TOPOLOGY MATH4121A/9021
LECTURE 3

Definition. The **interior** of a set A in a topological space X, denoted A° or $\mathrm{Int}(A)$, is defined as:

$$\mathrm{Int}(A) = \bigcup_{U \subset A,\, U \in \mathcal{T}} U.$$

Points of $\mathrm{Int}(A)$ are called **interior points** of A.

Theorem 1. *Let A be a subset of a topological space X. Then p is an interior point of A if and only if there exists an open set U with $p \in U \subset A$.*

Proof.

Exercise 2. Show that a set U is open in a topological space X if and only if every point of U is an interior point of U.

Solution.

A useful claim to prove before attempting the final theorem is the following:

Claim 3. $\mathrm{Int}(X \smallsetminus A) = X \smallsetminus \overline{A}$ and $\overline{X \smallsetminus A} = X \smallsetminus \mathrm{Int}(A)$.

Notice that this claim is very set theoretical in nature. It's useful to do as practice, but doesn't give any insight in regards to the material we are learning in this course.

Proof.

Definition. The **boundary** of A, denoted $\mathrm{Bd}(A)$ or ∂A, is defined to be $\overline{A} \cap \overline{X - A}$.

Theorem 4. *Let A be a subset of a topological space X. Then $\mathrm{Int}(A)$, $\mathrm{Bd}(A)$ and $\mathrm{Int}(X-A)$ are disjoint sets whose union is X. Draw an illustration of the theorem statement.*

Something to think about: One aspect of this proof is to show that a collection of three sets are all pairwise disjoint. Do you actually need to show that each pair is disjoint, or is there some symmetry to the argument allowing you to bypass showing that each pair is disjoint?

Proof.