AERO 626 HOMEWORK #3

(50 points)

1. Code up the spring-mass problem in Section 2.7.1 of the Notes. Plot the range and range-rate residuals at each iteration for four iterations. What is the reference state after the fourth iteration?

2. The equations of motion for the two-body problem in inertial, Cartesian coordinates under the assumption of Keplerian motion are

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{\|\boldsymbol{r}\|^3}\boldsymbol{r}\,.$$

Define two (vector) states to be the position, $\boldsymbol{r}$, and velocity, $\boldsymbol{v} = \dot{\boldsymbol{r}}$, of the object, and formulate a first-order nonlinear differential equation of the form $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t))$.

3. Determine the Jacobian of $\boldsymbol{f}(\boldsymbol{x}(t))$, given by $\boldsymbol{F}(\boldsymbol{x}(t))$.

4. Given the position of an observer, $\boldsymbol{r}_{\text{obs}}$, and the position of the object, $\boldsymbol{r}$, the right-ascension and declination angles may be computed as

$$\begin{aligned}\alpha &= \tan^{-1}\frac{y}{x}\\ \delta &= \tan^{-1}\frac{z}{\sqrt{x^2+y^2}}\,,\end{aligned}$$

where x, y, and z are the components of the relative position vector $\boldsymbol{r}-\boldsymbol{r}_{\text{obs}}$. If $\boldsymbol{h}(\boldsymbol{x}) = [\alpha \quad \delta]^T$, determine the Jacobian of $\boldsymbol{h}(\boldsymbol{x})$, given by $\tilde{\boldsymbol{H}}(\boldsymbol{x})$.

5. A simulation of the motion of an object in orbit about Earth ($\mu = 3.986004415 \times 10^5$ km^3/s^2) has been conducted to generate synthetic data of the right-ascension and declination angles with respect to a ground observer. The resulting data is provided is the data file, `data_HW03.mat`, and the following is a brief description of the data:

```
% DATA PROVIDED ARE:
%   T     = (m x 1)     measurement times [s]
%   Z     = (2 x m)     RA/DEC measurements [deg]
%   Pvv   = (2 x 2 x m) RA/DEC measurement noise covariances [deg^2]
%   Xobsv = (6 x m)     pos. and vel. of the observer [km and km/s]
%   Xtrue = (6 x m)     true pos. and vel. of the object [km and km/s]
```

where `m` is the number of times at which measurements are simulated.

   (a) Given a reference of $\boldsymbol{x}_0^*$ of

$$\boldsymbol{x}_0^* = \begin{bmatrix} 32500.0 \\ 24500.0 \\ 2000.0 \\ -2.0 \\ 2.5 \\ 0.2 \end{bmatrix},$$

   with units of km and km/s, apply the iterative batch processor for five iterations using the angles-only data in the provided data file. The first data point is at t_0, which is the same time for which the reference state is provided. Plot the right ascension and declination residuals at each iteration. What is the reference state after the fifth iteration? Has the process converged? Justify your assessment with explanation and numerical analysis. What is the error with respect to the true state of the object?

(b) Repeat the previous problem with

$$\boldsymbol{x}_0^* = \begin{bmatrix} 35000.0 \\ 25000.0 \\ 1000.0 \\ -1.0 \\ 3.0 \\ 1.0 \end{bmatrix},$$

including all plots and analyses. What, if any, differences do you observe between the results and findings of these two problems?