AERO 626 Homework #7

(50 points)

1. Consider a constant velocity, linear system of the form

$$\boldsymbol{x}_k = \boldsymbol{F}_{x,k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1}$$
$$\boldsymbol{z}_k = \boldsymbol{H}_{x,k}\boldsymbol{x}_k + v_k\,,$$

where

$$\boldsymbol{F}_{x,k-1} = \begin{bmatrix} 1.00 & 1.00 \\ 0.00 & 1.00 \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{H}_{x,k} = \begin{bmatrix} 1.00 & 0.00 \end{bmatrix}\,.$$

The states of this system are interpreted as the position (first state) and the velocity (second state), meaning that position-only measurements are obtained. The process and measurement noises are taken to be zero-mean Gaussian with covariances

$$\boldsymbol{P}_{ww,k-1} = \begin{bmatrix} 0.01 & 0.00 \\ 0.00 & 0.01 \end{bmatrix} \qquad \text{and} \qquad P_{vv,k} = 1.00\,,$$

respectively. The initial distribution of the state is taken to be a 4-component Gaussian mixture of the form

$$p(\boldsymbol{x}_0) = \sum_{\ell=1}^{L_{x,0}} w_{x,0}^{(\ell)} p_g(\boldsymbol{x}_0; \boldsymbol{m}_{x,0}^{(\ell)}, \boldsymbol{P}_{xx,0}^{(\ell)})\,,$$

where $L_{x,0} = 4$ and weights, means, and covariances given by

$$w_{x,0}^{(1)} = 0.20\,,\quad w_{x,0}^{(2)} = 0.30\,,\quad w_{x,0}^{(3)} = 0.10\,,\quad w_{x,0}^{(4)} = 0.40$$
$$\boldsymbol{m}_{x,0}^{(1)} = \begin{bmatrix} -2.50 \\ -0.50 \end{bmatrix}\,,\quad \boldsymbol{m}_{x,0}^{(2)} = \begin{bmatrix} -1.00 \\ 0.20 \end{bmatrix}\,,\quad \boldsymbol{m}_{x,0}^{(3)} = \begin{bmatrix} -0.50 \\ -0.30 \end{bmatrix}\,,\quad \boldsymbol{m}_{x,0}^{(4)} = \begin{bmatrix} 1.20 \\ 1.00 \end{bmatrix}$$
$$\boldsymbol{P}_{xx,0}^{(1)} = \begin{bmatrix} 0.20 & 0.00 \\ 0.00 & 0.10 \end{bmatrix}\,,\ \boldsymbol{P}_{xx,0}^{(2)} = \begin{bmatrix} 0.25 & 0.00 \\ 0.00 & 0.05 \end{bmatrix}\,,\ \boldsymbol{P}_{xx,0}^{(3)} = \begin{bmatrix} 0.20 & 0.00 \\ 0.00 & 0.10 \end{bmatrix}\,,\ \boldsymbol{P}_{xx,0}^{(4)} = \begin{bmatrix} 1.00 & 0.00 \\ 0.00 & 0.30 \end{bmatrix}.$$

Develop, apply, and analyze a Gaussian mixture filter for this problem using the data file provided (`data_HW07.m`). Plot the position and velocity estimation error (at each time step, before and after any updates) on separate plots, and include the corresponding 3σ intervals. Comment on your results. The data file contains the results of simulating the true states and measurements for $0 \le k \le 20$, where measurements are only generated for $k > 0$. The data file contains three variables: `x0`, `xk`, and `zk`. The initial true state, `x0`, is given as a 2×1 array. The rest of the true states are stored as a 2×20 array, where the k^{th} column corresponds to the true state at step k. The measurements are stored as a 1×20 array, where the k^{th} column corresponds to the measurement at step k.

2. Consider the linear system described by the state-space model

$$\boldsymbol{x}_k = \boldsymbol{F}_{x,k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1}$$
$$\boldsymbol{z}_k = \boldsymbol{H}_{x,k}\boldsymbol{x}_k + \boldsymbol{v}_k\,,$$

where the process noise is taken to be zero-mean, Gaussian with covariance $\boldsymbol{P}_{ww,k-1}$ and the measurement noise is taken to be zero-mean, Gaussian with covariance $\boldsymbol{P}_{vv,k}$. Given the posterior density at t_{k-1} to be

$$p(\boldsymbol{x}_{k-1} \,|\, \boldsymbol{z}_{1:k-1}) = \sum_{\ell=1}^{L^+_{x,k-1}} w^{(\ell)+}_{x,k-1} p_g(\boldsymbol{x}_{k-1}; \boldsymbol{m}^{(\ell)+}_{x,k-1}, \boldsymbol{P}^{(\ell)+}_{xx,k-1}) \,,$$

determine a closed-form expression for the *a priori* mean and covariance of $\boldsymbol{x}_k$, i.e., $\boldsymbol{m}^-_{x,k}$ and $\boldsymbol{P}^-_{xx,k}$. Express your answers solely in terms of $\boldsymbol{F}_{x,k-1}$, $\boldsymbol{P}_{ww,k-1}$, and the mean and covariance of the posterior density at t_{k-1}, i.e., $\boldsymbol{m}^+_{x,k-1}$ and $\boldsymbol{P}^+_{xx,k-1}$. Comment on your findings, especially with respect to other uncertainty propagation schemes.