Lab 8 - Decission Trees

An Introduction to Statistical Learning

```
pacman::p_load(tree, ISLR)
```

1. Fitting Classification Trees

The process has the following steps:

1. Fit the tree to the training data.
2. Find the optimal tree complexity using cross-validation.
3. Make predictions on the testing subset.

The `Carseats` dataset will be used.

Classification and Regression Trees are fitted using the `tree` library.

```
summary(Carseats)
```

```
##      Sales          CompPrice       Income        Advertising
##  Min.   : 0.000   Min.   : 77   Min.   : 21.00   Min.   : 0.000
##  1st Qu.: 5.390   1st Qu.:115   1st Qu.: 42.75   1st Qu.: 0.000
##  Median : 7.490   Median :125   Median : 69.00   Median : 5.000
##  Mean   : 7.496   Mean   :125   Mean   : 68.66   Mean   : 6.635
##  3rd Qu.: 9.320   3rd Qu.:135   3rd Qu.: 91.00   3rd Qu.:12.000
##  Max.   :16.270   Max.   :175   Max.   :120.00   Max.   :29.000
##    Population        Price        ShelveLoc        Age          Education
##  Min.   : 10.0   Min.   : 24.0   Bad   : 96   Min.   :25.00   Min.   :10.0
##  1st Qu.:139.0   1st Qu.:100.0   Good  : 85   1st Qu.:39.75   1st Qu.:12.0
##  Median :272.0   Median :117.0   Medium:219   Median :54.50   Median :14.0
##  Mean   :264.8   Mean   :115.8                Mean   :53.32   Mean   :13.9
##  3rd Qu.:398.5   3rd Qu.:131.0                3rd Qu.:66.00   3rd Qu.:16.0
##  Max.   :509.0   Max.   :191.0                Max.   :80.00   Max.   :18.0
##  Urban       US
##  No :118   No :142
##  Yes:282   Yes:258
##
##
##
##
```

```
attach(Carseats)
```

First we split the data in train and testing subsets:

```
set.seed(1)
train = sample(1:nrow(Carseats), nrow(Carseats)/2)
```

We will classify the data according to `Sales`; as it's a continuous variable, we'll encode it as a binary variable, with `ifelse()`, using the mean as an approximate threshold. The resulting array must be converted to a categorical variable using `factor()`:

```r
High = ifelse(Sales > 8, 'Yes', 'No')
High = factor(High)
```

We merge the new variable `High` in the dataset:

```r
Carseats$High = High
```

Fitting the model

We fit the decission tree using all the variables but `Sales`:

```r
tree.carseats = tree(High~.-Sales, data = Carseats, subset = train)
summary(tree.carseats)
```

```
##
## Classification tree:
## tree(formula = High ~ . - Sales, data = Carseats, subset = train)
## Variables actually used in tree construction:
## [1] "Price"       "Population"  "US"          "CompPrice"   "Advertising"
## [6] "Income"      "ShelveLoc"   "Age"
## Number of terminal nodes:  20
## Residual mean deviance:  0.4549 = 81.89 / 180
## Misclassification error rate: 0.105 = 21 / 200
```

The *residual mean deviance* is the deviance divided by $n - |T_0|$, being n the number of observations and T_0 the number of terminal nodes (reported by `summary()`). A small deviance indicates that the tree provides a god fit to the training data.

`summary()` also includes the *misclassification error rate*, which for *classification trees* is given by:

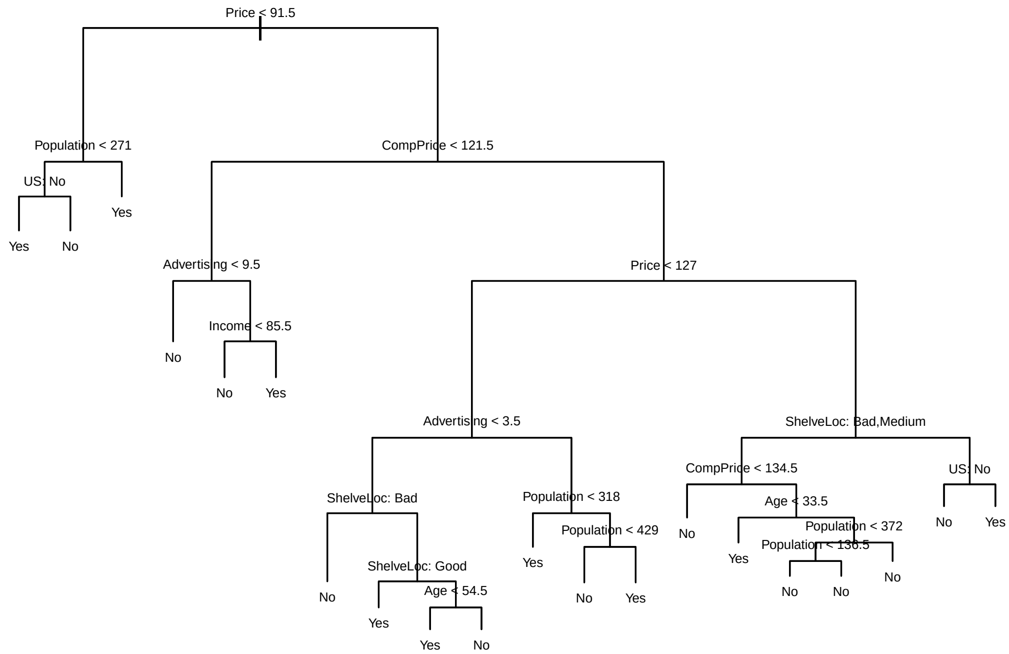
$$-2 \sum_m \sum_k n_{mk} \log \hat{p}_{mk}$$

where n_{mk} is the number of observations in the mth terminal node that belong to the kth class.

`tree()` selects the relevant variables and exclude the rest from the model (in this case, `Population`, `Education` and `Urban` have been excluded)

Plotting the tree

```r
plot(tree.carseats)
text(tree.carseats, pretty = 0, cex = .5)
```



ShelveLoc appears to be an important indicator for Sales, because the first branch differenciates Good locations from Medium and Bad locations.

Making predictions

For classification trees, `type="class"` tells `predict()` to return the *actual class prediction*:

```
tree.pred = predict(tree.carseats, newdata = Carseats[-train,], type = 'class')
table(tree.pred, High[-train])
```

```
##
## tree.pred No  Yes
##       No  84  37
##       Yes 35  44
```

The tree makes correct classifications for the 64% of the testing data:

```
mean(tree.pred == High[-train])
```

```
## [1] 0.64
```

Pruning the tree with Cross-Validation

Finding the optimal level of complexity

The optimal level of complexity for a fitted tree can be determined using cross-validation, with the `cv.tree()` function. The default metric used to guide cross-validation is the deviance, but we can specify that we want the missclassification error rate with FUN=prune.misclass:

```
set.seed(42)
cv.carseats = cv.tree(tree.carseats, FUN = prune.misclass)
```

The result includes the following fields:

```
cv.carseats
```

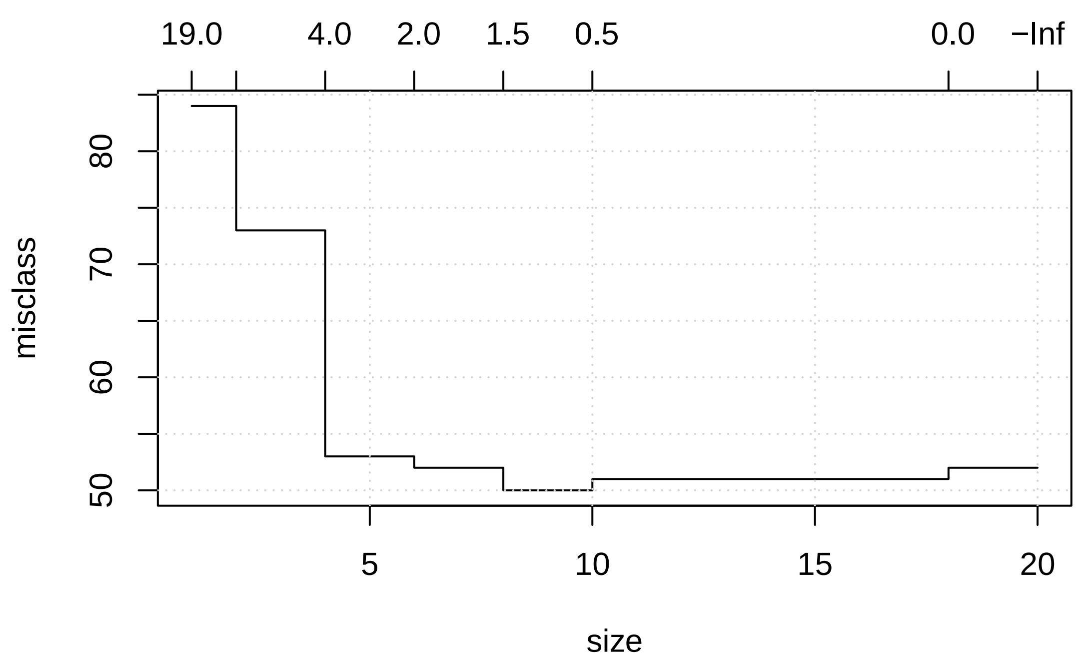
```
## $size
## [1] 20 18 10  8  6  4  2  1
##
## $dev
## [1] 52 52 51 50 52 53 73 84
##
## $k
## [1] -Inf  0.0  0.5  1.5  2.0  4.0 12.0 19.0
##
## $method
## [1] "misclass"
##
## attr(,"class")
## [1] "prune"         "tree.sequence"
```

size is the number of terminal nodes of each tree considered. dev is actually the cross-validation error rate, as indicated with FUN. k is the cost-complexity parameter used, which corresponds to α in equation 8.4.

The 8-node tree has the lowest error rate, so we will use it as the final tree.

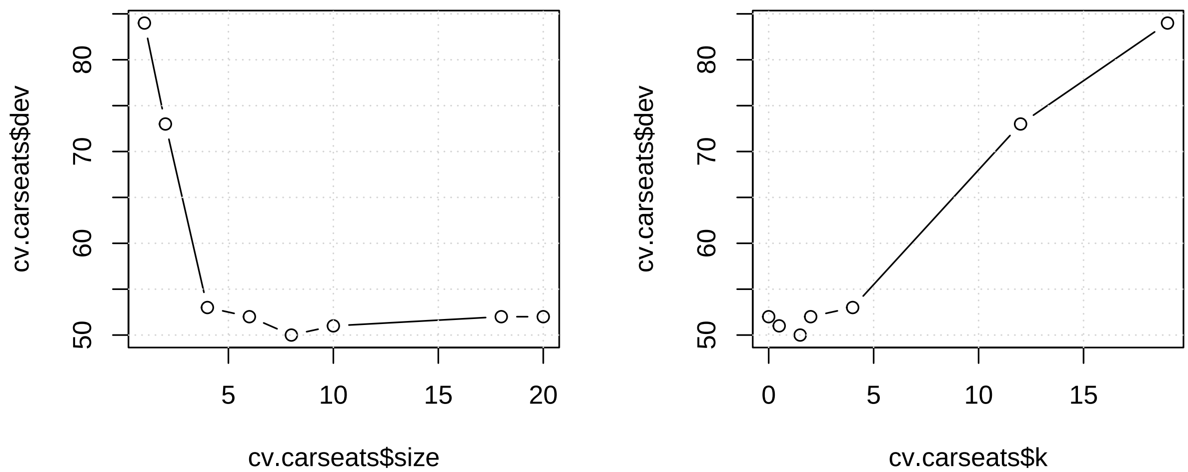
Plotting the cross-validation tree shows the relations between size, k and dev:

```
plot(cv.carseats)
grid()
```



We can also plot the error rate as a function of both `k` and `size`:

```
par(mfrow=c(1, 2))
plot(cv.carseats$size, cv.carseats$dev, type='b'); grid()
plot(cv.carseats$k, cv.carseats$dev, type='b'); grid()
```

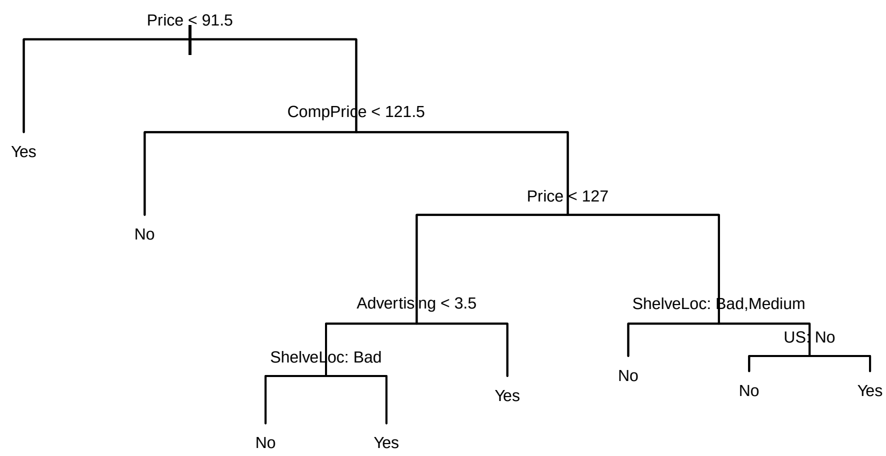


Pruning the tree

We apply the `prune.misclass()` function to prune the tree to obtain the 8-node tree:

```
prune.carseats = prune.misclass(tree.carseats, best = 8)
```

```
plot(prune.carseats)
text(prune.carseats, pretty = 0, cex = .5)
```



Making predictions using the pruned tree

```
tree.pred = predict(prune.carseats, newdata = Carseats[-train,], type='class')
table(tree.pred, High[-train])
```

```
##
## tree.pred No Yes
##       No  83  24
##       Yes 36  57
```

```
mean(tree.pred == High[-train])
```

```
## [1] 0.7
```

Now the tree correctly labels the 70% of the testing data.

2. Fitting Regression Trees

The process is the same than for classification trees:

1. Fit the tree to the training data.
2. Find the optimal tree complexity using cross-validation.
3. Make predictions on the testing subset.

The `Boston` dataset will be used.

```
pacman::p_load(MASS)
attach(Boston)
```

Training and test subsets are created:

```r
set.seed(1)
train = sample(1:nrow(Boston), nrow(Boston)/2)
```

Fitting the model

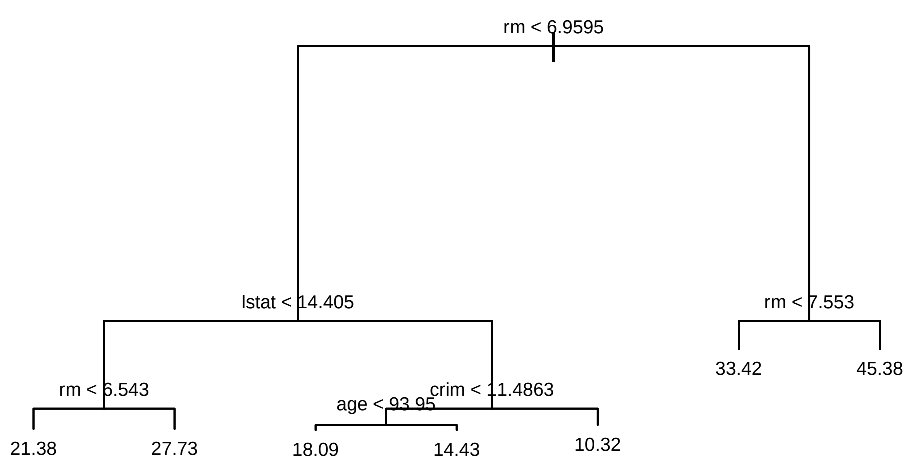
As the target variable is continuous, a *regression tree* is fitted:

```r
tree.boston = tree(medv ~ ., data = Boston, subset = train)
summary(tree.boston)
```

```
##
## Regression tree:
## tree(formula = medv ~ ., data = Boston, subset = train)
## Variables actually used in tree construction:
## [1] "rm"    "lstat" "crim"  "age"
## Number of terminal nodes:  7
## Residual mean deviance:  10.38 = 2555 / 246
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -10.1800  -1.7770  -0.1775   0.0000   1.9230  16.5800
```

`summary()` reports that only 3 variables have been used to fit the tree: `lstat`, `crim` and `age`.

```r
plot(tree.boston)
text(tree.boston, pretty = 0, cex = .6)
```

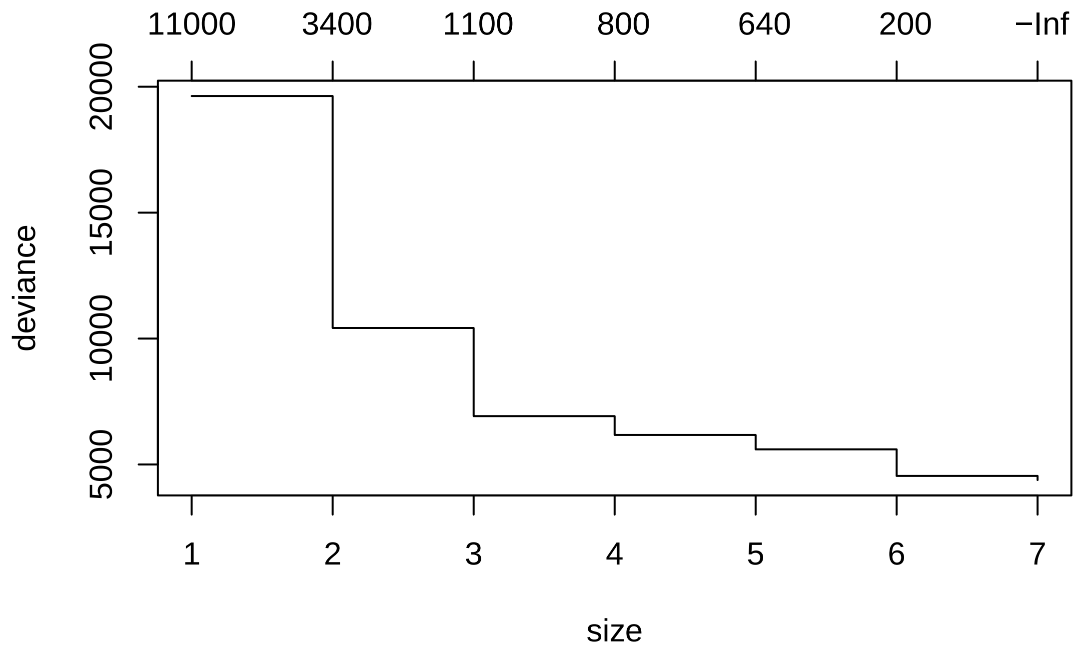


Pruning the tree with Cross-Validation

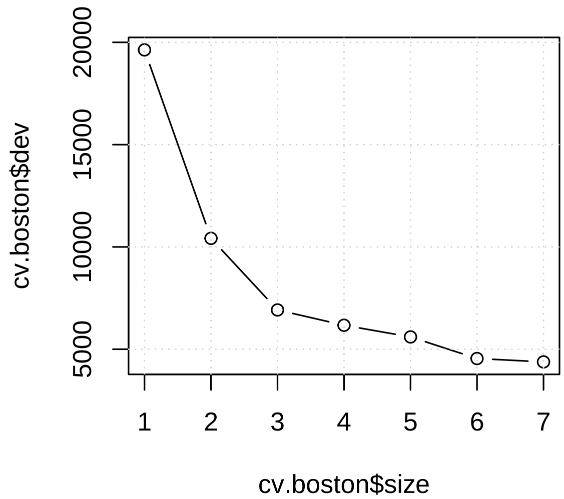
Finding the optimal level of complexity

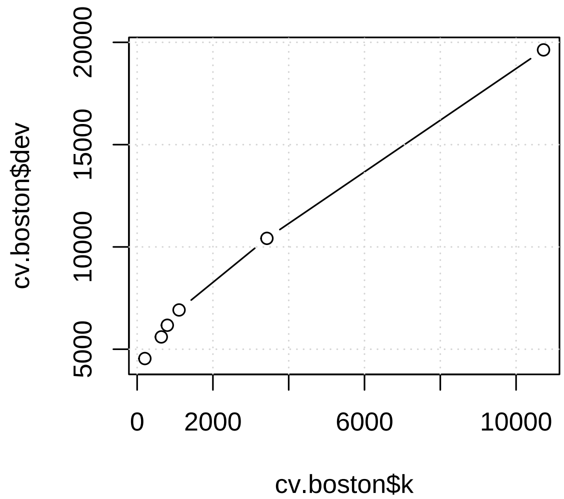
In this example deviance (the default) will be used to select the best model, as the `prune.misclass()` function only applies to classification trees:

```
cv.boston = cv.tree(tree.boston)
plot(cv.boston)
```




```
par(mfrow=c(1, 2))
plot(cv.boston$size, cv.boston$dev, type='b'); grid()
plot(cv.boston$k, cv.boston$dev, type='b'); grid()
```



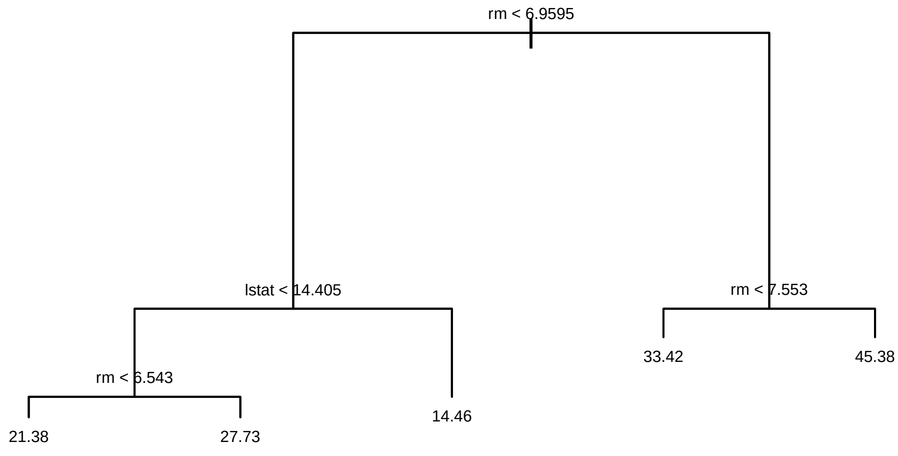


In this case the most complex tree has the lowest deviance.

Pruning the tree

We can anyway prune the tree to get a simpler one:

```
prune.boston = prune.tree(tree.boston, best = 5)
plot(prune.boston)
text(prune.boston, pretty = 0, cex=.5)
```



Making predictions

The complete tree is used, as it's the best model:

```
pred.boston = predict(tree.boston, newdata = Boston[-train,])
mean((pred.boston - medv[-train])^2)
```

```
## [1] 35.28688
```

The mean squared error is aproximatelly 35.3, and its square rooot is 5.9, meaning that the model leads to test predictions that are within around $6000 of the true median home value for the suburb, as can be seen in the following plot:

```
plot(pred.boston, Boston$medv[-train],
     xlab = 'Prediction (k$)', ylab = 'Real Value (k$)')
abline(0, 1, lty = 2)
grid()
```