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AERO 626 Homework #2

Spring 2023 David van Wijk

```
data = load('data_HW02.mat');
format long

% DATA PROVIDED ARE:
% T = (m x 1) array of measurement times [s]
% Z = (m x 1) array of position measurements [m]
% W = (m x 1) array of measurement weights [nd]
% R = (m x 1) array of measurement noise covariances [m^2]
```

Part A: Least-Squares Estimate of State

```
F = [0 1; -1 0];
H_tilde = [1 0];
H = [];
for i = 1:length(data.T)
    Phi_i = expm(F*(data.T(i) - data.T(1)));
    H_i = H_tilde*Phi_i;
    H = [H; H_i];
end
disp('Least-Squares Estimate of Initial State using matrix exponential for Phi:')
x_hat_0 = (H'*H)\(H'*data.Z)

H = [];
for i = 1:length(data.T)
    t_i = data.T(i);
    Phi_i = [cos(t_i) sin(t_i); -sin(t_i) cos(t_i)];
    H_i = H_tilde*Phi_i;
    H = [H; H_i];
end
disp('Least-Squares Estimate of Initial State using analytical solution for Phi:')
x_hat_0 = (H'*H)\(H'*data.Z)
```

Least-Squares Estimate of Initial State using matrix exponential for Phi:

x_hat_0 =

```
1.001143323295085
-0.005513909940539
```

Least-Squares Estimate of Initial State using analytical solution for Phi:

```
x_hat_0 =  
  
    1.001143323295085  
   -0.005513909940539
```

Part B: Weighted Least-Squares Estimate of State

```
W = diag(data.W);  
disp('Weighted Least-Squares Estimate of Initial State:')  
x_hat_0 = (H'*W*H)\(H'*W*data.Z)
```

Weighted Least-Squares Estimate of Initial State:

```
x_hat_0 =  
  
    1.004569504535051  
   -0.007651308799164
```

Part C: Weighted Least-Squares Estimate with prior information

```
x_bar = [1; 0];  
W_bar = [3 0; 0 3];  
disp('Weighted Least-Squares Estimate of Initial State using prior info:')  
x_hat_0 = (H'*W*H + W_bar)\(H'*W*data.Z + W_bar*x_bar)
```

Weighted Least-Squares Estimate of Initial State using prior info:

```
x_hat_0 =  
  
    1.004375010528215  
   -0.007340452622603
```

Part D: LUMVE of State

```
disp('LUMVE of Initial State:')  
P_vv = diag(data.R);  
x_hat_0 = (H'*P_vv^-1*H)\(H'*P_vv^-1*data.Z)  
disp('The uncertainty in our measurement can be evaluated using the covariance matrix of the estimate:')  
cov_matrix = (H'*P_vv^-1*H)^-1
```

LUMVE of Initial State:

```
x_hat_0 =  
  
    1.001143323295085  
   -0.005513909940539
```

The uncertainty in our measurement can be evaluated using the covariance matrix of the estimate:

cov_matrix =

1.0e-03 *

0.188443928965983	-0.006703912786495
-0.006703912786495	0.209123502754448