**Mathematical Modelling of Carbon Fee and Dividend**

Dave Waltham, March 4th 2021

I assume that household emissions, and hence household carbon-fees, increase with household income. A simple model which does this is

(1)

where *E0* is initial household emissions and *I* is household disposable income. Emissions, *E*, are assumed to change (with time, *t*) by a factor *R* so that emissions are

*E(t)* = *R(t)E0* (2)

Household carbon-fees are then

(3)

where *F* is the fee and *P* is the price.

The value of *a* can be found by averaging and rearranging eqn (1) to give

. (4)

where and are national averages of the quantities under the over-bars. Note that this approach effectively assumes that a single-occupancy household on average-earning emits the average amount of CO2.

The dividend, per person, is found from a similar assumption, i.e. that an individual with average emissions will receive a dividend equal to their increased costs. Hence, the dividend per person is

(5)

Emission reductions are assumed to be driven by the carbon price, i.e. *R* decreases as *P* increases. Many choices could be made for the form of this (e.g. linear or exponential) but, in this model I use the sigmoidal function

*P*<*Pmax* (6)

= 0 *P*≥*Pmax*

Where *Pmax* is the price needed to eliminate emissions entirely. This sigmoidal form fits the [REMI model](https://citizensclimatelobby.org/remi-report/) reasonably well for *Pmax*~$400/tonne CO2. We also need to introduce a price policy and, for this model, I assume a simple ramp

(7)

where *Pstart* is the initial price, ** is the price increase each year and *t* is time since carbon pricing was introduced.