AERO 626 HOMEWORK #8

(50 points)

Consider a one-dimensional, nonlinear, additive-noise model of the form

$$x_k = x_{k-1} - 0.01 \sin(x_{k-1}) + w_{k-1}$$
$$z_k = 0.5 \sin(2x_k) + v_k$$

where w_{k-1} is a zero-mean, white, Gaussian sequence with constant variance $P_{ww} = 0.01^2$ and v_k is a zero-mean, white, Gaussian sequence with constant variance $P_{vv} = 0.02$. The initial distribution of the state is Gaussian with mean and variance given by $m_{x,0} = 1.5$ and $P_{xx,0} = 0.15^2$, respectively. For each problem, use the data provided in the file `data_HW08.m`, which contains the results of simulating the true states and measurements for $0 \leq k \leq 500$, where measurements are only generated for $k > 0$. The data file contains three variables: `x0`, `xk`, and `zk`. The initial true state, `x0`, is given as a single number. The rest of the true states are stored as a 1×500 array, where the k^{th} column corresponds to the true state at step k. The measurements are stored as a 1×500 array, where the k^{th} column corresponds to the measurement at step k.

1. Develop and implement an EKF. Document the derivatives required for implementing the EKF. Plot the posterior state estimation error and 3σ interval as functions of k. Include your code at the end of your submission.

2. Develop and implement a bootstrap particle filter (BPF) without resampling using $N = 10,000$ particles. Plot the posterior state estimation error and 3σ interval as functions of k. Plot $\hat{N}_{\text{eff}}$ as a function of k. Include your code at the end of your submission.

3. Develop and implement a BPF with basic resampling using $N = 10,000$ particles and a threshold of $\hat{N}_{\text{eff}} < 0.1N$. Plot the posterior state estimation error and 3σ interval as functions of k. Plot $\hat{N}_{\text{eff}}$ as a function of k. How many times is resampling performed? Include your code at the end of your submission.

4. Compare the results of each filter by computing the root-mean-square error (RMSE) and mean absolute error (MAE) using the posterior estimation errors, and comment on the performance of each filter.

The remaining questions are optional.

5. Develop and implement an extended particle filter (EPF) with basic resampling using a threshold of $\hat{N}_{\text{eff}} < 0.1N$. Plot the posterior state estimation error and 3σ interval as functions of k. Plot $\hat{N}_{\text{eff}}$ as a function of k. How many times is resampling performed? Include your code at the end of your submission.

6. Compare the results of the EPF to the other filters by computing the RMSE and MAE using the posterior estimation errors, and comment on the performance of the EPF with respect to the other methods.