Solving Systems of Equations, Errors and Explorations

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Abstract

1 Introduction

2 The PA = LU factorization method for linear systems

2.1 Why is PA = LU needed for solving linear systems approximately?

When solving linear systems of the form $Ax = b$, we begin by gaussian elimination of the matrix A, followed by back substitution, and ultimately arrive at our solution. However, when a particular matrix A is being used for multiple iterations, the overhead involved can become quite an obstacle. This is because the process of Gaussian elimination is a computationally expensive process, with complexity on the order $O(n^2)$. But with $PA = LU$ factorization, we essentially remove the overhead involved with gassian elimination, for all but the first iteration, by rewriting the matrix A in terms of the upper and lower matrices L, and U, respectively. Thus, for every subsequent iteration involving the same matrix, we need not perform gaussian elimination, since L and U are all we need to begin performing back-substitution, which has complexity $O(n)$.