

# Systemics $\Sigma$ — Minimal Specification

## Charter

### Charter (*normative*)

This document gives a domain-agnostic, minimal formal specification of Systemics  $\Sigma$ . It treats any practice as a kernel-shaped contract that produces decisions from posted evidence under benign variation, with replayable records. No specific domain assumptions (physics, ML, audits, etc.) are required.

## Alphabet (Objects & Maps)

### Alphabet (*normative*)

- $U$ : Universe of artifacts.
- $\mathbb{V}$ : Valuation space (any measurable space; commonly  $\mathbb{R}^k \times \mathbb{B}^m$ ).
- $\mathbf{2}$ : Decision space  $\mathbf{2} = \{0, 1\}$ .
- $\Pi$ : Frames (benign contexts).
- $P_n$ : Probes (benign perturbations).
- $\Theta$ : Floors/thresholds (partially ordered set).
- $\beta$ : Invariance budgets (tolerances in a poset/lattice).
- $C$ : Capacity budgets (bits/time/energy constraints).
- $\Gamma$ : Envelope/meta (versions, seeds, numeric modes, commits).
- $\mathcal{R}$ : Records (canonical map  $\rightarrow$  bytes; hash/ledger optional).

## Definition: Systemic Kernel

### Systemic Kernel (*normative*)

A systemic kernel is the tuple

$$K_\mu^\Sigma = (v, \chi, \Pi, P_n, \Theta, \beta, C, \Gamma), \quad (1)$$

where  $v : U \rightarrow \mathbb{V}$  is a valuation map and  $\chi : \mathbb{V} \times \Theta \times \beta \rightarrow \mathbf{2}$  is a decision gate.

## Metrics & Order

### Wobble and orderings (*normative*)

- A divergence ("wobble")  $w : \mathbb{V} \times \mathbb{V} \rightarrow \mathbb{R}_{\geq 0}$  on decision-relevant coordinates.
- Orders:  $\theta \preceq \theta'$  means tightening floors;  $\beta' \preceq \beta$  means loosening invariance budgets;  $C' \preceq C$  means reduced capacity.
- Metrics:  $\rho$  is a metric on the decision space (e.g.,  $\mathbb{R}_{\geq 0}$  for costs,  $\mathbf{2}$  for errors).
- A (pseudo-)distance on systemic kernels is  $d_\mu(K, K') = \rho(v, v') + \theta(\chi, \chi') + \beta(C, C')$ .

## Axioms

### Axiom 1: Existence of Valuation (*normative*)

$\forall u : U, \exists! v : V, \text{val}(u, v)$ .

### Axiom 2: Universal Decision Gate (*normative*)

$\forall \theta : \Theta, \exists \chi : \mathbb{V} \times \Theta \times \beta \rightarrow \mathbf{2}, \text{gate}(\chi, \theta)$ .

### Axiom 3: Invariance under Reparameterization (*normative*)

If  $\beta' \preceq \beta$  and  $\theta' \preceq \theta$ , then  $\chi'(\cdot, \theta')$  is equivalent to  $\chi(\cdot, \theta)$ .

### Axiom 4: Capacity Constraints (*normative*)

$\forall t, \exists c : C, \text{cap}(t, c)$ .

### Axiom 5: Envelope Uniqueness (*normative*)

If  $\gamma : \mathcal{R}$  is a record, then  $\exists! \mu : \Gamma, \text{envelope}(\mu, \gamma)$ .

### Axiom 6: Record-keeping (*normative*)

$\forall r : \mathcal{R}, \exists e : U \rightarrow \mathbb{V}, \text{record}(r, e)$ .

### Axiom 7: Bounded Wobble (*normative*)

$\forall v, v' : V, w(v, v') < \infty$ .

### Axiom 8: Tightening Floors (*normative*)

$\forall \theta, \theta' : \Theta, \theta \preceq \theta' \Rightarrow \chi(\cdot, \theta') \preceq \chi(\cdot, \theta)$ .

## Conformance

### Conformance (*normative*)

A systemic kernel  $K$  conforms to a contract  $C$  if  $\forall t, \text{exec}(K, t) \in C$ .

## Morphisms

### Morphism (*normative*)

A morphism between systemic kernels  $K, K'$  is a tuple

$$f = (f_v, f_\chi, f_\beta, f_C, f_\Gamma), \quad (2)$$

where -  $f_v : V \rightarrow V'$  is a valuation map, -  $f_\chi : \mathbb{V} \times \Theta \times \beta \rightarrow \mathbb{V}' \times \Theta' \times \beta'$  is a compatible decision map, -  $f_\beta : \beta \rightarrow \beta'$  and  $f_C : C \rightarrow C'$  are invariance and capacity transformations, -  $f_\Gamma : \Gamma \rightarrow \Gamma'$  is an envelope transformation.

### Equivalence of Morphisms (*normative*)

Morphisms  $f$  and  $g$  are equivalent, written  $f \approx g$ , if

- $f_v = g_v$ ,

- $f_\chi$  and  $g_\chi$  are equivalent decision maps,
- $f_\beta = g_\beta$  and  $f_C = g_C$ ,
- $f_\Gamma = g_\Gamma$ .

## Recipe

### Systemics $\Sigma$ Recipe (*normative*)

To specify a Systemics  $\Sigma$  instance:

1. Select a universe  $U$  of artifacts.
2. Define a valuation map  $v : U \rightarrow \mathbb{V}$ .
3. Specify decision-relevant coordinates using floors  $\theta$  and invariance budgets  $\beta$ .
4. Define capacity budgets  $C$ .
5. Specify an envelope/meta description  $\mu$ .
6. Define a decision gate  $\chi$ .

## Notes

### Notes (*informative*)

- See also the separate specification for `Composition`.
- This specification is a work in progress; see GitHub for updates.

## References

- GraphFrame K0 (GF0) ()
- SpecFrame K1 ()
- Composition (separate spec) ()