**MATHS**

Over all real numbers, find the minimum value of a positive real number, y such that

y = sqrt((x + 6)2 + 25) + sqrt((x - 6)2 + 121)

Solution

y = √((x + 6)2 + 25) + √((x - 6)2 + 121)

y = √((x + 6)(x + 6) + 25) + √((x - 6)(x - 6) + 121)

y = √((x2 + 12x + 36) + 25) + √(x2 – 12x + 36) + 121)

y = √(x2 + 12x + 61) + √(x2 – 12x + 157)

To find critical points, find derivative of y

Differentiate part by part

Using chain rule, d/dx = du/dx \* d/du

Let u = x2 + 12x + 61

du/dx = 2x2-1 + 12x1-1 + 0

du/dx = 2x + 12

d/du(√u) = d/du(u1/2) = ½ \* u1/2 – 1

d/du = ½ \* u-1/2

d/du = ½ \* 1/√u

d/du = 1/2√u

d/dx = du/dx \* d/du

d/dx = (2x + 12) \* 1/2√u

d/dx = (2x + 12)/2√u

d/dx = (x + 6)/ √u

d/dx = x + 6/ √(x2 + 12x + 61)

Using chain rule, d/dx = du/dx \* d/du

Let u = x2 - 12x + 157

du/dx = 2x2-1 - 12x1-1 + 0

du/dx = 2x - 12

d/du(√u) = d/du(u1/2) = ½ \* u1/2 – 1

d/du = ½ \* u-1/2

d/du = ½ \* 1/√u

d/du = 1/2√u

d/dx = du/dx \* d/du

d/dx = (2x - 12) \* 1/2√u

d/dx = (2x - 12)/2√u

d/dx = (x - 6)/ √u

d/dx = x - 6/ √(x2 - 12x + 157)

dy/dx = ((x + 6)/ √(x2 + 12x + 61)) + ((x – 6)/ √(x2 - 12x + 157))

Solve for x,

((x + 6)/ √(x2 + 12x + 61)) + ((x – 6)/ √(x2 - 12x + 157)) = 0 ------- (1)

((x – 6)/ √(x2 - 12x + 157)) = -((x + 6)/ √(x2 + 12x + 61))

Square both sides,

(((x – 6)/ √(x2 - 12x + 157)))2 = (-((x + 6)/ √(x2 + 12x + 61)))2

(x – 6)2/(x2 – 12x + 157) = (x + 6)2/(x2 + 12x + 61)

Cross multiply,

(x + 6)2(x2 – 12x + 157) = (x – 6)2(x2 + 12x + 61)

x2 + 12x + 36 (x2 – 12x + 157) = x2 – 12x + 36 (x2 + 12x + 61)

Solve LHS,

x4 – 12x3 + 157x2 + 12x3 – 144x2 + 1884x + 36x2 – 432x + 5652

x4 – 12x3 + 12x3 + 157x2 – 144x2 + 36x2 + 1884x– 432x + 5652

x4 + 49x2 + 1452x + 5652

Solve RHS,

x4 +12x3 + 61x2 – 12x3 – 144x2 – 732x + 36x2 + 432x + 2196

x4 + 12x3 – 12x3 + 61x2 – 144x2 + 36x2 – 732x + 432x + 2196

x4 – 47x2 – 300x + 2196

x4 + 49x2 + 1452x + 5652 = x4 – 47x2 – 300x + 2196

x4 + 49x2 + 1452x + 5652 – x4 + 47x2 + 300x – 2196 = 0

x4 – x4 + 49x2 + 47x2 + 1452x + 300x + 5652– 2196 = 0

96x2 + 1752x + 3456 = 0

Divide through by 24 (LCM),

4x2 + 73x + 144 = 0

Using quadratic formula, a = 4, b = 73, c = 144,

x1 =

x1 =

x1 =

x1 =

x1 =

x1 =

x1 =

x2 =

x2 =

x2 =

x2 =

x2 =

x2 =

x2 =

Substituting the values for x into eqn (1),

When x = ,

+ = 0

+ = 0

+ = 0

+ = 0

+ = 0

+ = 0

+ = 0

+ = 0

x = satisfies the equation.

When x = -16,

+ = 0

+ = 0

+ = 0

+ = 0

+ = 0

+ = 0

+ = 0

= 0

x = does not satisfy the equation.