Approximating open quantum systems

Using Stinespring dilation

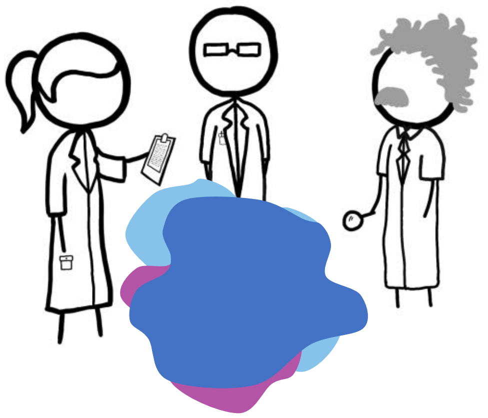
BFP Applied Physics, Applied Mathematics
David Chen, 1742477

My project in one minute

Create a model

(Most scientists)

Compare to experiment



My project in one minute

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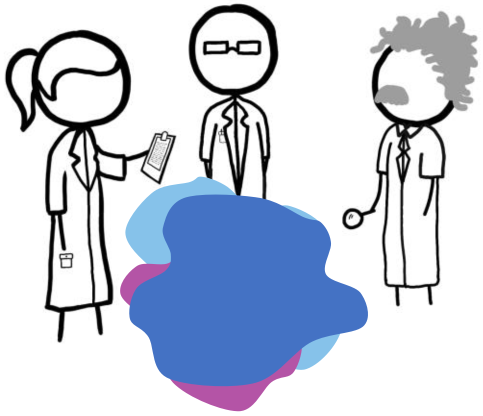


Infer model

(Me)

Make measurements

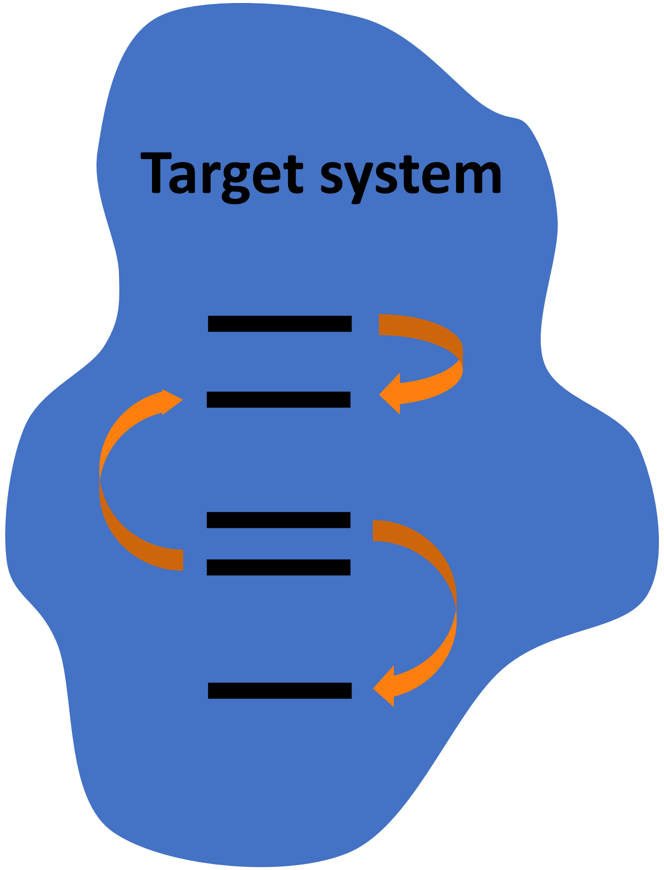
Agenda

Background

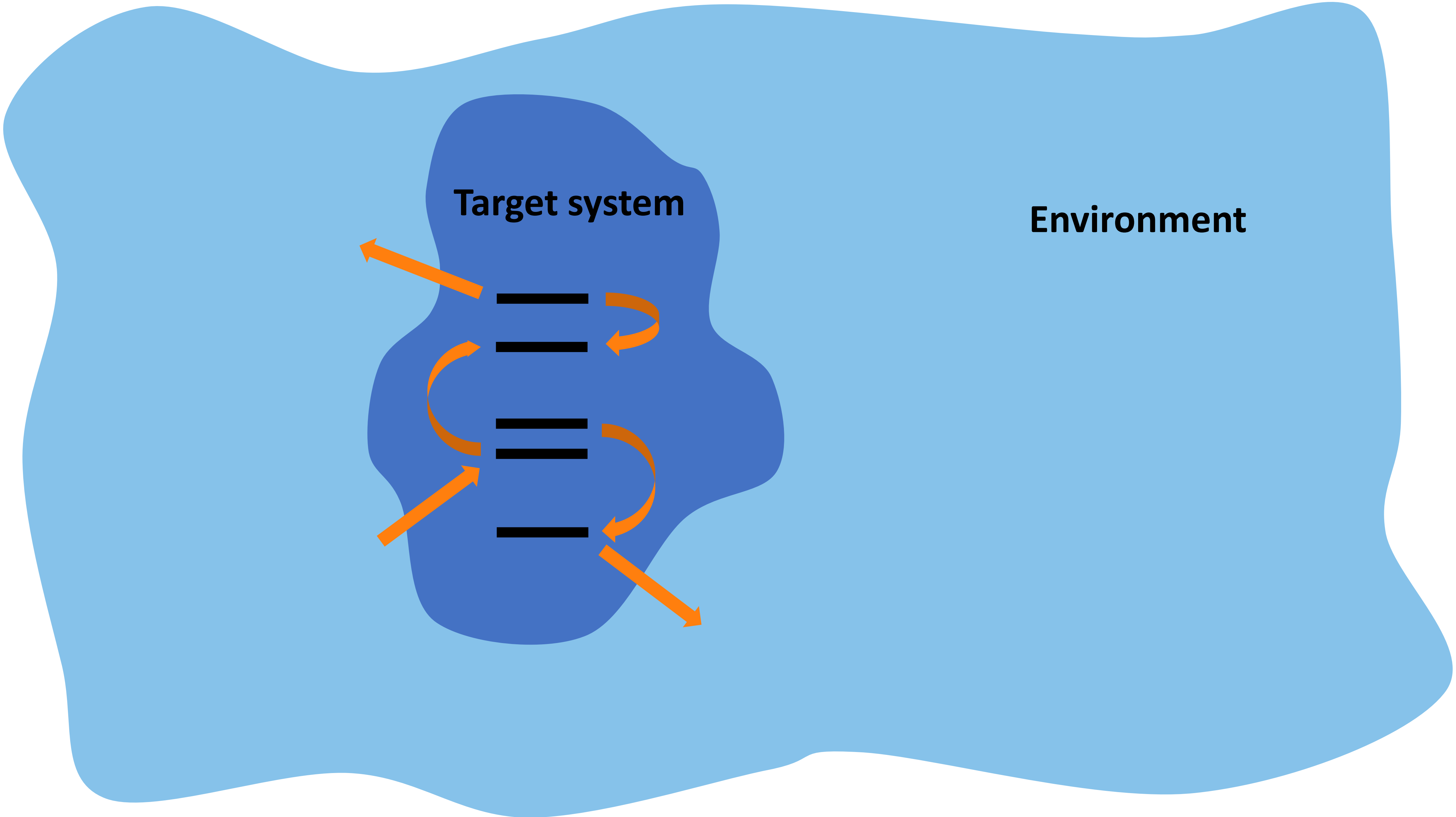
- Quantum systems
- Density matrix formalism
- Evolution equations
- Quantum channels
- Combining qubits
- Stinespring’s dilation theorem

My project

What is a quantum system?



What is an open quantum system?



Density matrix formalism

$$|\psi\rangle = \alpha_n \, |e^n\rangle$$

$$= \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix}$$

???

Pure state

$$\varrho = |\psi\rangle\langle\psi|$$

$$= \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} \begin{bmatrix} \overline{\alpha}_1 & \dots & \overline{\alpha}_N \end{bmatrix} = \begin{bmatrix} \alpha_1\overline{\alpha}_1 & \dots & \alpha_1\overline{\alpha}_N \\ \vdots & \ddots & \vdots \\ \alpha_N\overline{\alpha}_1 & \dots & \alpha_N\overline{\alpha}_N \end{bmatrix}$$

Mixed state

$$\rho = \sum_k p_k |\psi^k\rangle\langle\psi^k|, \quad \sum_k p_k = 1$$

Density matrix formalism

Pure state

$$|\psi\rangle = \alpha_n \left|e^n\right\rangle = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix}$$

$\xrightarrow{N}$

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Mixed state

???

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$$\rho = \sum_k p_k |\psi^k\rangle\langle\psi^k|, \quad \sum_k p_k = 1$$

Evolution equations

Closed system (Schrödinger equation)

(Von Neumann equation)

$$\frac{d}{dt}|\psi\rangle = -\frac{i}{\hbar}H|\psi\rangle$$

Density matrix formalism

N

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho]$$

Open system (Lindblad equation)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_k \gamma_k \left(A_k \rho A_k^\dagger - \frac{1}{2}\{A_k^\dagger A_k, \rho\} \right)$$

Evolution equations

Closed system (Schrödinger equation)

$$\frac{d}{dt}|\psi\rangle = -\frac{i}{\hbar}H|\psi\rangle$$

Density matrix formalism $\longrightarrow$

(Von Neumann equation)

$$\frac{d}{dt}\rho = \underbrace{-\frac{i}{\hbar}[H,\rho]}_{\text{“Hamiltonian”}}$$

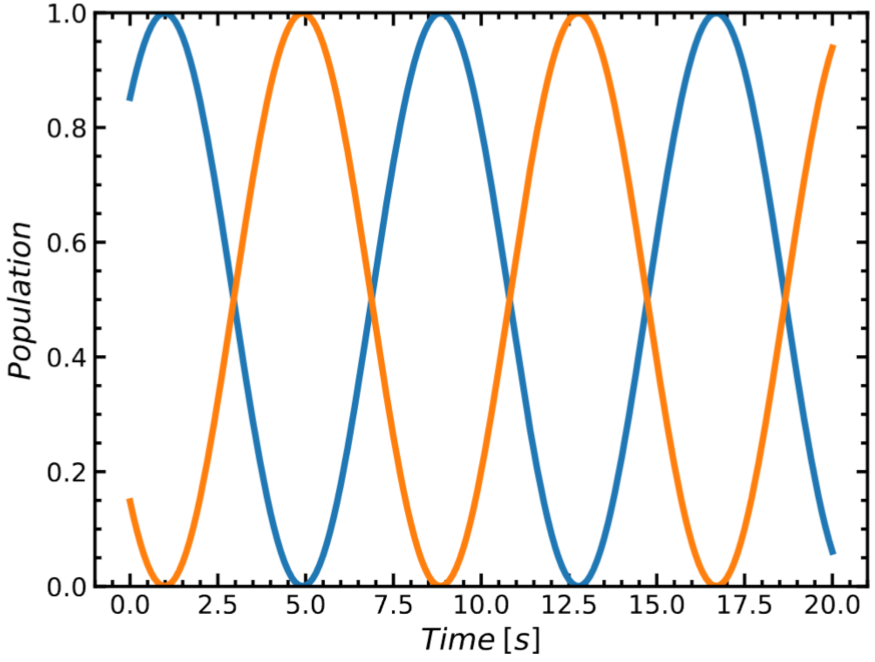
Open system (Lindblad equation)

$$\frac{d}{dt}\rho = \underbrace{-\frac{i}{\hbar}[H,\rho]}_{\text{“Hamiltonian”}} + \underbrace{\sum_k \gamma_k \left(A_k \rho A_k^\dagger - \frac{1}{2}\{A_k^\dagger A_k, \rho\} \right)}_{\text{“Interactions”}}$$

Evolution equations

Closed system (Von Neumann)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho]$$



Open system (Lindblad)

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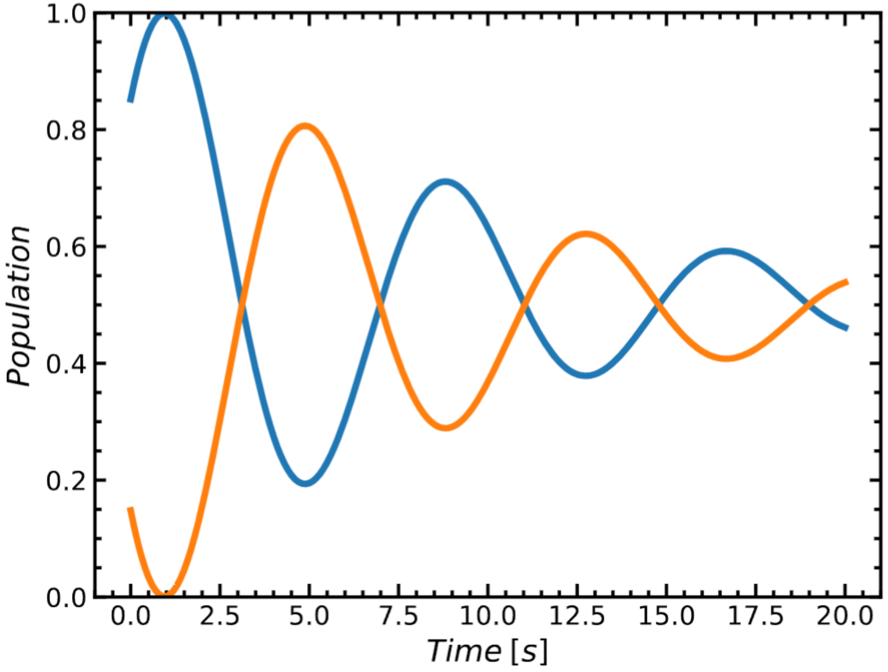

Quantum channels

Closed system (Von Neumann)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho]$$

$$U_t \rho_0 U_t^\dagger = \rho_t,$$

$$U_t = \exp\left(-\frac{i}{\hbar}Ht\right)$$

Open system (Lindblad)

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Goal: approximate $\Phi_t(\rho_0) = \rho_t$

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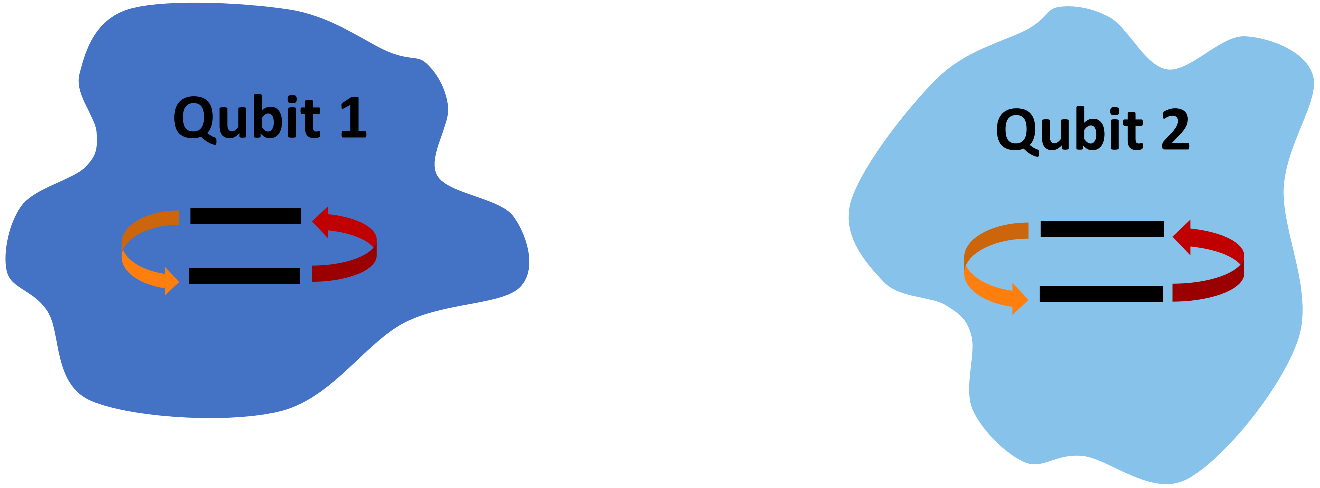
Combining qubits



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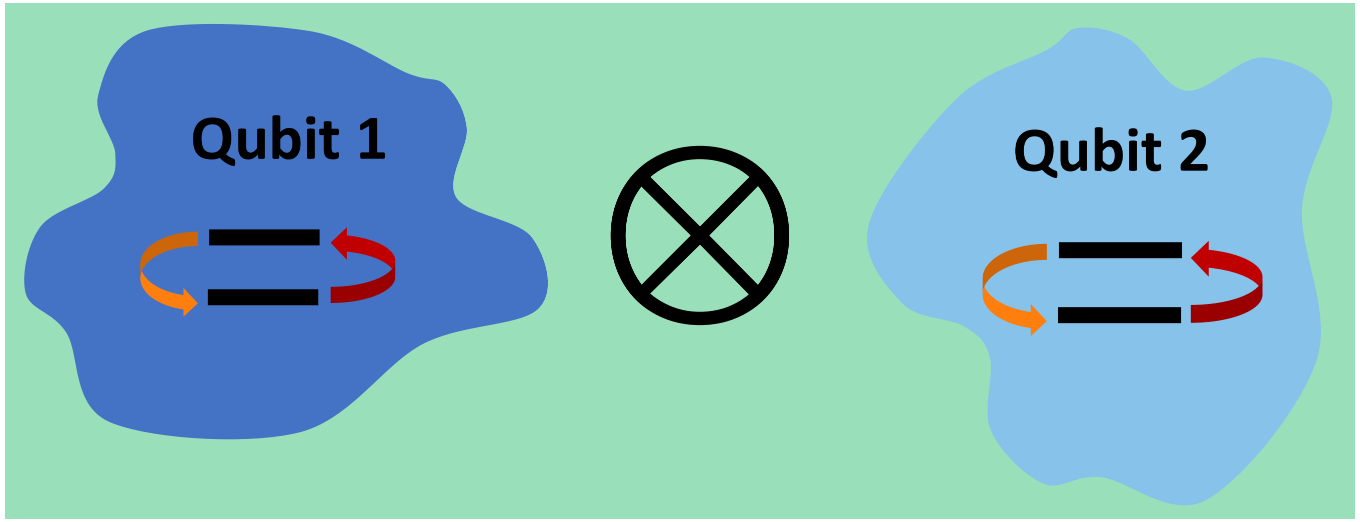
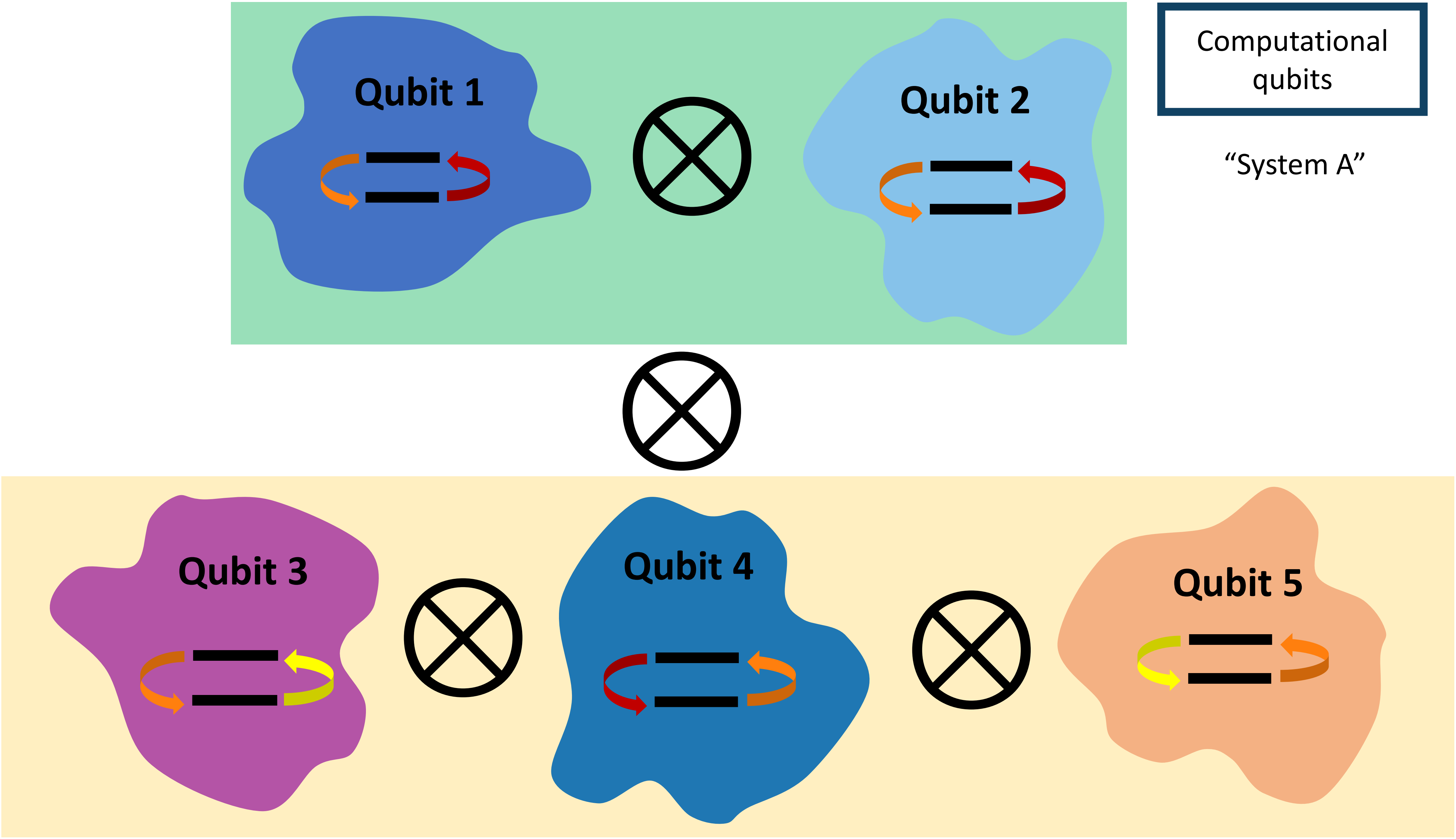


Computational qubits

Combining qubits



Computational qubits

“System A”

Ancilla qubits

“System B”

Stinespring's dilation theorem

There exists U_t on dilated space (computational qubits) $\otimes$ (ancilla qubits) such that:

$$\boxed{\Phi_t(\rho_0) = \mathrm{Tr}_B[U_t\,(\rho_0 \otimes |0\rangle_B\langle 0|_B)\,U_t^\dagger]}$$

Stinespring's dilation theorem

There exists U_t on dilated space (computational qubits) $\otimes$ (ancilla qubits) such that:

$$\Phi_t(\rho_0) = \mathrm{Tr}_B\left[U_t\left(\rho_0 \otimes |0\rangle_B\langle 0|_B\right)U_t^\dagger\right]$$

"Stinespring unitary"

Agenda

My project

- Approximation method
- Parametrization of Stinespring unitary
- Results

Approximation method

Gather training data

(Initial state) $\rho_0 \xrightarrow{\text{Evolve for time } t} \rho_t \xrightarrow{\text{Measure expectations of } O_1, O_2, \ldots}$ (Expectation values) $E_1, E_2, \ldots$

Repeat for different initial states

Approximate quantum channel

$$\Phi_t(\rho_0) = \mathrm{Tr}_B[U_t\,(\rho_0 \otimes |0\rangle_B\langle 0|_B)\,U_t^\dagger]$$

$\Phi_t(\rho_0) \approx \rho_t$

Optimize parameters

$$J(\theta) = \sum_\ell (E_\ell - \hat{E}_\ell)^2$$

Approximation method

Gather training data

(Initial state) ρ_0 → Evolve for time t → ρ_t → Measure expectations of $O_1, O_2, \ldots$ → (Expectation values) $E_1, E_2, \ldots$

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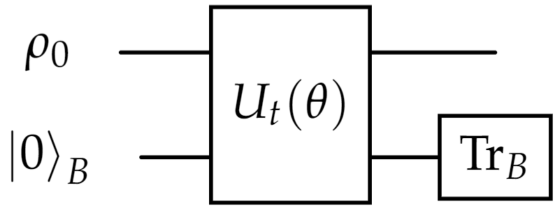
Parametrize Stinespring unitary →

"Quantum circuit"



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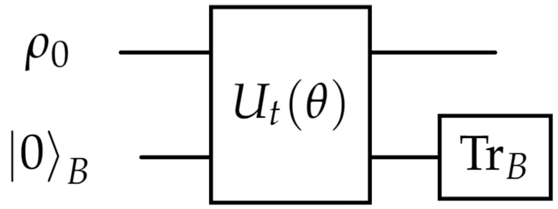


$\Phi_t(\rho_0) \approx \rho_t$

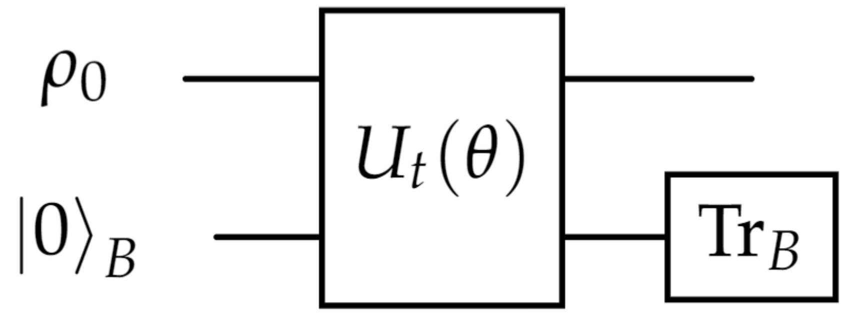
Optimize parameters

$$J(\theta) = \sum_\ell (E_\ell - \hat{E}_\ell)^2$$

Measured

Approximated

Parametrization of Stinespring unitary

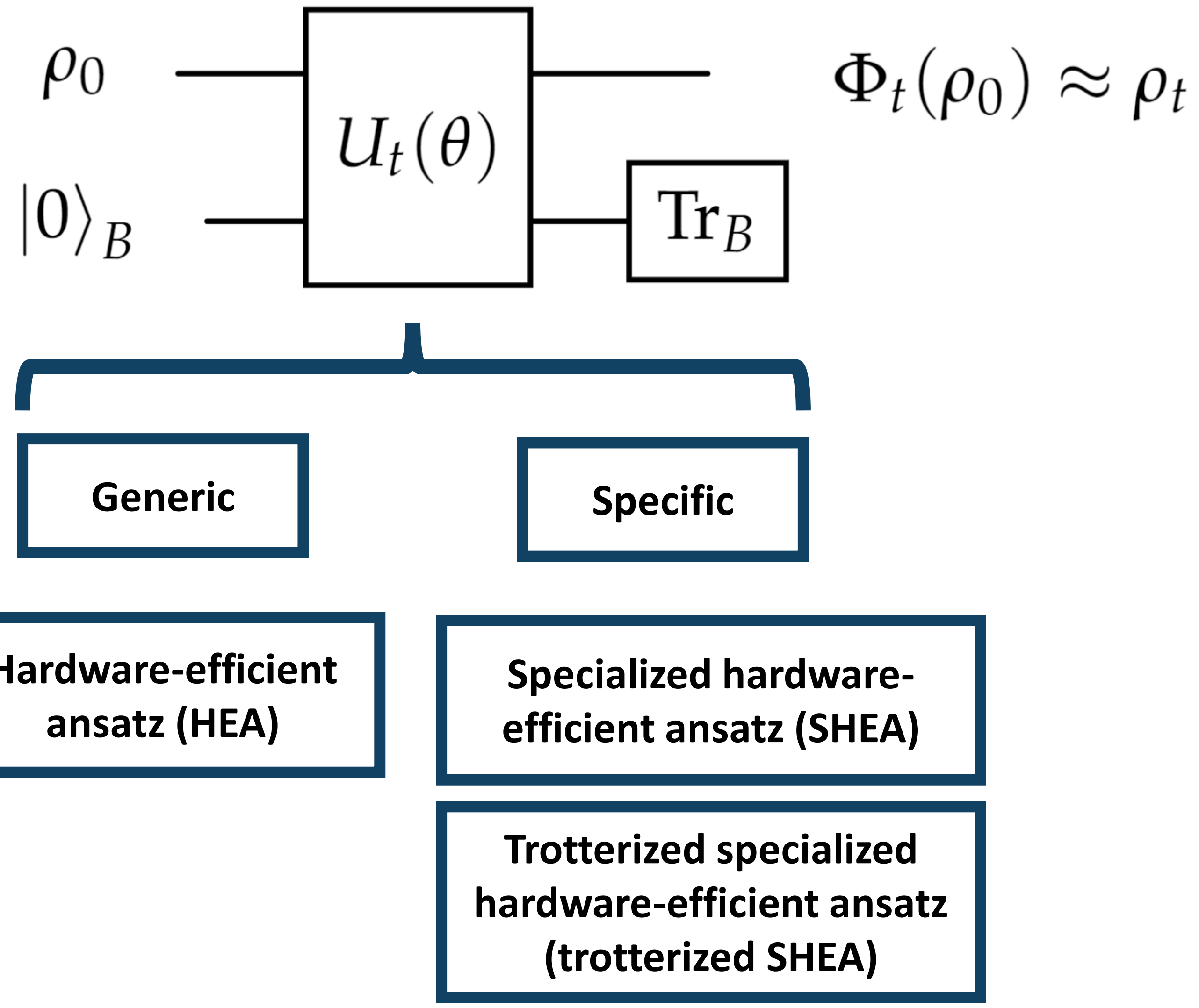


$$\Phi_t(\rho_0) \approx \rho_t$$

Generic

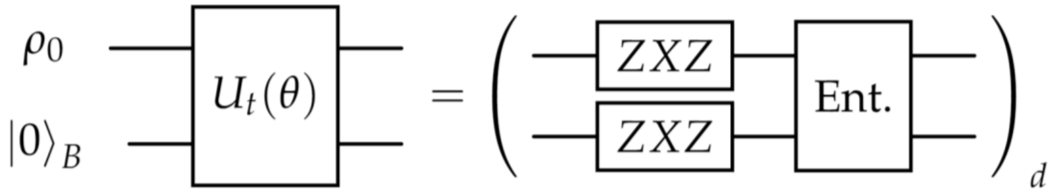
Hardware-efficient ansatz (HEA)

Parametrization of Stinespring unitary



Parametrization of Stinespring unitary

Hardware-efficient ansatz (HEA)



Specialized hardware-efficient ansatz (SHEA)

Interlude: trotterization

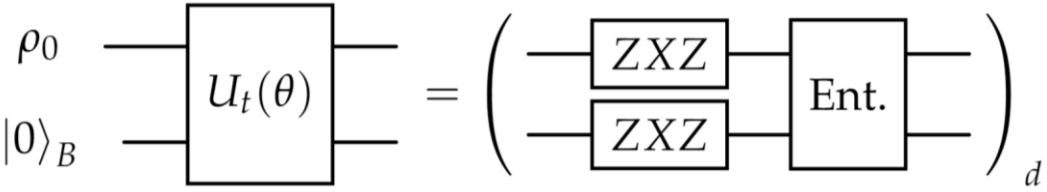
For Hermitian operators A and B we have:

$$\lim_{n\to\infty}\left(e^{-iA\frac{t}{n}}e^{-iB\frac{t}{n}}\right)^n = e^{-i(A+B)t}$$

Trotterized specialized hardware-efficient ansatz (trotterized SHEA)

Parametrization of Stinespring unitary

Hardware-efficient ansatz (HEA)



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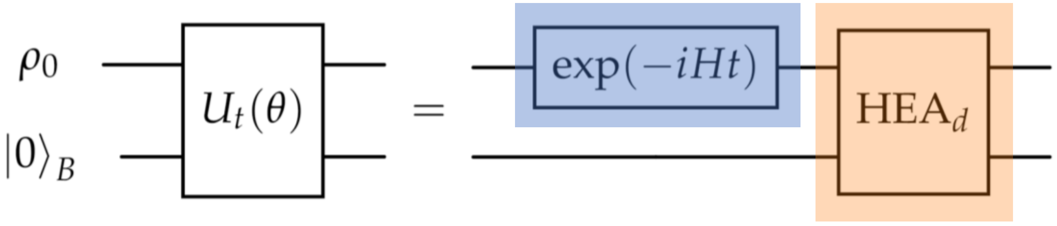

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_k \gamma_k \left(A_k \rho A_k^\dagger - \frac{1}{2}\{A_k^\dagger A_k, \rho\} \right)$$

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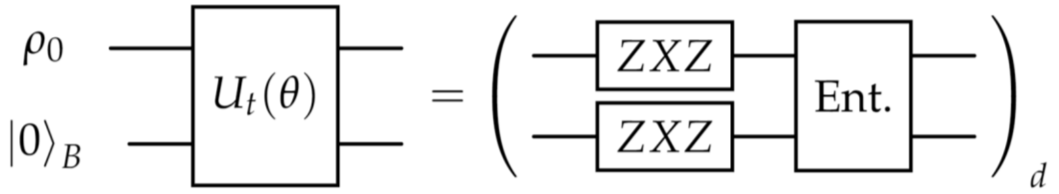

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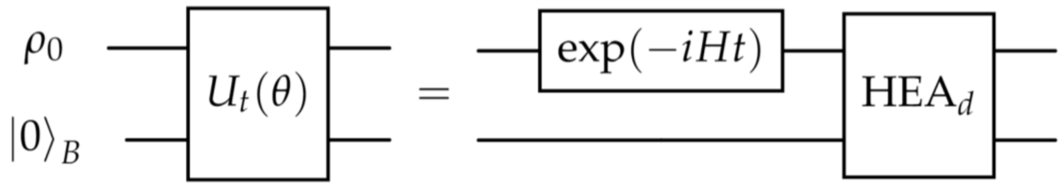

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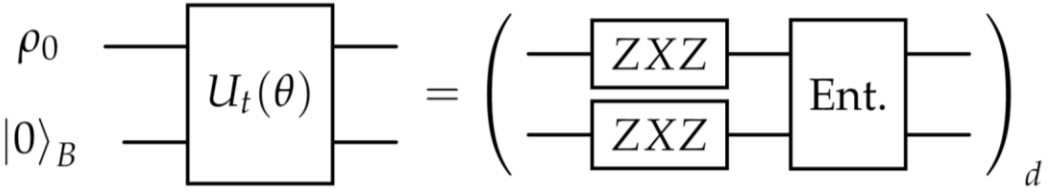

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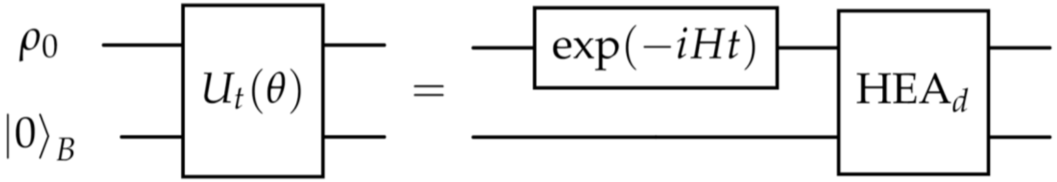

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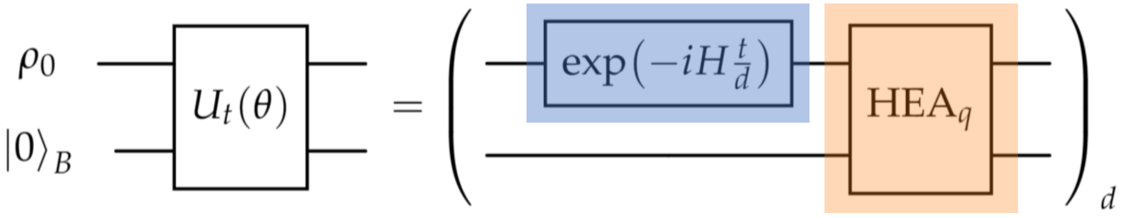
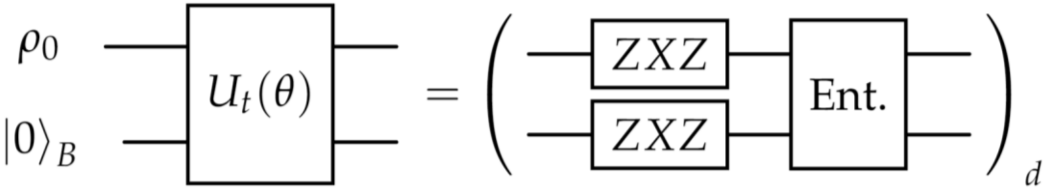


Parametrization of Stinespring unitary

Hardware-efficient ansatz (HEA)



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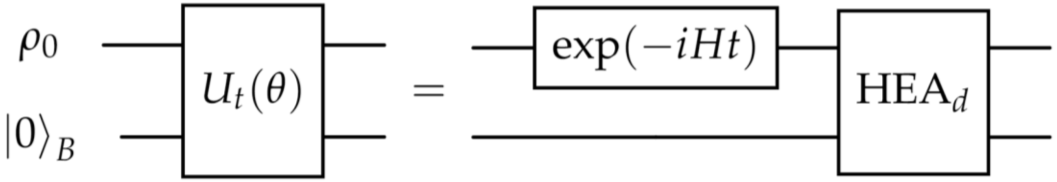

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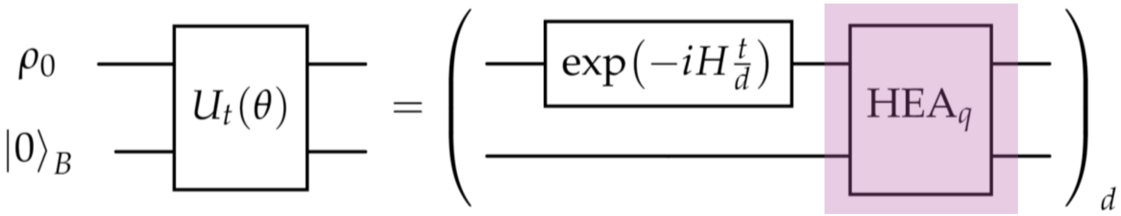
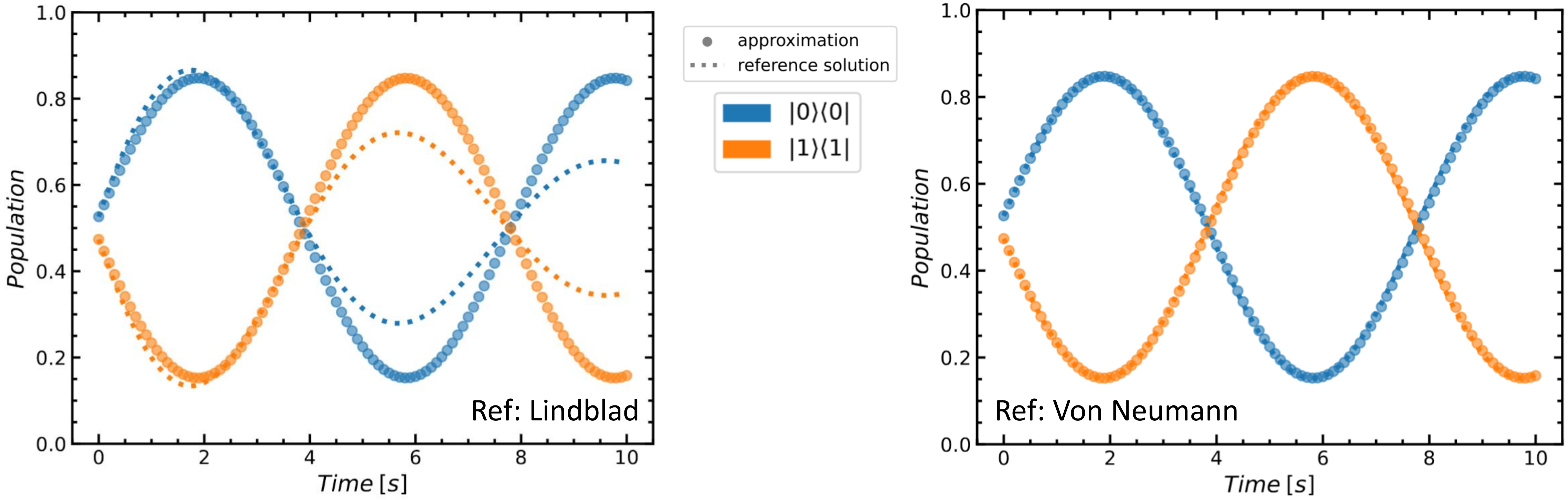
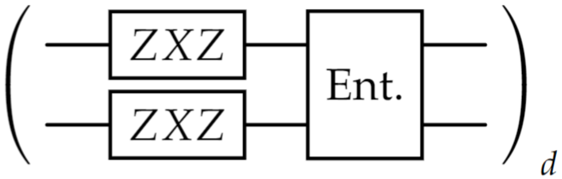


"Same parameters"

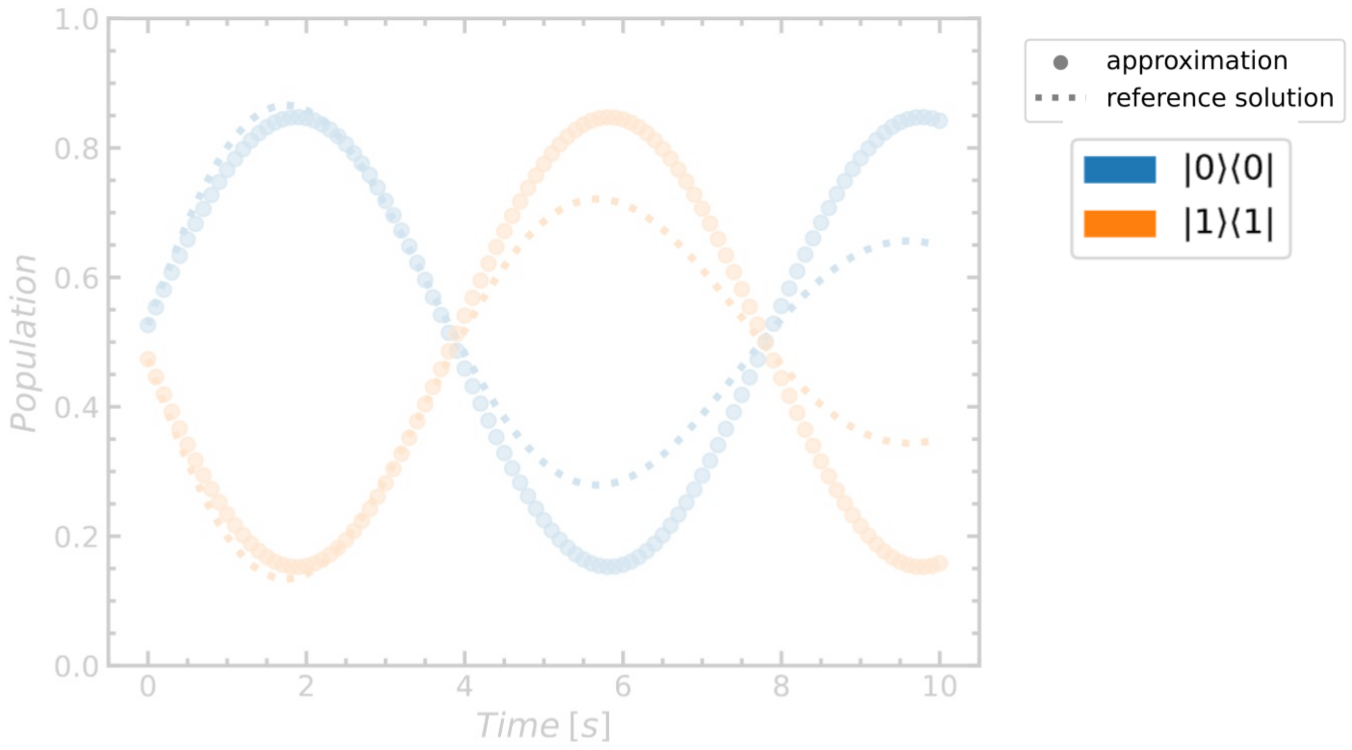
Results: approximating Rabi oscillation (one qubit)

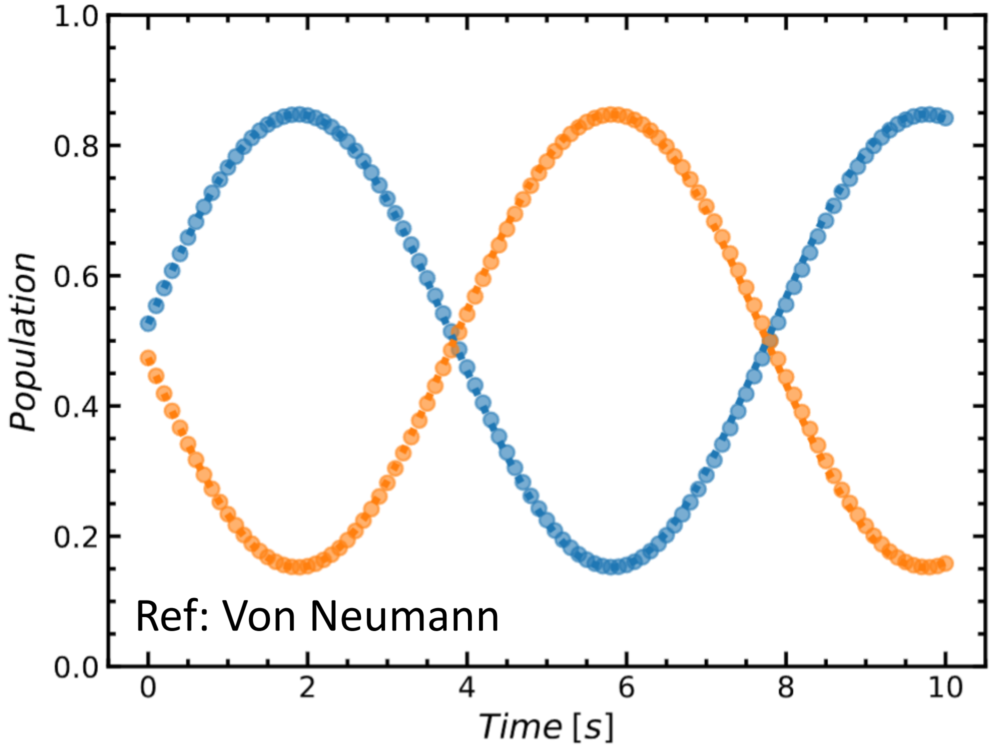


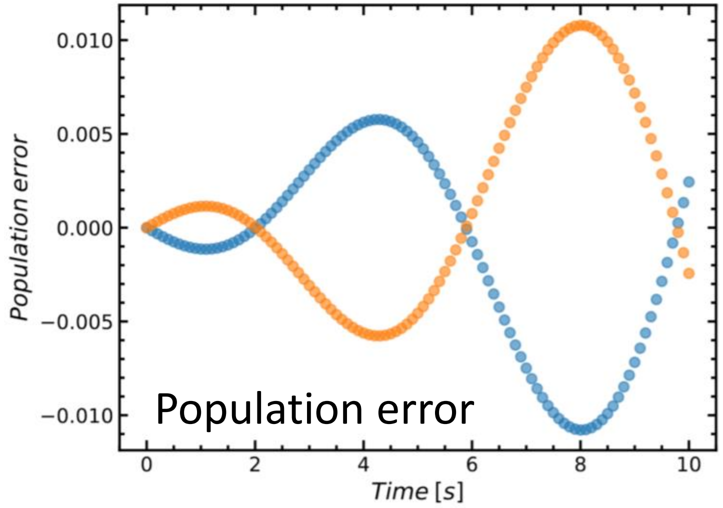


HEA

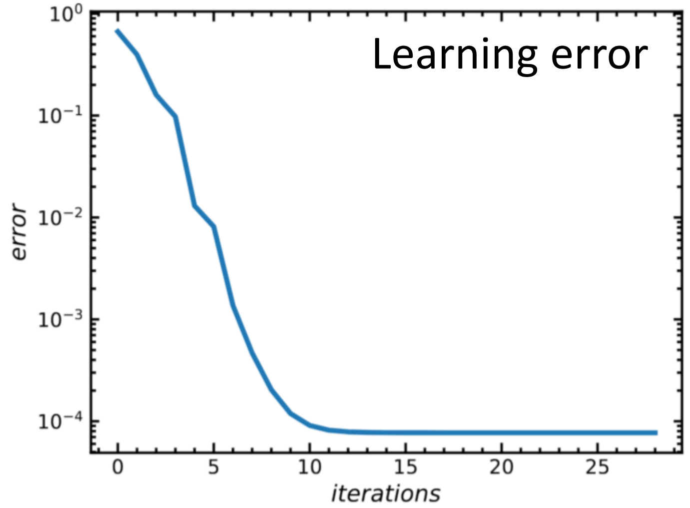
Results: approximating Rabi oscillation (one qubit)







Results: approximating Rabi oscillation (one qubit)



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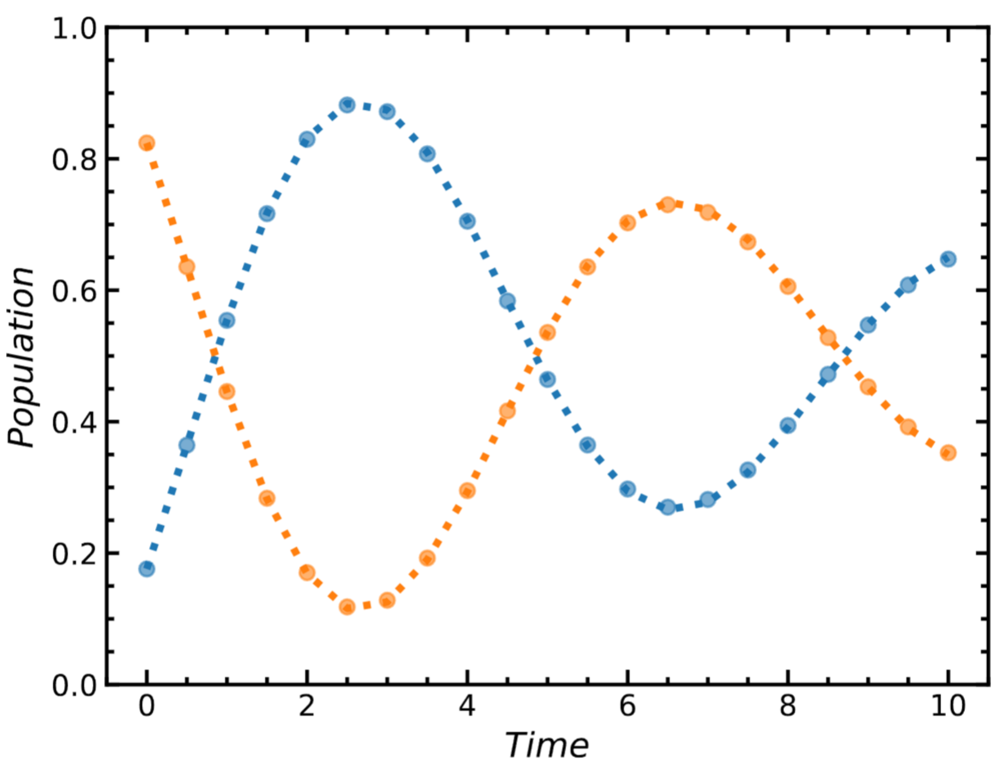
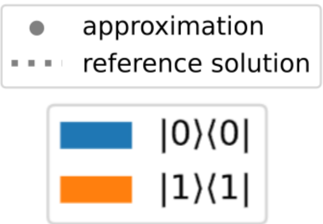
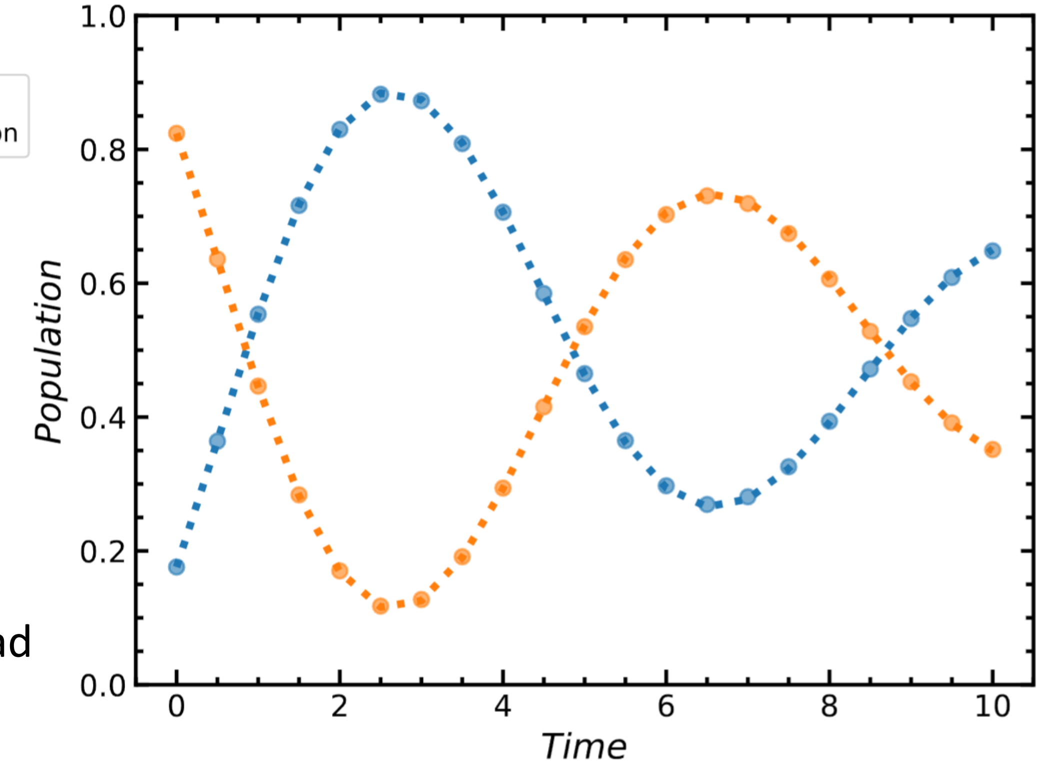
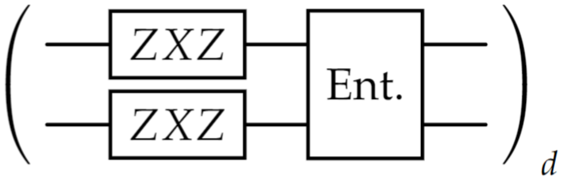


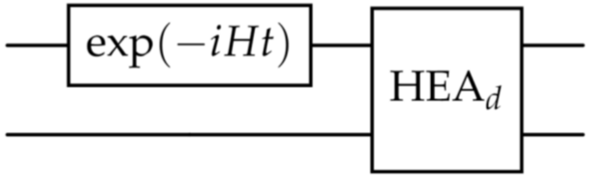




Ref: Lindblad

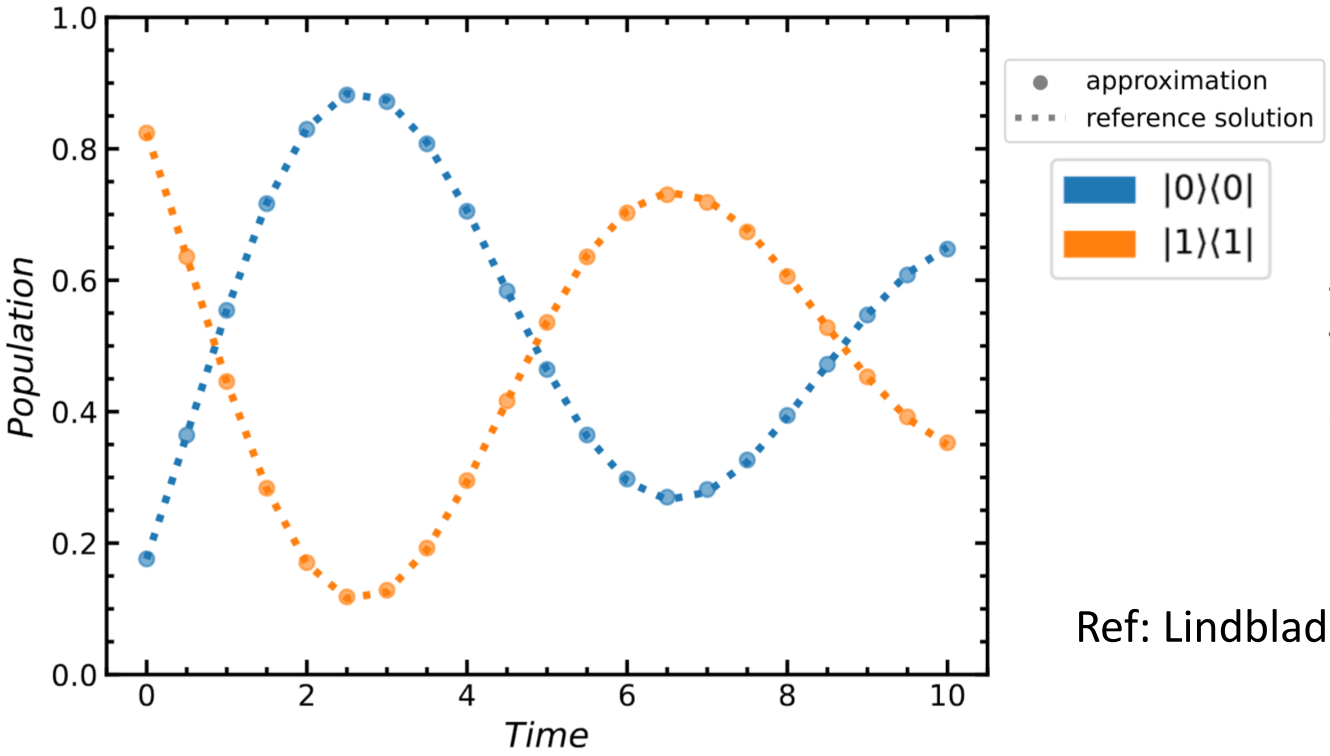


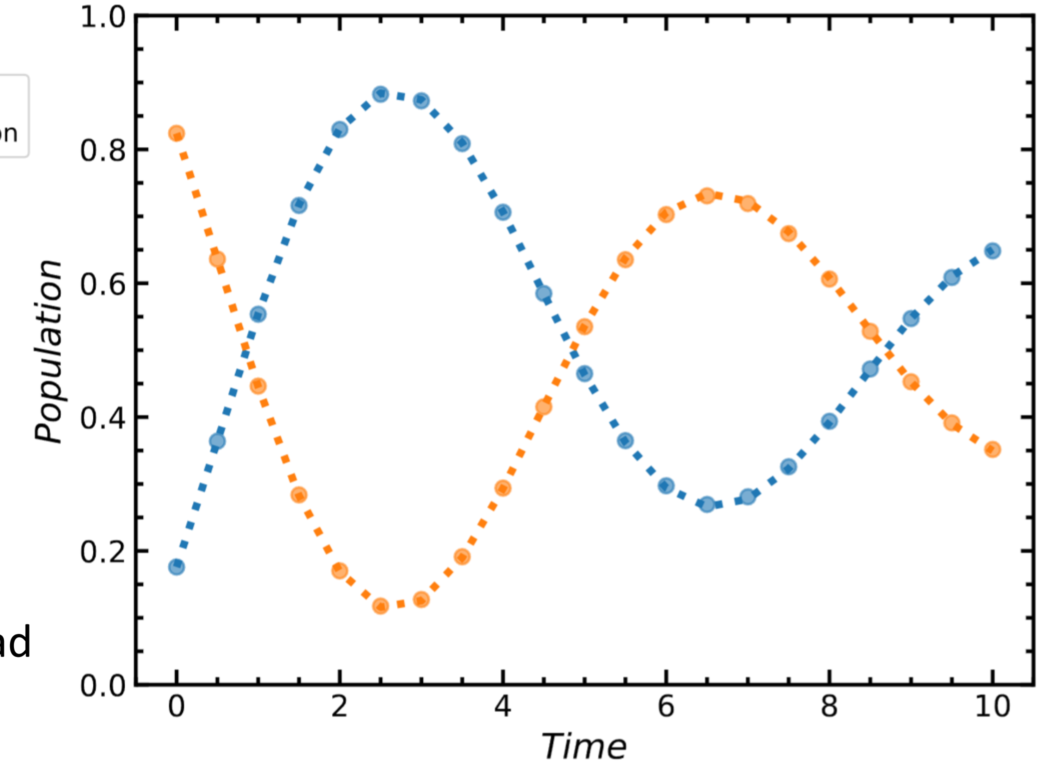
HEA



SHEA

Results: approximating Rabi oscillation (one qubit)





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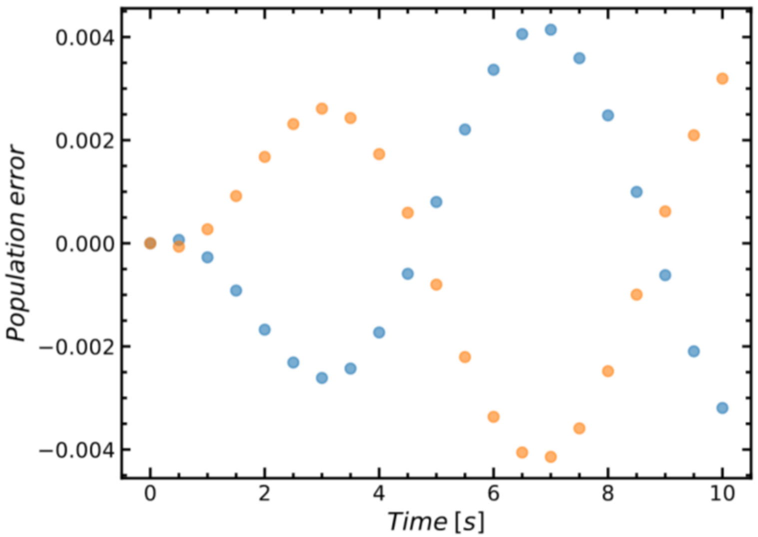
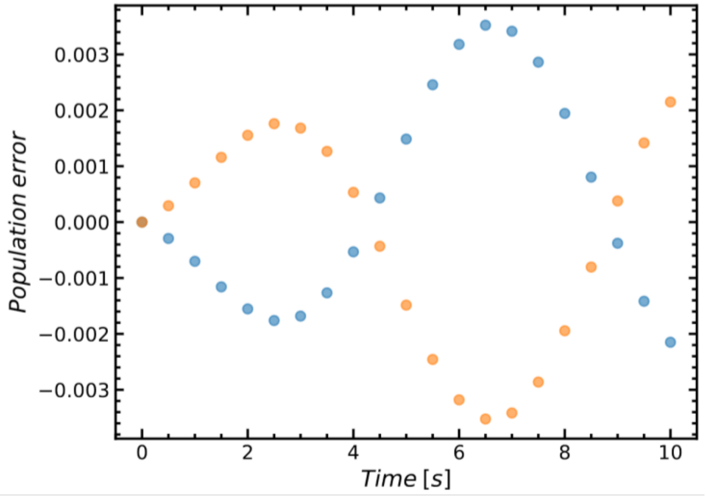
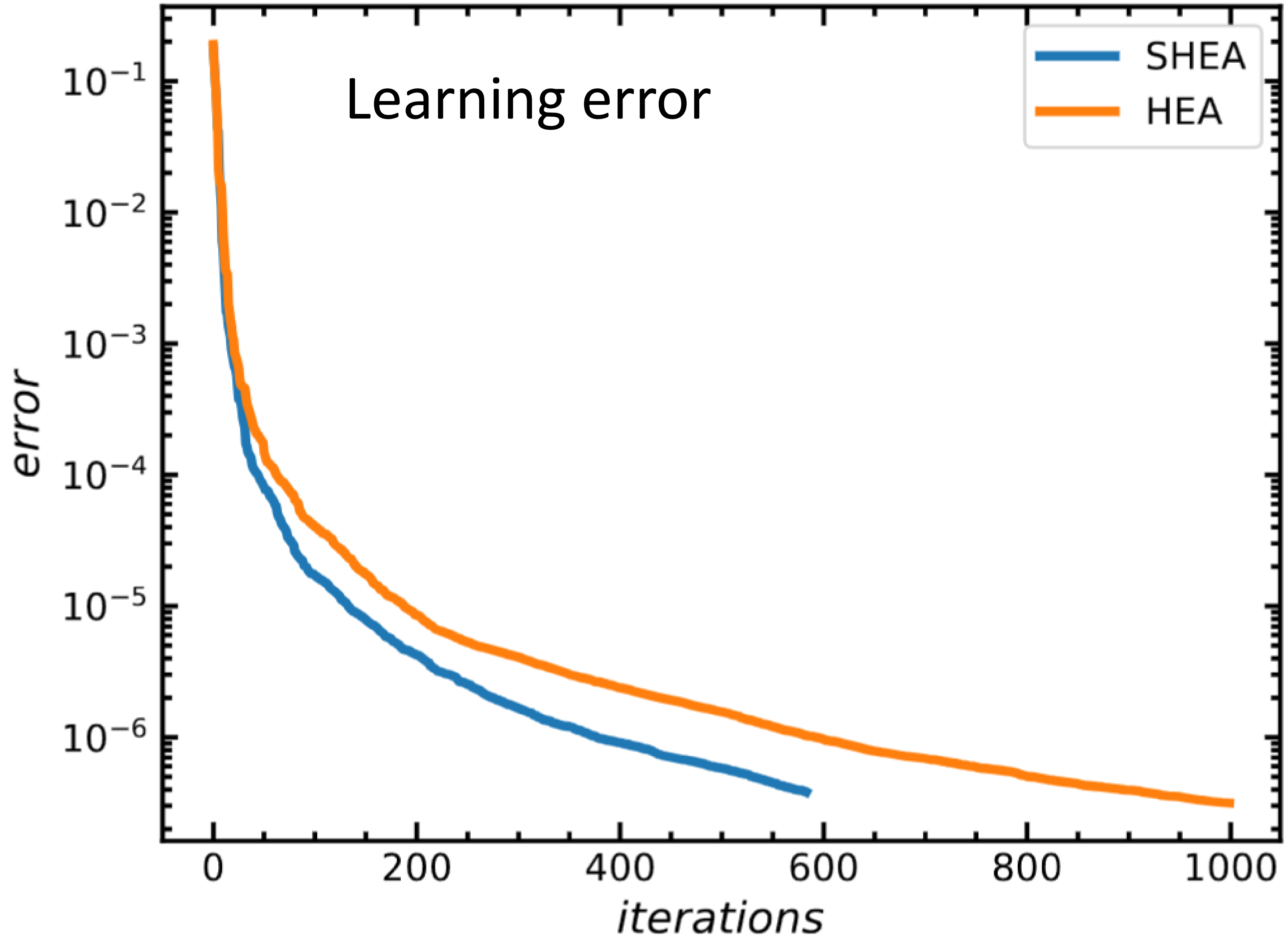
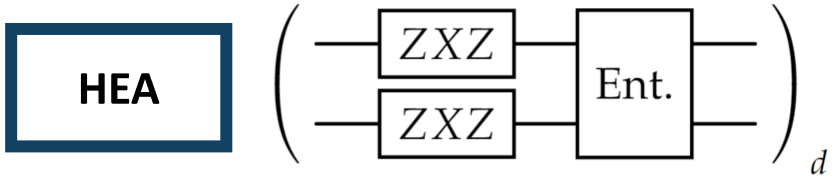


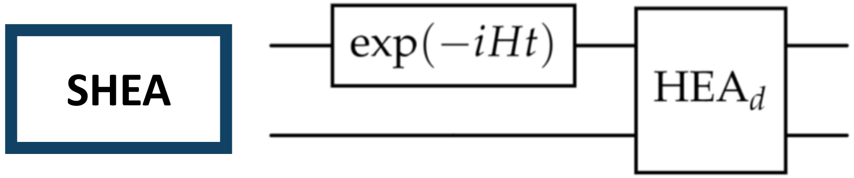


Population error

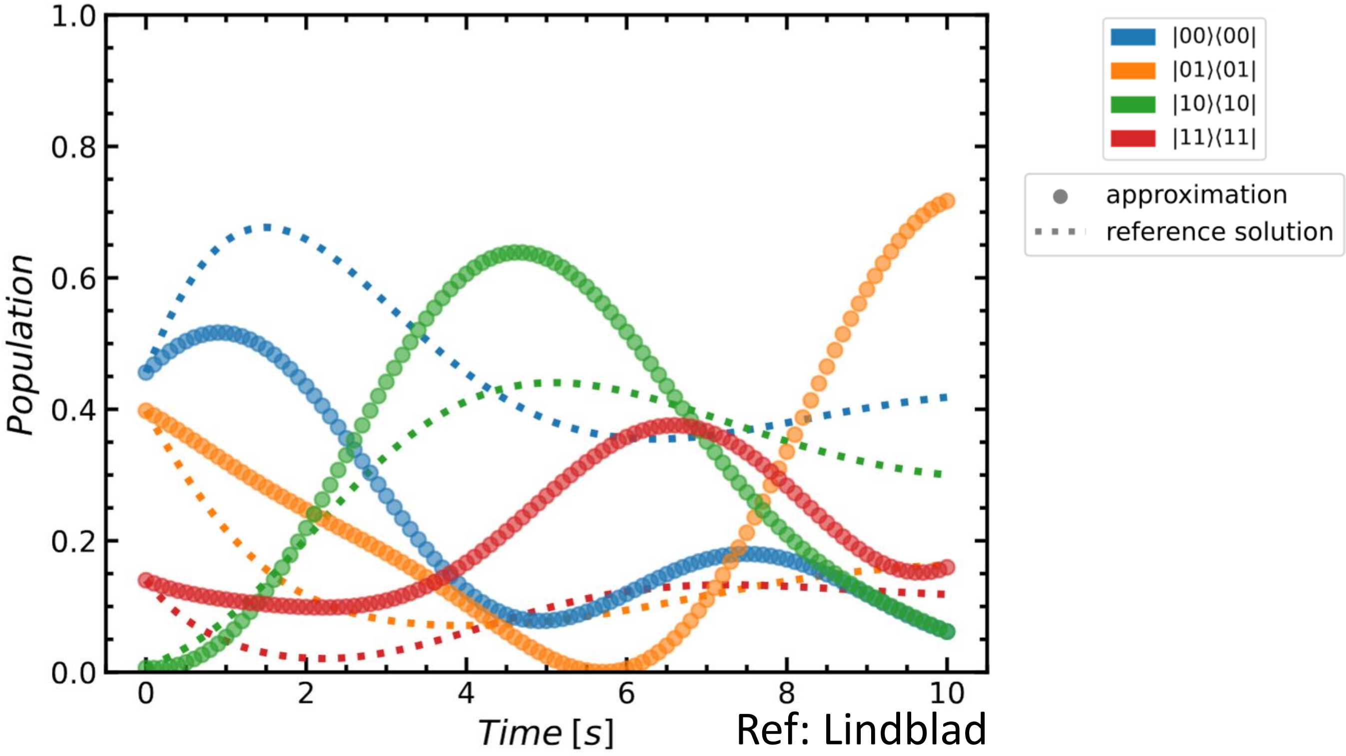
Results: approximating Rabi oscillation (one qubit)



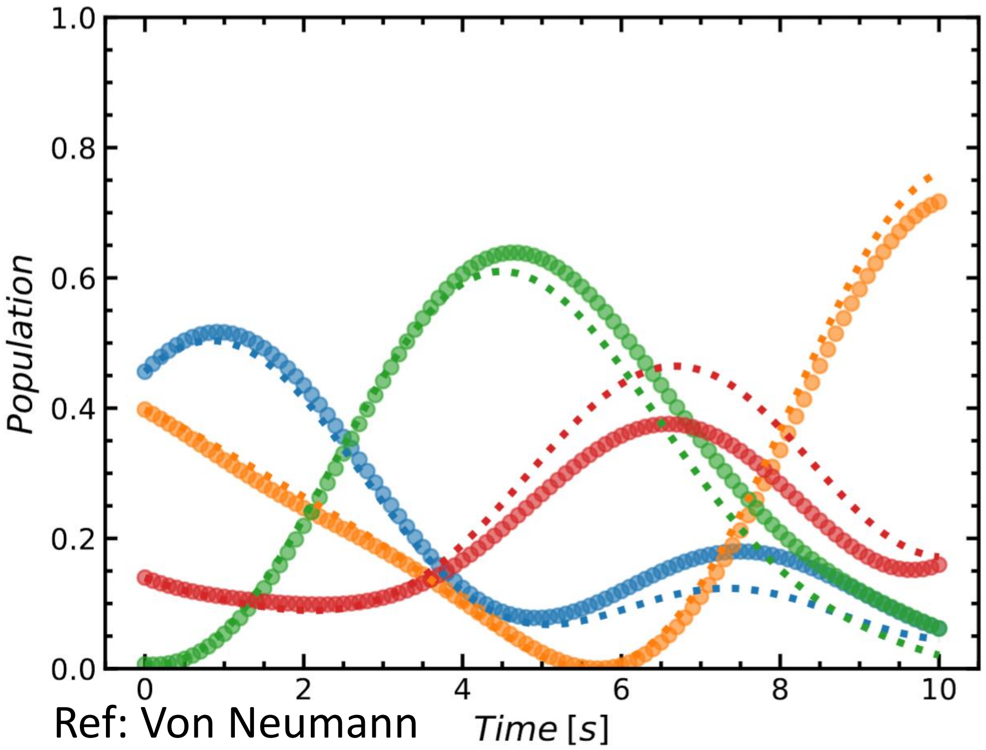




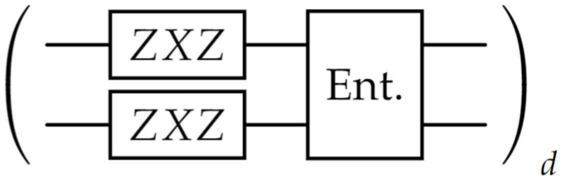
Results: approximating Rabi oscillation (two qubits)



Ref: Lindblad

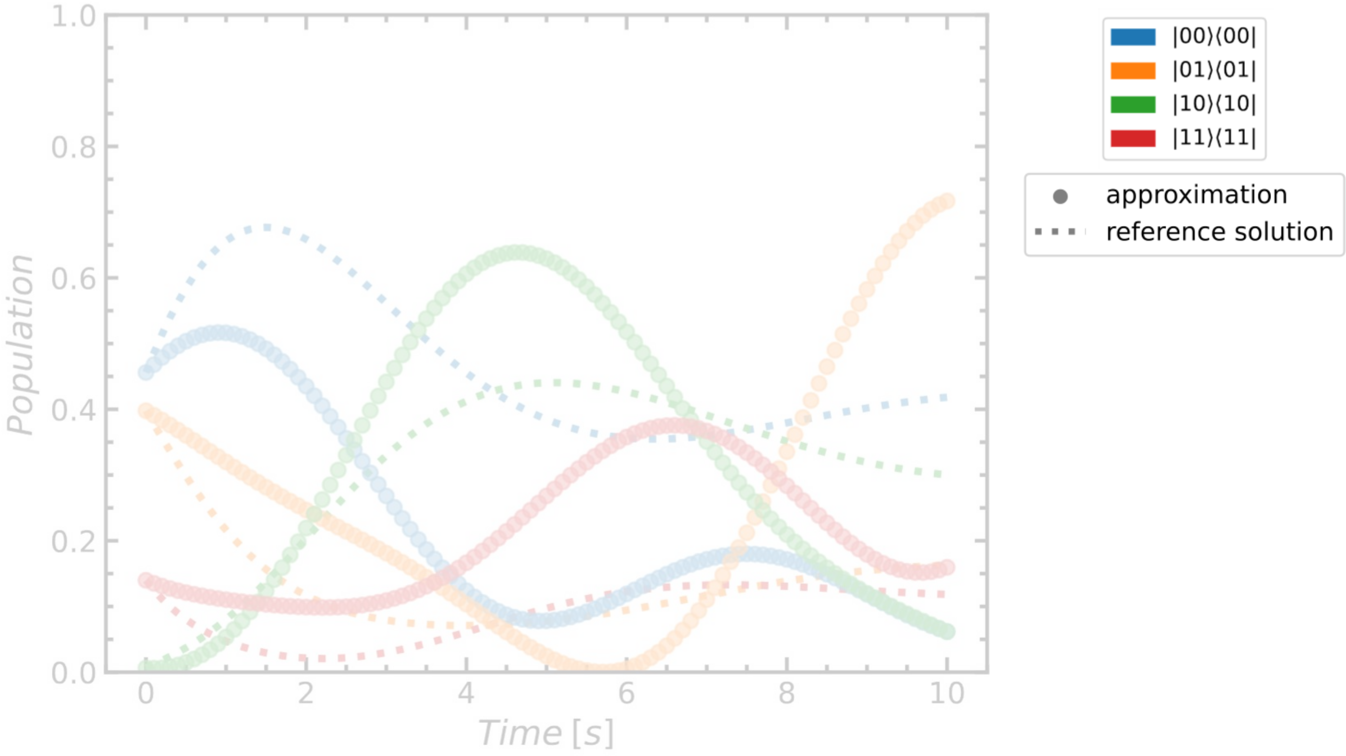


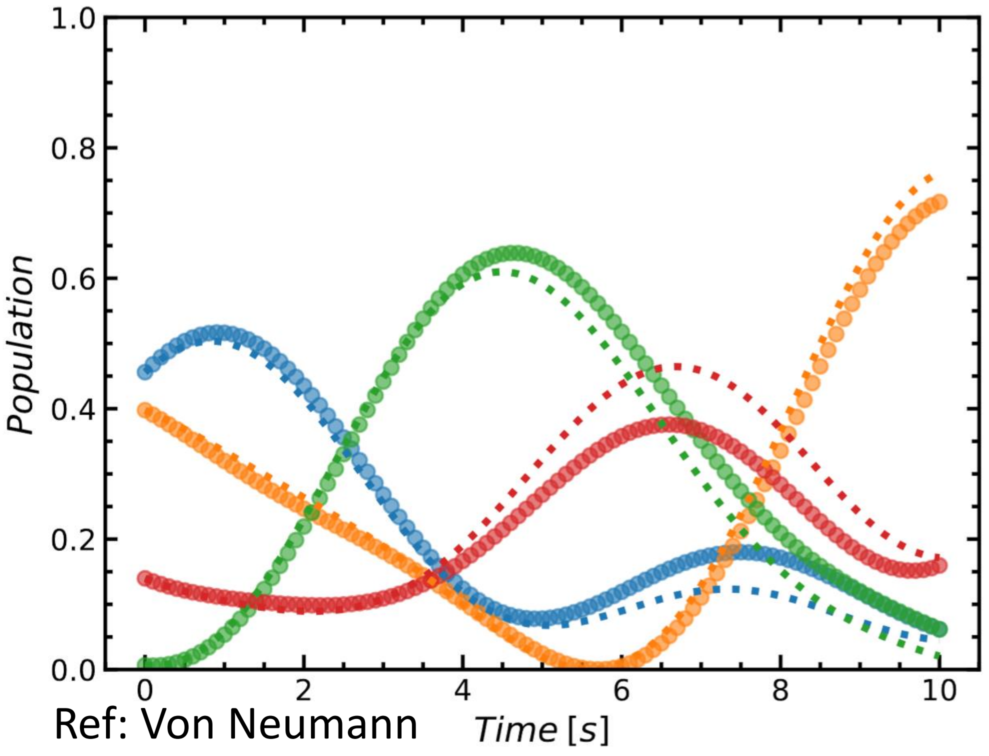
Ref: Von Neumann

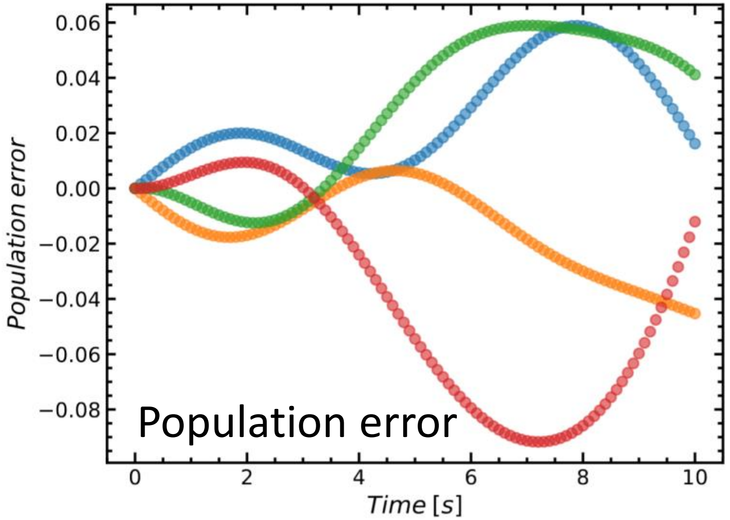


HEA

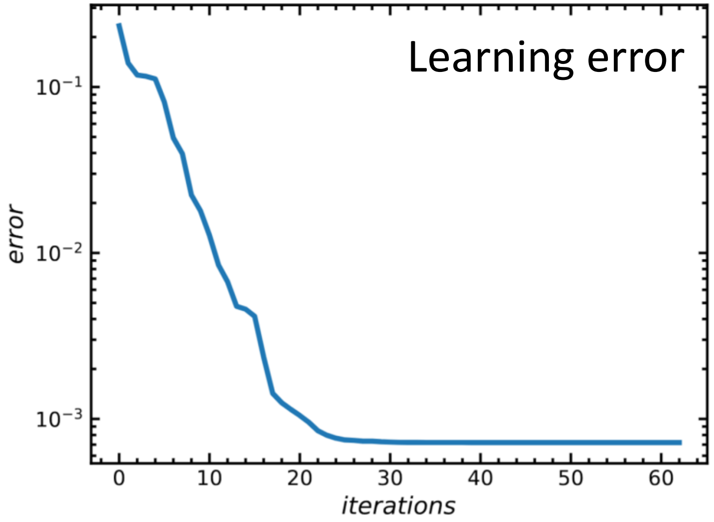
Results: approximating Rabi oscillation (two qubits)



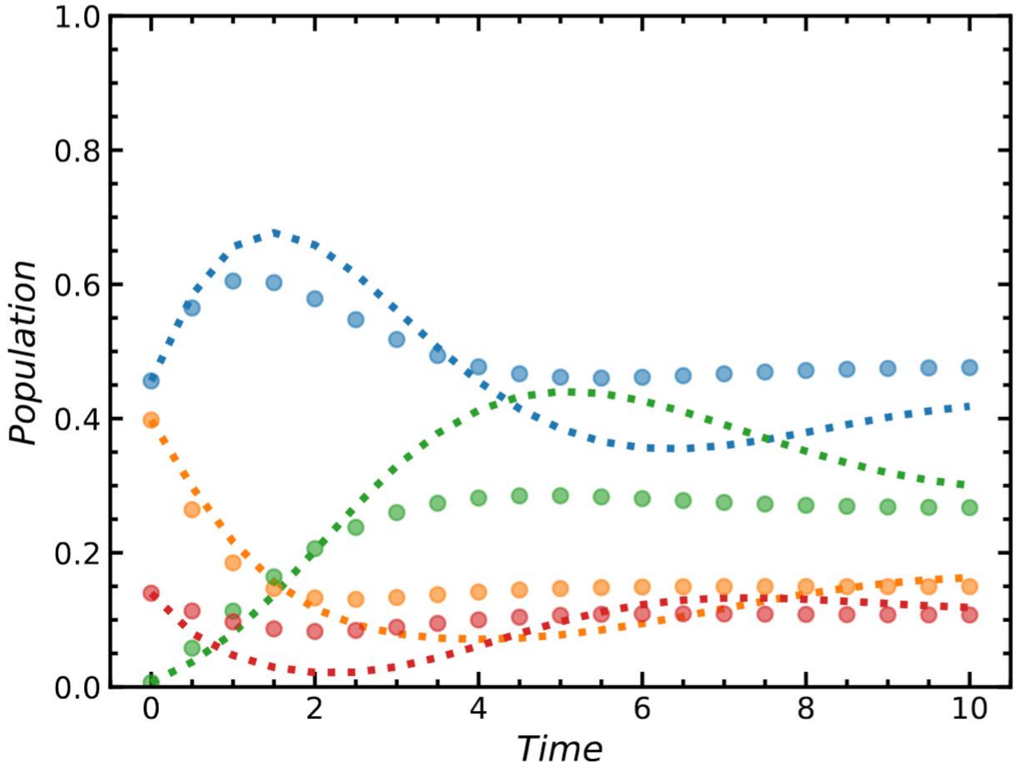


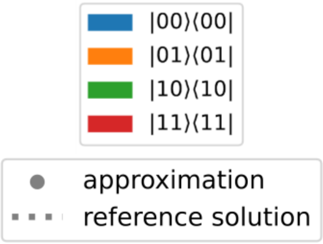


Results: approximating Rabi oscillation (two qubits)

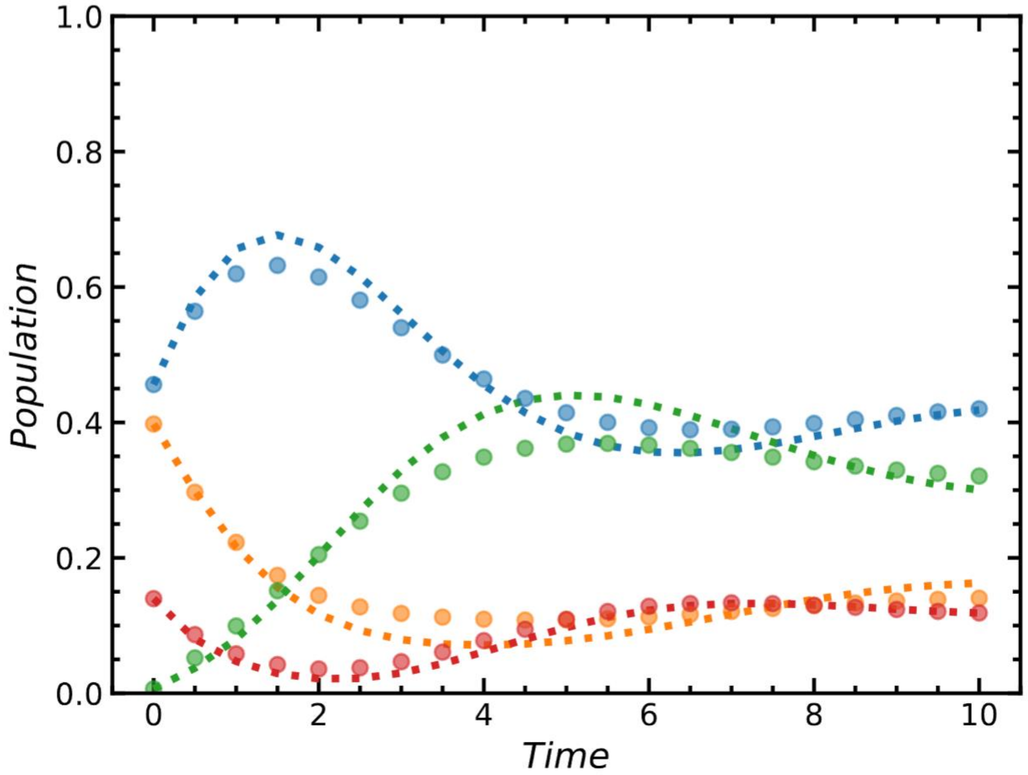


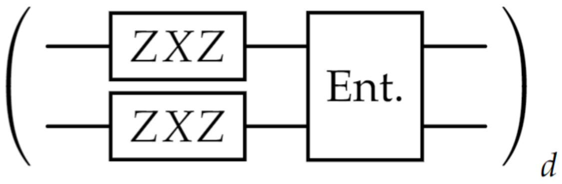
Results: approximating Rabi oscillation (two qubits)



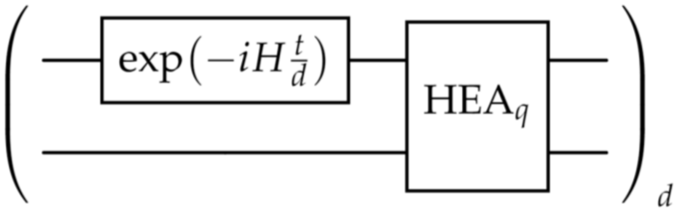


Ref: Lindblad



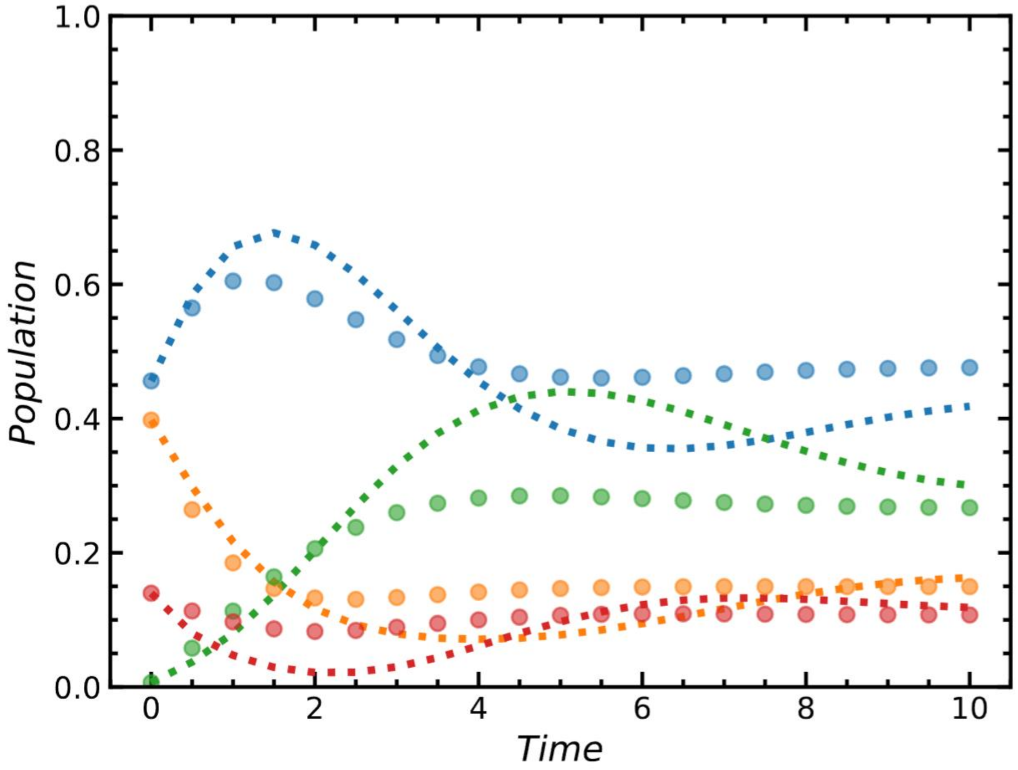


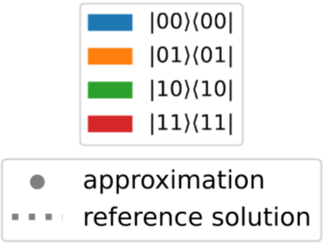
HEA

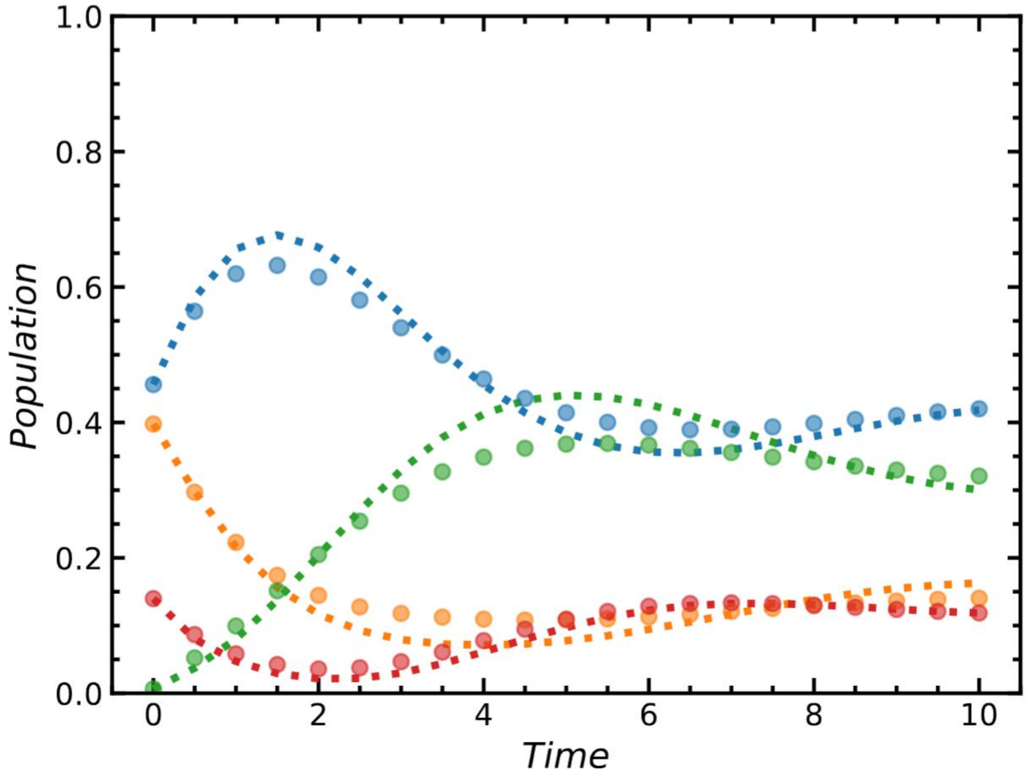


Trotterized SHEA

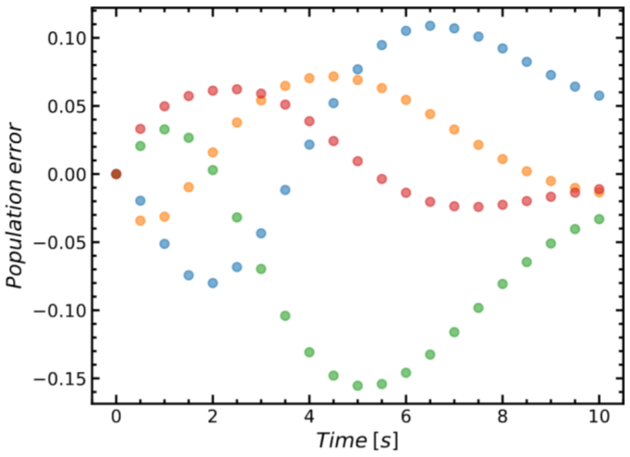
Results: approximating Rabi oscillation (two qubits)



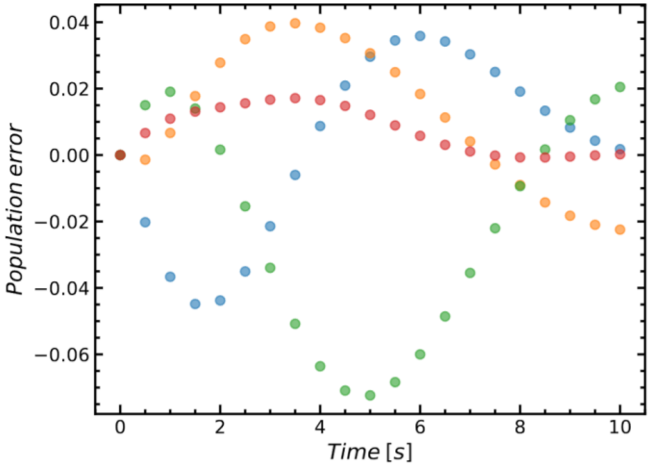




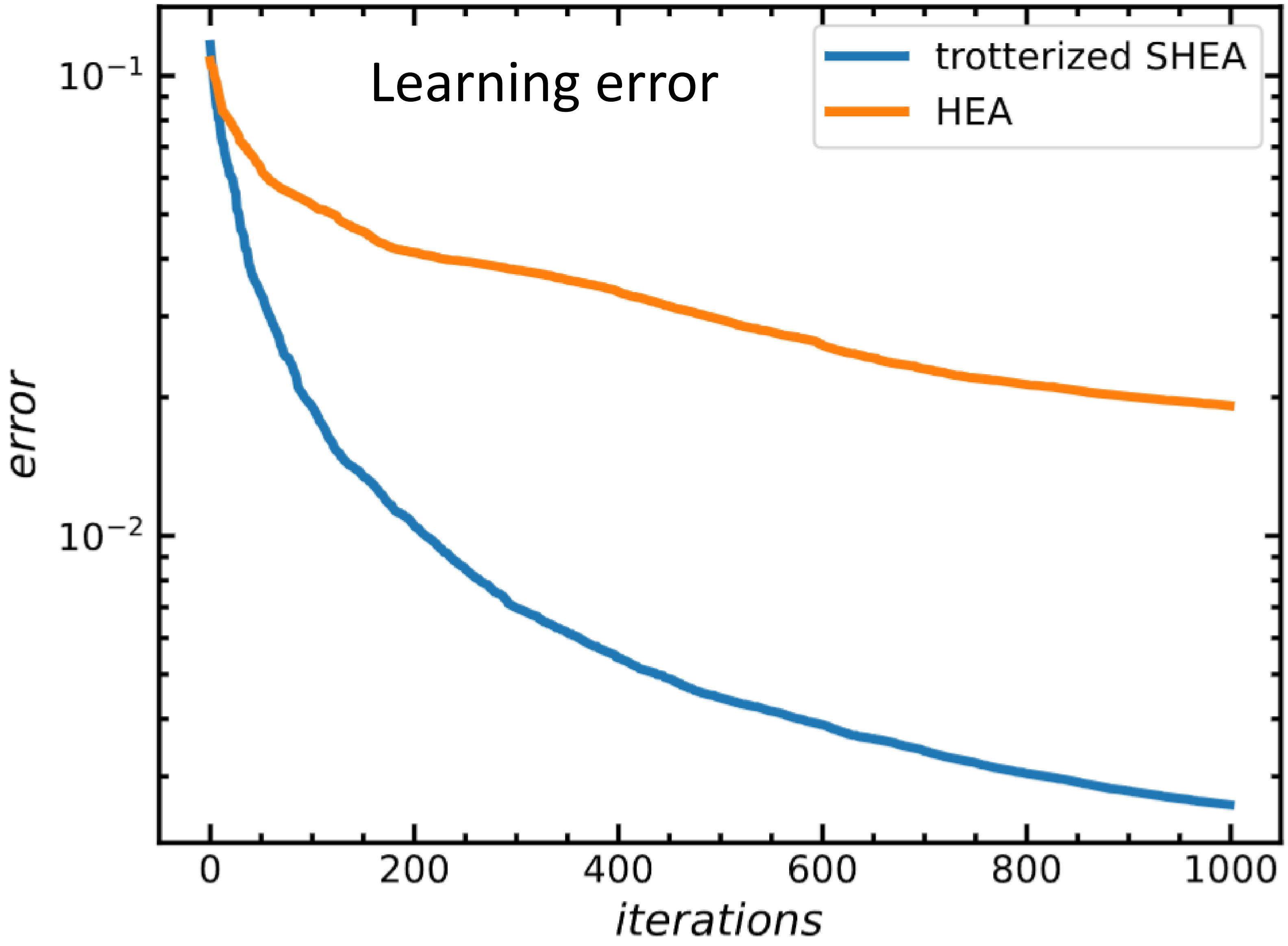
Ref: Lindblad



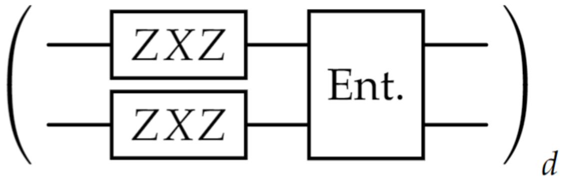
Population error



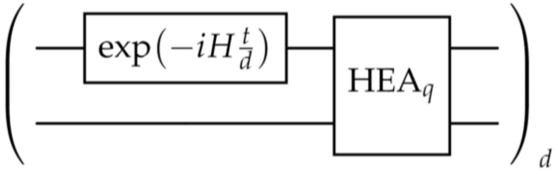
Results: approximating Rabi oscillation (two qubits)



HEA



Trotterized SHEA



Future research

- **How much training data is needed?**
- **What is the relation between depth and expressibility?**

- **Implement on quantum computer**