AERO 626 HOMEWORK #6

(50 points)

1. Given a transformation of the form

$$\boldsymbol{z} = \boldsymbol{H}_x \boldsymbol{x}\,,$$

where the mean and covariance of $\boldsymbol{x}$ are given as $\boldsymbol{m}_x$ and $\boldsymbol{P}_{xx}$, respectively, use the unscented transform – with arbitrary, non-zero values of α, κ, and β – to derive closed-form expressions for the mean and covariance of $\boldsymbol{z}$. It may help to know that, for any $\boldsymbol{S}_{xx}$, such that $\boldsymbol{S}_{xx}\boldsymbol{S}_{xx}^T = \boldsymbol{P}_{xx}$, with $\boldsymbol{s}_i$ denoting the i^{th} column of $\boldsymbol{S}_{xx}$, such that $\boldsymbol{S}_{xx} = [\boldsymbol{s}_1 \;\; \boldsymbol{s}_2 \;\; \cdots \;\; \boldsymbol{s}_n]$,

$$\boldsymbol{P}_{xx} = \boldsymbol{S}_{xx}\boldsymbol{S}_{xx}^T = \sum_{i=1}^{n} \boldsymbol{s}_i \boldsymbol{s}_i^T\,,$$

where n is the dimension of $\boldsymbol{x}$. Your results should be expressed entirely in terms of $\boldsymbol{m}_x$, $\boldsymbol{P}_{xx}$, and $\boldsymbol{H}_x$. Comment on your results.

2. Consider a one-dimensional, nonlinear, additive-noise model of the form

$$\begin{aligned} x_k &= x_{k-1} - 0.01\sin(x_{k-1}) + w_{k-1} \\ z_k &= 0.5\sin(2x_k) + v_k \end{aligned}$$

where w_{k-1} is zero-mean and white with constant variance $P_{ww} = 0.01^2$ and v_k is zero-mean and white with constant variance $P_{vv} = 0.02$. The initial mean and variance of the state are $m_{x,0} = 1.5$ and $P_{xx,0} = 0.15^2$, respectively.

   (a) Develop a simulation that generates a random truth, propagates the truth, and creates synthetic measurements for a sequence of 500 steps. The random truth should be generated at $k = 0$; measurements should be simulated $\forall\ k > 0$. Assume that all distributions are Gaussian with their respective means and variances as given. Provide a plot of the true trajectory and measurements for one set of randomly generated values. Plot the true trajectory as a solid line, and plot the measurements using only markers. Include your code at the end of your submission.

   (b) Develop and implement an EKF. Document the derivatives required for implementing the EKF. Provide plots of the innovations and state estimation errors. Include appropriate standard deviation intervals on your plots. Include your code at the end of your submission.

   (c) Develop and implement a UKF. Apply the UKF for two different sets of parameters, given by

      i. $\alpha = 1$, $\beta = 0$, and $\kappa = 2$, and

      ii. $\alpha = 0.5$, $\beta = 2$, and $\kappa = 2$.

   For both implementations of the UKF, use Cholesky decomposition. Provide plots of the innovations and state estimation errors. Analyze the UKF results separately for each set of parameters. Include appropriate standard deviation intervals on your plots. Include your code at the end of your submission.

   (d) Compare the results of each filter, and comment on the performance of each filter. Use numerical analysis methods to compare the filters. Do not rely only on visual assessment of the performance. Provide any plots that are necessary to complete the comparison.

(e) Choose one of the three filter implementations and perform a Monte Carlo simulation for $N = 500$ trials. Compute the average filter error, the average filter state estimation error variance, and the Monte Carlo state estimation error variance. Plot the average error and appropriate standard deviation intervals. Comment on the performance of the selected filter.