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AERO 626
HW1

1.

$$m = \int_{-\infty}^{\infty} x\, p(x)\, dx \quad \text{where} \quad p(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0 \end{cases}$$

$$m = \int_{-\infty}^{\infty} x \left(\frac{1}{b-a}\right) dx = \cancelto{0}{\int_{-\infty}^{a} x \left(\frac{1}{b-a}\right) dx} + \int_{a}^{b} x \left(\frac{1}{b-a}\right) dx + \cancelto{0}{\int_{b}^{\infty} x \left(\frac{1}{b-a}\right) dx}$$

$$m = \frac{x^2}{2}\left(\frac{1}{b-a}\right)\Bigg|_a^b = \frac{b^2}{2}\left(\frac{1}{b-a}\right) - \frac{a^2}{2}\left(\frac{1}{b-a}\right) \rightarrow \boxed{m = \frac{1}{2(b-a)}\left(b^2 - a^2\right)}$$

$$P = \int_a^b (x-m)^2 \left(\frac{1}{b-a}\right) dx = \int_{a-m}^{b-m} u^2 \left(\frac{1}{b-a}\right) du = \frac{u^3}{3}\left(\frac{1}{b-a}\right)\Bigg|_{a-m}^{b-m}$$

$u = x - m$

$du = dx$

$$\boxed{P = \frac{1}{3(b-a)}\left((b-m)^3 - (a-m)^3\right)}$$

with m given above (constant)

2.

When I generate 1×10^6 uniform random numbers w/ $a=0$ & $b=1$ vs. generating these random numbers using the mean and covariance, I notice that both "methods" result in a mean and covariance that are close to the true underlying values of the distibution. As $n \to \infty$ these values would get closer and closer to the true underlying distribution. In the case of $a=0$, $b=1$ according to m obtained above, this mean value should approach $m = \frac{1}{2(1-0)}(1^2 - 0^2) = \frac{1}{2}$, which is exactly what we observe. Similarly, the covariance P, should approach $P = \frac{1}{3(1)}\left(\left(1-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^3\right) = \frac{1}{12}$, which again is what we observe in matlab.

$\boxed{3.}$

$$\epsilon_1 = z_1 - h_1 = z_1 - [\hat{a} + \hat{b}t_1] = z_1 - \left[\frac{z_1t_2 - z_2t_1}{t_2 - t_1} + \frac{z_2 - z_1}{t_2 - t_1}t_1\right]$$

$$= \frac{z_1(t_2 - t_1)}{t_2 - t_1} + \frac{-z_1t_2 + z_2t_1}{t_2 - t_1} + \frac{-z_2t_1 + z_1t_1}{t_2 - t_1}$$

$$= \frac{\cancel{z_1t_2} - \cancel{z_1t_1} - \cancel{z_1t_2} + \cancel{z_2t_1} - \cancel{z_2t_1} + \cancel{z_1t_1}}{t_2 - t_1}$$

$$\boxed{\epsilon_1 = 0}$$

✓

$$\epsilon_2 = z_2 - h_2 = z_2 - \left[\frac{z_1t_2 - z_2t_1}{t_2 - t_1} + \frac{z_2 - z_1}{t_2 - t_1}t_2\right] = \frac{\cancel{z_2(t_2 - t_1)}}{t_2 - t_1} - \left(\frac{\cancel{z_1t_2} - \cancel{z_2t_1}}{t_2 - t_1} + \frac{\cancel{z_2t_2} - \cancel{z_1t_2}}{t_2 - t_1}\right)$$

$$\boxed{\epsilon_2 = 0}$$

✓

$\boxed{4.}$

$$\left.\begin{array}{l} \ddot{x}(t) = -\omega_n^2 x(t) \quad , \quad \text{here assume } \omega_n = 1 \\ \ddot{x}(t) = -x(t) \end{array}\right\} \dot{x}_2 = -x_1$$

a) $$\dot{\underline{x}}(t) = \begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \therefore \quad \boxed{\underline{F}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}$$

b) Solution of system given by $\underline{x}(t) = \Phi(t, t_0)x_0$

By inspection, the solution to $\ddot{x}(t) = -x(t)$ is

$$x(t) = A\cos t + B\sin t$$
$$\dot{x}(t) = -A\sin t + B\cos t$$
$$\ddot{x}(t) = -A\cos t - B\sin t$$

Apply IC's...

$$x(0) = A\cancel{\cos t}^{1} + \cancel{B\sin t}^{0}$$
$$\dot{x}(0) = -A\cancel{\sin t}^{0} + B\cancel{\cos t}^{1}$$
$$A = x(0)$$
$$B = \dot{x}(0)$$

$$\boxed{\Phi(t_i, t_0) = \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}}$$

$$\therefore \quad x_1 = x_1(0)\cos t + x_2(0)\sin t$$
$$x_2 = -x_1(0)\sin t + x_2(0)\cos t$$

c) $\underline{h}_i = \tilde{\underline{H}}_i \underline{x}_i$

Assuming we can only measure the position, $x_1(t)$

$$[h_i] = [1 \;\; 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad , \quad \boxed{\tilde{H}_i = [1 \;\; 0]}$$