) + s(X)$ , we obtain
$$e(X) = m(X) + b(X) = c(X)(q'(X) - q(X))$$
- Theorem2: *Undetected errors correspond to error patterns which are multiples of  $c(X)$*
- Theorem3: *If  $c(X)$  has 2 or more nonzero coefficients—i.e., at least 1 feedback term—then it can detect all single-bit errors*
  - Proof: all nonzero multiples of  $c(X)$  must have at least 2 nonzero coefficients. Therefore, any error with only 1 nonzero coefficient cannot be a multiple of  $c(X)$  and must be detectable.

# Aliasing Probability

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- Theorem4: for a  $k$ -bit response sequence, if all possible error patterns are equally likely, then the probability of failing to detect an error (i.e., the *aliasing probability*) by the LFSR of length  $r$  is

$$P_{al} = \frac{2^{k-r} - 1}{2^k - 1}$$

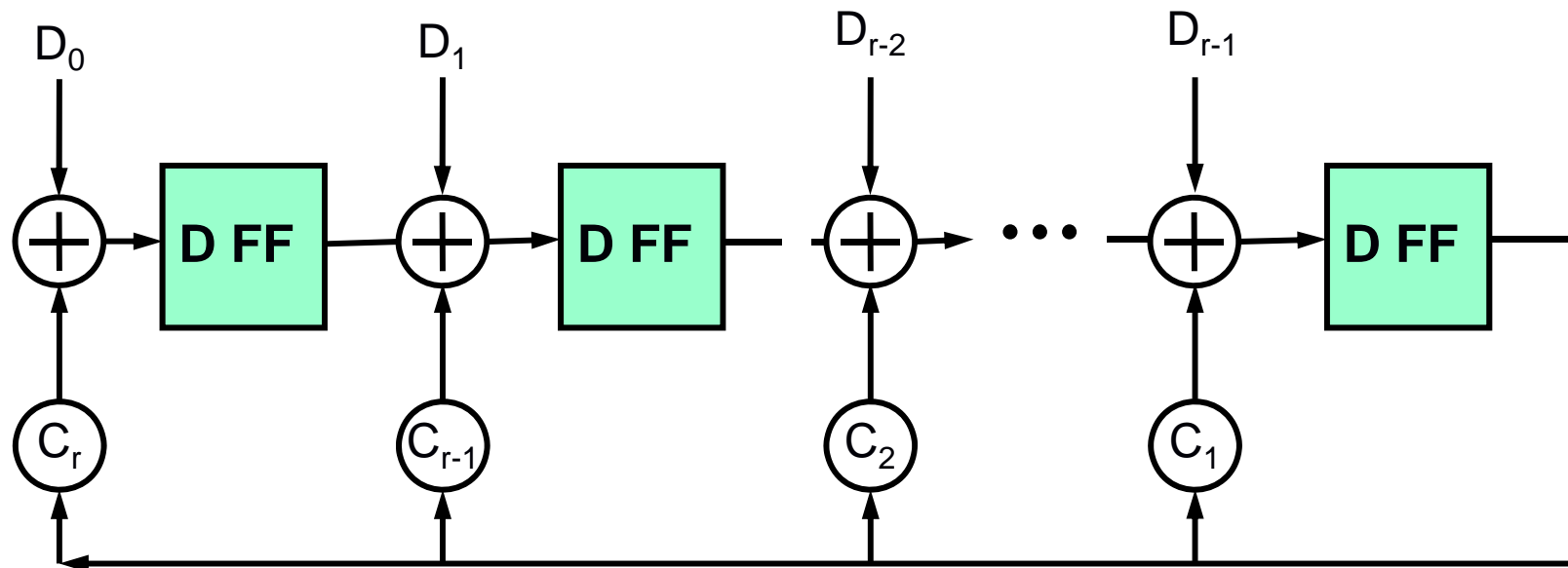
- Proof: For a  $k$ -bit response,  $\deg(m(X)) = k-1$ , and  $\deg(e(X)) \leq k-1$ . Therefore, the number of possible error polynomial is represented by  $e(X) = c(X)p(X)$  for some nonzero  $p(X)$ . Since  $\deg(c(X)) = r$ , the number of possible  $p(X)$ 's is  $2^{k-r}-1$ . Thus

$$P_{al} = \frac{2^{k-r} - 1}{2^k - 1}$$

- For a long sequence,  $k \gg r \rightarrow P_{al} \sim 1/2^r$

# Multiple-Input Signature Register

- The structure of multiple-input signature register (MISR)



- The mathematical theory is a direct extension of the results shown above
- For equally likely error patterns and long data streams, the aliasing probability for an MISR of  $r$  stages also is  $P_{al} \approx 1/2^r$ .

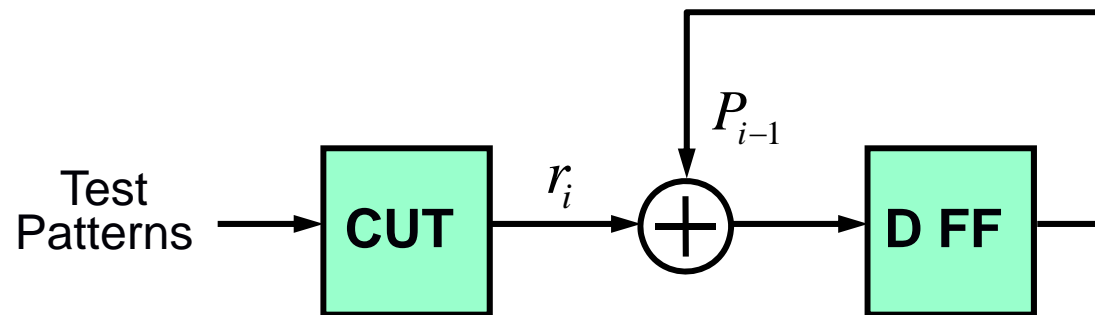
# Response Compaction

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- Usually, we think of *data compression* as a process that preserves data integrity. This is why we given more attention here to *data compaction*, which may result in some losses
- There are several compaction testing techniques
  - Parity testing
  - One counting
  - Transition counting
  - Syndrome calculation
  - Signature analysis

# Parity Testing

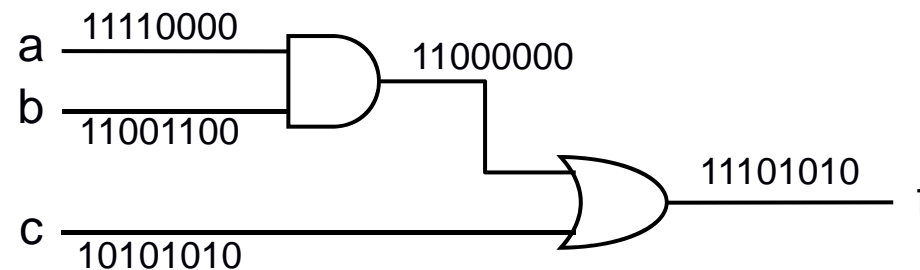
- This is the simplest of all techniques but also the most lossy
- The parity of responses to the test patterns is calculated as
  - $P = \sum_{i=1}^{i=L} r_i$ , where  $L$  is the length of the test and  $r_i$  is the response for the  $i$ th test pattern
- The response of the circuit under test (CUT) to pattern  $i$  and the partial product  $P_{i-1}$  is illustrated as below



# One Counting

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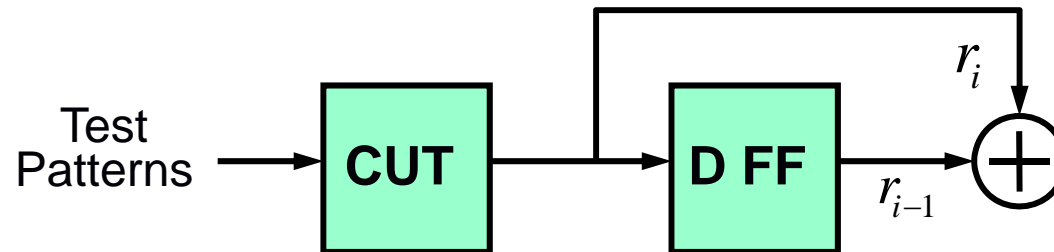
- The number of 1's in the response stream is calculated and compared to the number of 1's in the fault-free responses
- Consider the circuit shown below



- If we have a test of length  $L$  and the fault-free count is  $m$ , the possibility of aliasing is  $[C(L, m) - 1]$  patterns out of a total number of possible strings of length  $L$ ,  $(2^L - 1)$

# Transition Counting

- In transition counting compaction, it is only the number of transition  $0 \rightarrow 1$  and  $1 \rightarrow 0$  that are counted. Thus the signature is given by
  - $\sum_{i=1}^{i=L-1} r_i \oplus r_{i+1}$ , where the summation is ordinary addition and  $\oplus$  is XOR operation
- The compaction scheme is shown below





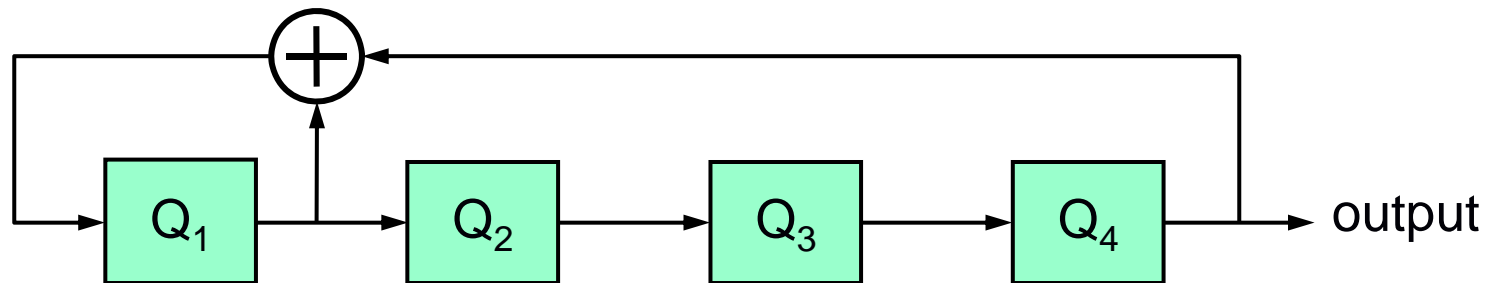
# Pseudorandom Pattern Generator

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- ❑ Logic BIST uses mostly pseudorandom (PR) tests. They are usually much longer than deterministic tests, but are definitely less costly to generate
- ❑ PR tests are generated using a LFSR or cellular automata
- ❑ By means of a simple circuit called an *autonomous linear feedback shift register* (ALFSR)
- ❑ Definition: an ALFSR is a LFSR with no external inputs
- ❑ Faults that are hard to detect with PR tests are called *random pattern resistant* faults

# Pseudorandom Pattern Generator (PRPG)

- Example: the following ALFSR generates the pseudorandom sequence shown in the table below

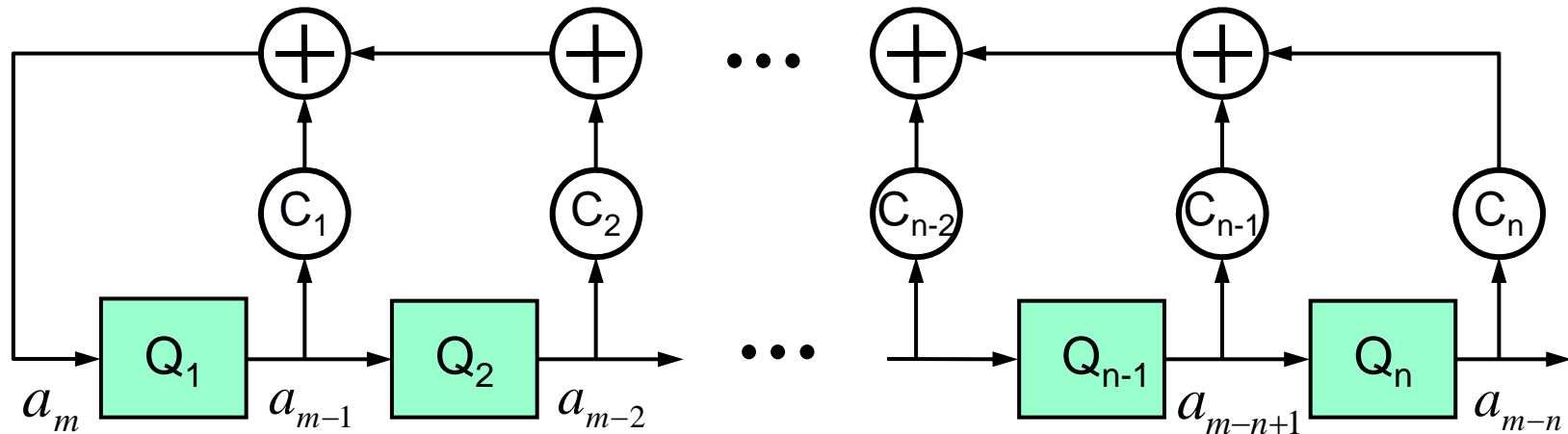


State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15=0
Q <sub>1</sub>	1	1	1	1	0	1	0	1	1	0	0	1	0	0	0	1
Q <sub>2</sub>	0	1	1	1	1	0	1	0	1	1	0	0	1	0	0	0
Q <sub>3</sub>	0	0	1	1	1	1	0	1	0	1	1	0	0	1	0	0
Q <sub>4</sub>	0	0	0	1	1	1	1	0	1	0	1	1	0	0	1	0

- The output sequence is 000111101011001, which repeats after  $15(2^n-1)$  clocks
- Max period for an n-stage ALFSR =  $2^n-1$
- All-0 state of the register cannot occur in the max-length cycle

# Mathematical Foundation of PRPG

- A generic structure of ALFSR



- A sequence of bits  $\{a_m\} = a_0, a_1, \dots, a_m, \dots$  can be associated with a polynomial—its **generation function**:

$$G(X) \equiv a_0 + a_1X + \dots + a_mX^m + \dots = \sum_{m=0}^{\infty} a_mX^m$$

- In the above figure, assume that the current state of  $Q_i$  is  $a_{m-i}$ ,  $i=1, 2, \dots, n$ , and the initial state of  $Q_i$  is  $a_{-i}=0$ ,  $i=1, 2, \dots, n$ , but  $a_{-n}=1$ , then  $a_m = \sum_{i=1}^n c_i a_{m-i}$

# Mathematical Foundation of PRPG

$$\begin{aligned} G(X) &= \sum_{m=0}^{\infty} a_m X^m = \sum_{m=0}^{\infty} \left( \sum_{i=1}^n c_i a_{m-i} \right) X^m = \sum_{i=1}^n c_i X^i \sum_{m=0}^{\infty} a_{m-i} X^{m-i} \\ &= \sum_{i=1}^n c_i X^i [a_{-i} X^{-i} + \cdots + a_{-1} X^{-1} + \sum_{m=i}^{\infty} a_{m-i} X^{m-i}] \\ &= \sum_{i=1}^n c_i X^i [a_{-i} X^{-i} + \cdots + a_{-1} X^{-1} + \sum_{m=0}^{\infty} a_m X^m] \\ &= \sum_{i=1}^n c_i X^i [a_{-i} X^{-i} + \cdots + a_{-1} X^{-1} + G(X)] \\ &= \sum_{i=1}^n c_i X^i G(X) + \sum_{i=1}^n c_i X^i (a_{-i} X^{-i} + \cdots + a_{-1} X^{-1}) \\ \Rightarrow 1 + \sum_{i=1}^n c_i X^i G(X) &= \sum_{i=1}^n c_i X^i (a_{-i} X^{-i} + \cdots + a_{-1} X^{-1}) \\ \Rightarrow G(X) &= \frac{\sum_{i=1}^n c_i X^i (a_{-i} X^{-i} + \cdots + a_{-1} X^{-1})}{1 + \sum_{i=1}^n c_i X^i} \end{aligned}$$

# Mathematical Foundation of PRPG

- Now  $c(X) = 1 + \sum_{i=1}^n c_i X^i$  is the characteristic polynomial of the LFSR as defined above. Since  $a_{-1}=0, i=1,2,\dots,n-1$ , and  $a_{-n}=1$ , we have

$$G(X) = \frac{1}{c(X)} = \sum_{m=0}^{\infty} a_m X^m$$

- The sequence  $\{a_m\}$  is cyclic, with the period assumed to be  $p$

$$\begin{aligned} \therefore G(X) &= \frac{1}{c(X)} = (a_0 + a_1 X + \dots + a_{p-1} X^{p-1}) \\ &\quad + X^p (a_0 + a_1 X + \dots + a_{p-1} X^{p-1}) \\ &\quad + X^{2p} (a_0 + a_1 X + \dots + a_{p-1} X^{p-1}) \\ &\quad + \dots \\ &= (a_0 + a_1 X + \dots + a_{p-1} X^{p-1})(1 + X^p + X^{2p} + \dots) \\ &= \frac{(a_0 + a_1 X + \dots + a_{p-1} X^{p-1})}{1 - X^p} \end{aligned}$$

$$\therefore \frac{1 - X^p}{c(X)} = a_0 + a_1 X + \dots + a_{p-1} X^{p-1} \quad \text{i.e., } c(X) \text{ evenly divides into } 1 - X^p$$

# Theorems

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- Theorem: If the initial state of an  $n$ -stage LFSR is  $a_i=0$ ,  $i=1,2,\dots,n-1$ , and  $a_n=1$ , then the LFSR sequence  $\{a_m\}$  is periodic with a period that is the smallest integer  $p$  for which  $c(X)$  divides  $1-X^p$ 
  - The period  $p \leq 2^n - 1$
  - For a given  $n$ , we want to find a  $c(X)$  that maximizes  $p$
- Definition: The sequences produced by max-length LFSRs are called *pseudorandom sequences* or *m-sequences*. The characteristic polynomial associated with an m-sequence is called a *primitive polynomial*. An *irreducible polynomial* is one that cannot be factored
  - Pseudorandom sequences (or m-sequences) are not really random since they are produced by a fixed circuit.

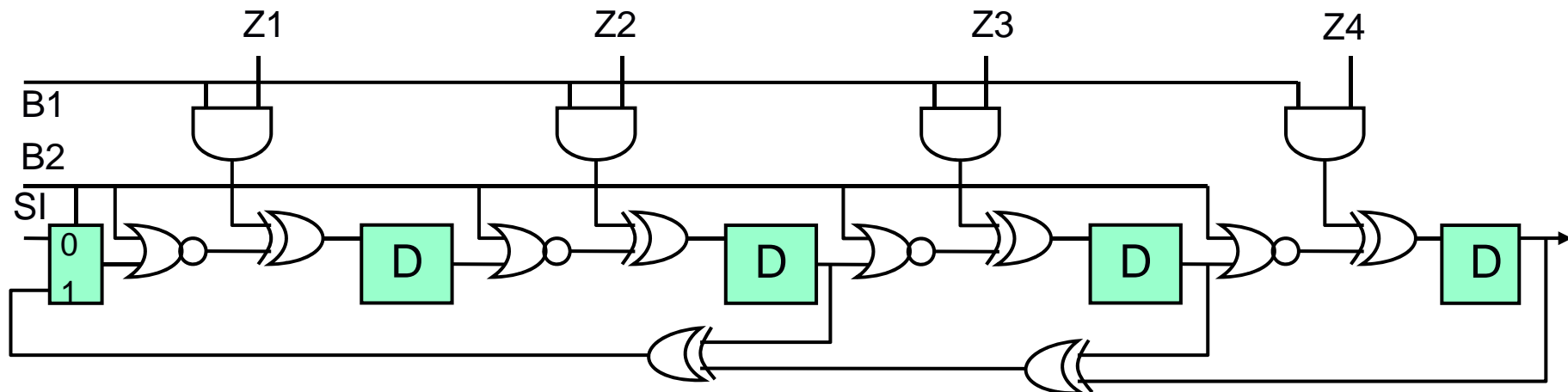
# Theorems

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- Theorem: An *irreducible* polynomial  $c(X)$  satisfies the following 2 conditions:
  - It has an odd number of terms including the constant term
  - If its degree  $n > 3$ , then  $c(X)$  must divide  $1 + X^p$ , where  $p = 2^n - 1$
- Theorem: A primitive polynomial is irreducible if the smallest positive integer  $p$  that allows the polynomial to divide evenly into  $1 + X^p$  occurs for  $p = 2^n - 1$ , where  $n$  is the degree of the polynomial

# Built-In-Logic-Block-Observer (BILBO)

- A BILBO is a multi-purpose test module which serves as a test generator or a signature analyzer. It is composed of a row of FFs and some additional gates for shift and feedback operation

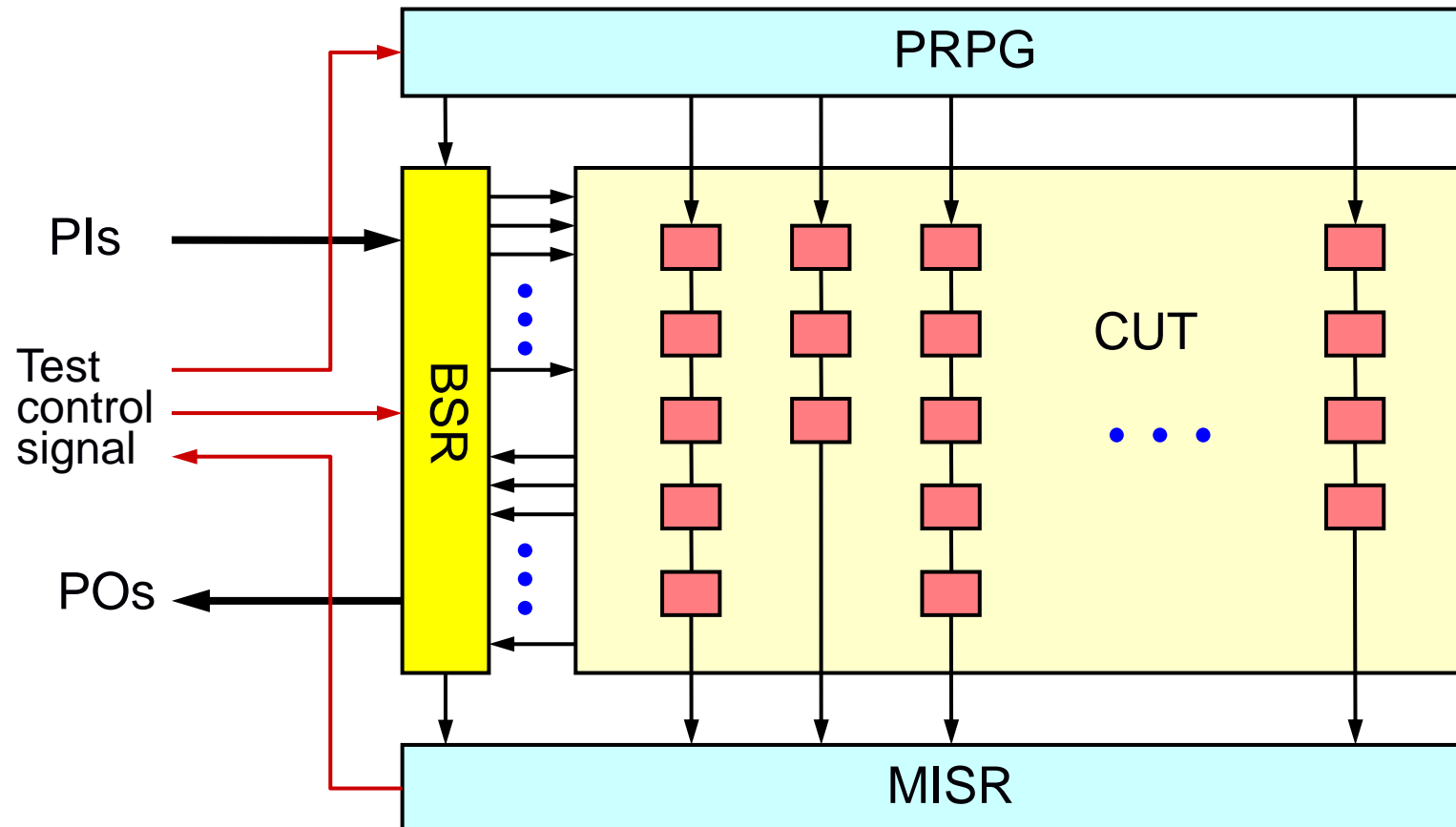


B1	B2	Function
0	1	All FFs are reset
1	1	Behaves as separate latches—normal mode
0	0	A linear shift register—SR mode
1	0	MISR/PRPG—test mode



# STUMPS Architecture

- Logic BIST with STUMPS architecture



# Summary

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- Design-for-testability techniques
  - Ad-hoc techniques
  - Scan
  - LSSD
  - Random access scan
  - Syndrome-testable
  - C-testability
- Scan is a popular DFT technique in modern IC design
- DFT can increase the controllability and observability of the circuit under test

# Summary

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- Built-in self-test methodology is more and more important for deep submicron designs
- Two key components of BIST
  - Test pattern generator
    - E.g., LFSR
  - Response evaluator
    - E.g., BILBO