

# Relationship between Schneider's and Jarvis's Shear-Shear Correlation Function

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From Jarvis (Page 3):

$$\xi_+ = \langle \gamma_i \gamma_j^* \rangle = \text{xip} + i(\text{xip\_im}) \quad (1)$$

$$\xi_- = \langle \gamma_i \gamma_j e^{-4i\alpha} \rangle = \text{xim} + i(\text{xim\_im}) \quad (2)$$

where  $\alpha$  is the angle between the two objects  $i, j$  and each  $\gamma$  is given by  $\gamma_n = |\gamma_n| e^{2i\theta_n}$  in polar form.

From Schneider (Page 92):

$$\xi_{\pm} = \langle \gamma_{it} \gamma_{jt} \rangle \pm \langle \gamma_{ix} \gamma_{jx} \rangle \quad (3)$$

$$\xi_x = \langle \gamma_{it} \gamma_{jx} \rangle \quad (4)$$

where  $\gamma_{nt} = -\text{Re}(\gamma_n e^{-2i\alpha})$  and  $\gamma_{nx} = -\text{Im}(\gamma_n e^{-2i\alpha})$ .

## Schneider's $\xi_+$ to Jarvis's xip

Starting with Schneider's definition, observe that

$$\begin{aligned} \xi_+ &= \langle \gamma_{it} \gamma_{jt} \rangle + \langle \gamma_{ix} \gamma_{jx} \rangle \\ &= \langle \text{Re}(\gamma_i e^{-2i\alpha}) \cdot \text{Re}(\gamma_j e^{-2i\alpha}) \rangle + \langle \text{Im}(\gamma_i e^{-2i\alpha}) \cdot \text{Im}(\gamma_j e^{-2i\alpha}) \rangle \\ &= \langle \text{Re}(|\gamma_i| e^{2i(\theta_i - \alpha)}) \cdot \text{Re}(|\gamma_j| e^{2i(\theta_j - \alpha)}) \rangle + \langle \text{Im}(|\gamma_i| e^{2i(\theta_i - \alpha)}) \cdot \text{Im}(|\gamma_j| e^{2i(\theta_j - \alpha)}) \rangle \\ &= \langle |\gamma_i| \cdot |\gamma_j| \cos(2(\theta_i - \alpha)) \cos(2(\theta_j - \alpha)) \rangle + \langle |\gamma_i| \cdot |\gamma_j| \sin(2(\theta_i - \alpha)) \sin(2(\theta_j - \alpha)) \rangle. \end{aligned}$$

Using the trig identities

$$\cos(u) \cos(v) = \frac{1}{2} [\cos(u - v) + \cos(u + v)], \quad (5)$$

$$\sin(u) \sin(v) = \frac{1}{2} [\cos(u - v) - \cos(u + v)], \quad (6)$$

and the linearity of the expectation operator, it is straightforward to show that

$$\xi_+ = \langle |\gamma_i| \cdot |\gamma_j| \cos(2(\theta_i - \theta_j)) \rangle.$$

Now using the identity

$$\cos(u \pm v) = \cos(u) \cos(v) \mp \sin(u) \sin(v), \quad (7)$$

the previous equation can be written as

$$\begin{aligned} \xi_+ &= \langle |\gamma_i| \cdot |\gamma_j| \cdot [\cos(2\theta_i) \cos(2\theta_j) + \sin(2\theta_i) \sin(2\theta_j)] \rangle \\ &= \langle \text{Re}(\gamma_i) \cdot \text{Re}(\gamma_j) + \text{Im}(\gamma_i) \cdot \text{Im}(\gamma_j) \rangle. \end{aligned}$$

Using the complex number identities

$$\text{Re}(z) = \frac{1}{2} [z + z^*], \quad (8)$$

$$\text{Im}(z) = \frac{1}{2i} [z - z^*], \quad (9)$$

it follows that

$$\begin{aligned}
\xi_+ &= \left\langle \frac{1}{4} [(\gamma_i + \gamma_i^*)(\gamma_j + \gamma_j^*) - (\gamma_i - \gamma_i^*)(\gamma_j - \gamma_j^*)] \right\rangle \\
&= \left\langle \frac{1}{2} [\gamma_i \gamma_j^* + (\gamma_i \gamma_j^*)^*] \right\rangle \\
&= \langle \text{Re}(\gamma_i \gamma_j^*) \rangle = \mathbf{xip}.
\end{aligned}$$

## Schneider's $\xi_-$ to Jarvis's $\mathbf{xim}$

The first few steps are identical to the previous section besides the negative in the definition of  $\xi_-$ , giving

$$\begin{aligned}
\xi_- &= \langle \gamma_{it} \gamma_{jt} \rangle - \langle \gamma_{ix} \gamma_{jx} \rangle \\
&= \langle |\gamma_i| \cdot |\gamma_j| \cos(2(\theta_i - \alpha)) \cos(2(\theta_j - \alpha)) \rangle - \langle |\gamma_i| \cdot |\gamma_j| \sin(2(\theta_i - \alpha)) \sin(2(\theta_j - \alpha)) \rangle \\
&= \langle |\gamma_i| \cdot |\gamma_j| \cos(2(\theta_i + \theta_j - 2\alpha)) \rangle.
\end{aligned}$$

Now using Equation (7) twice, first letting  $u = 2(\theta_i + \theta_j)$  and  $v = -4\alpha$ , and then letting  $u = 2\theta_i$  and  $v = 2\theta_j$ , this equation becomes

$$\begin{aligned}
\xi_- &= \left\langle |\gamma_i| \cdot |\gamma_j| \left( [\cos(2\theta_i) \cos(2\theta_j) - \sin(2\theta_i) \sin(2\theta_j)] \cos(4\alpha) + [\sin(2\theta_i) \cos(2\theta_j) + \cos(2\theta_i) \sin(2\theta_j)] \sin(4\alpha) \right) \right\rangle \\
&= \left\langle [\text{Re}(\gamma_i) \cdot \text{Re}(\gamma_j) - \text{Im}(\gamma_i) \cdot \text{Im}(\gamma_j)] \cos(4\alpha) + [\text{Im}(\gamma_i) \cdot \text{Re}(\gamma_j) + \text{Re}(\gamma_i) \cdot \text{Im}(\gamma_j)] \sin(4\alpha) \right\rangle.
\end{aligned}$$

Using the identities (8) and (9), and simplifying the leftover terms, this equation can be shown to equal

$$\xi_- = \langle \text{Re}(\gamma_i \gamma_j) \cos(4\alpha) + \text{Im}(\gamma_i \gamma_j) \sin(4\alpha) \rangle.$$

Now observe that for two complex numbers  $a$  and  $b$ , it is true that

$$\text{Re}(a \cdot b^*) = \text{Re}(a) \cdot \text{Re}(b) + \text{Im}(a) \cdot \text{Im}(b).$$

Then setting  $a = \gamma_i \gamma_j$  and  $b = e^{4i\alpha}$ , it must be true that

$$\text{Re}(\gamma_i \gamma_j e^{-4i\alpha}) = \text{Re}(\gamma_i \gamma_j) \cos(4\alpha) + \text{Im}(\gamma_i \gamma_j) \sin(4\alpha).$$

Combining this with our previous result, this means that

$$\xi_- = \langle \text{Re}(\gamma_i \gamma_j) \cos(4\alpha) + \text{Im}(\gamma_i \gamma_j) \sin(4\alpha) \rangle = \langle \text{Re}(\gamma_i \gamma_j e^{-4i\alpha}) \rangle = \mathbf{xim}.$$

## Schneider's $\xi_\times$ to Jarvis's $\frac{1}{2}(\mathbf{xim\_im} - \mathbf{xip\_im})$

Starting from Schneider's definition of  $\xi_\times$ ,

$$\begin{aligned}
\xi_\times &= \langle \gamma_{it} \gamma_{jx} \rangle \\
&= \left\langle \text{Re}(|\gamma_i| e^{2i(\theta_i - \alpha)}) \cdot \text{Im}(|\gamma_j| e^{2i(\theta_j - \alpha)}) \right\rangle \\
&= \langle |\gamma_i| \cdot |\gamma_j| \cos(2(\theta_i - \alpha)) \sin(2(\theta_j - \alpha)) \rangle.
\end{aligned}$$

Using the trig identity

$$\cos(u) \sin(v) = \frac{1}{2} [\sin(u + v) - \sin(u - v)], \quad (10)$$

the previous equation becomes

$$\xi_{\times} = \left\langle |\gamma_i| \cdot |\gamma_j| \cdot \frac{1}{2} [\sin(2(\theta_i + \theta_j - 2\alpha)) - \sin(2(\theta_i - \theta_j))] \right\rangle.$$

Next, applying the identities

$$\sin(u \pm v) = \sin(u) \cos(v) \pm \cos(u) \sin(v) \quad (11)$$

along with (7) iteratively until each trig function only has a single term, the equation becomes

$$\begin{aligned} \xi_{\times} &= \left\langle |\gamma_i| \cdot |\gamma_j| \cdot \frac{1}{2} \left( [\sin(2\theta_i) \cos(2\theta_j) + \cos(2\theta_i) \sin(2\theta_j)] \cos(4\alpha) \right. \right. \\ &\quad \left. \left. - [\cos(2\theta_i) \cos(2\theta_j) - \sin(2\theta_i) \sin(2\theta_j)] \sin(4\alpha) \right. \right. \\ &\quad \left. \left. - \sin(2\theta_i) \cos(2\theta_j) + \cos(2\theta_i) \sin(2\theta_j) \right) \right\rangle \\ &= \left\langle \frac{1}{2} \left( [\operatorname{Im}(\gamma_i) \cdot \operatorname{Re}(\gamma_j) + \operatorname{Re}(\gamma_i) \cdot \operatorname{Im}(\gamma_j)] \cos(4\alpha) \right. \right. \\ &\quad \left. \left. - [\operatorname{Re}(\gamma_i) \cdot \operatorname{Re}(\gamma_j) - \operatorname{Im}(\gamma_i) \cdot \operatorname{Im}(\gamma_j)] \sin(4\alpha) \right. \right. \\ &\quad \left. \left. - \operatorname{Im}(\gamma_i) \operatorname{Re}(\gamma_j) + \operatorname{Re}(\gamma_i) \cdot \operatorname{Im}(\gamma_j) \right) \right\rangle. \end{aligned}$$

Using the identities (8) and (9), this can be simplified to

$$\xi_{\times} = \left\langle \frac{1}{2} [\operatorname{Im}(\gamma_i \gamma_j) \cos(4\alpha) - \operatorname{Re}(\gamma_i \gamma_j) \sin(4\alpha) - \operatorname{Im}(\gamma_i \gamma_j^*)] \right\rangle.$$

Consider again two complex numbers  $a$  and  $b$ . Observe that

$$\operatorname{Im}(a \cdot b^*) = \operatorname{Im}(a) \cdot \operatorname{Re}(b) - \operatorname{Re}(a) \cdot \operatorname{Im}(b).$$

Then setting  $a = \gamma_i \gamma_j$  and  $b = e^{4i\alpha}$ , it must be true that

$$\operatorname{Im}(\gamma_i \gamma_j e^{-4i\alpha}) = \operatorname{Im}(\gamma_i \gamma_j) \cos(4\alpha) - \operatorname{Re}(\gamma_i \gamma_j) \sin(4\alpha).$$

Combing these results give

$$\begin{aligned} \xi_{\times} &= \left\langle \frac{1}{2} [\operatorname{Im}(\gamma_i \gamma_j) \cos(4\alpha) - \operatorname{Re}(\gamma_i \gamma_j) \sin(4\alpha) - \operatorname{Im}(\gamma_i \gamma_j^*)] \right\rangle \\ &= \left\langle \frac{1}{2} [\operatorname{Im}(\gamma_i \gamma_j e^{-4i\alpha}) - \operatorname{Im}(\gamma_i \gamma_j^*)] \right\rangle \\ &= \frac{1}{2} [\text{xim\_im} - \text{xip\_im}]. \end{aligned}$$

as desired.