N-15

From Physics 6510/4410 Wiki

Contents

Measurement of the Mean Lifetime of Cosmic Ray Muons

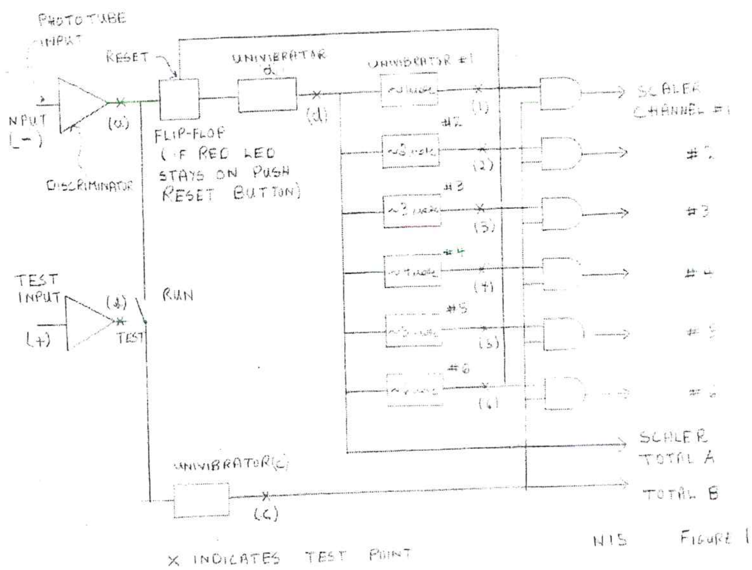
Introduction

The objective of this lab is to measure the mean lifetime of mu mesons (on the order of a couple microseconds), which are primarily produced in the atmosphere as the by-products of cosmic rays which bombard the upper atmosphere.

This experiment is designed to teach the techniques of particle detection, calibration of equipment, and the statistical analysis of data. Much like its computer-controlled cousin, N-17, the experimental aspect of this lab is very straightforward. However, the analysis component of this lab is much more extensive than most labs in 4410 / 6510 and requires a significant amount of thought and effort. This lab is recommended for those students who have good familiarity with some kind of data analysis software (like MATLAB or Igor).

Experimental Apparatus

The experimental apparatus is comprised of 2 main parts. The first is the particle detector itself, comprising of some scintillator blocks (covered to prevent ambient light) coupled to a photomultiplier tube and its high voltage power supply. The output from the PMT is then fed into the second part, which is an electronic circuit with a series of gates whose logic circuitry forms a series of time "bins" which allow us to determine the muon lifetime. Negative pulses from the photomultiplier tube are fed into the negative input connector. Since the light output from the scintillator is a fixed function of the energy deposited by the incident particle, the current pulse from the photomultiplier can be varied only by changing the high voltage and hence the gain of the photomultiplier. The tube used is a 10 stage tube and as a general rule the gain changes by roughly a factor of two per 100 volt increase in the high voltage. Because the cosmic ray rate is low and the fluctuations in energy loss in the scintillator are high, it is difficult to observe this behavior directly with an oscilloscope. The pulses from the photomultiplier are discriminated by using a comparator. If the pulse amplitude exceeds a preset amount, the comparator gives a standard width and amplitude output pulse (TTL). This eliminates the noise pulses from the photomultiplier if the high voltage is properly set.



A block diagram of the logic is shown at right. A more detailed detailed version of this figure can be found here. The data sheets for some of the circuits are also included. Refer to the figure to right when reading the description below. Pulses from a photomultiplier tube are fed into the negative input connector. These pulses are discriminated by using a comparator; if the pulse amplitude exceeds a present amount the discriminator makes a standard width and amplitude pulse. This pulse can be seen at test point (a). The output from (a) is used to seta flip-flop such that no other pulses can be accepted until the flip-flop is reset by the trailing edge of the last bin (univibrator #6). The flip-flop is followed by univibrator (d) which serves to delay the gate starting time by a fraction of a microsecond. This prevents the stopping-muon.l pulse from counting in the gates. Univibrator (d) is followed by the gate-forming univibrators #1 to #6. The "gates" generated by the univibrators form one set of inputs to the six coincidence circuits, the other input to each two-fold coincidence is the input line (through the TEST/RUN switch and univibrator (c)). Thus, if there is a delayed pulse on the input line within the time interval of one of the gates, the corresponding coincidence will register a count.

The outputs of the coincidence gates drive scalars. In addition the total number of pulses of univibrator (d) is scales as "TOTAL A", and those of univibrator (c) as "TOTAL B". In the run mode these two totals are roughly equal; the real utility of having two total scalars is in the test mode. A pulse generator may be used to introduce a known rate of positive pulses through the TEST input (switch set to TEST). These make coincidences with the randomly triggered gates, triggered by random muon incidences. If the number of muon triggers is known (TOTAL A), as well as the total number of positive test pulses (TOTAL B), then the number of random coincidences in each channel enables you to calculate the "width" of each channel. The resolving time of each coincidence is the sum of the pulse widths of the two pulse inputs.

Procedure

The decay rate of cosmic-ray muons is studied by detecting those muons that stop in a scintillation counter, and by timing a delayed pulse from the decay of the muon into an electron and neutrinos. Most cosmic-ray muons have enough energy to pass right through the scintillator without stopping, but those having a kinetic energy less than about 100 MeV are stopped. The stopping of a muon and its subsequent decay into an electron (and neutrinos) is signaled by a double-pulse from a photo multiplier viewing the scintillator. The time lapse between stopping and decay pulses is sorted in discrete "bins" by the electronic logic circuitry. The bins formed by the electronics are all started at the same time and have widths which are roughly multiples of 1 microsecond. From the electronic bins one can determine the number of decays in consecutive time intervals. A *rough* measure of the decay lifetime is then obtained by plotting the number of decays observed in consecutive time bins. The overall shape of the decay curve will be found to be a simple exponential, i.e., if plotted on semi-log paper, the points will fall on a straight line. The slope of the line then represents the mean lifetime.

1. Using a pulse generator, observe the time relationships between the various pulses in the electronics.
2. Choose the photomultiplier operating voltage by measuring the total counting rate and the pulse counting rate versus photomultiplier voltage.
3. Determine the widths and starting times of individual gates to comparable accuracy using random coincidences. Obtaining accurate measurements of the gates is essential for obtaining good results for the muon lifetime.
4. Measure the distribution of decay times for the muon decays. Accumulate enough data to limit the statistical uncertainty to a few percent. This will typically take a week, but the longer the better.
5. Analyze the data using a least squares fit to obtain the muon lifetime and the error of the result. You should correct your result for nuclear capture using the published literature on capture cross-sections. You should leave yourself at least a week to analyze the data. You may want to read off your numbers a few days early to get a preliminary set of data to analyze before doing the analysis of the final data.

To choose a suitable operating point for the high voltage for the photomultiplier, turn on the apparatus and wait a few minutes for the electronics to stabilize. If the high voltage is too low it will mean poor efficiency (and your experiment will take a long time to run) for detecting the stopping muons and their decay products while a high operating voltage will lead to excess noise and a large accidental coincidence rate. Because the apparatus must operate for at least a day to obtain a sufficient number of events. it is important to choose a stable operating point so that small changes in the high voltage do not cause large changes in the counting rate. To find that "voltage plateau", measure the counting rate as a function of voltage. A step size of 25 volts should be sufficient and run for long enough so that the statistical accuracy for the rate is better than 10%. You will select an operating voltage near the center of the plateau (flat portion) of the curve.

Analysis

As mentioned earlier, the analysis of the data is the most challenging part of the lab. In addition to these notes in the wiki, you should also read the references by Rossi and Orear to get a complete picture of the data analysis you will need to perform. An accurate determination of the mean lifetime requires accounting for the bin overlaps and widths. A least-squares fitting procedure should be used. A further correction to the observed lifetime is required, to extract the actual decay lifetime in vacuum. This is because although muons decay away with the characteristic "decay" lifetime, stopped negative muons disappear slightly more rapidly due to the competing process of nuclear capture in the surrounding matter (in this case, the scintillator). The ratio of muons in the cosmic ray shower has been measured at sea level. It should thus be possible to obtain the true decay lifetime of muons from your measurements, using known capture rates in carbon.

Reporting Uncertainties

This lab includes a multitude of interacting uncertainties, both statistical and systematic. You are encouraged to report uncertainties that are solely systematic, and a uncertainty that is solely statistical. For instance, both the count of each bin as well as the bin widths have an associated *statistical* uncertainty associated with them. Direct methods exist to determine uncertainty in a fit parameter from "vertical" errors, and Orear (you really should read it) has suggestions for how to handle "horizontal" errors. Systematic uncertainties include parameters obtained in other experiments (capture rate, for example), and circuit characteristics that are measured directly on the oscilloscope. One should think what uncertainties are most directly coupled, and what can be treated as independent within a particular fitting routine.

Statistics

As the detection of a muon incidence is a stochastic process independent of all other detections, it is described well as a Poisson process. Thus, the counts in each of the scalars will obey Poisson statistics. A general review of uncertainties within Poisson statistics will be useful. The notes by Jay Orear concerning statistics for physicists is available in the lab, and it offers a succinct treatment of the least-squares method, as well as the determination of statistical errors in fit parameters. This is particular importance when determining the statistical uncertainty in the lifetime you measure.

A common practice in performing the least squares fit is to assign a particular time to the count within a bin, and then to fit the resulting function N(t) as an exponential decay. When doing so, it is important to realize that the time assignment to the count should occur at the average time within the bin, which for an exponential decay is NOT given by the midpoint of the bin. The exponential distribution weighs earlier times more heavily, and thus an expectation value of the time within a bin should be taken. However, methods exist that do not require the assignment of a time within a bin, such as one that calculates the expectation value of the total number of decays within each bin.

There is a nonzero probability that two incident muons can cause a coincidence within a particular bin, rather than an incident muon and its decay. This a "background count", and one can calculate the probability that within a certain time a background count has been detected via Poisson statistics. The background counts can then be subtracted directly from the raw data.

Determining the Average Time of Decay Within a Bin

The midpoint of the bin is (t2+t1)/2. This is the intuitive choice for where to plot the counting rate as derived from your data. t1 is the time at which the previous gate ends (or the dead time ends, in the case of gate # 1) and t2 is the time at which the gate of interest ends. So, for example, the counts you see on the scaler # 3 should be: (counts in scalar #2 + counts between t1 and t2). This is because the gates overlap in the way they do, each gate being t2-t1 = Δt longer than the one before it. (The value of Δt depends on the gate. They are not all exactly equal.)

The choice of the midpoint is not, strictly speaking exactly correct, since the muon exponential decays tend to weight the decay time slightly closer to the start of the gate, t1 than the midpoint. Calculating a weighted average of $< t >= \frac{\int te^{-\lambda t}dt}{\int e^{-\lambda t}dt}$, with both integrals running from t1 to t2. The exact answer is :

$$< t >= \frac{e^{\lambda t_1}(1+\lambda t_2) - e^{\lambda t_2}(1+\lambda t_1)}{\lambda(e^{\lambda t_1} - e^{\lambda t_2})}$$

Since the bin width $\Delta t = t2 - t1$ is short compared to the muon lifetime, we expect the correction to the midpoint time to be small. If the muon had a very long lifetime, λ approaches 0, and we expect the correction would go to zero. Therefore, expand in a power series in λ :

$$< t >= \frac{t_1 + t_2}{2} - \frac{\lambda}{12}(\Delta t)^2 + O(\lambda^3)$$

The correction gets smaller as λ goes to 0, as we expect. Notice that there is no λ squared term, so if $\lambda\ \Delta t \sim 0.5 < 1$, this will be a good approximation to <t>. You could assume a value for λ, fit the data and iterate, although this is a small correction. You could also use the published value for λ in the literature and correct for it by the negative muon correction, giving you $\lambda = 1/\tau + \Lambda/2$ (where Λ is the capture rate for negative muons in carbon).

If all the gate width differences Δt were equal, no correction to the measured lifetime would be occur, because all midpoints would shift the average decay time by the same amount in all six cases.

Negative Muon Capture

Since approximately half of the muons decaying in the apparatus are negative muons, there will need to be a correction for the fact that the negative muons are disappearing slightly faster than the positive muons, hence a measured muon lifetime in which only a single exponential is fit to the data will appear slightly shorter than the published vacuum lifetimes (These are equal for both types of muons-particle and anti-particle- due to a fundamental theorem of physics.)

Negative muons inside a material can be captured by a proton of a host nuclei. This capture can release both electrons and photons, which can be detected at the photomultiplier. As nearly half of all the muons incident on our detector are negative, the correction for the negative muon capture is significant; a calculation of the lifetime without such a correction will be too short. Both the muon capture rate (in carbon and elsewhere) and the fraction of +/- muons at sea level has been measured, and have associated uncertainties. To account for the nuclear capture of negative muons, your function N(t) can be split into the sum of positive and negative muon contributions. The negative muon contribution should decay at a rate increased by the capture rate, and each term should be weighted by the fraction of each species. Another suggested method is to initially fit the data with a single exponential, and then to apply appropriate perturbations iteratively, considering that the experiment is averaging over to approximately equal exponentials.

You can use published values for the negative muon capture rate in carbon, and for the +/- ratio. Our experiments are so accurate that these corrections are necessary. A separate uncertainty should be quoted for the uncertainty in these experimentally determined quantities. It should not be combined with the statistical uncertainties from your fit to the data.

You should fit the data to a single exponential (after subtracting calculated background from accidental coincidences). This will give an effective decay rate, λeff, which includes both signs of muon charge. To make the correction, assume the experiment is averaging over two nearly equal exponentials, and that

$$\lambda_{\text{effective}} \equiv \lambda_{\text{measured}} = \text{weighted average of } \lambda_{\text{vacuum}} \text{ and } \lambda_{\text{vacuum}} + \Lambda_{\text{capture}}$$

If the plus/minus ratio, N+ / N- is defined as "r", then you should show that the number you are trying to measure, is

$$\lambda_{\text{vacuum}} = \lambda_{\text{effective}} - \frac{\Lambda}{1+r}$$

So it should be easy to use the published values of Λ and r to make the necessary correction. There is a paper by Suzuki listed in the references which lists more than a dozen experimental values for the capture rate. A 1998 paper by Mukhopadhyay et al, combines all of these and the results of two other papers referred to therein to give a "best" value. The experimental capture rate in carbon 12 is λ = 3.76 +/- .04 x 10^4 Hz. (It agrees with the theoretical number within a few percent, for whatever that is worth.)

Since r is close to 1, the relative correction will be, very approximately, $\frac{1}{2}\frac{3.76 \times 10^4}{5 \times 10^5} \approx 3\%$, but you should calculate it exactly, using your own data.

What value of r should you use? That is a slightly tricky question. There are more pi+ mesons produced high in the atmosphere during the reactions of cosmic rays than pi-mesons. Hence we expect $\frac{N^+}{N^-} > 1$. In fact the ratio depends on the momentum of the muons, and published values range from 1.06 to 1.3 [3]. We really need a way to measure this number for muons which actually stop and decay or capture in our apparatus. As a "stop-gap" measure, just take the average, r = 1.18 ± .12 for your calculations. You should check to see how important that error actually is.

References

1. The accepted value (as of the 1998 Particle Data Book) for the lifetime of the muon is : τ = 2.19703 +/- 0.00004 microseconds
2. Jay Orear, Notes on Statistics for Physicists, Revised, 1982
3. Morewitz and Shamos, The Variation of the Ratio of Positive to Negative Cosmic-Ray μ Mesons with Momentum and Altitude, Physical Review 1953
4. Suzuki and Measday, Total nuclear capture rates for negative muons, Physical Review 1987
5. Rossi, High Energy Particles, Chapter 2, pp 10-20.
6. N.C. Mukhopadhyay, et al, Physics Letters B, v434, n 1-2, 20 Aug. 1998, p7-13

N-1517Notes

From Physics 6510/4410 Wiki

Contents

Muon Capture Correction To Lifetime in N15 and N17

Since approximately half of the muons decaying in the apparatus are negative muons, there will need to be a correction for the fact that the negative muons are disappearing slightly faster than the positive muons, hence a measured muon lifetime in which only a single exponential is fit to the data will appear slightly shorter than the published vacuum lifetimes [1], (These are equal for both types of muons-particle and anti-particle- due to a fundamental theorem of physics.)

You can use published values for the negative muon capture rate in carbon, and for the +/- ratio. Our experiments are both so accurate that a correction is necessary. A separate uncertainty should be quoted for the uncertainty in these experimentally determined quantities. It should not be combined with the statistical uncertainties from your fit to the data.

However, the method for making the correction will be somewhat different depending on whether the experiment is N15 or N17.

N-15 Experiment

You should fit the data to a single exponential (after subtracting calculated background from accidental coincidences). This will give a "$\lambda_{effective}$", an effective decay rate which includes both signs of muon charge. To make the correction, assume the experiment is averaging over two nearly equal exponentials, and that

$$\lambda_{\text{effective}} \equiv \lambda_{\text{measured}} = \text{weighted average of } \lambda_{\text{vacuum}} \text{ and } \lambda_{\text{vacuum}} + \Lambda_{\text{capture}}$$

If the plus/minus ratio, $r \equiv \frac{N^+}{N^-}$ you should prove to yourself that the number you are trying to measure, is λ_{vacuum}, is

$$\lambda_{\text{vacuum}} = \lambda_{\text{effective}} - \frac{\Lambda}{1+r}$$ [OBJ]

So it should be easy to use the published values of Λ and r to make the necessary correction.

In the back of your lab writeup, there is a paper by Suzuki et al which lists more than a dozen experimental values for the capture rate. A 1998 paper by Mukhopadhyay et al [2], combines all of these and the results of two other papers referred to therein to give a "best" value:

The experimental capture rate in ^{12}C is $\Lambda = 3.76 \pm .04 \times 10^4 \text{sec}^{-1}$. (It agrees with the theoretical number within a few percent, for whatever that is worth.) Since r is close to 1, the relative correction will be, very approximately, $\frac{1}{2}\frac{3.76 \times 10^4}{5 \times 10^5} \approx 3\%$, but you should calculate it exactly, using your own data.

What value of r should you use? That is a slightly tricky question. There are more pi+ mesons produced high in the atmosphere during the reactions of cosmic rays than pi-mesons. Hence we expect $\frac{N^+}{N^-} > 1$. In fact the ratio depends on the momentum of the muons, and published values range from 1.06 to 1.3 [3]. We really need a way to measure this number for muons which actually stop and decay or capture in our apparatus. As a "stop-gap" measure, just take the average, r = 1.18 ± .12 for your calculations. Let's see how important that error is!

N17 Experiment

The basic numbers you will need are quoted in the previous section. And, the method of estimating the additional error from the capture process can be done the same way. But, since you are using maximum likelihood in a nonlinear fit to your data, you can tell it to maximize the likelihood that the numbers in the bins ($\Delta t = 1$ microseconds) are ($t = t_{\text{bin}}$)

$$N_{\text{bin}}(t) = B - \Delta t \frac{d}{dt} N_0 \left\{ \frac{re^{-\lambda t}}{1+r} + \frac{e^{-(\lambda+\Lambda)t}}{1+r} \right\}$$

Since you have the full power of the maximum likelihood method at your disposal, you can use *all* of the information from the data to fit N_0,λ and B. (B is the background rate/bin.) You do not have to make the *ad hoc* assumption that the measured rate is the average of the two rates. The fit will take care of all that. And, what is more, you do not have to rely on a calculation of B, although you should calculate it as a check.

If you don't understand where the formula above comes from, or how to use Poisson statistics in the maximum likelihood calculation, consult the Orear notes, or ask your instructor to explain.

References

[1] The best value of the muon lifetime from the world data (1998 Particle Data Book) is: $\tau = 2.19703 \pm .00004$ microseconds. Thus $\lambda_{\text{vacuum}} = 4.55160 \pm .00008 \times 10^5$ Hz.
[2] N.C. Mukhopadhyay, et al, Physics Letters B, v434, n 1-2, 20 Aug. 1998, p7-13

[3] Thanks to Jonathan Blender of P410 for supplying these numbers, and for a lively discussion of how to treat them in our special case.

Experiment N-15

Measurement of the Mean Lifetime of Cosmic Ray Muons

This experiment is designed to teach the techniques of particle detection, calibration of equipment, and the statistical analysis of the data.

1. Using a double pulser, observe the time relationships between the various pulses in the electronics.
2. Choose the photomultiplier operating voltage by measuring the total counting rate and the double pulse counting rate versus photomultiplier voltage.
3. Measure the distribution of decay times for μ decays. Accumulate enough data to limit the statistical uncertainty to a few percent.
4. Determine the widths and starting times of individual gates to comparable accuracy using random coincidences.
5. Analyze the data using a least squares fit to obtain the μ[redacted] lifetime and the error of the result. You should correct your result for nuclear capture using the published literature on capture cross-sections.

References:

1. B. Rossi, *High Energy Particles*, pp 10-29 (Ionization losses).
2. H. A. Morewitz and M. H. Shamos, Physics Review 92, 134 (1953) (μ^+/μ^- ratio and capture cross-sections)
3. J. Orear, "Notes on Statistics for Physicists, Revised", CLNS 82/511

Notes on Experiment N-15

The decay rate of cosmic-ray μ (muons) [redacted] is studied by detecting those muons that stop in a scintillation counter and by requiring a delayed pulse that signals the decay of the muon into an electron and neutrinos. e.g. $(\mu \rightarrow e + \nu_{\mu} + \nu_{e})$.

Most cosmic-ray muons have enough energy to pass right through the scintillator without stopping, but those having a kinetic energy less than ~ 100 MeV are stopped. The stopping of a μ [redacted] and its subsequent decay into an electron (and neutrinos) is signaled by a **double-pulse** from a photo multiplier viewing the scintillator. The time lapse between stopping and decay pulses is sorted in discrete "bins" by the electronic logic circuitry. The bins formed by the electronics are all started at the same time and have widths which are roughly multiples of 1 μsec. From the electronic bins one can determine the number of decays in consecutive time intervals. A rough measure of the decay lifetime is then obtained by plotting the number of decays observed in consecutive time bins. The overall shape of the decay curve will be found to be a simple exponential, i.e., if plotted on semi-log paper, the points will fall on a straight line. The slope of the line then represents the mean lifetime.

A more accurate determination of the mean lifetime requires accounting for the bin overlaps and widths. A least-squares fitting procedure should be used. A further correction to the observed lifetime is required, to extract the actual decay lifetime in vacuum. This is because although μ^{+} mesons decay away with the characteristic "decay" lifetime, stopped μ^{-} [redacted] disappear slightly more rapidly due to the competing process of nuclear capture in the surrounding matter (in this case, the scintillator). The ratio of μ^{+}/μ^{-} [redacted] in the cosmic rays has been measured. It should thus be possible to obtain the true decay lifetime of muons from your measurements, using known capture rates in carbon.

Description of the Electronics

A block diagram of the logic is shown in fig. 1 and a detailed drawing in fig. 2. The data sheets for some of the circuits are also included. Refer to fig. 1 when reading the description below.

Pulses from a photo multiplier tube are fed into the negative input connector. These pulses are discriminated by using a comparator; if the pulse amplitude exceeds a present amount the discriminator makes a standard width and amplitude pulse. This pulse can be seen at test point (a). The output from (a) is used to set a flip-flop such that no other pulses can be accepted until the flip-flop is reset by the trailing edge of the last bin (univibrator #6). The flip-flop is followed by univibrator (d) which serves to delay the gate starting time by a fraction of a microsecond. This prevents the stopping-μ pulse from counting in the gates. Univibrator (d) is followed by the gate-forming univibrators #1 to #6.

The "gates" generated by the univibrators form one set of inputs to the six coincidence circuits, the other input to each two-fold coincidence is the input line (through the TEST/RUN switch and univibrator (c)). Thus, if there is a delayed pulse on the input line within the time interval of one of the gates, the corresponding coincidence will register a count.

The outputs of the coincidence gates drive scalars. In addition the total number of pulses of univibrator (d) is scaled as "TOTAL A", and those of univibrator (c) as "TOTAL 8". In the run mode these two totals are roughly equal; the real utility of having two "total" scalars is in the test mode.

A pulse generator may be used to introduce a known rate of positive pulses through the TEST input (TEST/RUN switch on TEST). These make coincidences with the randomly triggered gates, triggered by μ-meson pulses. If the number of μ[redacted] triggers is known (TOTAL A counts), as well as the total number of positive test pulses (TOTAL B counts), then the number of random coincidences in each channel enables you to calculate the "width" of each channel. The resolving time of each coincidence is the sum of the pulse widths of the two pulse inputs.