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Linking Halo Mass to Galaxy Luminosity

Statistical Model

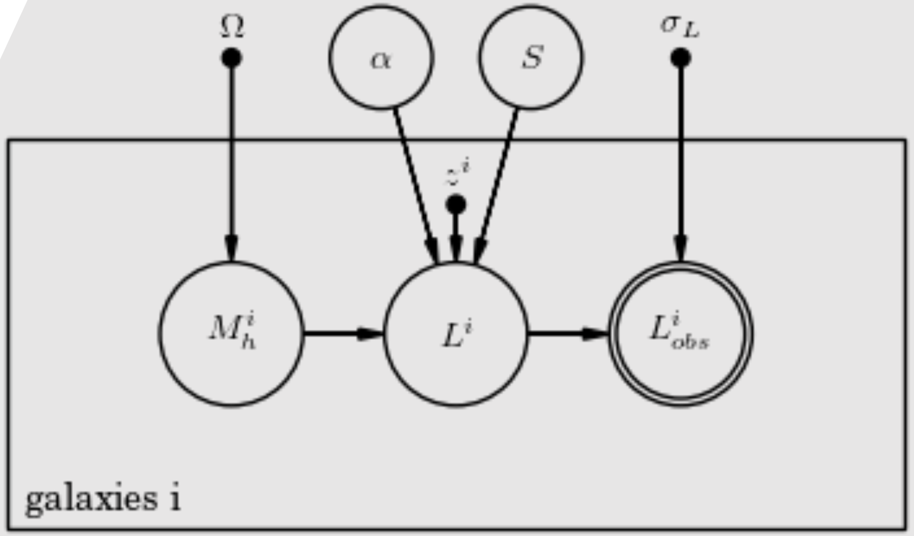
$$P(\mathbf{M}_h) \propto \frac{dn}{dM} = f(\sigma)\frac{\rho_m}{M}\frac{dln\sigma^{-1}}{dM}$$

$$P(\mathbf{L}|\alpha, S, \mathbf{z}, \mathbf{M}_h) \sim \frac{1}{\sqrt{2\pi S^2 \mathbf{L}^2}} \exp\left(-\frac{(\ln \mathbf{L} - \ln \mathbf{u}_L)^2}{2S^2}\right);$$

$$\ln \mathbf{u}_L = \alpha_1 + \alpha_2 \ln(\frac{\mathbf{M}_h}{\alpha_3}) + \alpha_4 \ln(1+\mathbf{z})$$

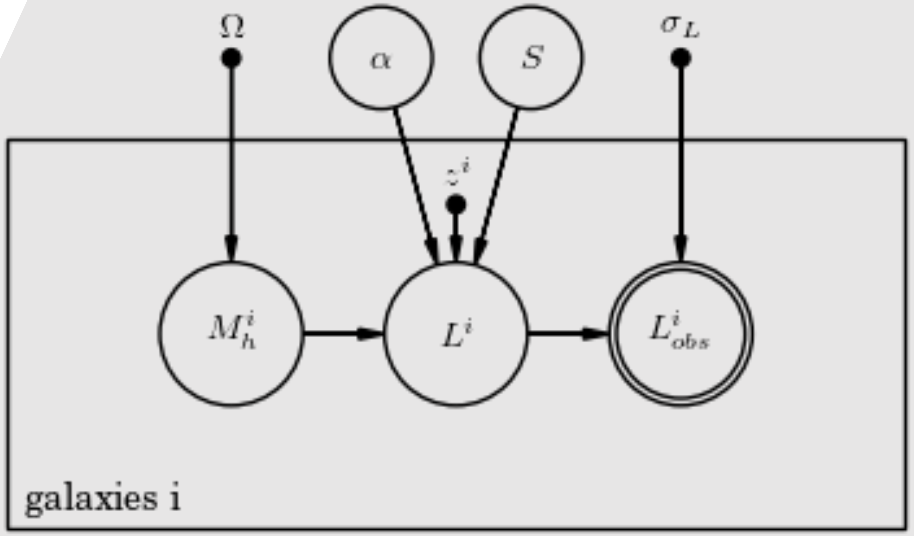
$$P(\mathbf{L}_{obs}|\mathbf{L}) \sim \frac{1}{\sqrt{2\pi\sigma_L^2 \mathbf{L}_{obs}^2}} \exp\left(-\frac{(\ln \mathbf{L}_{obs} - \ln \mathbf{L})^2}{2\sigma_L^2}\right)$$

$$P(\alpha, S) \sim \text{Uniform}$$



Statistical Inference

$$
\begin{aligned}
\underbrace{P(\alpha, S|\mathbf{L}_{obs}, \mathbf{z})}_{posterior} &= \frac{P(\mathbf{z})P(\alpha, S)P(\mathbf{L}_{obs}|\alpha, S, \mathbf{z})}{P(\mathbf{L}_{obs}, \mathbf{z})}; \text{ (Bayes Theorem)} \\
&\propto P(\alpha, S)\Big[P(\mathbf{L}_{obs}|\alpha, S, \mathbf{z})\Big]; \text{ (Normalization)} \\
&\propto \underbrace{P(\alpha, S)}_{prior} \underbrace{\int\int P(\mathbf{L}_{obs}|\mathbf{L})P(\mathbf{L}|\alpha, S, \mathbf{z}, \mathbf{M}_h)P(\mathbf{M}_h)d\mathbf{M}_h d\mathbf{L}}_{likelihood}
\end{aligned}
$$



Importance Sampling

- Approximate sampling a distribution by sampling from a more convenient distribution in a ‘fair’ way.

$$\begin{aligned}\mathcal{L} &= \int\int P(\mathbf{L}_{obs}|\mathbf{L})P(\mathbf{L}|\alpha, S, \mathbf{z}, \mathbf{M}_h)P(\mathbf{M}_h)d\mathbf{M}_h d\mathbf{L} \\ &= \int\int \frac{P(\mathbf{L}_{obs}|\mathbf{L})P(\mathbf{L}|\alpha, S, \mathbf{z}, \mathbf{M}_h)P(\mathbf{M}_h)}{Q(\mathbf{M}_h, \mathbf{L})} Q(\mathbf{M}_h, \mathbf{L})d\mathbf{M}_h d\mathbf{L}; \text{ (Insert Q/Q)} \\ &\approx \frac{1}{N_s}\sum_{s\sim Q} \frac{P(\mathbf{L}_{obs}|\mathbf{L}_s)P(\mathbf{L}_s|\alpha, S, \mathbf{z}, \mathbf{M}_{hs})P(\mathbf{M}_{hs})}{Q(\mathbf{M}_{hs}, \mathbf{L}_s)}; \text{ (Discretize)}\end{aligned}$$

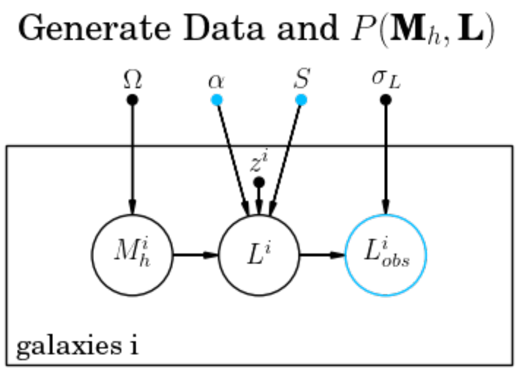
- Go from iterating through $[\min(M_h), \max(M_h)]^{100,000} \times [\min(L), \max(L)]^{100,000}$ to sampling from $\hat{P}(\mathbf{M}_h, \mathbf{L})$

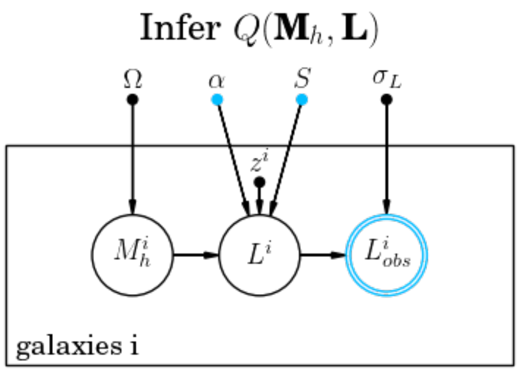
Generating Q Similar To P

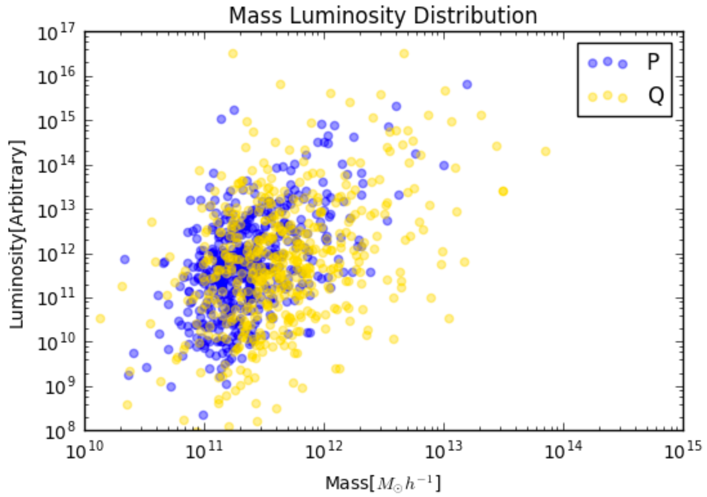
$$
\begin{aligned}
Q(\mathbf{M}_h, \mathbf{L}) &= \hat{P}(\mathbf{M}_h, \mathbf{L}) \\
&= \int \hat{P}(\mathbf{M}_h|\mathbf{L})\hat{P}(\mathbf{L}|\mathbf{L}_{obs})d\mathbf{L}_{obs} \\
&\approx \frac{1}{N_s}\sum_{s\sim\mathrm{L}_{obs}} \hat{P}(\mathbf{M}_h|\mathbf{L})\hat{P}(\mathbf{L}|\mathbf{L}_{obs})
\end{aligned}
$$

$$
\hat{P}(\mathbf{M}_h|\mathbf{L}) \sim \mathrm{Delta}\left[\exp(\alpha_1')\left(\frac{\mathbf{M}_h}{\alpha_3'}\right)^{\alpha_2'}(1+z)^{\alpha_4'}\right]
$$

$$
\hat{P}(\mathbf{L}|\mathbf{L}_{obs}) \sim \frac{1}{\sqrt{2\pi\sigma_L^2\mathbf{L}^2}}\exp\left(-\frac{(\ln\mathbf{L}-\ln\mathbf{L}_{obs})^2}{2\sigma_L^2}\right)
$$







Validating Q Against P