AERO 626 HOMEWORK #4

(50 points)

1. Consider the noiseless, scalar, continuous-time dynamical system

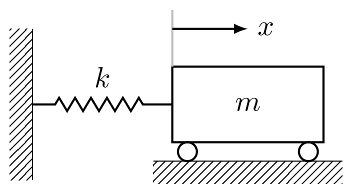
$$\dot{x}(t) = 0$$

that is accompanied by scalar, discrete-time measurements of the form

$$z_k = x_k + v_k\,,$$

where v_k is a zero-mean measurement noise with constant variance P_{vv}. If the initial mean and variance of the state are known to be $m_{x,0}$ and $P_{xx,0}$, respectively, derive closed-form expressions for the Kalman gain and posterior variance at the ℓ^{th} update time, where $k = 1, 2, \ldots, \ell$. Your results should be expressed entirely in terms of $P_{xx,0}$, P_{vv}, and ℓ.

2. Consider the unforced spring-mass system



which has the first-order equations of motion

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}\,.$$

The initial mean and covariance of the position and velocity of the mass are known to be

$$\boldsymbol{m}_{x,0} = \begin{bmatrix} 1.0\text{ m} \\ 0.0\text{ m/s} \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{P}_{xx,0} = \begin{bmatrix} (2.0\text{ m})^2 & 0.0 \\ 0.0 & (1.0\text{ m/s})^2 \end{bmatrix}\,.$$

It is also known that noisy measurements of the position of the mass that take the form

$$z_k = x_k + v_k\,,$$

where v_k is zero mean with constant covariance $P_{vv} = (0.1\text{ m})^2$, can be taken. Assuming that measurements are taken every one second, starting at $t_0 = 0$ (i.e., the time that the initial mean and covariance are known) and continuing through $t = 20$ sec (for a total of 21 measurements), determine the covariance history through all of the measurements. Plot the $\pm 1\sigma$ of the position and velocity covariance at each time, prior to and after each measurement. Plot the $\pm 1\sigma$ of the measurement noise covariance and innovation covariance (on the same figure) at each time. Comment on your results. For instance, consider the final standard deviation of the position with respect to the measurement noise standard deviation, and comment on any interesting characteristics.