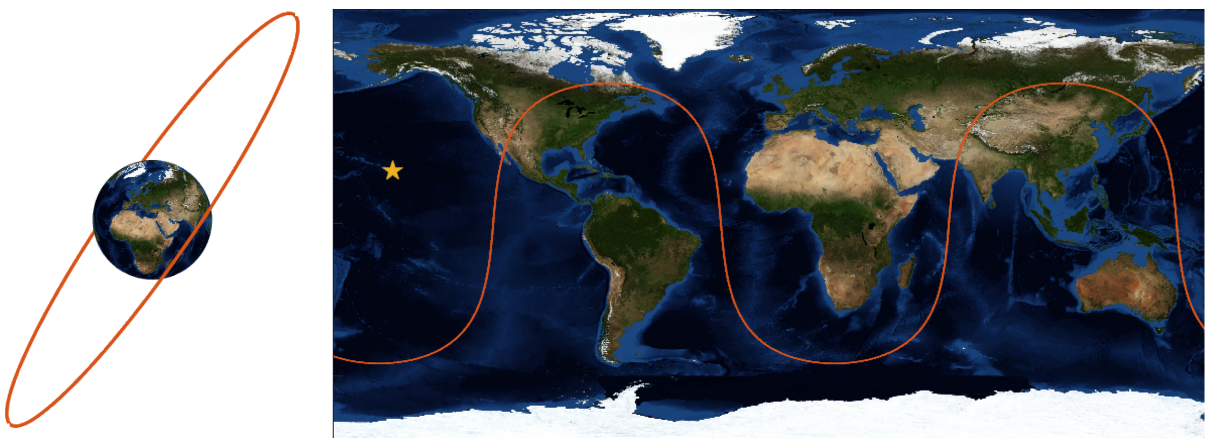
AERO 626 HOMEWORK #6

(50 points)

The Block III-5 GPS satellite is a currently operational GPS satellite that was launched on June 17, 2021 on a Falcon 9 Block 5 from Cape Canaveral Space Force Station (Space Launch Complex 40). It is the second newest member of the GPS constellation. As of March 7, 2023, the TLE for this satellite is

```
1 48859U 21054A   23065.67636231 -.00000083  00000+0  00000+0 0  9996
2 48859  55.2917  20.5084 0008784 219.6548 320.3658  2.00559429 12710
```

from which it is seen that this GPS satellite is in an orbit that has an inclination of approximately 55° and a period of about 12 hours. The geometry of the orbit is shown in Fig. 1a.



(a) Orbit Geometry　　(b) Ground Track and Location of the Maui GEODSS Site

Figure 1: GPS Block III-5 Orbit and Ground Track

The Ground-based Electro-Optical Deep Space Surveillance (GEODSS) is an optical system that forms a core element of the Space Surveillance Network. One of the GEODSS sites is located on Haleakalā on the island of Maui. The telescope is located at a latitude of 20.7088°N and a longitude of 156.2578°W. A ground track of the Block III-5 orbit, along with the location of the GEODSS site in Maui, is illustrated in Fig. 1b.

Measurements of right-ascension and declination of the Block III-5 satellite are simulated to be taken from the Maui GEODSS site. Measurements can only be taken when the satellite is above a 20° elevation mask and when it is locally nighttime at the station. For the purposes of this study, night is taken to be any time after 22:00:00 and before 04:00:00 at the station (where times are local solar time). A plot of the elevation of the Block III-5 satellite from the Maui GEODSS site, along with the elevation mask and the daytime periods, is shown for a duration of ten days past the TLE epoch in Fig. 2, from which it is seen that the first available time that a measurement can be acquired is 22 hours, 1.5 minutes after the TLE epoch.

Arcs of data, as simulated for this study, have an average length of 120 measurements, where measurements are taken at a rate of one every 10 seconds. Only one arc of data is available each night, and data are simulated for a sequence of 10 nights. The data are corrupted by zero-mean, white noise with a standard deviation of 3 arcseconds on both the right-ascension and the declination measurements. The right-ascension and declination measurements, as well as the true elevation of

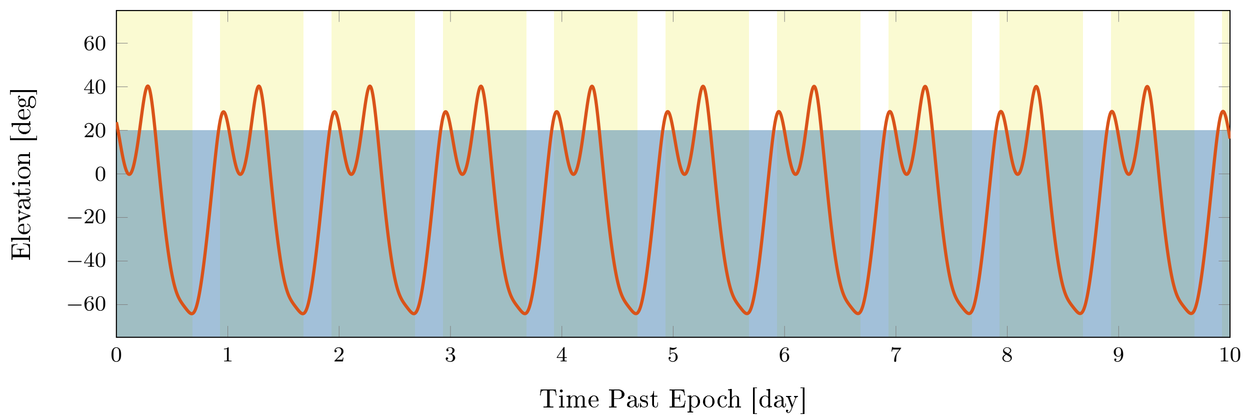


Figure 2: Elevation as a function of time for the Block III-5 satellite with the daytime periods illustrated in yellow and the elevation mask illustrated in blue.

the object with respect to the Maui GEODSS station, are illustrated in Fig. 3. The observation times, simulated observations, measurement noise covariances, inertial location of the observer, and true object state are all provided in the data file `data_HW05.mat`.

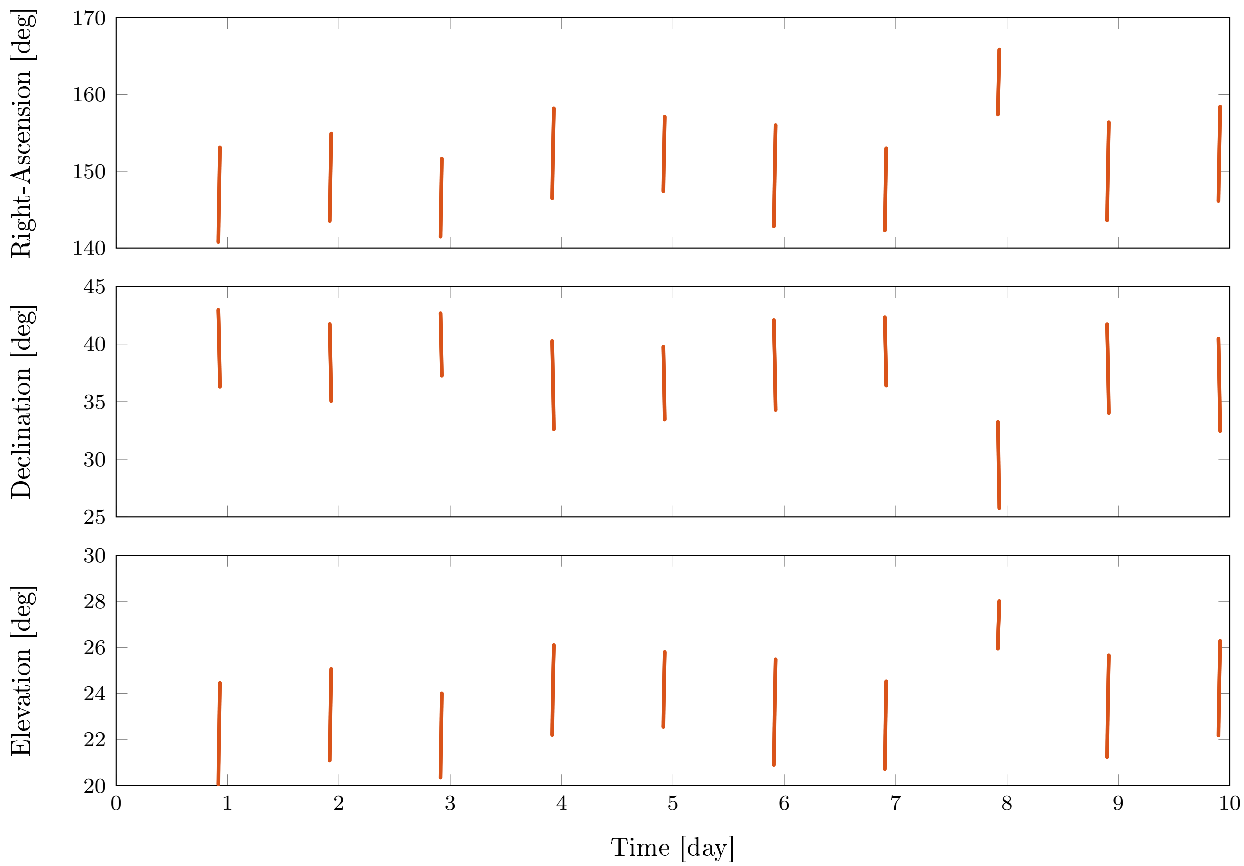


Figure 3: Right-ascension and declination measurements and true elevation as a function of time for the Block III-5 satellite.

The following is a brief description of the data:

```
% DATA PROVIDED ARE:
%   T     = (m x 1)     measurement times [s]
%   Z     = (2 x m)     RA/DEC measurements [arcsec]
%   Pvv   = (2 x 2 x m) RA/DEC measurement noise covariances [arcsec^2]
%   Robsv = (3 x m)     inertial pos. of the observer [km]
%   Xtrue = (6 x m)     true pos. and vel. of the object [km and km/s]
%
% m = 1199 for this dataset
% There are 123, 118, 101, 128, 106, 137, 108, 110, 134, 134 measurements on
% each of the ten consecutive nights, respectively
```

Using the provided data,

1. Let the state be given by the position and velocity of the GPS Satellite, such that the state and dynamics of the problem are

$$\boldsymbol{x}(t) = \begin{bmatrix} \boldsymbol{r}(t) \\ \boldsymbol{v}(t) \end{bmatrix} \qquad \text{and} \qquad \dot{\boldsymbol{x}}(t) = \begin{bmatrix} \boldsymbol{v}(t) \\ -\frac{\mu}{\|\boldsymbol{r}(t)\|^3}\boldsymbol{r}(t) \end{bmatrix}$$

   where $\mu = 3.986004415 \times 10^5 \text{ km}^3/\text{s}^2$ is the gravitational parameter of the Earth. Assume that there is no process noise. Using an initial estimate and covariance of

$$\boldsymbol{m}_{x,0} = \begin{bmatrix} -24864.5000 \text{ km} \\ -9288.5000 \text{ km} \\ 16.5000 \text{ km} \\ 0.7726 \text{ km/s} \\ -2.0677 \text{ km/s} \\ -3.1867 \text{ km/s} \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{P}_{xx,0} = \begin{bmatrix} (0.2 \text{ km})^2 \boldsymbol{I}_3 & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & (2 \times 10^{-3} \text{ km/s})^2 \end{bmatrix}$$

   develop and apply the extended Kalman filter making use of the provided data. Process the data in units of arcseconds. No results are required for this problem. Include your code at the end of your submission.

2. Plot the right-ascension and declination innovations (in units of arcseconds) as a function of the measurement number (not as a function of time). Include the standard deviations (3σ) of the measurement noise covariance and innovation covariance in your plots.

3. Plot the squared Mahalanobis distance for the innovation as a function of the measurement number (not as a function of time). Include χ^2 thresholds for measurement rejection according to probability gates of $P_G = 95\%$, $P_G = 99\%$, and $P_G = 99.9\%$ in your plot.

4. Plot the prior and posterior estimation error for each state (in units of m and mm/s) as a function of the measurement number (not as a function of time). Include the prior and posterior standard deviations (3σ) of the state estimation error covariance in your plots.

5. Compute the standard deviations of the posterior covariance after the last measurement has been processed. Report your values in units of m and mm/s, respectively.
   You should find results close to: $\sigma_x = 33.529$ *m*, $\sigma_y = 23.379$ *m*, $\sigma_z = 33.133$ *m*
   $\sigma_{\dot{x}} = 23.167$ *mm/s*, $\sigma_{\dot{y}} = 13.665$ *mm/s*, $\sigma_{\dot{z}} = 16.389$ *mm/s*

6. Comment on the performance of the extended Kalman filter. Indicate if any interesting behaviors are observed.