

# Solving Systems of Equations, Errors and Explorations

David Tran and Spencer Kelly

March 9, 2024

**Abstract**

## **1 Introduction**

## **2 The $PA = LU$ factorization method for linear systems**

### **2.1 Why is $PA = LU$ needed for solving linear systems approximately?**

When solving linear systems of the form  $Ax = b$ , we begin by gaussian elimination of the matrix  $A$ , followed by back substitution, and ultimately arrive at our solution. However, when a particular matrix  $A$  is being used for multiple iterations, the overhead involved can become quite an obstacle. This is because the process of Gaussian elimination is a computationally expensive process, with complexity on the order  $O(n^2)$ . But with  $PA = LU$  factorization, we essentially remove the overhead involved with gaussian elimination, for all but the first iteration, by rewriting the matrix  $A$  in terms of the upper and lower matrices  $L$ , and  $U$ , respectively. Thus, for every subsequent iteration involving the same matrix, we need not perform gaussian elimination, since  $L$  and  $U$  are all we need to begin performing back-substitution, which has complexity  $O(n)$ .

2.2 How to identify systems  $Ax = b$  for which  $PA = LU$  is not suited

2.3 Larger applications of  $PA = LU$  factorization

### 3 Iterative solution of systems of linear equations

3.1 Solving an equation for  $n = 100,000$

3.2 Comparison of  $PA = LU$  and Jacobi Iteration

3.3 Why is solving such large systems important in applications?

### 4 Implement Newton's method for multiple variables

4.1 Implement Newton's method for systems using vectorization

4.2 Testing

4.3 Challenging Example

### 5 Summary

### 6 Appendices

6.1 Code

6.2 Plots

### 7 Code

### 8 Summary

8.1 Results

8.2 Team Description

8.3 Future Explorations

8.4 References

### Appendix