Approximating open quantum systems

Using Stinespring dilation

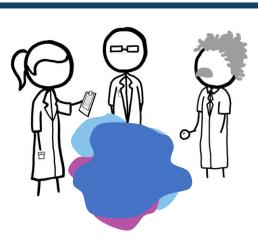
My project in one minute



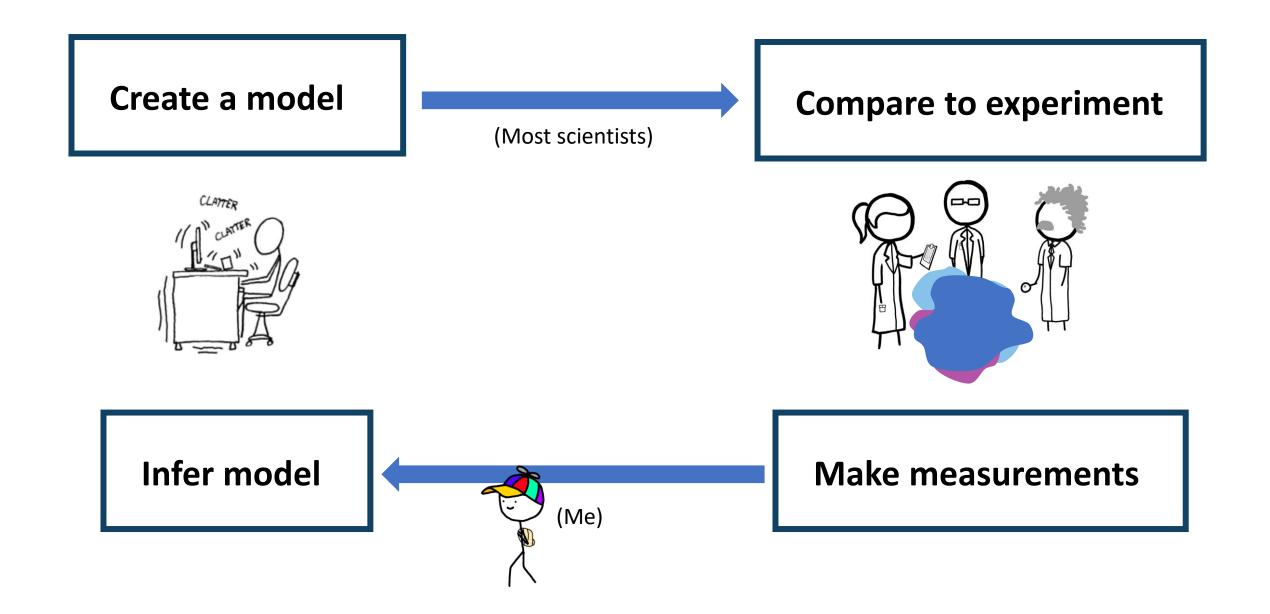
(Most scientists)

Compare to experiment





My project in one minute



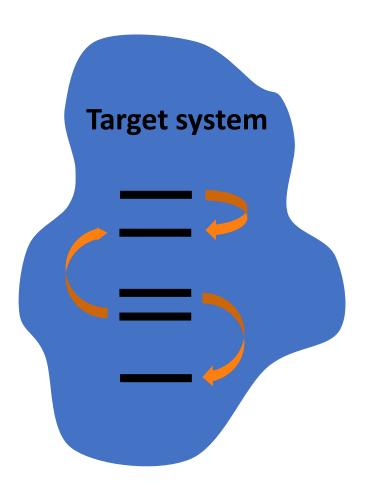
Agenda

Background

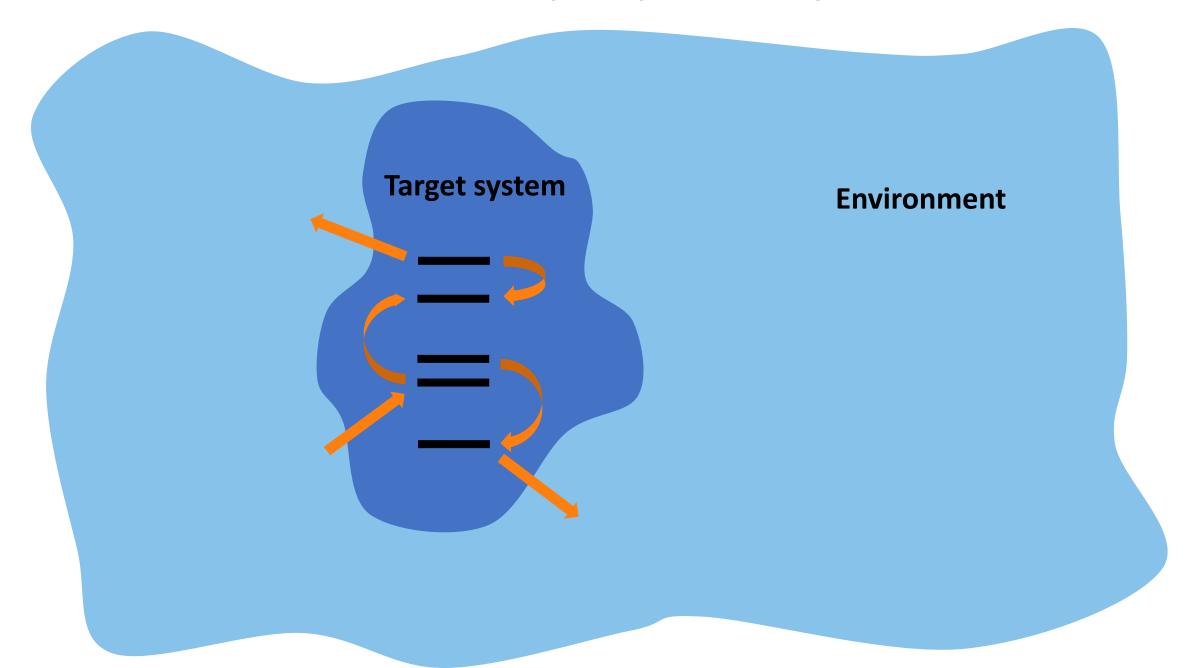
- Quantum systems
- Density matrix
- Evolution equations
- Quantum channels
- Combining qubits
- Stinespring's dilation theorem

My project

What is a quantum system?



What is an open quantum system?



Density matrix formalism

$$|\psi\rangle = \alpha_n |e^n\rangle$$

$$= \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix}$$

Pure state

$$\varrho = |\psi\rangle\langle\psi|$$

$$= \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} [\overline{\alpha}_1 \dots \overline{\alpha}_N] = \begin{bmatrix} \alpha_1\overline{\alpha}_1 & \dots & \alpha_1\overline{\alpha}_N \\ \vdots & \ddots & \vdots \\ \alpha_N\overline{\alpha}_1 & \dots & \alpha_N\overline{\alpha}_N \end{bmatrix}$$

Mixed state

$$\rho = \sum_{k} p_{k} |\psi^{k}\rangle \langle \psi^{k}|, \quad \sum_{k} p_{k} = 1$$

Density matrix formalism

Pure state

$$|\psi\rangle = \alpha_n |e^n\rangle$$

$$egin{aligned} arrho &= ig|\psi
angle\langle\psiig| \ &= ig|ar{lpha_1} ig|_{ar{lpha}_N} ig|_{ar{lpha}_1 \ lpha_N} ig|_{ar{lpha}_1 \ lpha_Nar{lpha}_1 \ lpha_Nar{lpha}_1 \ lpha_Nar{lpha}_1 \ lpha_Nar{lpha}_N \ lpha_Nar{lpha}_N \ ar{lpha}_N ar{lpha}_N \ ar{lpha}_N ar{lpha}_N \ a$$

Mixed state

$$\rho = \sum_{k} p_k |\psi^k\rangle\langle\psi^k|, \quad \sum_{k} p_k = 1$$

Evolution equations

Closed system (Schrödinger equation)

(Von Neumann equation)

$$rac{d}{dt}\ket{\psi} = -rac{i}{\hbar}H\ket{\psi}$$

Density matrix formalism

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho]$$

Open system (Lindblad equation)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_{k} \gamma_{k} \left(A_{k}\rho A_{k}^{\dagger} - \frac{1}{2} \{ A_{k}^{\dagger} A_{k}, \rho \} \right)$$

Evolution equations

Closed system (Schrödinger equation)

(Von Neumann equation)

$$\frac{d}{dt}\ket{\psi} = -\frac{i}{\hbar}H\ket{\psi}$$

Density matrix formalism

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho]$$

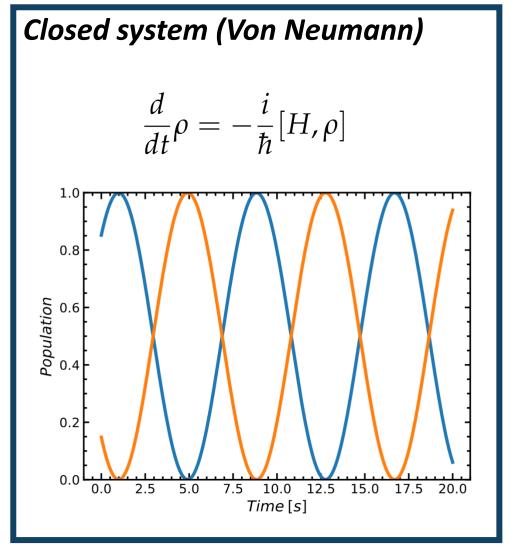
"Hamiltonian"

Open system (Lindblad equation)

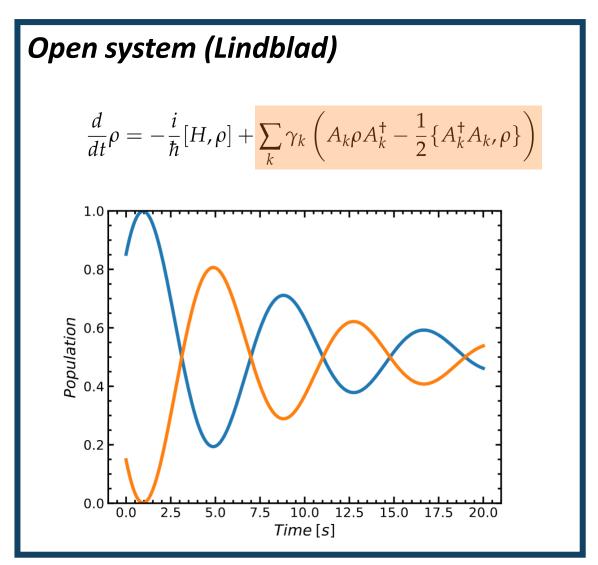
$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_{k} \gamma_{k} \left(A_{k}\rho A_{k}^{\dagger} - \frac{1}{2} \{ A_{k}^{\dagger} A_{k}, \rho \} \right)$$
"Hamiltonian"

"Interactions"

Evolution equations







Quantum channels

Closed system (Von Neumann)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho]$$

$$U_t \rho_0 U_t^{\dagger} = \rho_t,$$

$$U_t = \exp\left(-\frac{i}{\hbar}Ht\right)$$

Open system (Lindblad)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_{k} \gamma_{k} \left(A_{k}\rho A_{k}^{\dagger} - \frac{1}{2} \{ A_{k}^{\dagger} A_{k}, \rho \} \right)$$

Goal: approximate
$$\Phi_t(
ho_0)=
ho_t$$

Quantum channels

Closed system (Von Neumann)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho]$$

$$U_t \rho_0 U_t^{\dagger} = \rho_t,$$

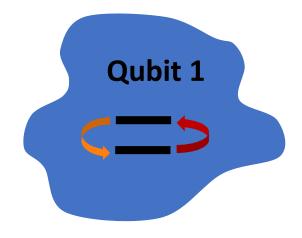
$$U_t = \exp\left(-\frac{i}{\hbar}Ht\right)$$

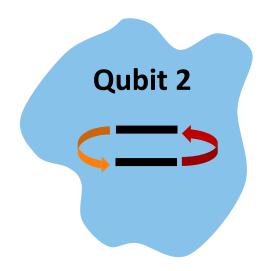
Open system (Lindblad)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_{k} \gamma_{k} \left(A_{k}\rho A_{k}^{\dagger} - \frac{1}{2} \{ A_{k}^{\dagger} A_{k}, \rho \} \right)$$

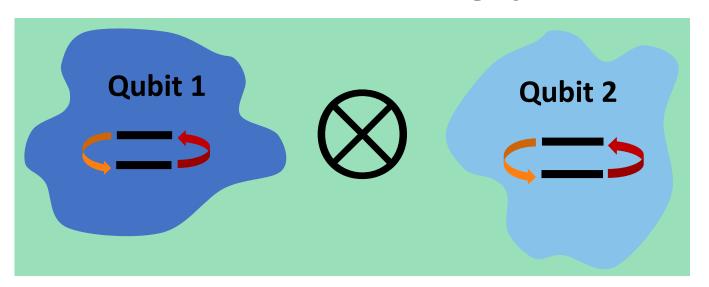
Goal: approximate $\Phi_t(
ho_0)=
ho_t$

Combining qubits



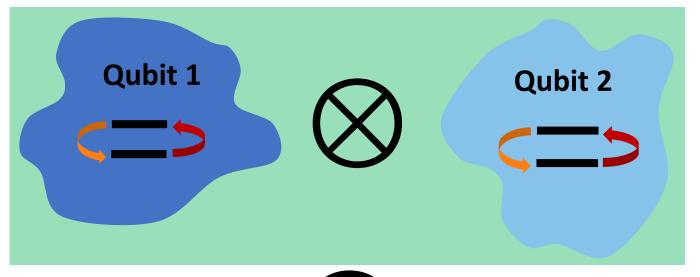


Combining qubits



Computational qubits

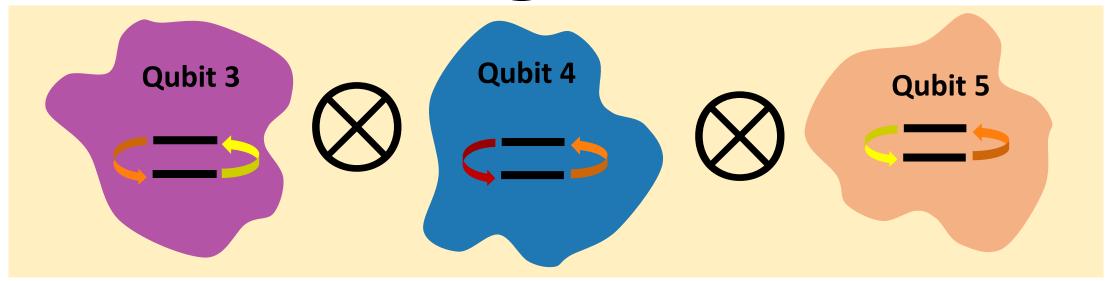
Combining qubits



Computational qubits

"System A"





Ancilla qubits

"System B"

Stinespring's dilation theorem

There exists U_t on dilated space (computational qubits) \otimes (ancilla qubits) such that:

$$\Phi_t(\rho_0) = \operatorname{Tr}_B[U_t(\rho_0 \otimes |0\rangle_B\langle 0|_B) U_t^{\dagger}]$$

Stinespring's dilation theorem

There exists $|U_t|$ on dilated space (computational qubits) \otimes (ancilla qubits) such that:

$$\Phi_t(
ho_0) = \operatorname{Tr}_B[U_t(
ho_0 \otimes |0\rangle_B\langle 0|_B)U_t^{\dagger}]$$
"Stinespring unitary"

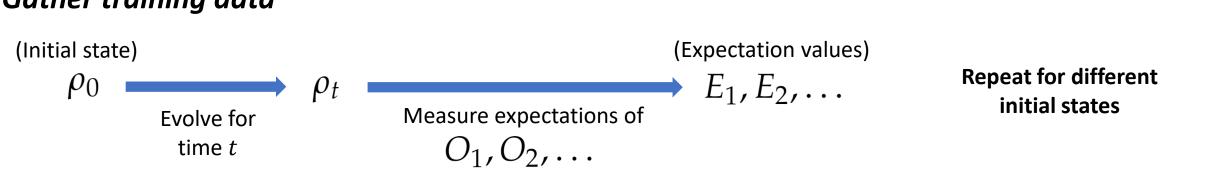
Agenda

My project

- Approximation method
- Parametrization of Stinespring unitary
- Results

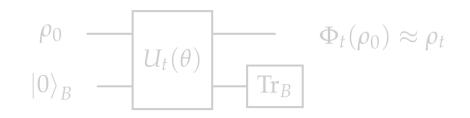
Approximation method





Approximate quantum channel

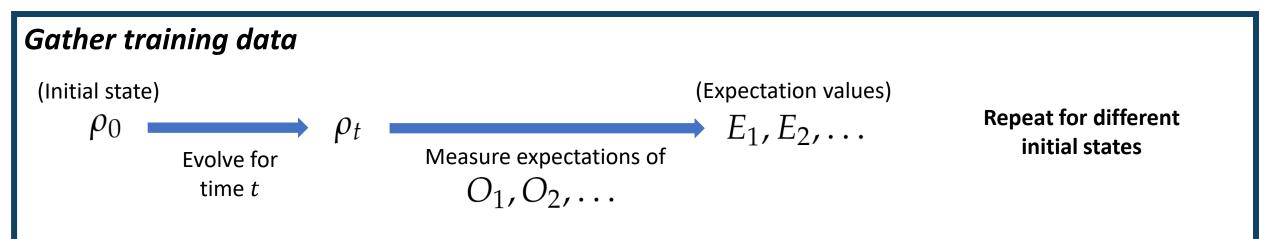
$$\Phi_t(\rho_0) = \operatorname{Tr}_B[U_t(\rho_0 \otimes |0\rangle_B\langle 0|_B) U_t^{\dagger}]$$

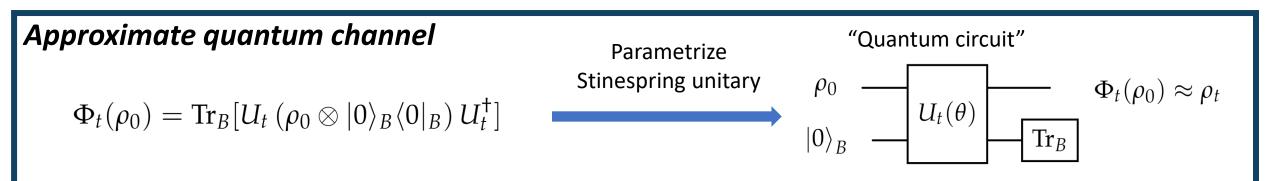


Optimize parameters

$$J(\theta) = \sum_{\ell} (E_{\ell} - \hat{E}_{\ell})^2$$

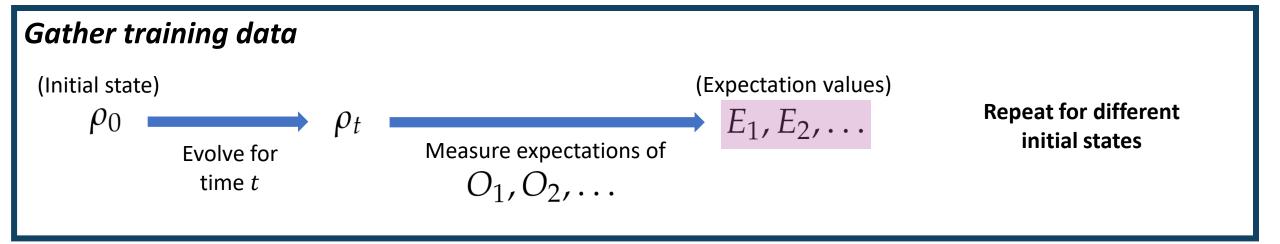
Approximation method

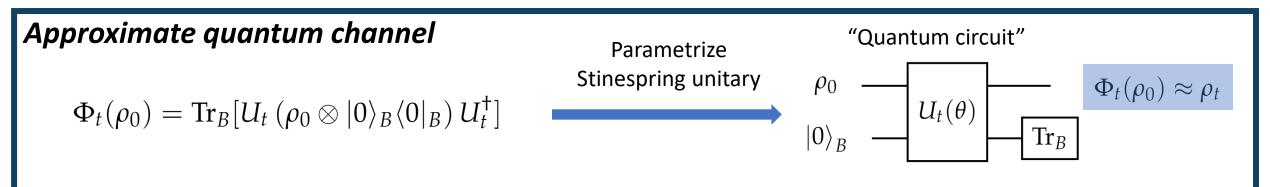




$$J(\theta) = \sum_{\ell} (E_{\ell} - \hat{E}_{\ell})^2$$

Approximation method

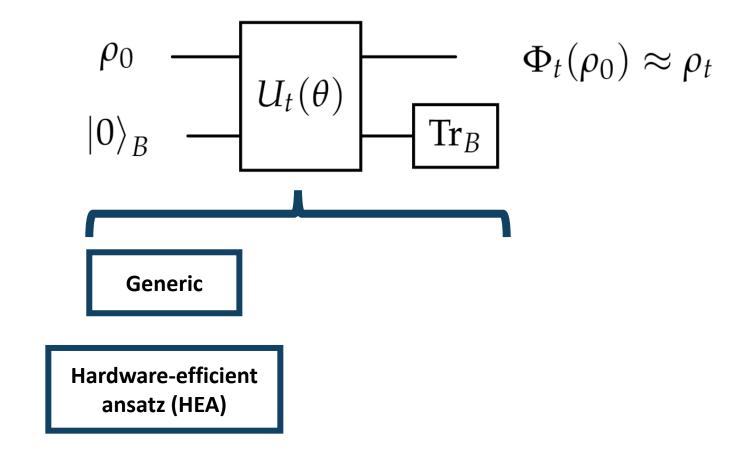


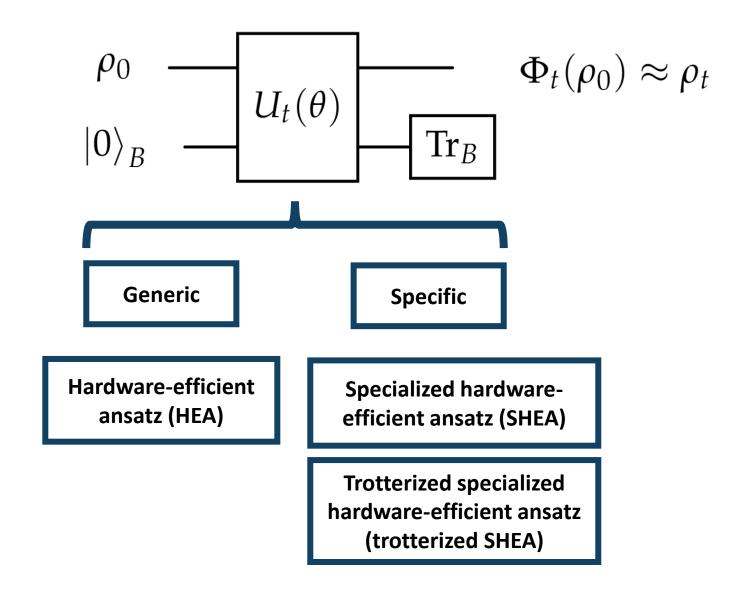


$$J(\theta) = \sum_{\ell} (E_{\ell} - \hat{E}_{\ell})^2$$

Measured

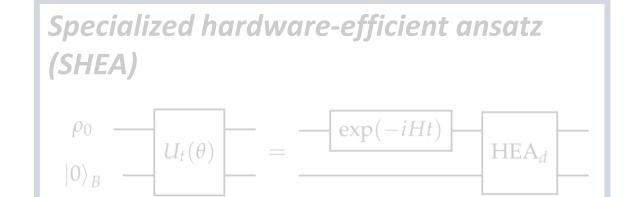
Approximated





Hardware-efficient ansatz (HEA)

$$\begin{array}{c|c}
\rho_0 & & \\
|0\rangle_B & & & \\
\end{array}$$
 $= \left(\begin{array}{c}
ZXZ & \\
ZXZ & \\
\end{array}\right)_d$



Interlude: trotterization

For Hermitian operators A and B we have:

$$\lim_{n \to \infty} (e^{-iA\frac{t}{n}}e^{-iB\frac{t}{n}})^n = e^{-i(A+B)t}$$

$$\begin{vmatrix} \rho_0 \\ |0\rangle_B \end{vmatrix} = \left(\frac{\exp(-iH\frac{t}{d})}{\text{HEA}_q} \right)_d$$

Hardware-efficient ansatz (HEA)

$$\begin{array}{c|c}
\rho_0 \\
|0\rangle_B
\end{array}$$
 $=$
 $\left(\begin{array}{c}
ZXZ \\
ZXZ
\end{array}\right)$
Ent.

Interlude: trotterization

For Hermitian operators A and B we have:

$$\lim_{n \to \infty} (e^{-iA\frac{t}{n}}e^{-iB\frac{t}{n}})^n = e^{-i(A+B)t}$$

Specialized hardware-efficient ansatz (SHEA)

$$ho_0 = U_t(\theta) = \frac{\exp(-iHt)}{|0\rangle_B}$$

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_{k} \gamma_{k} \left(A_{k}\rho A_{k}^{\dagger} - \frac{1}{2} \{ A_{k}^{\dagger} A_{k}, \rho \} \right)$$

$$\begin{vmatrix}
\rho_0 & & \\
|0\rangle_B & & \\
\end{vmatrix} U_t(\theta) = \begin{pmatrix}
-\exp(-iH\frac{t}{d}) & \\
& \\
\end{bmatrix} HEA_q$$

Hardware-efficient ansatz (HEA)

$$\begin{array}{c|c}
\rho_0 & & \\
|0\rangle_B & & U_t(\theta)
\end{array} = \left(\begin{array}{c}
ZXZ & Ent.
\end{array}\right)_d$$

Interlude: trotterization

For Hermitian operators A and B we have:

$$\lim_{n\to\infty} (e^{-iA\frac{t}{n}}e^{-iB\frac{t}{n}})^n = e^{-i(A+B)t}$$

Specialized hardware-efficient ansatz (SHEA)

$$\begin{array}{c|c}
\rho_0 & \hline \\
|0\rangle_B & \hline
\end{array} =
\begin{array}{c|c}
\exp(-iHt) & \hline \\
\text{HEA}_d & \hline
\end{array}$$

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_{k} \gamma_{k} \left(A_{k}\rho A_{k}^{\dagger} - \frac{1}{2} \{ A_{k}^{\dagger} A_{k}, \rho \} \right)$$

$$|0\rangle_B$$
 $U_t(\theta)$ $=$ $\left(\begin{array}{c} \exp\left(-iH\frac{t}{d}\right) \\ \text{HEA}_q \end{array}\right)_{a}$

Hardware-efficient ansatz (HEA)

$$\begin{array}{c|c}
\rho_0 \\
|0\rangle_B
\end{array} = \left(\begin{array}{c}
ZXZ \\
ZXZ
\end{array}\right) \quad \text{Ent.}$$

Interlude: trotterization

For Hermitian operators A and B we have:

$$\lim_{n \to \infty} (e^{-iA\frac{t}{n}}e^{-iB\frac{t}{n}})^n = e^{-i(A+B)t}$$

Specialized hardware-efficient ansatz (SHEA)

$$\begin{array}{c|c}
\rho_0 & \hline \\
|0\rangle_B & \hline
\end{array} =
\begin{array}{c|c}
\exp(-iHt) & \hline \\
\text{HEA}_d & \hline
\end{array}$$

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_{k} \gamma_{k} \left(A_{k}\rho A_{k}^{\dagger} - \frac{1}{2} \{ A_{k}^{\dagger} A_{k}, \rho \} \right)$$

Hardware-efficient ansatz (HEA)

$$\begin{array}{c|c}
\rho_0 & & \\
|0\rangle_B & & U_t(\theta)
\end{array} = \left(\begin{array}{c}
ZXZ & Ent.
\end{array}\right)_d$$

Specialized hardware-efficient ansatz (SHEA)

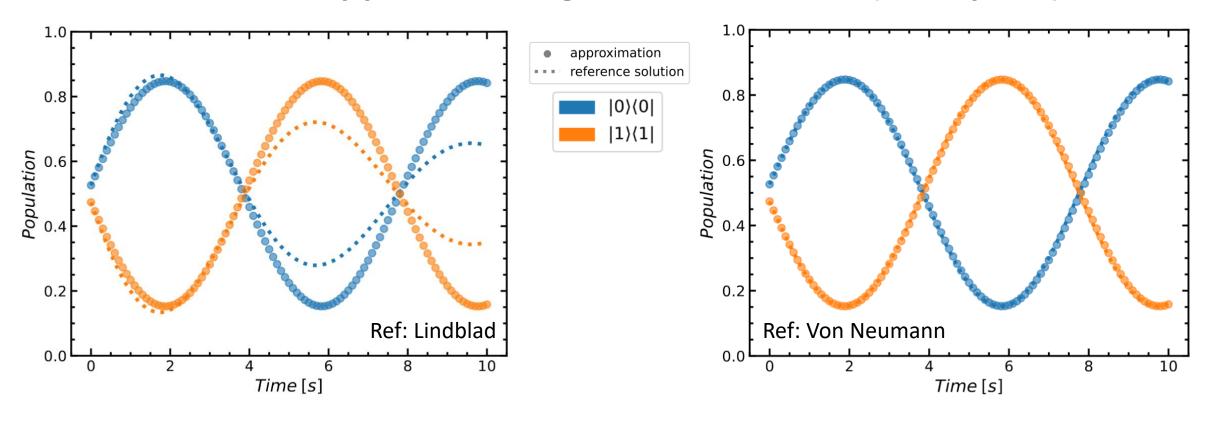
$$\begin{array}{c|c}
\rho_0 & \hline \\
|0\rangle_B & \hline
\end{array} =
\begin{array}{c|c}
\exp(-iHt) & \hline \\
\text{HEA}_d & \hline
\end{array}$$

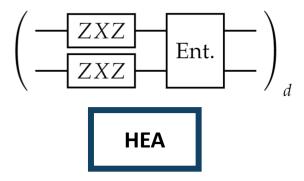
Interlude: trotterization

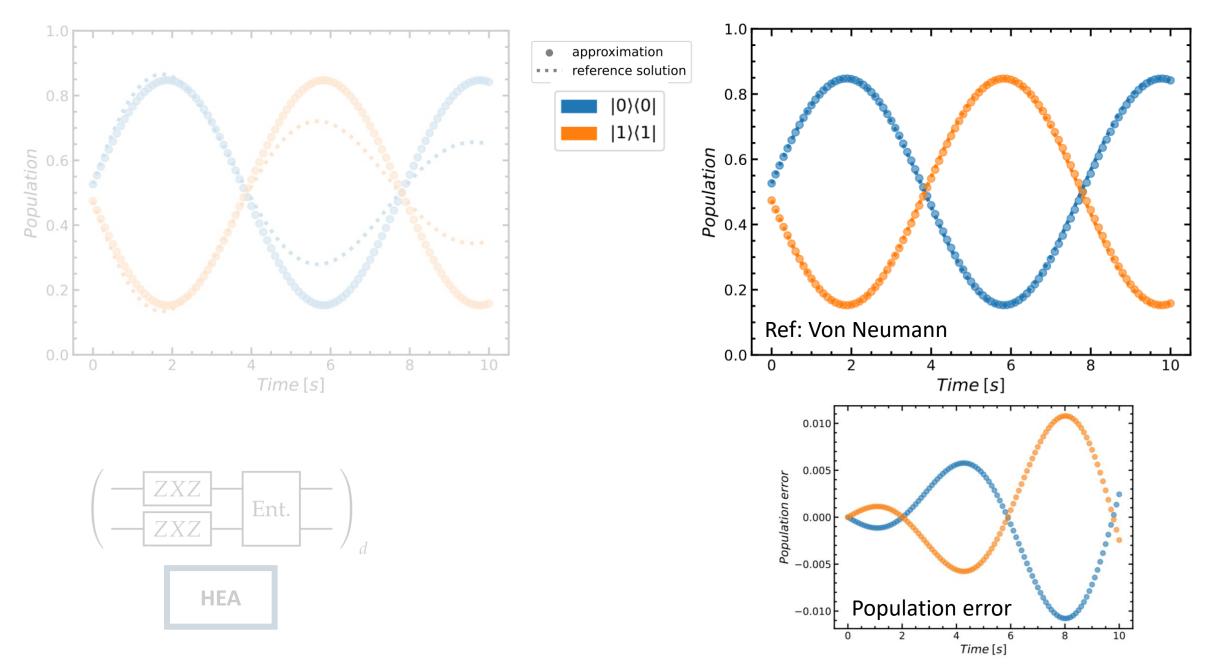
For Hermitian operators *A* and *B* we have:

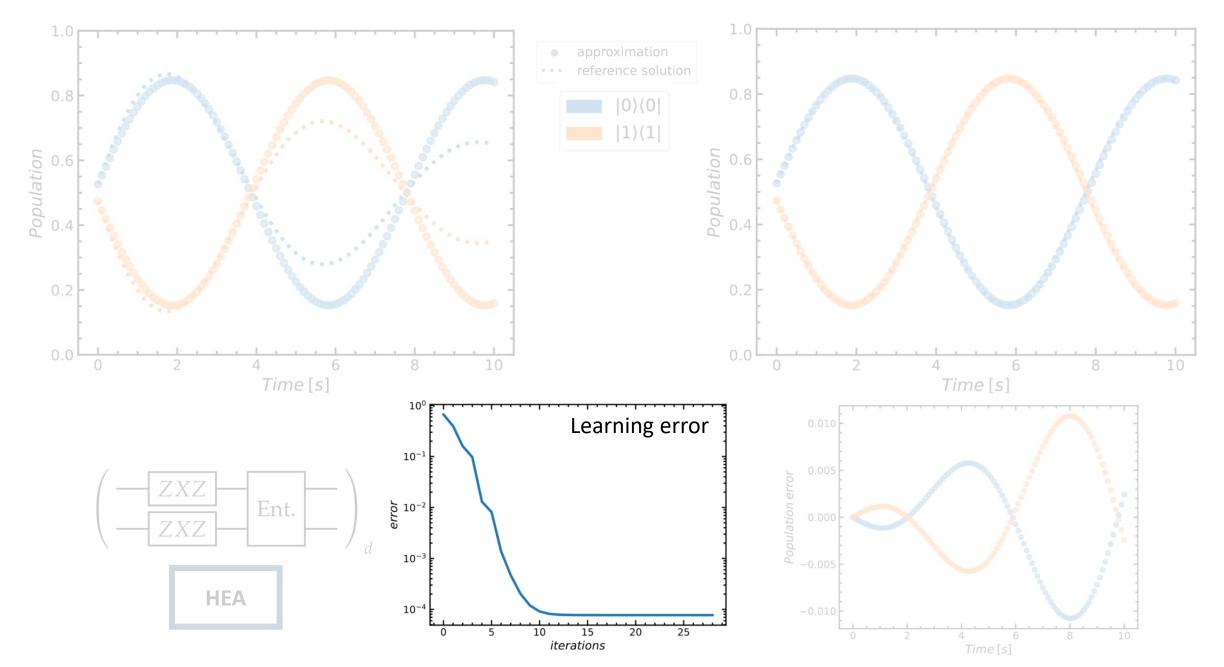
$$\lim_{n \to \infty} (e^{-iA\frac{t}{n}}e^{-iB\frac{t}{n}})^n = e^{-i(A+B)t}$$

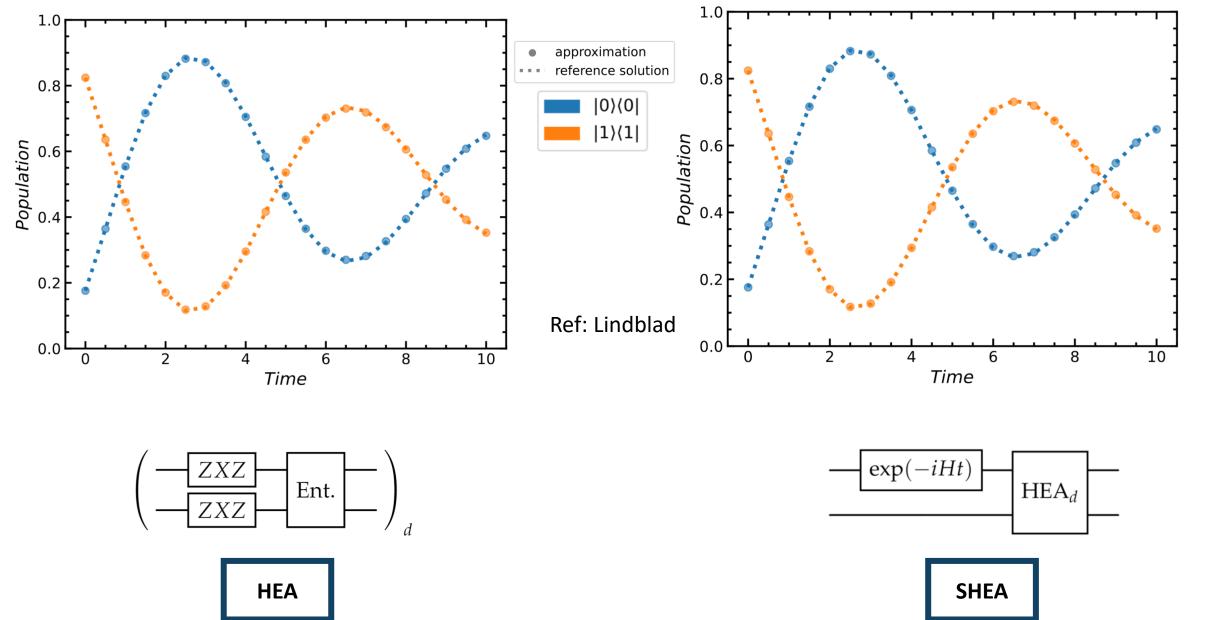
$$\begin{array}{c|c}
\rho_0 & & \\
|0\rangle_B & & \\
\end{array}$$
 $\begin{array}{c|c}
U_t(\theta) & = \\
\end{array}$
 $\begin{array}{c|c}
\exp(-iH\frac{t}{d}) & \\
\end{array}$
"Same parameters"

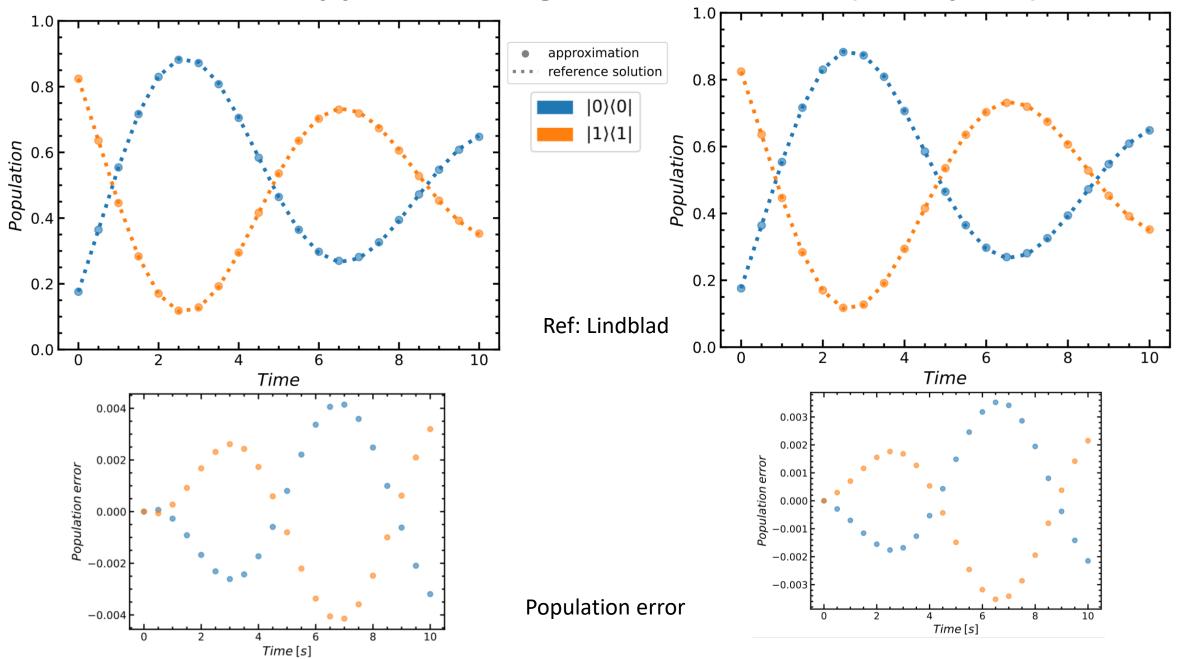


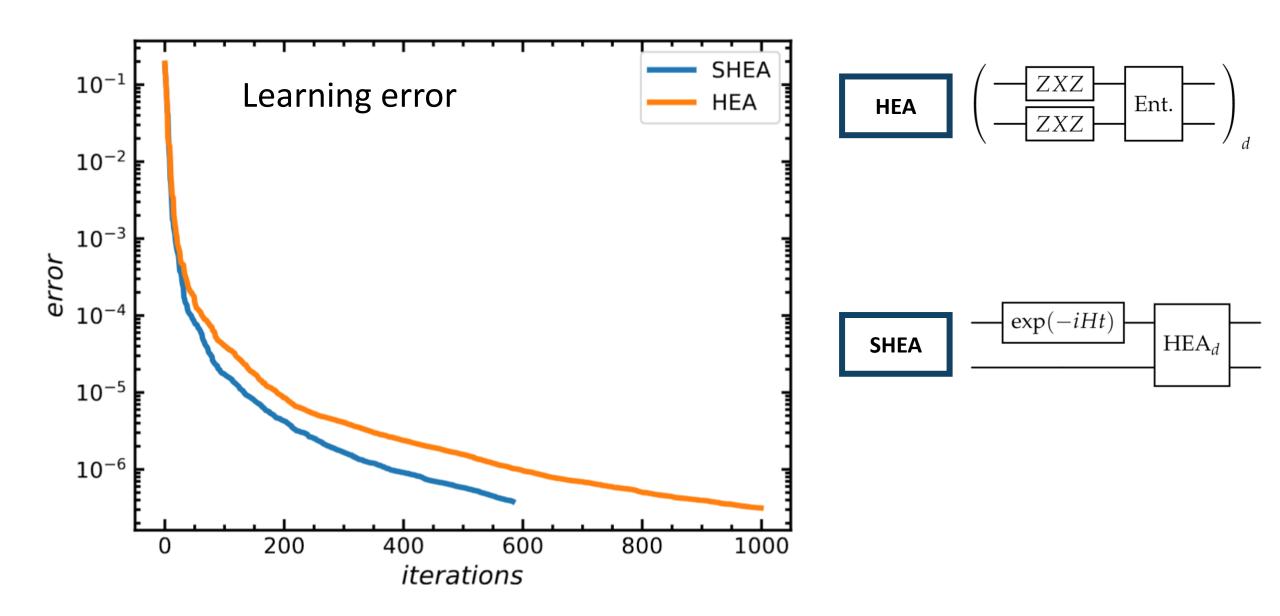


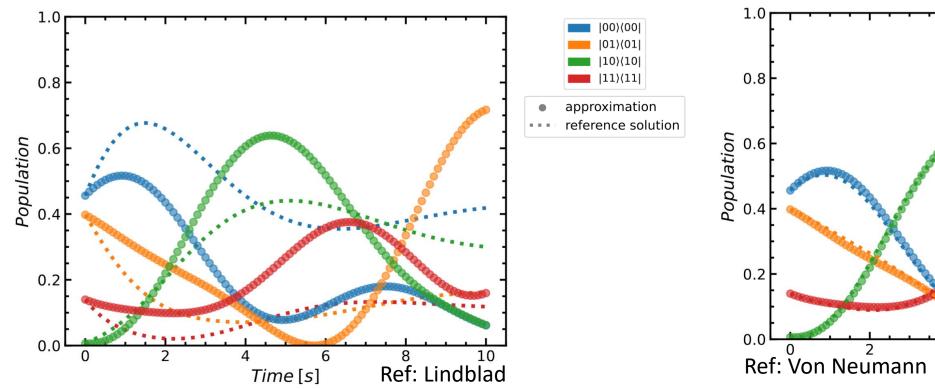


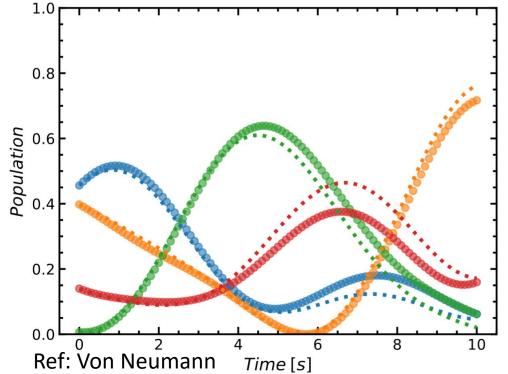


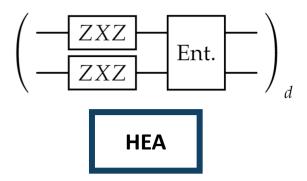


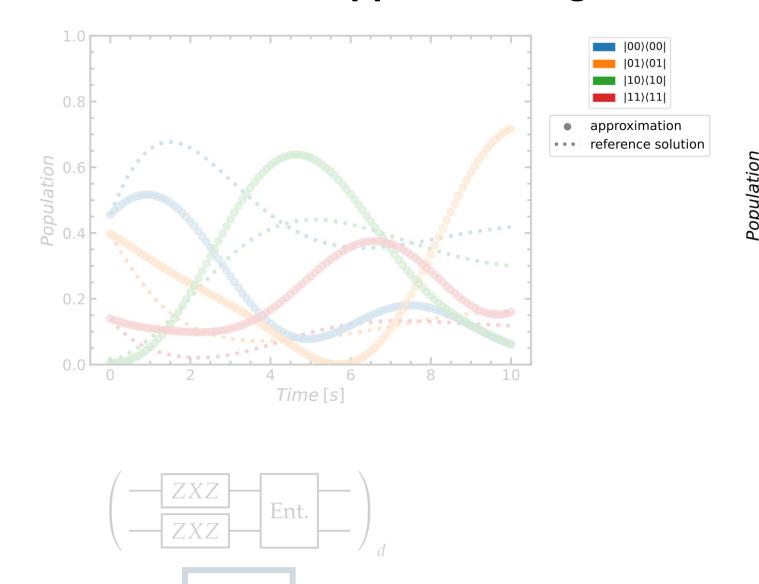




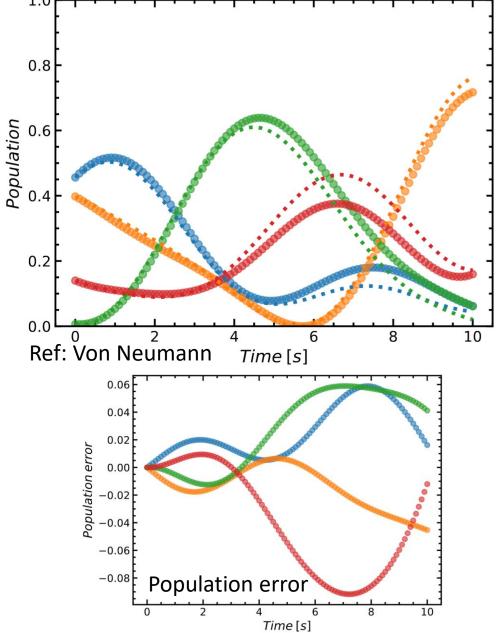


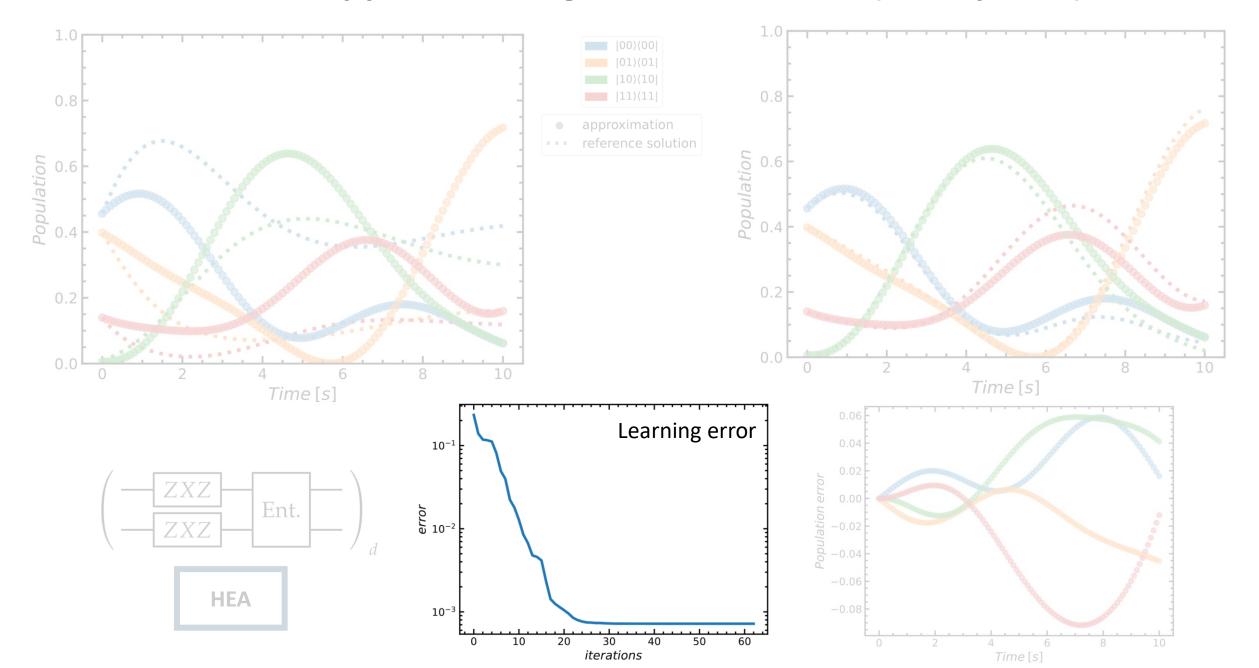


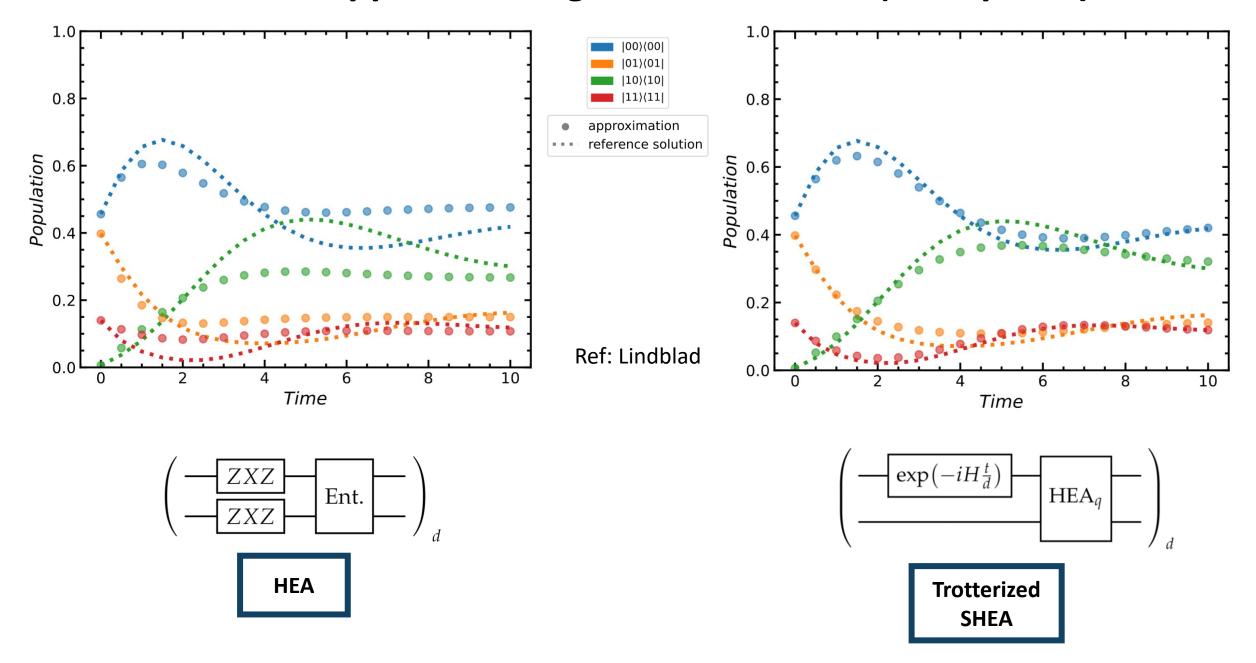


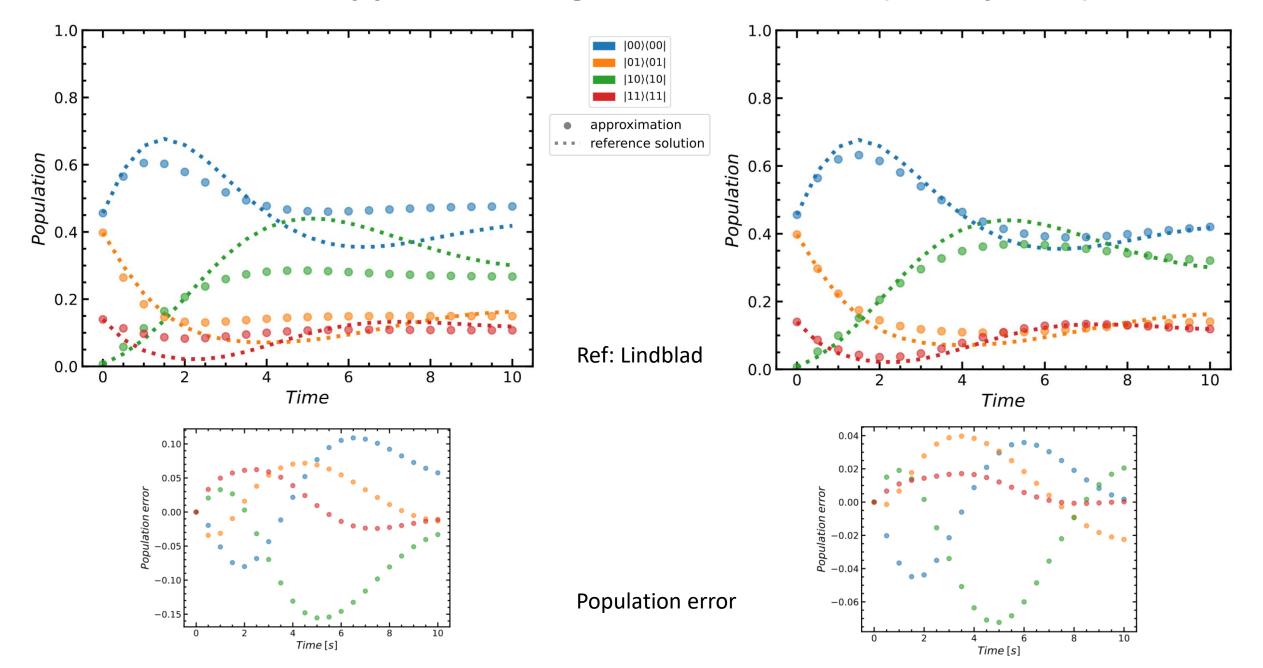


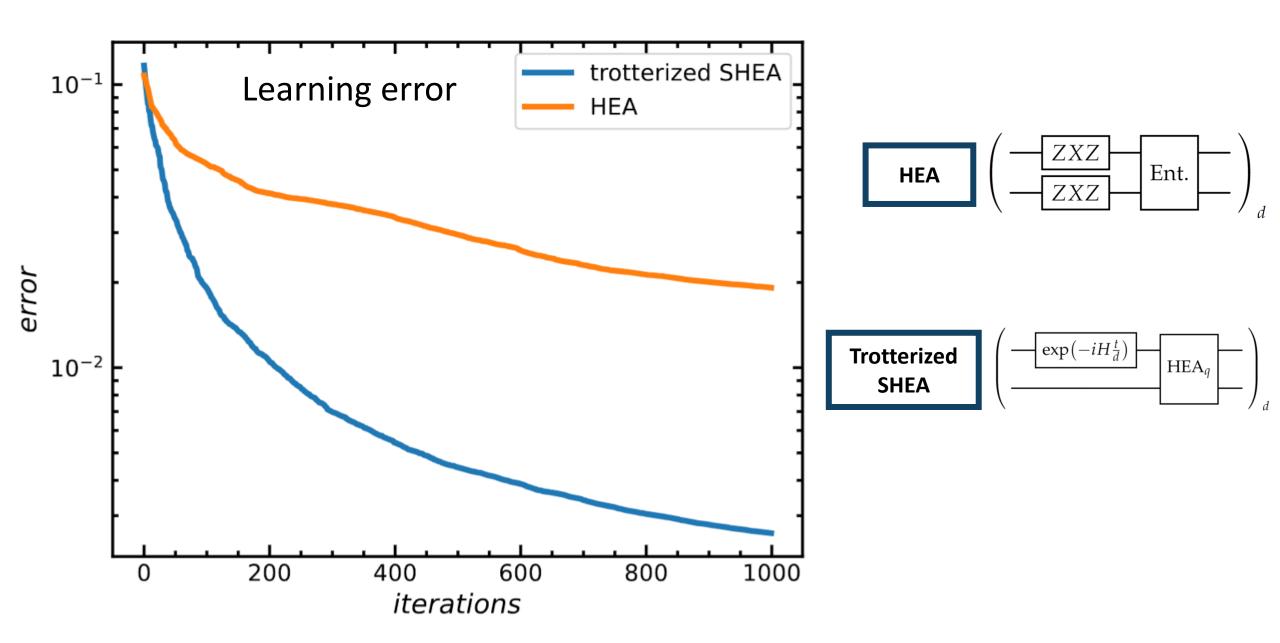
HEA











Future research

- How much training data is needed?
- What is the relation between depth and expressibility?
- Implement on quantum computer