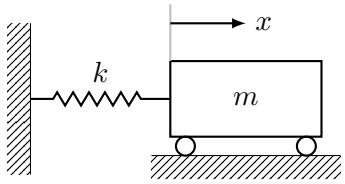
AERO 626 Homework #2

(50 points)

1. Consider the unforced spring-mass system



which has the equation of motion

$$\ddot{x}(t) = -\omega_n^2 x(t) \quad \text{where} \quad \omega_n^2 = k/m\,.$$

Assume that $\omega_n = 1$ for this problem. Define two states to be $x_1(t) = x(t)$ and $x_2(t) = \dot{x}(t)$, such that the dynamics of the system can be written in the form $\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t)$, where $\boldsymbol{x}(t) = [x_1(t) \;\; x_2(t)]^T$. Assume that the position of the mass cart at time t_i, $x(t_i)$, can be observed, such that the measurement can be modeled as $\boldsymbol{h}_i = \tilde{\boldsymbol{H}}_i \boldsymbol{x}_i$.

A simulation of the true motion of the system has been conducted to generate synthetic data of the position measurements. The resulting data is provided is the data file, `data_HW02.mat`, and the following is a brief description of the data:

```
% DATA PROVIDED ARE:
%   T = (m x 1) array of measurement times [s]
%   Z = (m x 1) array of position measurements [m]
%   W = (m x 1) array of measurement weights [nd]
%   R = (m x 1) array of measurement noise covariances [m^2]
```

where `m` is the number of measurements.

(a) Given the data, `Z`, determine the least-squares estimate, $(\hat{\boldsymbol{x}}_0)_{\text{LS}}$.

(b) Using the weights provided in `W`, determine the weighted least-squares estimate, $(\hat{\boldsymbol{x}}_0)_{\text{WLS}}$.

(c) Assuming that you are given prior knowledge of the state, represented by

$$\bar{\boldsymbol{x}}_0 = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} \quad \text{with weight} \quad \bar{\boldsymbol{W}}_0 = \begin{bmatrix} 3.0 & 0.0 \\ 0.0 & 3.0 \end{bmatrix},$$

determine the weighted least-squares estimate, including prior information $(\hat{\boldsymbol{x}}_0)_{\text{WLSP}}$.

(d) Using the measurement noise covariance provided in `R`, determine the linear, unbiased, minimum variance estimate, $(\hat{\boldsymbol{x}}_0)_{\text{LUMVE}}$. What is the uncertainty in this estimate?

2. **Project Proposal**: Develop a proposal for your project. The project should be focused on *Estimation of Dynamic Systems*, in that the proposed effort should seek to develop and apply strategies to the estimation of the states (and potentially parameters) of a dynamic system. The system selected should have nonlinear behaviors in the dynamics and measurements. The estimators used should include at least two options from the set of: least-squares methods, Kalman filters, and nonlinear Bayesian filters. You are encouraged to align your proposal to your research and/or personal interests, but the project must be original work. You are not permitted to re-use work from another class or from any previous research you have conducted, including, but not limited to, prior technical reports, conference papers, presentations, and journal papers. Your proposal should include, but is not limited to, the following elements.

Problem Statement Write a problem statement that introduces the problem you are trying to solve and the methods that you intend to apply.

Literature Review Conduct a brief literature review that includes at least five references. Your literature review should provide details on existing methods that address the proposed work and may include approaches up to the current state of the art.

Expected Results Discuss what outcomes you expect from your project, including how your project might advance or alter the current state of the art.

Work Plan Provide and discuss a timeline for how you will carry out your proposed project with reasonable milestones that culminate with a final report at the end of the semester.

Challenges Discuss any challenges that you anticipate might arise in carrying out your proposed effort. Describe mitigation strategies for overcoming these challenges.