Digital Lab

David Galbraith

March 3, 2014

Abstract

For this lab, we covered a wide range of topics relevant to digital radio. I don't know what the hell they were yet though.

1 Introduction

The foremost, most fundamental piece of physics relevant to digital signal sampling is the Nyquist criterion. A signal is a continuously-varying quantity, but a computer can only read its value a finite number of times. The Nyquist criterion states that in order to get sample data that approximates the actual shape of the signal, one must sample at at least twice the frequency of the signal.

Another piece of physics often relevant to digital radio is signal mixing. To mix signals, one uses a signal mixer. The mixer takes as input two signals and outputs their product in the time domain. This turns out to be their convolution in the frequency domain by the Convoolution Theorem. A mixer can be either Single Side Band (SSB) or Double Side Band (DSB). A Double Sideband mixer outputs a wave given by the product of its two input waves. By the equation $\cos a \cos b = 1/2 * (\cos(a-b) + \cos(a+b))$, this output is equivalent to the sum of two waves whose frequencies are equal to the sum and difference of the frequencies of the input waves. A Single Sideband Mixer takes the additional step of filtering out one of these terms, so the output frequency is just one of $a-b$ or $a+b$.

2 Experiments, Observations, Analysis and Interpretation

2.1 The Nyquist Frequency

The first thing we did in this lab was try to understand and visualize the Nyquist criterion. To do this, we sampled a signal at frequencies ranging from 0.1 times the Nyquist frequency to 3 times the Nyquist frequency and compared how effectively they seemed to illustrate the underlying sinusoid. First we sampled at less than the Nyquist frequency: the resulting graphs appear in Figure 1. Then, we sampled at the Nyquist frequency and triple the Nyquist frequency: the resulting graphs appear in Figure 2. As you can see, the samples at less than the Nyquist frequency are periodic, but it is not easy to see that the underlying signal is a sinusoid or determine its amplitude or frequency. At the Nyquist frequency, the signal is clearly sinusoidal, and its amplitude and frequency can be easily determined. Finally, at triple the Nyquist frequency, the picture smooths out even further and looks almost like a continuous waveform.

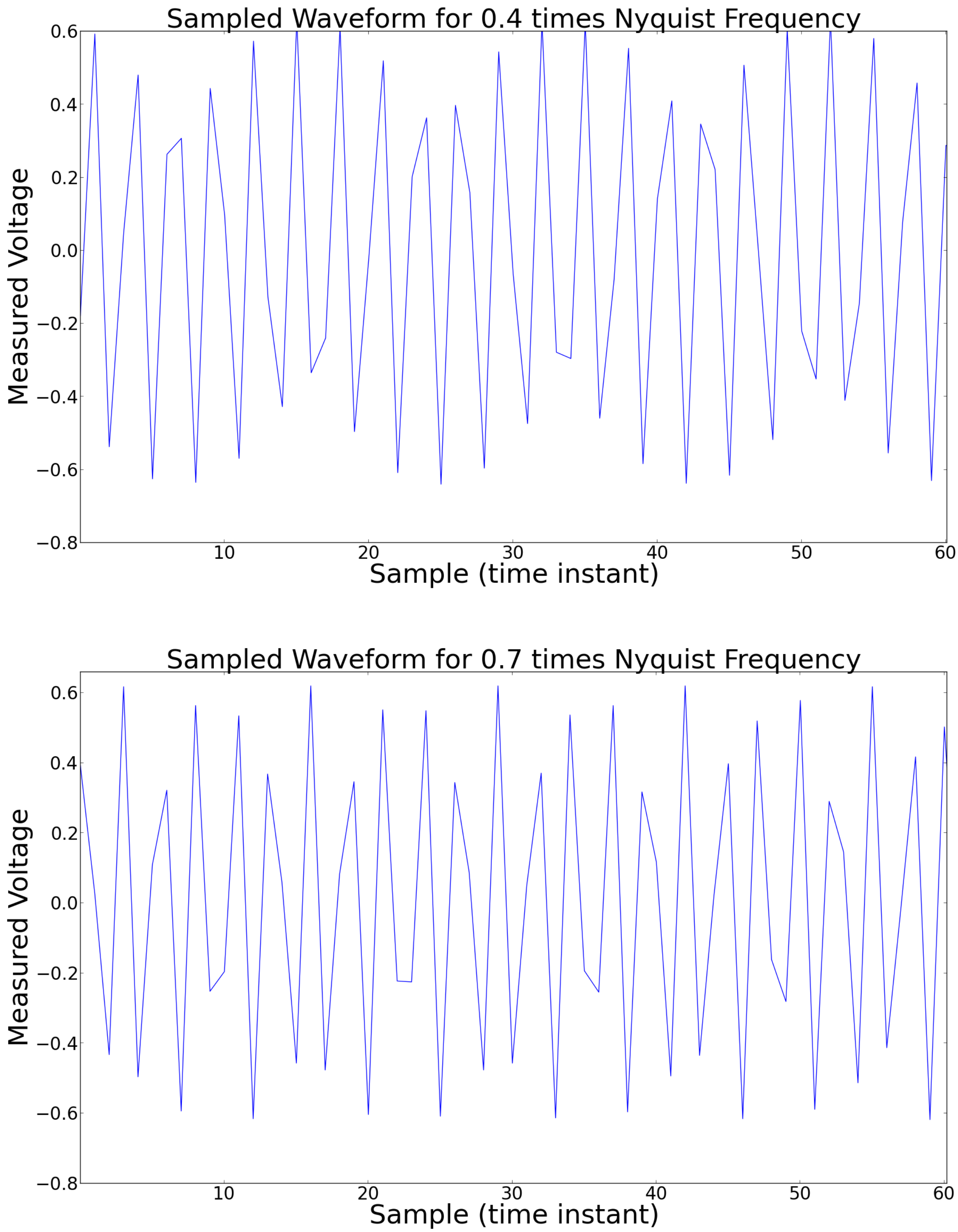

Figure 1: Sinusoidal signal sampled at 0.4 and 0.7 times Nyquist frequency.

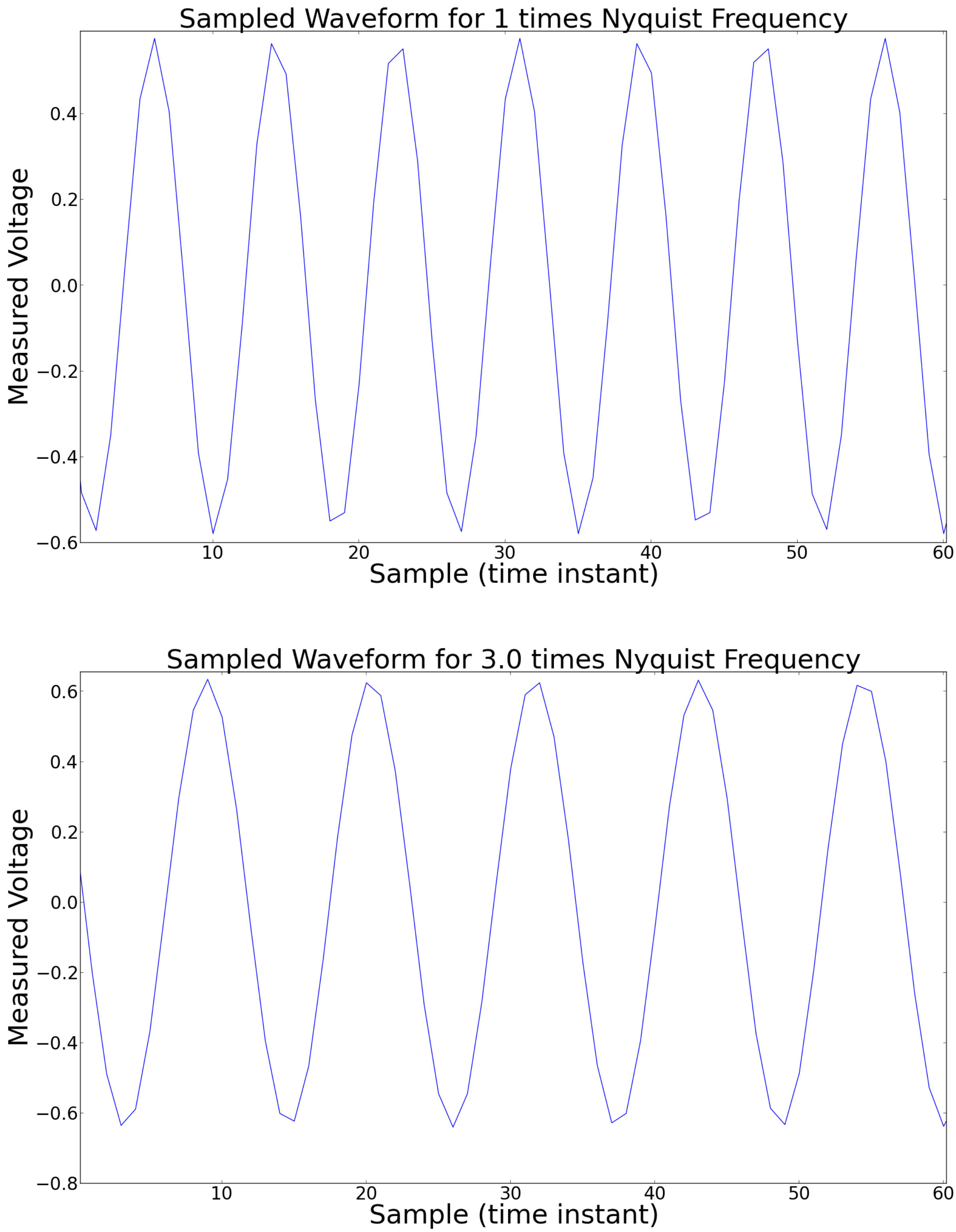

Figure 2: Sinusoidal signal sampled at 1 and 3 times Nyquist frequency.

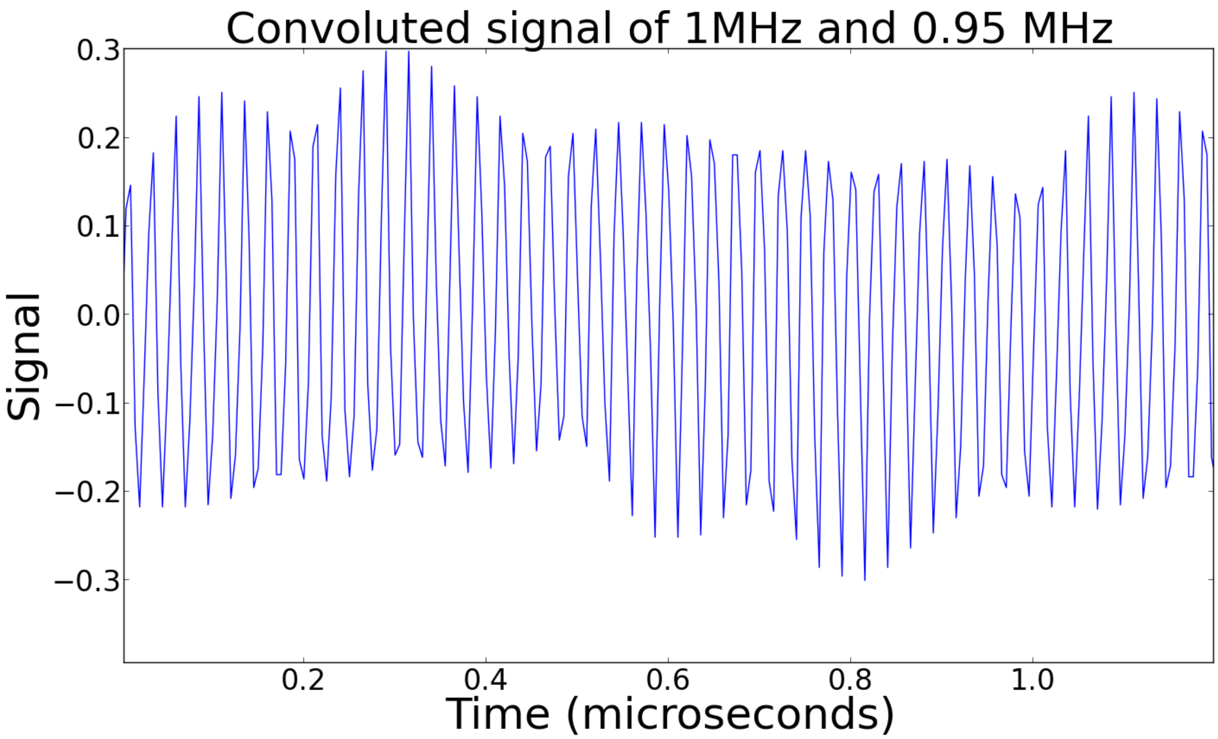

Figure 3: Convolved signal of a 0.95 MHz wave and a 1.0 MHz wave

2.2 Mixing

In the second week of the lab, we used analog and digital mixers to combine waveforms generated by SRS generators and compared the outputs. First, we used an analog DSB to combine signals of 0.95 and 1.0 MHz as well as signals of 1.05 and 1.0 MHz. The waveform for the first case appears in Figure 3.

From the waveforms, we calculated the power spectrum using a Fourier transform: the results appear in Figure 4.

As you can see, the power amplitude coefficients strongly peak around 2 MHz, because that is roughly the sum of the frequencies we mixed. Looking closely, one can see that the highest peak of the mixture of 0.95 MHz and 1.0 MHz is at slightly less than 2 MHz, while the highest peak of the mixture of 1.05 and 1.0 MHz is slightly greater than 2 MHz, as expected. These peaks form the "upper sideband", the part of the spectrum centered at the sum of the input frequencies. In addition, you can see the "lower sideband" in the vicinity of 0 MHz, where the frequency is the difference of the input frequencies. Because we inadvertantly used too much power, the lower sideband is lower than the upper sideband, and some extra coefficients appear at ± 4 MHz. Disregard these features. In addition, the whole distribution is reproduced on the negative side of the x-axis, because actual waveforms are complex exponentials, so the sine wave we mixed with is actually represented by $\sin x = 1/2 * i * (e^{-ix} - e^{ix})$, so we ended up convolving our 1.0 MHz local oscillator with waves of frequency 0.95 and -0.95 MHz in the first case, and 1.05 and -1.05 MHz in the second case. Then, we applied a Fourier filter to the convolved waveforms of Figure 4 for the case of 0.95 MHz, zeroing out everything except the term at 0.05 MHz. The result appears in Figure 5.

The IFFT is roughly a sinusoid with frequency 0.05 MHz. This makes sense because it is approximately the IFFT of a delta-function spectrum centered at that frequency. Finally, we made

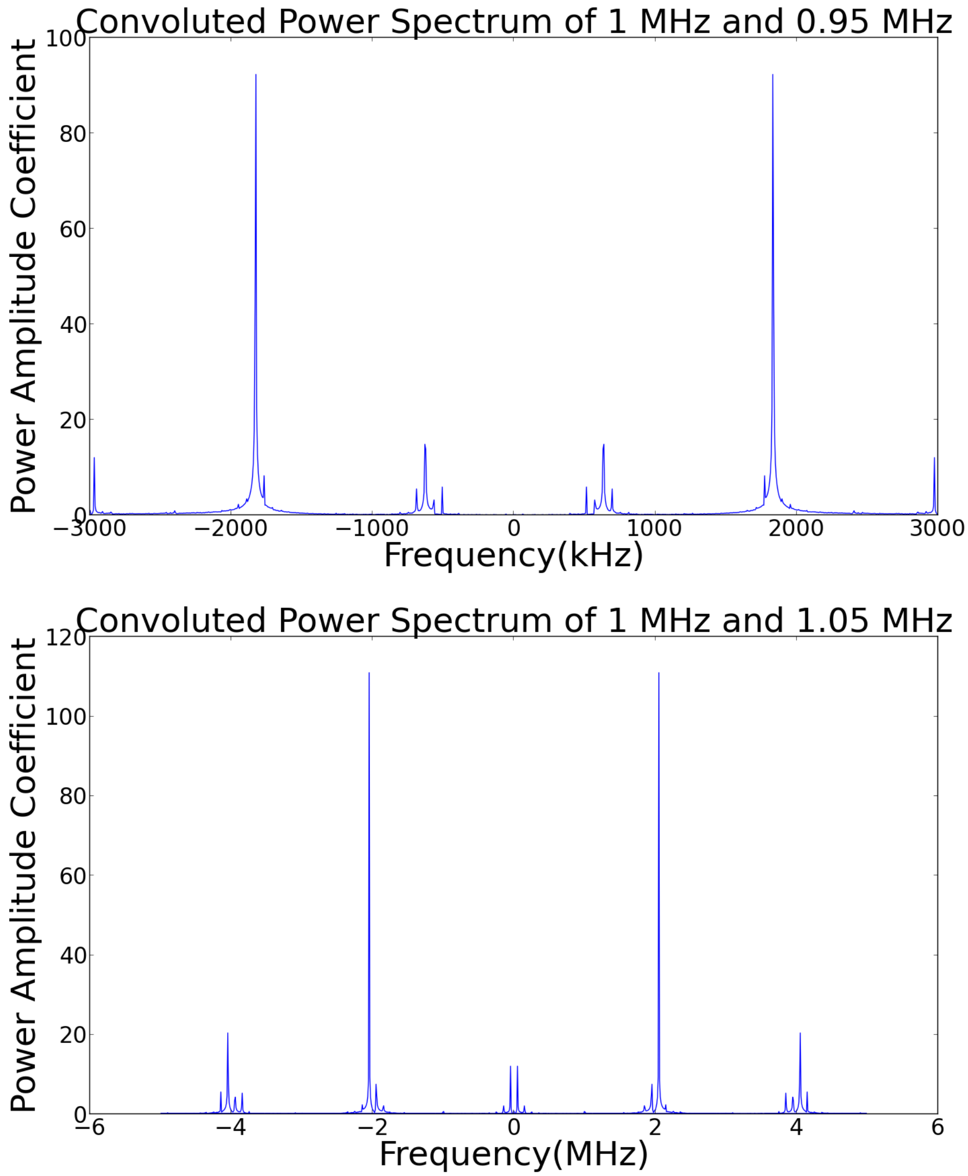

Figure 4: Analog DSB convolution power spectra of 0.95 MHz and 1.0 MHz (top), and 1.05 MHz and 1.0 MHz (bottom).

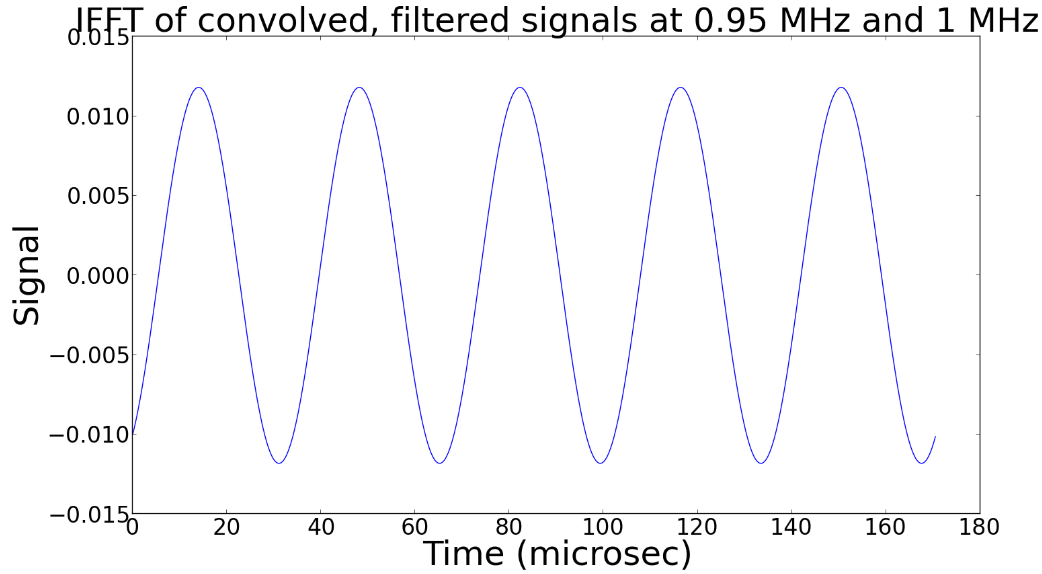

Figure 5: IFFT of 0.05 MHz term from convolved 0.95 and 1.0 MHz waves.

use of a digital mixer to explore DSB and SSB digital mixing. First, we used the digital DSB mixer to mix the same signals we mixed with the analog mixer above, and we compared the results. The digitally-mixed output waveform appears in Figure 6, while the digitally-mixed power spectrum appears in Figure 7.

Even though we mixed the same signals in the digital DSB mixer as in the analog DSB mixer, the graphs look different because of differences between digital and analog mixers. There are two primary differences relevant to this discussion. First, the digital mixer has a sort of implicit input in the form of a DC offset, which it mixes with the input signals. Second, the digital mixer has a resolution given by ν_s/N, where ν_s is the sampling frequency and N is the number of samples. Since the ROACH gives 2048 samples at 200 MHz, its resolution is roughly $200000000/2000 = 100$kHz. Signals whose frequencies are closer together than this resolution will be indistinguishable to the digital mixer. The digital graphs show five separate peaks. The center peak, at a frequency of 0, corresponds to the DC offset. In addition, since the difference frequencies $|1 - 1.05|$ and $|1 - 0.95|$ are equal to 50kHz, which is less than the resolution, their power amplitudes are added to this center peak as well. The next peak to the right, around 1 MHz, is the sum of the input frequencies with the DC offset, whose frequency is zero. The resulting output signal has the same frequency as the input signal. Finally, around 2 MHz, there appears a spike corresponding to the output signal whose frequency is the sum of the two input frequencies. For the same complex-exponential reasons as in the analog case, the whole spectrum is reproduced on the negative side of 0.

Then, we used the SSB mixing functionality of our digital mixer to mix an input signal at 1 MHz with a local oscillator at 1.56 MHz, specified by a low_freq register of 2. The resulting waveform and power spectrum appear in Figure 8.

The power spectrum shows three peaks. In the middle, there is a peak at 6.25 MHz. This is from the mixing of the DC offset with the local oscillator. On the right, there is a peak at 16.25 MHz. This is from the output wave whose frequency is the sum of the input frequencies, $6.25+10$. On the left, there is a peak at -4.75 MHz. This is from the output wave whose frequency is the difference of the two input frequencies, $6.25 - 10$. In the DSB filter, we would have seen two additional frequencies, one for $-(6.25 + 10)$ and one for $-(6.25 - 10)$; since we used an SSB, we multiplied by

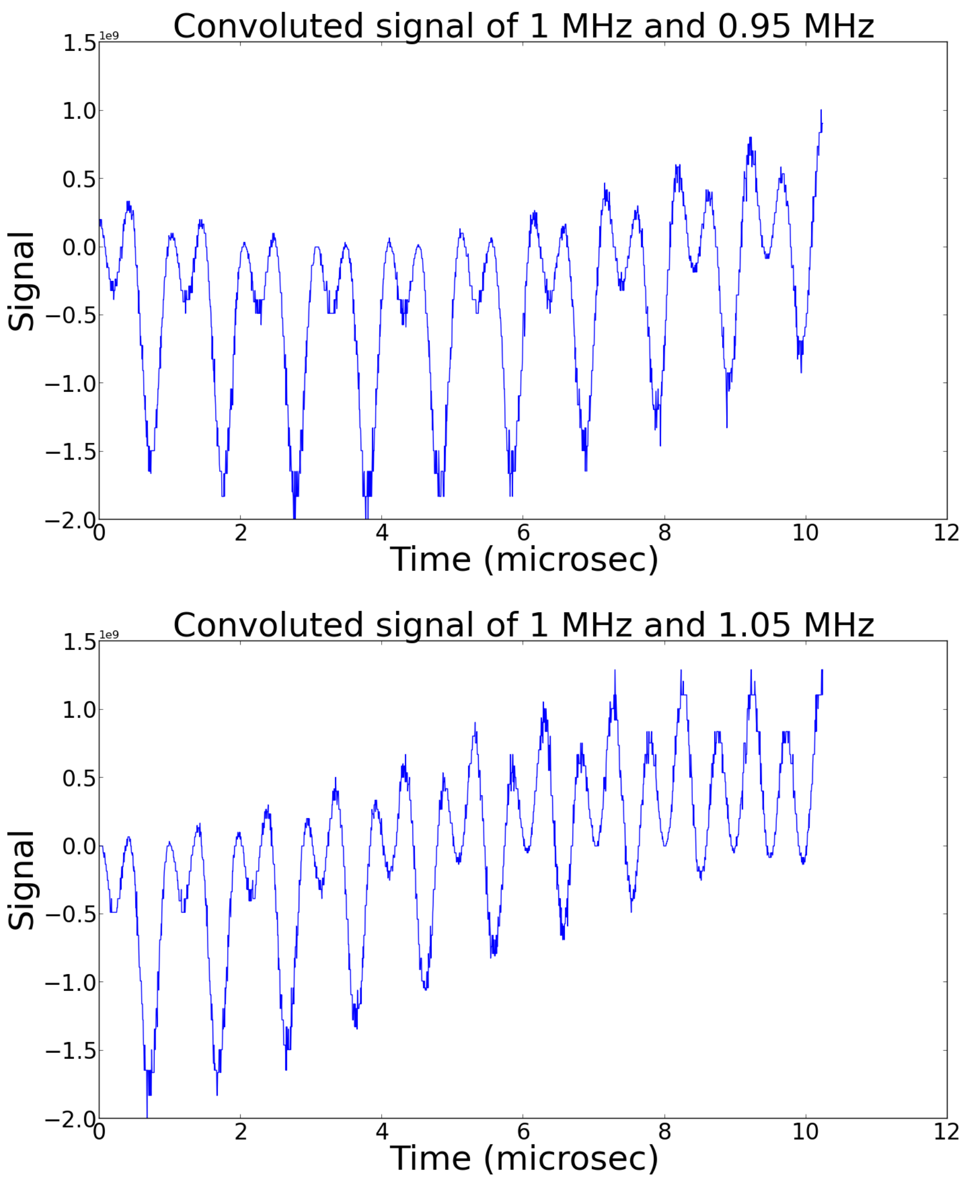

Figure 6: Output waveform for digitally mixed signals of 1 MHz and 0.95 MHz (top); 1 MHz and 1.05 MHz (bottom)

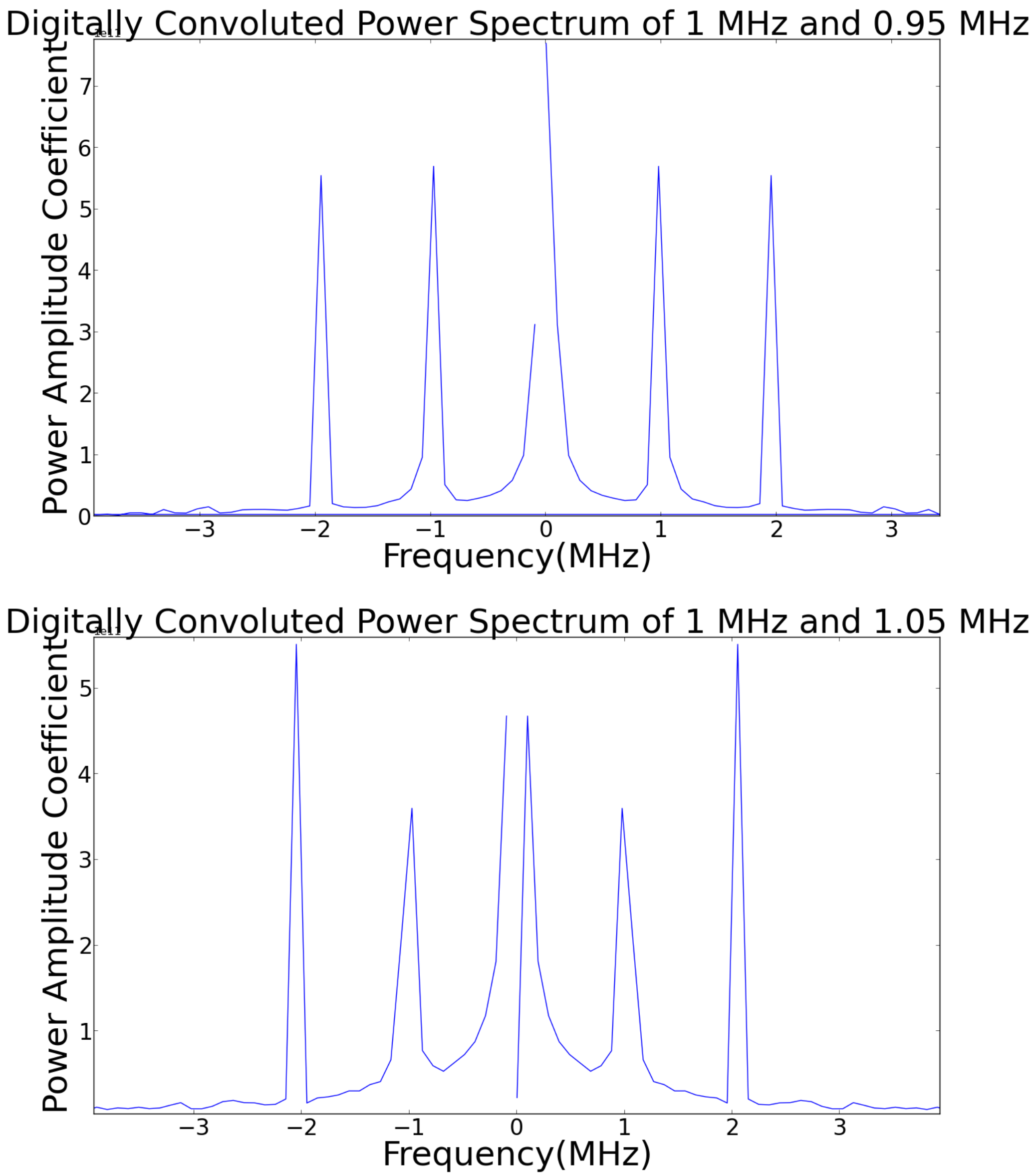

Figure 7: Power spectra for digitally mixed signals of 1 MHz and 0.95 MHz (top); 1 MHz and 1.05 MHz (bottom)

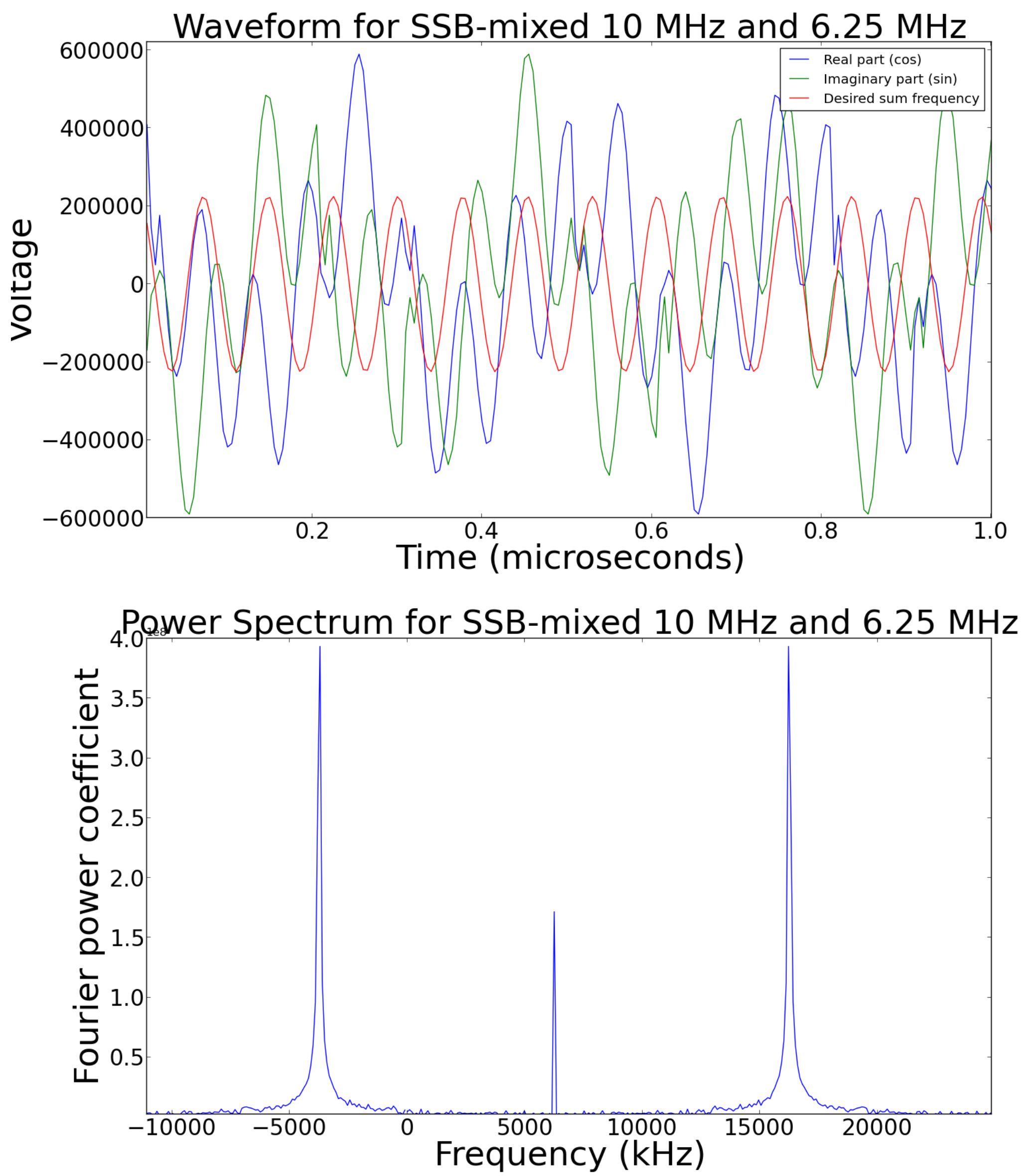

Figure 8: Waveform and power spectrum for SSB-mixed signals of 10 MHz and 6.25 MHz.

Register	Coefficient	Binary representation
-4	0.239389	0.00111101010010001
-3	0.508846	0.10000010010000111
-2	0.759187	0.11000010010110100
-1	0.936155	0.11101111101001111
0	1.0	0.11111111111111111
1	0.936155	0.11101111101001111
2	0.759187	0.11000010010110100
3	0.508846	0.10000010010000111

Figure 9: Coefficients for our FIR filter.

a single complex waveform instead of the sum of two, so these frequencies do not appear. In the plot of the waveform, the red line shows the Fourier-filtered version of the output signal, generated by setting all terms in the FFT of the time series to zero except the one corresponding to the 16.25 MHz frequency. It has a positive frequency, since $16.25 > 0$.

Analog and digital DSB and SSB mixers have lots of advantages and disadvantages. With an analog mixer, you are free of many of the restraints imposed by the digital mixer. For instance, the analog mixer does not suffer from the resolution issues that the digital mixer has: it can measure and output frequencies in a more or less continuous spectrum. Also, it does not have the DC offset that features in our discussions of the digital results: this can make the results easier to understand and interpret. However, digital mixers are good too because they do exact math on their inputs to calculate the outputs, instead of having to worry about the physics of a potentially messy mixing process. On the other hand, you do have to do an analog-to-digital conversion to use the digital mixer, which introduces errors including the resolution problem mentioned above, because you can't fully capture a continuous signal with a finite digital representation. Both of them are prone to sometimes breaking and giving weird outputs, as we experienced in this lab.

DSB mixers are apparently cheaper and easier to manufacture than SSB mixers, because an SSB mixer basically has to perform two separate convolution operations every time in order to kill the undesired sideband. If you have an application where having the extra sideband does not distract you too much, or you have the wherewithal to filter it out, you can use the DSB without worrying. But if not, then you can use the SSB and get only the terms you want in your product expansion.

3 FIR Filter

In the last week of the lab, we designed an FIR filter to be a 5/8 bandpass. In the frequency domain, such a filter ideally has nonzero coefficients in the five frequencies closest to zero and zero coefficients elsewhere. We approximated these features with a sinc function, $\sin x/x$, which rapidly drops off except in the vicinity of 0. By computing the inverse FFT of the sinc function on eight points about zero, we found the time-domain coefficients that would Fourier transform into it, creating the 5/8 bandpass we desired. The resulting coefficients and their binary representations appear in Figure 9.

The actual effect of our FIR filter appears in Figure 10.

4 Conclusion

This was a fun lab report and I love radios and stuff!

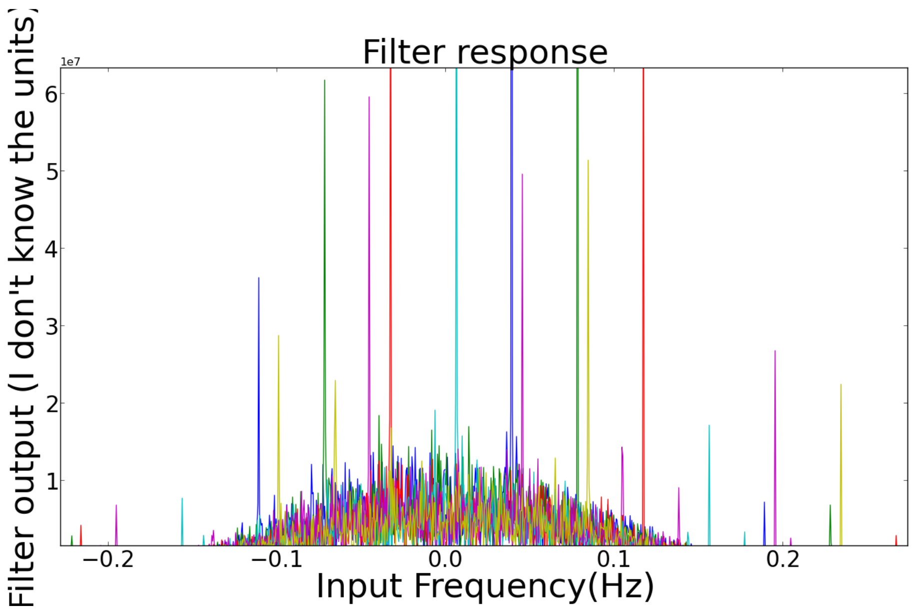

Figure 10: Output of our FIR filter.