N-15
Measuring the Muon Lifetime

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The N-15 Experiment allows students to get familiar with techniques of particle detection, equipment calibration, tracking uncertainty, and statistical analysis. The goal of N-15 is to measure the lifetime of the muon, an elementary particle. After setting up the apparatus and analyzing the observed data, the final calculation for the muon's life was $(2.23 \pm 0.066_{stat} \pm 0.010_{syst}) \times 10^{-6}$ seconds. For comparison, the current accepted value for the lifetime is $(2.19703 \pm 0.0004 \times 10^{-6}$ seconds, which occurs within the uncertainty bound of the calculation value.

I. INTRODUCTION

The goal of the N-15 experiment is to measure the lifetime of a muon. The muon is an elementary particle with an electric charge of $-1e$ and a spin of $\frac{1}{2}$. Like all elementary particles, the muon has a corresponding antiparticle of opposite charge but equal mass and spin.

The muons measured in this experiment near Earth's surface are primarily produced when cosmic rays bombard the upper atmosphere. Traveling near the speed of light, relativistic effects kick in to allow the muon to make it to the Earth's surface, as the muon's lifetime is on the order of a couple of microseconds.

An apparatus is set up to detect and stop the muons as they approach the Earth. This is then connected to a counting circuit that separately counts muons with different decay times. Running this experiment for around a week will give muon counts for size different bins. Using testing equipment, the time width of each bin can be calculated.

Because of the internal circuits of the counter, analysis needs to be done to calculate the time width of the bin. When the counter is test to 'TEST' mode, we are able to use the pulse generator to introduce a known rate of positive test pulse. These pulses will make coincidences with the trigger gates caused by the random muon incidences. Using the repetition rate of the pulse generator, the bin counts, and both the scalar counts, we are able to calculate the time-width of each bin.

Once presented with the bin widths and observed count data, will fit the data to function using the Least Squares method. As the detection of a muon is a stochastic process independent of all other detections, it will be modeled as a Poisson distribution.

Function fitting aside, the calculations necessary to arrive at the muon lifetime are straightforward. However, there is a multitude of uncertainties, both statistical and systematic, that must be tracked. Moreover, when doing arithmetic with values that carry uncertainty, proper care must be taken to calculate the resulting value's uncertainty, which depends both on the type of mathematical operation and whether two combined values are independent or not [1].

After fitting the observed data using the least squares approach, there is a final correction necessary to arrive at the muon's lifetime. Due to the way in which the experiment is capturing the muons, the negative muons will decay slightly faster than the positive muons. Negative muons inside a material can be captured by a proton of a host nuclei. This capture can release both electrons and photons, which can be detected by the apparatus. As nearly half the muons captured are negative, the observed lifetime is slightly shorter and a correction is necessary.

II. EXPERIMENT DESIGN

A. Scintillator Blocks and Photomultiplier Tube

A scintillator is a material, in our case an organic material (polymer), that exhibits scintillation, the property of luminescence, when excited by ionizing radiation [Citation]. [As muons decend to earth], the particles are stopped inside the scintillator, and two pulses are emitted. The first is caused from the muon's loss of kinetic energy, and the second is caused by the energy released when the muon decays. The time delay of these two pulses is how we will measure the muon's lifetime.

The pulses created inside of the scintillator are then fed into a photomultiplier tube (PMT). Because the components in the counting circuit have an operating voltage of around 5V, a PMT must be used to amplify the pulses created inside of the scintillator. The PMT is powered with a high-voltage power supply. Some care must be taken when choosing the operating voltage of the PMT. If the voltage it too high, the PMT output will have excess noise; too low, the detection of the stopped muon and their decay will be inefficient, leading to a slow collection rate. In balancing these conditions, an operating voltage of 1,100 V was chosen for the PMT.

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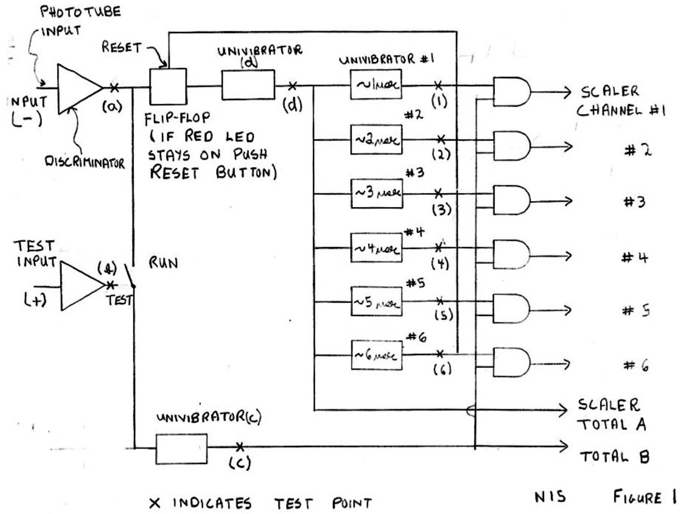

FIG. 1. Counting Circuit Block Diagram. The block diagram is a high-level design showing the components in the counter. The circuit has two inputs, a negative and a positive, and two modes, RUN and TEST.

B. Counting Circuit

Once amplified, the pulses are fed into the negative input of the counting circuit. A block diagram of the logic is shown in Figure X. We will first review the individual types of components used, and then how they operate together.

There are two discriminators present in the circuit. If fed a pulse that exceeds a preset amount, the discriminator will output a pulse of standard width and amplitude. There is a single D flip-flop. When the flip-flop is set, it does not allow any pulses to pass through it. There are a total of 8 univibrators. Univibrators are a circuit component that, when given an input, can produce a non-sinusoidal waveform with specific delay and pulse width. Finally, there are 6 AND gates, which produce a standard 5V output when both the inputs are given 5V.

The counting circuit has two mode, RUN and TEST. We will describe the RUN mode functionality now, and address the TEST mode functionality in the next section.

To begin, let us imagine that the first pulse from the kinetic energy of a muon is sent through the PMT and into the negative input of the counting circuit. The pulse is passed through a discriminator. The output follows two direction: through the bottom of the circuit, and through the top. After the top path passes through the D flip-flop, both paths are fed into different univibrators. The delay of univibrator (d) is longer than the delay of univibrator (c), so the bottom path can be ignored as it passes through the circuit with no interactions. Then top path then moves to set an array of 6 other univibrators, each with output pulses having approximately integer values of pulse widths, ranging from 1 μsec to 6 μsec. During this period where the array of univibrator pulses are passing, the muon decays and another pulse is sent into the negative input of the counting circuit. This second pulse passes through the discriminator, but can only follow the bottom path as the D flip-flop is still activated by the pulse of univibrator (6). Once the second pulse passes through univibrator (c), it can work to activate the AND gates of whichever univibrator pulse is still active, adding one to the count associated with that univibrator. As there are 6, there are 6 counter tracking these collision. Moreover, each pulse through the top of the circuit is counted, called Scalar A, and each pulse through the botttom of the circuit is counted, called Scalar B.

C. Testing Equipment

As seen in the block diagram in Figure X, there is a positive testing input. When the counting circuit is set to TEST mode, we are able to introduce a test pulse of a known rate to at as the second pulse from the muon decay, which will coincidences with the AND gates triggered by random muon incidences. This TEST mode is necessary to achieve accuracy results, as the exact timing of the array of univibrators is unknown.

III. UNCERTAINTY ANALYSIS

When collecting test data to calculated the bin widths, statistical uncertainties appear. The repetition rate shown on the oscilloscope from the pulse generator fluctuates due to electrical noise. Moreover, each of the bin and scalar counts have statistical uncertainties equal to the square root of the count. To determine which of these values are independent or dependent to the other values, thinking about which values are inputs to an abstract function to produce other values will help. For example, the count of Scalar A is produced from the random incidences of muons hitting the scintillator, which is not related to the repetition rate of the pulse generator. However, the counts of bins 1 through 6 are calculated from a function (the counter circuit) with the random muon incidences and the repetition rate as inputs, implying that the values from the bins are dependent with the Scalar A and repetition rate values. Finally, the different bin counts are all independent of one another. After the bin widths have been calculated, the values will carry statistical uncertainty.

There is also statistical uncertainty associated with the bin counts observed in RUN mode in the same fashion as before. However, a different problem arises when there is uncertainty in both the independent variable (the bin widths) and the dependent variable (the bin counts) when using least squares to fit a function.

Equation 1 shows the Chi Squared optimizer function that is used when fitting a function with the least squares method. Summed over the different observations, y_i is the observed outcome, $f(t_i)$ is the fitting function, and

σ_i is the uncertainty associated with the observed outcome y_i. If the data does not have any uncertainty in the independent variable, then Equation 1 needs no modification. However, the independent variable (bin width) does carry uncertainty, and the optimizer function therefore needs modification.

$$S(\alpha) \equiv \sum_{i=1}^{p} \left[\frac{y_i - f(t_i)}{\sigma_i} \right]^2 \tag{1}$$

Equations 2 and 3 are the solution. To incorporate the bin width uncertainty into the fitting process, the uncertainty associated with the bin counts will be exchanged.

$$S(\alpha) = \sum_{i=1}^{p} \left[\frac{y_i - f(t_i)}{\delta_i} \right]^2 \tag{2}$$

The new uncertainty is derived from uncertainty arithmetic on functions. The δ_i is the combination of the dependent variable uncertainty and the uncertainty of the value produced if the function was applied to the independent variable.

$$\delta_i^2 \equiv \left[\frac{\partial f}{\partial t} \right]^2 [\delta t_i]^2 + [\delta y_i]^2 \tag{3}$$

After this modified optimizer function is used in the least squares fit, the produced muon lifetime will carry a statistical uncertainty [2].

Final attention for uncertainty must be spend when corrections for the negative muon capture are made. Systematic uncertainties arise when parameters obtained from other experiments are included. In this case, both uncertainties carried with the negtive muon capture rate in carbon and the positive to negative muon ratio are statistical.

IV. RESULTS

A. Bin Width

Once, the counter is set to TEST mode, a pulse generator with a known repetition rate is connected to the positive input of the counter. The results of the test can be seen in Table 1.

To get the bin width for each Bin i, we can use the following formula:

$$\text{Bin Width (i)} = \frac{\text{Bin Count (i)}}{\text{Scalar A} * \text{Repetition Rate}} \tag{4}$$

The application of Equation 4, using the data observed during testing in Table 1, can be seen in Table 2.

Item	Value
Repetition Rate (Hz)	$(100.9 \pm 0.2_{stat}) \times 10^3$
Bin 1 (Count)	$(695 \pm 26_{stat})$
Bin 2 (Count)	$(512 \pm 44_{stat})$
Bin 3 (Count)	$(569 \pm 55_{stat})$
Bin 4 (Count)	$(541 \pm 64_{stat})$
Bin 5 (Count)	$(548 \pm 72_{stat})$
Bin 6 (Count)	$(637 \pm 80_{stat})$
Scalar A (Count)	$(6755 \pm 82_{stat})$
Scalar B (Count)	$(102987859 \pm 10148_{stat})$

TABLE I. This table shows the necessary values to calculate the time-width of each bin. To collect this data, the counter is set to TEST mode, with the positive input attached to a pulse generator with the associated pulse repetition rate. The other values are then read of the counter display.

Bin	Width (seconds)
1	$(1.02 \pm 0.052_{stat}) \times 10^{-6}$
2	$(0.75 \pm 0.090_{stat}) \times 10^{-6}$
3	$(0.83 \pm 0.12_{stat}) \times 10^{-6}$
4	$(0.8 \pm 0.14_{stat}) \times 10^{-6}$
5	$(0.8 \pm 0.17_{stat}) \times 10^{-6}$
6	$(0.9 \pm 0.20_{stat}) \times 10^{-6}$

TABLE II. This table shown the calculated bin widths, using the data in Table 1 and Equation 1. Note the uncertainties are purely statistic.

B. Fitting Observed Data

To account for both the start times of the bins and the bin widths, the fitting function of Equation 5 is used.

$$f(i; \lambda, N) = \frac{\int_{t_i}^{t_{i+1}} N e^{-\lambda t}}{\int_{t_1}^{t_7} e^{-\lambda t}} \tag{5}$$

If this function is summed over i from 1 to 6, it will result in N, the total number of muon detected. In the function, i is the bin number, λ is the muon decay rate (the reciprocal of the lifetime), t_i is the i^{th} bin's start time, with t_{i+1} being its end time. $(t_{i+1} - t_i)$ is the i^{th} bin's width.

The intuition for Equation 5 comes from the fact that the detection of a muon incidence is a stochastic process, governed by a Poisson distribution. Because the fit function, when summed over all the bins, needs to sum to N, the denominator is necessary to properly normalize the function to correct fit.

In order to fit Equation 5 to the observed data, the coding language Python [3] is used with the SciPy library [4].

With the modified standard deviations of error (uncertainty) for the uncertainty in the the fitting function, the calculated muon lifetime is $(2.16 \pm 0.061_{stat}) \times 10^{-6}$ seconds. This measured lifetime needs to corrected for the slightly faster decay rate associated with captured negative muons.

Bin	Count
1	$(10507 \pm 103_{stat})$
2	$(5420 \pm 163_{stat})$
3	$(4261 \pm 190_{stat})$
4	$(2368 \pm 206_{stat})$
5	$(1682 \pm 216_{stat})$
6	$(1326 \pm 223_{stat})$

TABLE III. This table shows the observed bin counts after running the experiment for around 11 days. Note the statistical uncertainties associated with each count.

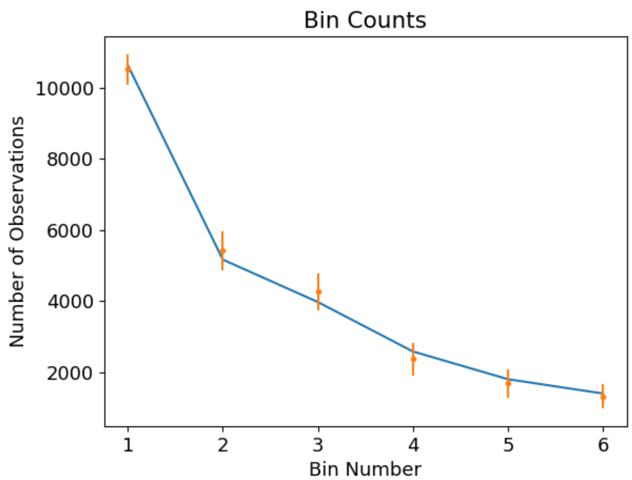

FIG. 2. The orange dots and vertical uncertainty lines represent the observed bin counts after 11 days of collection. The associated uncertainties are a combination of the statistical uncertainty from the observed counts and the calculated statistical uncertainty seen in the bin widths, combined with the use of Equation 3. The blue line represents the values calculated from the fitting function shown in Equation 5.

C. Negative Muon Correction

To make the correction, we fit the data to the single exponential, achieving a measured decay rate of $\lambda_{measured}$. We then assume that the experiment is averaging over nearly two equal exponentials, with $\lambda_{measured}$ being a weighted average of λ_{vacuum} for the positive muons and $\lambda_{vacuum} + \Lambda_{capture}$ for the negative. After some algebraic manipulation, we arrive at Equation 6,

$$\lambda_{vacuum} = \lambda_{measured} - \frac{\Lambda_{capture}}{1+r} \tag{6}$$

were r is the ratio of positive to negative muons, and $\Lambda_{capture}$ is capture rate of negative muons in a given material.

Published values for both $\Lambda_{capture}$ and r will be used. A paper by Suzuki [5] lists multiple experimental values for the capture rate. A 1998 paper by Mukhobpadhyay et al [6] combines these values and more to conclude an experimental capture rate in carbon 12 of $(3.76 \pm 0.04_{syst}) \times 10^4$ Hz. The exact ratio depends on the momentum of the muons, which have published values that range from 1.06 to 1.33 [7] [8]. However, instead of measuring this ratio for our exact experimental setup, we will use the average value of $r = (1.18 \pm 0.12_{syst})$.

Using Equation 6, the published values of r and $\Lambda_{capture}$, and the measured decay rate from fitting the observed data, the calculated muon lifetime is $(2.23 \pm 0.066_{stat} \pm 0.010_{syst}) \times 10^{-6}$ seconds.

V. CONCLUSION

When comparing the calculated value to the accepted muon's lifetime of $(2.19703 \pm 0.0004 \times 10^{-6}$ seconds, it is concluded that the accepted value is within the uncertainty bounds of the calculated value.

The accuracy of this experiment relatively high for the simple set up and analysis. Many of the values that carry uncertainty are purely statistical, which have diminishing returns if the experiment is run for a longer time or for more iterations. When the analysis was being done, variation in bin width and observed count uncertainties did affect the fitted lifetime value before the negative muon correction. The fitting process is done through a computer program, which means the only real systematic error that might occur is during the calculation of the uncertainties used in the fitting process. Calculating the bin width uncertainty (incorrectly determining that two variables are independent, or vice versa) or calculating the adjustments for the optimizer function are two possible stops of error during this experiment and associated analysis. Determining the derivative of the fitting function with respect to the independent is difficult as bin indices are used as the independent variable in Equation 5. Transforming Equation 5 to take the width width as the input allows the derivative to be taken, but creates new associated with different start and finish times for each bin.

Appendix A: Code

Here is the link to the Github reposotory `https://github.com/davidthuman/PHYS4410-N15`

[1] MIT, Propagation of uncertainty through mathematical operations (2022).
[2] J. Orear, Notes on statistics for physicists, revised (1982).
[3] Python (2022).

[4] Scipy (2022).
[5] T. Suzuki, D. F. Measday, and J. P. Roalsvig, Total nuclear capture rates for negative muons, Phys. Rev. C **35**, 2212 (1987).
[6] N. C. Mukhopadhyay, H. ching Chiang, S. K. Singh, and E. Oset, Inclusive muon capture in light nuclei, Physics Letters B **434**, 7 (1998).
[7] H. A. Morewitz and M. H. Shamos, The variation of the ratio of positive to negative cosmic-ray μ mesons with momentum and altitude, Phys. Rev. **92**, 134 (1953).
[8] C. Collaboration, Measurement of the charge ratio of atmospheric muons with the cms detector, Physics Letters B **692**, 83 (2010).