Iteration and Error

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Abstract

Introduction

Particular aspects of this Lab

Taylor series and error

Problem 0

Let $f(x) = e^{-x}$. Note that $f^{(1)}(x) = -e^{-x}, f^{(2)}(x) = e^{-x}$, and in general, $f^{(n)}(x) = (-1)^n e^{-x}$. So, $f^{(n)}(0) = (-1)^n$, and the Taylor series expansion around $x_0 = 0$ is given by

$$\begin{aligned} T_n(x) &= \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!} x^k \\ &= \sum_{k=0}^{n} \frac{(-1)^k}{k!} x^k \end{aligned}$$

Similarily, the remainder term is

$$\begin{aligned} R_n &= \frac{f^{(n+1)}(z)}{(n+1)!}(x - x_0)^{n+1} \\ &= \frac{(-1)^{n+1} e^{-c}}{(n+1)!} x^{n+1}. \end{aligned}$$

for some $0 \leq c \leq x$.

Problem 1

a. We have

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

so

$$\begin{aligned} F(x) &= \frac{e^{-x} - 1 + x}{x^2} \\ &= \frac{1}{2!} - \frac{x}{3!} + \frac{x^2}{4!} - \dots \\ &= \sum_{k=0}^{\infty} (-1)^k \frac{x^k}{(k+2)!} \end{aligned}$$

and its Taylor series up to $n+1$ terms is

$$T_n(x) = \sum_{k=0}^{n} (-1)^k \frac{x^k}{(k+2)}$$

To find the remainder term R_n, we use the fact that $R_n = F(x) - T_n(x)$. TODO.

b. TODO.

Problem 2

a. We compute and plot $F(x)$ from above with the following.

```
% Evaluate F(x) using the first n terms of its Taylor series.
function T = F(x, n)
    % Initialize sum as 0
    T = 0;
    % Loop over terms in series
    for k = 0:n
        T += (-1)^k * x.^k ./ factorial(k + 2);
    end
end

% Plot F(x).
```

F.m

b. TODO.

Problem 3

a. TODO.

b. TODO.

c. TODO.

d. TODO.

Problem 4

a. TODO.

b. TODO.

c. TODO.

Summary and Conclusions

References

Teamwork Statement

Code Appendix

Plot Appendix