Digital Lab

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Abstract

For this lab, we covered a wide range of topics relevant to digital radio. I don't know what the hell they were yet though.

1 Introduction

The foremost, most fundamental piece of physics relevant to digital signal sampling is the Nyquist criterion. A signal is a continuously-varying quantity, but a computer can only read its value a finite number of times. The Nyquist criterion states that in order to get sample data that approximates the actual shape of the signal, one must sample at at least twice the frequency of the signal.

Another piece of physics often relevant to digital radio is signal mixing. To mix signals, one uses a signal mixer. The mixer takes as input two signals and outputs their product in the time domain. This turns out to be their convolution in the frequency domain by the Convoolution Theorem. A mixer can be either Single Side Band (SSB) or Double Side Band (DSB). A Double Sideband mixer outputs a wave given by the product of its two input waves. By the equation $\cos a \cos b = 1/2 * (\cos{(a-b)} + \cos{(a+b)})$, this output is equivalent to the sum of two waves whose frequencies are equal to the sum and difference of the frequencies of the input waves. A Single Sideband Mixer takes the additional step of filtering out one of these terms, so the output frequency is just one of $a-b$ or $a+b$.

2 Experiments, Observations, Analysis and Interpretation

2.1 The Nyquist Frequency

The first thing we did in this lab was try to understand and visualize the Nyquist criterion. To do this, we sampled a signal at frequencies ranging from 0.1 times the Nyquist frequency to 3 times the Nyquist frequency and compared how effectively they seemed to illustrate the underlying sinusoid. First we sampled at less than the Nyquist frequency: the resulting graphs appear in Figure 1. Then, we sampled at the Nyquist frequency and triple the Nyquist frequency: the resulting graphs appear in Figure 2. As you can see, the samples at less than the Nyquist frequency are periodic, but it is not easy to see that the underlying signal is a sinusoid or determine its amplitude or frequency. At the Nyquist frequency, the signal is clearly sinusoidal, and its amplitude and frequency can be easily determined. Finally, at triple the Nyquist frequency, the picture smooths out even further and looks almost like a continuous waveform.

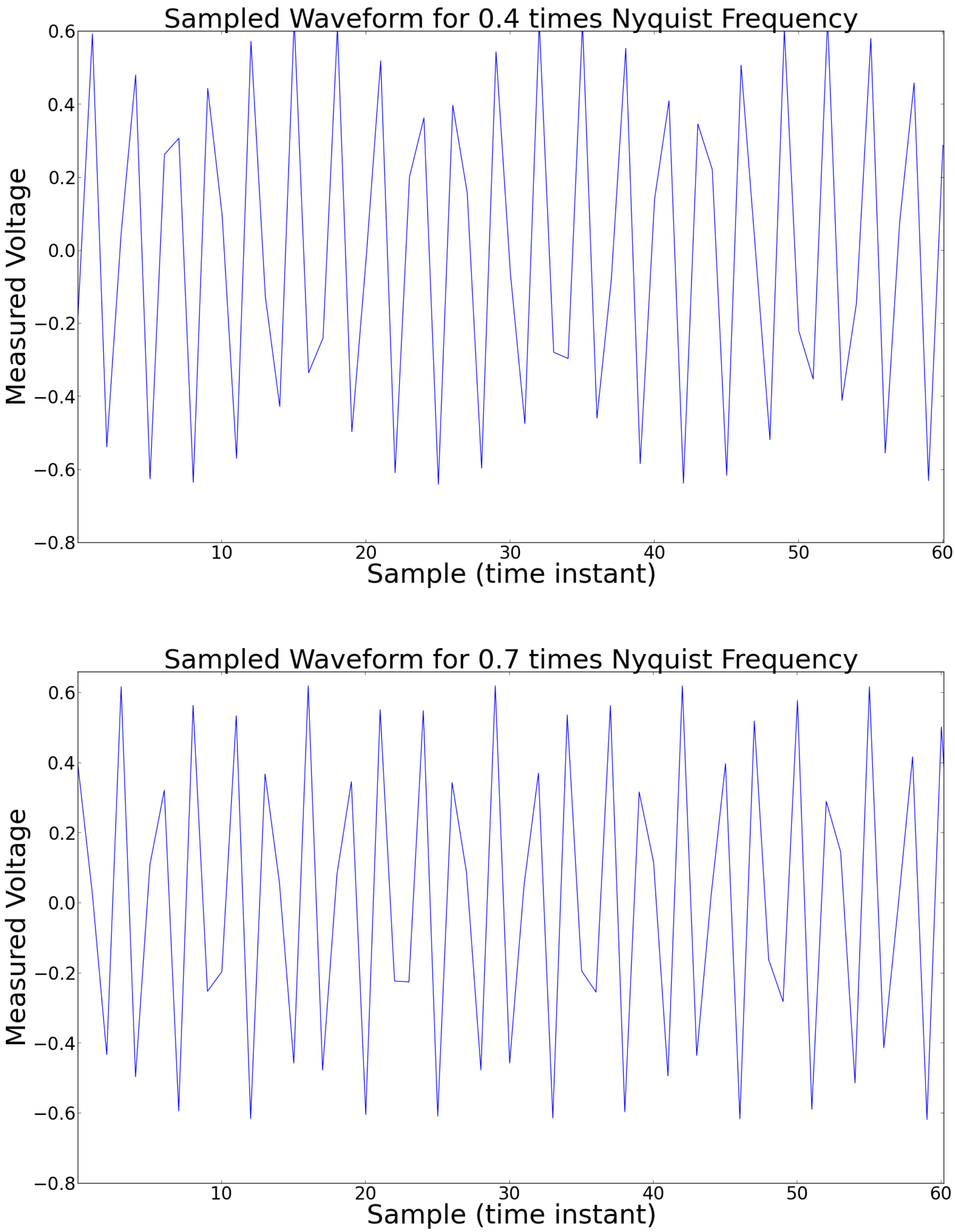

Figure 1: Sinusoidal signal sampled at 0.4 and 0.7 times Nyquist frequency.

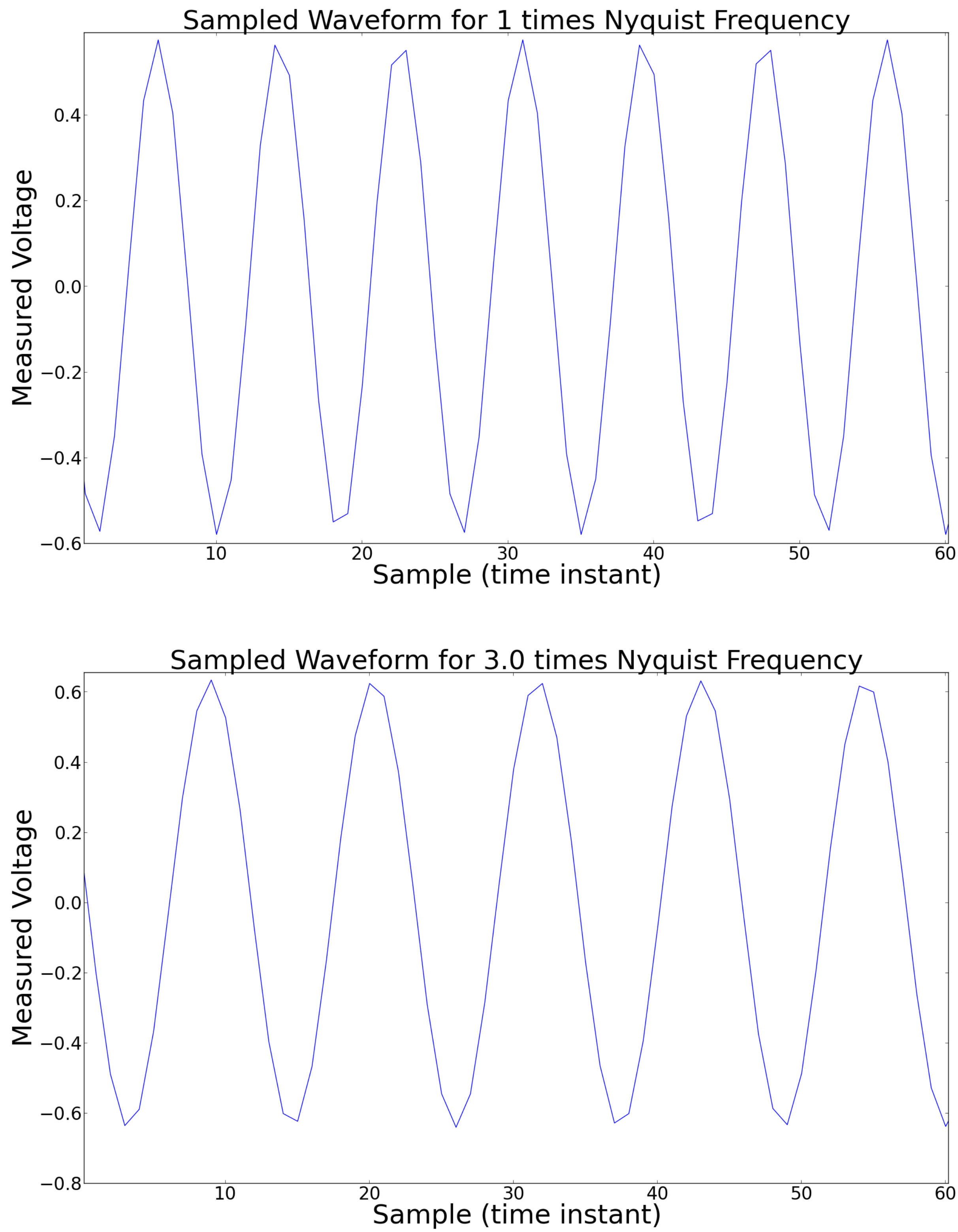

Figure 2: Sinusoidal signal sampled at 1 and 3 times Nyquist frequency.

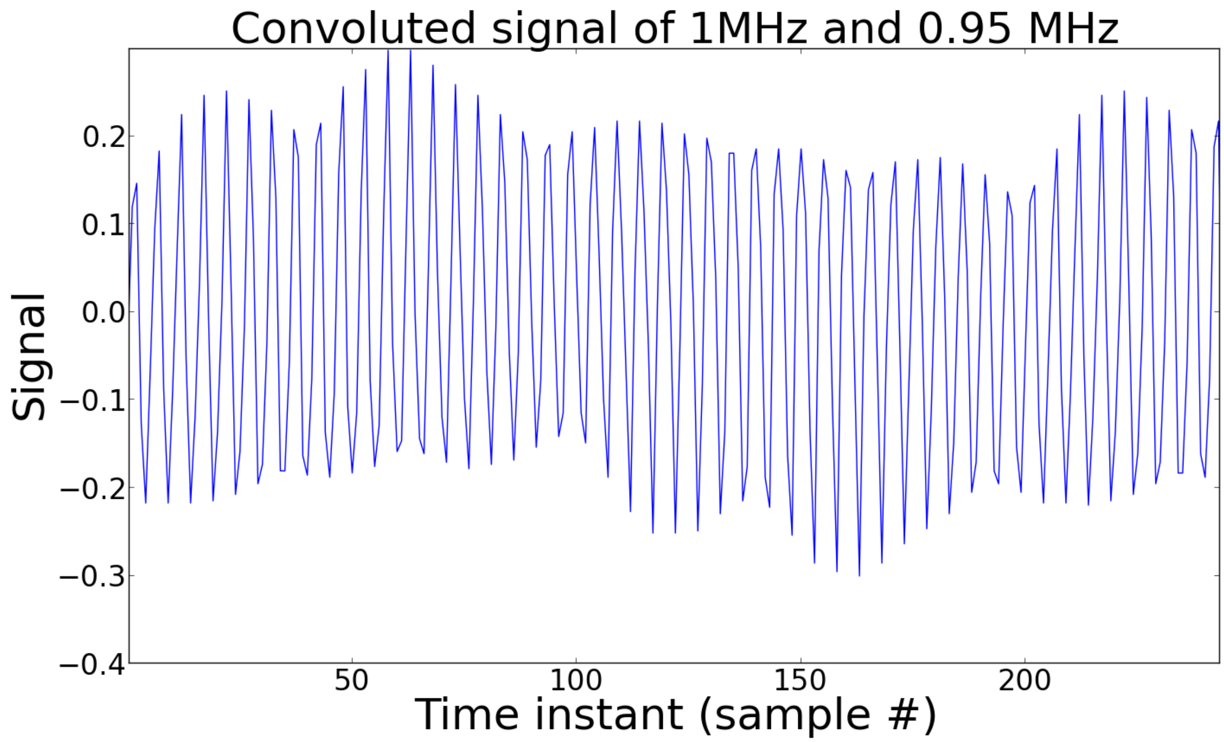

Figure 3: Convoluted signal of a 0.95 MHz wave and a 1.0 MHz wave

2.2 Mixing

In the second week of the lab, we used analog and digital mixers to combine waveforms generated by SRS generators and compared the outputs. First, we used an analog DSB to combine signals of 0.95 and 1.0 MHz as well as signals of 1.05 and 1.0 MHz. The waveform for the first case appears in Figure 3.

From the waveforms, we calculated the power spectrum using a Fourier transform: the results appear in Figure 4.

As you can see, the power amplitude coefficients strongly peak around 2 MHz, because that is roughly the sum of the frequencies we mixed. Looking closely, one can see that the highest peak of the mixture of 0.95 MHz and 1.0 MHz is at slightly less than 2 MHz, while the highest peak of the mixture of 1.05 and 1.0 MHz is slightly greater than 2 MHz, as expected. These peaks form the “upper sideband”, the part of the spectrum centered at the sum of the input frequencies. In addition, you can see the “lower sideband” in the vicinity of 0 MHz, where the frequency is the difference of the input frequencies. Because we inadvertantly used too much power, the lower sideband is lower than the upper sideband, and some extra coefficients appear at ± 4. Disregard these features. In addition, the whole distribution is reproduced on the negative side of the x-axis, because actual waveforms are complex exponentials, so the sine wave we mixed with is actually represented by $\sin x = 1/2 * i * (e^{-ix} - e^{ix})$, so we ended up convolving our 1.0 MHz local oscillator with waves of frequency 0.95 and -0.95 MHz in the first case, and 1.05 and -1.05 MHz in the second case. Then, we filtered out the upper sideband of the convolved waveforms of Figure 4 for the case of 0.95 MHz and found the IFFT of the lower sideband. The result appears in Figure 5.

The IFFT of the lower sideband is roughly a sinusoid with frequency 0.5 MHz. This makes sense because in the frequency domain it is sharply peaked about that frequency. The higher-order

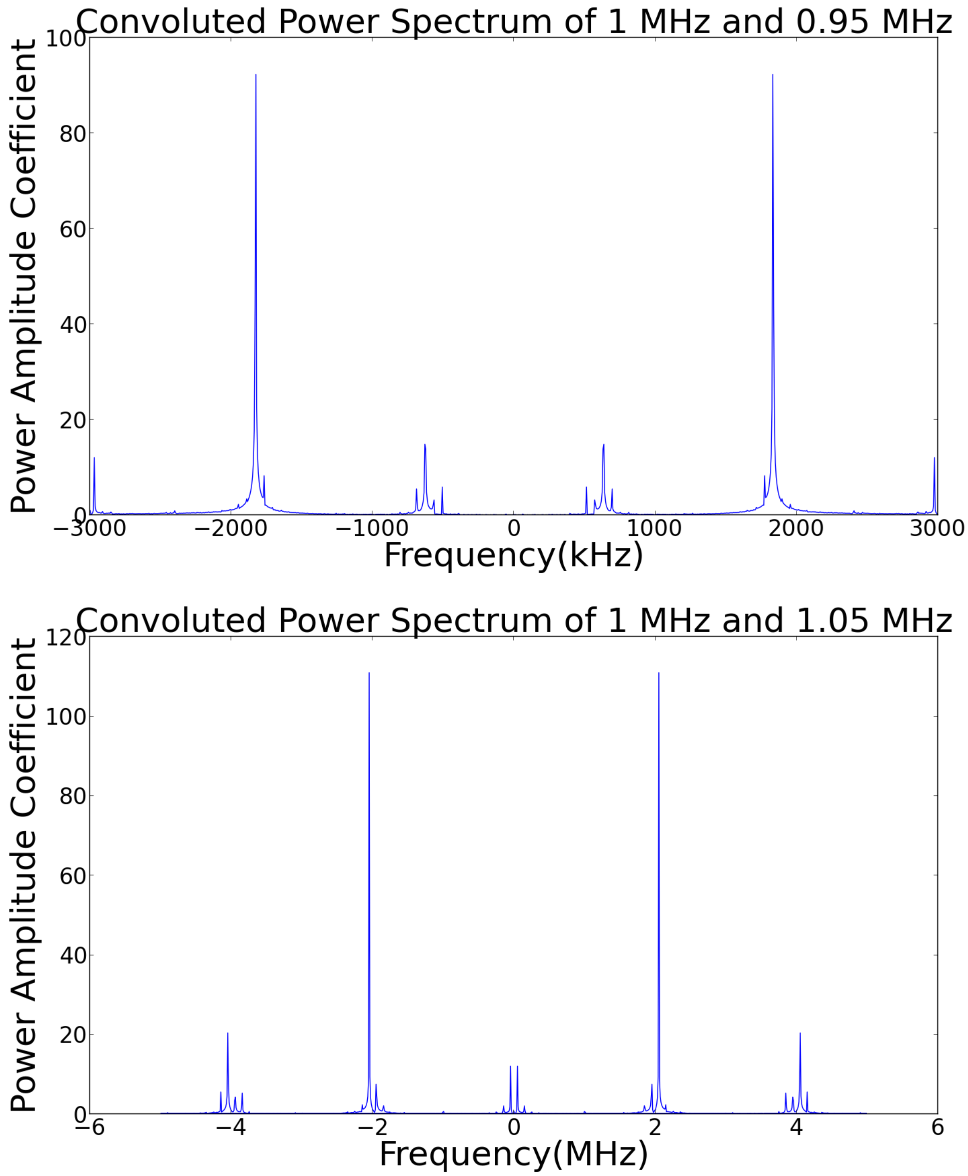

Figure 4: Analog DSB convolution power spectra of 0.95 MHz and 1.0 MHz (top), and 1.05 MHz and 1.0 MHz (bottom).

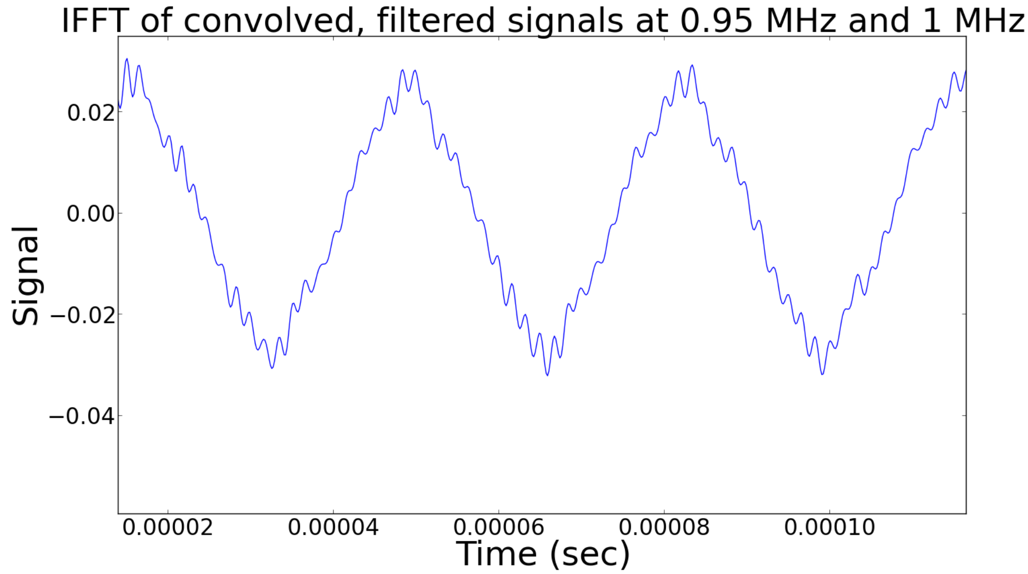

Figure 5: IFFT of lower sideband from convolved 0.95 and 1.0 MHz waves.

terms and noise errors contribute to small oscillations within the main 0.5 MHz oscillation. Finally, we made use of a digital mixer to explore DSB and SSB digital mixing. First, we used the digital DSB mixer to mix the same signals we mixed with the analog mixer above, and we compared the results. The digitally-mixed power spectrum appears in Figure 6.

Even though we mixed the same signals in the digital DSB mixer as in the analog DSB mixer, the graphs look different because of differences between digital and analog mixers. There are two differences relevant to this discussion. First, the digital mixer has a sort of implicit input in the form of a DC offset, which it mixes with the input signals. Second, the digital mixer has a resolution given by ν_s/N, where ν_s is the sampling frequency and N is the number of samples. Since the ROACH gives 2048 samples at 200 MHz, its resolution is roughly $200000000/2000 = 100$kHz. Signals whose frequencies are closer together than this resolution will be indistinguishable to the digital mixer. The digital graphs show five separate peaks. The center peak, at a frequency of 0, corresponds to the DC offset. In addition, since the difference frequencies $|1 - 1.05|$ and $|1 - 0.95|$ are equal to 50kHz, which is less than the resolution, their power amplitudes are added to this center peak as well. The next peak to the right, around 1 MHz, is the sum of the input frequencies with the DC offset, whose frequency is zero. The resulting output signal has the same frequency as the input signal. Finally, around 2 MHz, there appears a spike corresponding to the output signal whose frequency is the sum of the two input frequencies. For the same complex-exponential reasons as in the analog case, the whole spectrum is reproduced on the negative side of 0. Finally, the

Then, we used the SSB mixing functionality of our digital mixer to mix an input signal with a local oscillator.

3 FIR Filter

In the last week of the lab, we designed an FIR filter to be a 5/8 bandpass. In the frequency domain, such a filter ideally has nonzero coefficients in the five frequencies closest to zero and zero coefficients elsewhere. We approximated these features with a sinc function, $\sin x/x$, which rapidly drops off except in the vicinity of 0. By computing the inverse FFT of the sinc function on eight points about

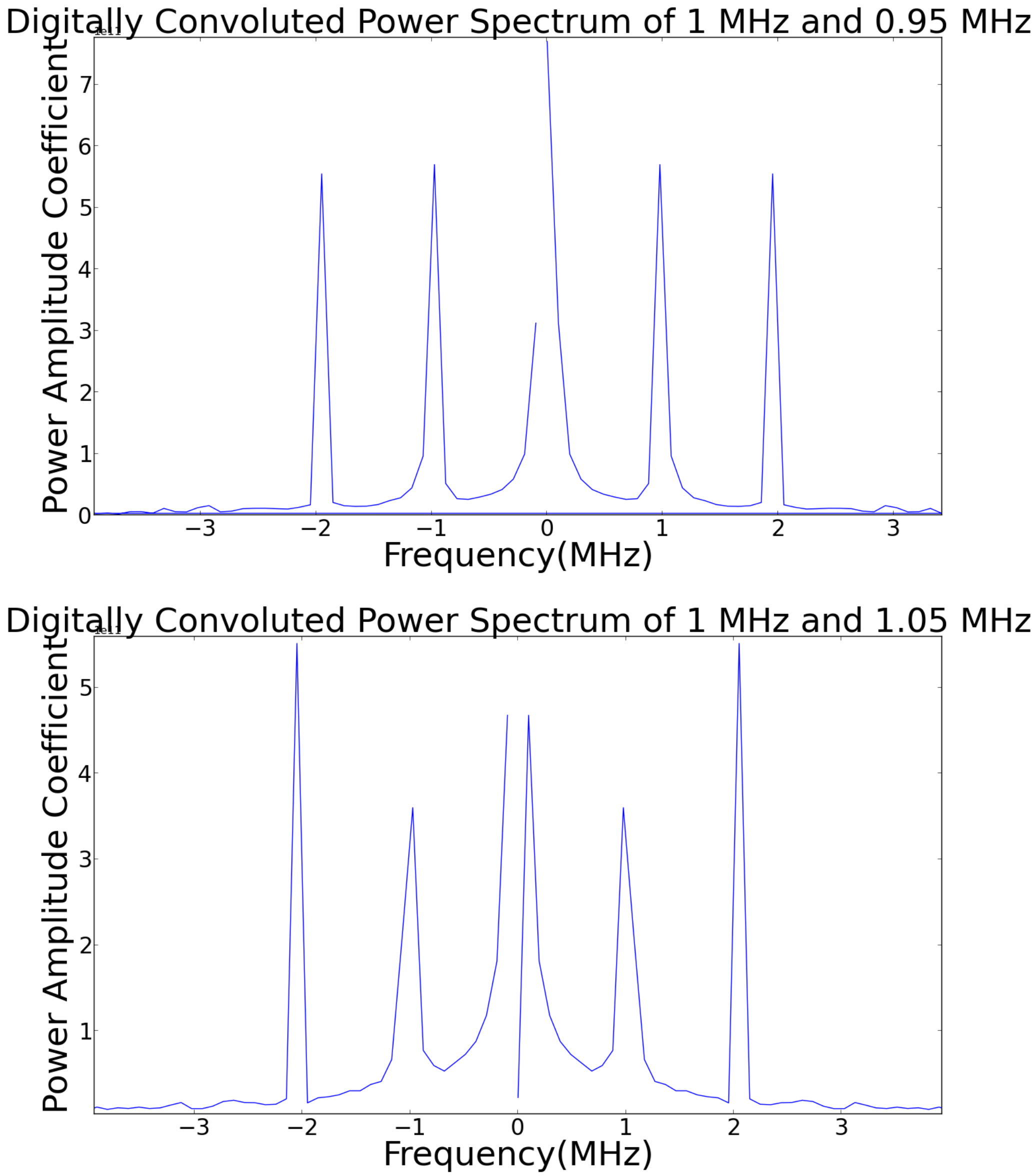

Figure 6: Power spectra for digitally mixed signals of 1 MHz and 0.95 MHz (top); 1 MHz and 1.05 MHz (bottom)

Register	Coefficient	Binary representation
-4	0.239389	0.00111101010010001
-3	0.508846	0.10000010010000111
-2	0.759187	0.11000010010110100
-1	0.936155	0.11101111101001111
0	1.0	0.11111111111111111
1	0.936155	0.11101111101001111
2	0.759187	0.11000010010110100
3	0.508846	0.10000010010000111

Figure 7: Coefficients for our FIR filter.

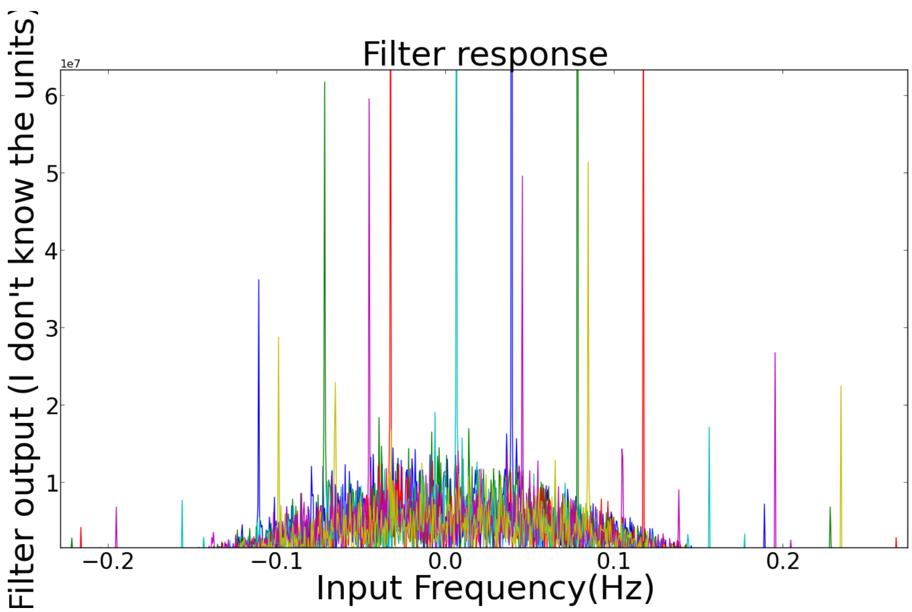

Figure 8: Output of our FIR filter.

zero, we found the time-domain coefficients that would Fourier transform into it, creating the 5/8 bandpass we desired. The resulting coefficients and their binary representations appear in Figure 7.

The actual effect of our FIR filter appears in Figure 8.

This was a fun lab report and I love radios and stuff!