

# The calibrating stars of the Mira $P$ – $L$ relation

I. S. Glass<sup>1</sup><sup>★</sup> and T. Lloyd Evans<sup>2</sup>

<sup>1</sup>*South African Astronomical Observatory, PO Box 9, Observatory 7935, South Africa*

<sup>2</sup>*School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews, Fife KY16 9SS*

Accepted 2003 March 15. Received 2003 March 11; in original form 2003 January 27

## ABSTRACT

Improved light curves extracted from the MACHO data base are used to discuss the periods and amplitudes of the calibrating stars of the Mira  $P$ – $L$  relations. Previously unpublished spectral types and positions are given for several of those that lie in the S Dor variable star field. It is found that the periods derived from the discovery observations were generally sufficiently accurate that the  $K$ - and  $m_{\text{bol}}\text{--}\log P$  relations are not significantly changed by the MACHO results. Furthermore, the periods have remained essentially constant over two to three decades. The MACHO  $r$ -band amplitudes of the oxygen-rich Large Magellanic Cloud (LMC) Miras are similar to those in galactic (Baade’s window) fields at a given period but there appear to be more shorter-period stars as a proportion of the whole than in the galaxy. In O-rich stars, the distinction in amplitude between the Miras and the semiregulars is not as conspicuous at shorter periods ( $<200$  d) as at longer ones. The LMC carbon Miras have smaller amplitudes than their O-rich counterparts. Of the six stars with  $P > 420$  d that were found to be too luminous to fit the  $P$ – $L$  relation, two are now known to be Li-rich. This tends to confirm the suggestion that they are hot-bottom burners.

**Key words:** stars: AGB and post-AGB – stars: variables: other.

## 1 INTRODUCTION

The first evidence that a Mira  $P$ – $L$  relation holds in the near-infrared (near-IR) depended on a small number of stars found in the bar of the Large Magellanic Cloud (LMC) (Glass & Lloyd Evans 1981). Following the discovery of additional Mira variables in the LMC by Glass & Reid (1985, hereafter GR), Wood, Bessell & Paltoglou (1985, hereafter WBP) and Reid, Glass & Catchpole (1988) a second programme was started to define the relation more exactly by observing a larger sample with better phase coverage. The preliminary result of this work was discussed by Glass et al. (1987). The final report, which added a possible colour term to the relation, is given in Feast et al. (1989, hereafter FGWC). The complete data were published by Glass et al. (1990).

The least scatter in the relationship is observed in the  $K$  band for oxygen-rich Miras. However, both oxygen-rich and carbon-rich Miras appear to obey the same relation (Glass et al. 1987; FGWC). The source of the scatter, which amounts to only  $\pm 0.13$  mag for the O-rich stars, has not yet been definitively established, although it can be caused by a small range in mass (see, e.g., fig. 6 of Hughes & Wood 1990). Differing values of  $T_{\text{eff}}$ , such as might be caused by a range of metal abundances (e.g. fig. 8 of Wood 1990), could also cause an effect of this kind.

In this paper, additional information on the periods and light curves from the MACHO project is considered. The Miras discussed

by Glass & Lloyd Evans (1981) were found by blinking a series of 44  $V$ -band Newtonian plates taken at the dark of moon over a period of 5 years at the 1.9-m telescope of the Radcliffe Observatory in Pretoria in the years 1966–1971 (Lloyd Evans 1972, 1992). The periods were derived from the epochs of the peaks in their light curves. A run of plates taken at 25 successive new moons in the latter part of this interval gave confidence that these initial periods were broadly correct; this facilitated the later determination of more precise periods from the near-infrared observations. The plates also included some taken in the colours  $B$  and  $I$ . The  $V$ ,  $I$  pairs enabled the red stars to be identified by their large colour indices, so that blue variables such as Cepheids could be eliminated from further consideration. Carbon stars could be identified by their weakness in the  $B$  band, provided that the  $V$  image was sufficiently far above the plate limit. Many variable stars were found by examining the red stars on the  $V$  plates, as well as by general blinking of these plates. Stars which first came to notice by their red colour have a suffix R and those that turned up in the blink microscope survey are denoted by C. The Mira and other long-period variables of GR (1985) and WBP (1985) were found on a series of 23  $I$  plates taken in 1977–1983 at the UK Schmidt telescope.

The MACHO (see, e.g., Alcock et al. 2000) project provides a data base of two-colour photometry of many millions of stars in the LMC that have been observed of the order of 1000 times each over a period of about 8 yr, from 1992 July to 2000 January. Seasonal gaps in the MACHO LMC data are minimal, offering the advantage that period aliasing is nearly absent. The (non-standard) filter bands

<sup>★</sup>E-mail: isg@sao.ac.za

**Table 1.** MACHO identifications of calibrating Miras, with amplitudes.

Name	Sp	MACHO id	MACHO period	FGWC per	Orig per	$r$ amp	$\Delta r$ amp	IR amp	Comments, additional periods
C38	M2	78.6345.30	130.4	128	131	2.4	1.2	0.49	
C11	M6	None		202	200				
C20	M4	79.5864.44	209.9	210	207	4.0	1.15	0.74	
R120	K5e	79.5744.141	216.7	217	213	3.5	0.75	0.63	
R141	M1e	79.5742.23	258.2	255	260	3.5	0.93	0.65	
R110	M2, 5	None		261	266				
C7	Ce	80.6468.18	326.7	326	314	1.37	0.12	0.26	$p = 170.77$ d
R153	C, 4e	80.6468.77	346.7	370	360	3.5	2.1	0.55	$p = 179.04$ d
R105	M6Se	None		420	430				
W132	M	77.7914.16	156.2	155		2.2	0.9	0.4	
W151	M	77.7916.24	173.8	172		2.7	0.6	0.66	
W148	(M)	77.7917.61	183.1	185		3.8	1.1	0.70	
W158	(M)	82.8039.2633	193.6	185		2.4	0.24	0.48	
W19	M	77.7431.18	193.8	189		0.3		0.24	SRa; $p = 103.77$ ; 240.92 d
W77	S	77.7795.15	212.5	217		1.6	0.45	0.21	
W94	M	None		220					
W74	M	77.7667.943	231.2	227		4.0	0.9	0.5	
W1	M	77.7429.303	234.9	233		4.4	1.2	0.8	
W140	(M)	77.7913.369	242.9	244		3.2	0.6	1.0	
W48	(M)	77.7554.11	278.8	279		4.5	0.7	0.67	
W46	C	77.7550.22	285.2	286		0.9		0.24	SRa; $p = 145.25$ d
W126	K?	77.7790.310	317.7	323		5.1	1.2	0.77	
W103	C	77.7791.104	362.7	351		1.8	0.8	0.79	
W30	C	77.7555.2818	183.7	400		0.12		0.2	SRa
0517–6551	(M)	59.6157.262	116.1	117		2.1	0.7	0.42	
0512–6559	M	59.5308.655	140.7	141	140	3.5	0.7	0.65	
0530–6437	C	None		157					
0526–6754	M	4.7578.16	157.2	160		2.7	0.9	0.76	
0528–6531	M	65.7977.11	194.7	195		2.3	0.4	0.62	
GR13	(M)	None		202					
0507–6639	M	56.4330.282	207.6	208	211	4.7	0.8	0.74	
0515–6617	C	58.5666.27	225.9	211	223	2.6	0.9	0.32	
0528–6520	C	65.7858.2	229.4	231	229	1.5	0.3	0.48	
0520–6528	C	63.6647.27	233.1	234	235	2.0	0.5	0.32	$p = 269.19$
0519–6454	(C)	None		242					
0533–6807	M	None		247					
0529–6759	(C)	4.7940.12	282.8	274		1.5	0.4	0.33	
0515–6451	(C)	None		284					
0537–6607	(M)	None		284	285				
0514–6605	(C)	59.5548.26	307.5	305		2.0	0.9	0.30	
0505–6657	(M)	53.4084.14	307.4	311		2.8	0.3	0.59	
0534–6531	(C)	65.8823.24	307.7	312		2.2	0.6	1.09	
0524–6543	(M)	63.7248.11	314.9	312		3.9	0.9	0.73	
0529–6739	(C)	None		319	319;				
0541–6631	(C)	66.9897.12	342.5	328		2.8	0.9	1.04	
0515–6438	(C)	None		365	340				
W220	C	77.8150.42	280.9	286		1.5	0.4	0.50	
0502–6711	(C)	25.3717.1133	321.9	308	168;	1.8	0.4	0.44	
0537–6740	(C)	50.9275.24	367.1	418	213	1.9	0.7	0.15?	
C2	CSe(Li)	80.6593.14	548.7	573	>500	2.0	0.3	0.65	
0515–6510		None		438					
0503–6620		55.3851.16	595.5	597		4.6	0.5	1.3	
0515–6608		59.5790.13	539.1	555		5.3	0.8	0.8	
0523–6644	S(Li)	60.7112.15	638.5	649	648	4.4	0.7	0.86	
GR17		67.9649.46	728.5	780		5.0	0.9	1.25	(IR period)

Notes: in the ‘Name’ column, the order is the same as in FGWC.

In the ‘Sp’ column, a spectral type in parentheses was inferred from photometry; the others were determined directly.

In the ‘Original period’ column, the discovery period is given, if different from that used in FGWC.

that it used are  $b$  (4500–6300 Å) and  $r$  (6300–7600 Å). In this work, the MACHO World Wide Web site has been searched for the light curves of the Miras that were used to calibrate the period–luminosity relation.

## 2 SPECTROSCOPY

Spectroscopic observations suitable for spectral classification were undertaken for the Miras used in the initial determination of the  $P$ – $L$  relation for LMC Miras (Glass & Lloyd Evans 1981), as well as several that were found later, with the Unit Spectrograph and RPCS detector (Jorden, Read & van Breda 1982) on the 1.9-m telescope at Sutherland. Observations in 1983 March used the 300 g mm<sup>−1</sup> grating to give a resolution of 7 Å over the range 4300–6800 Å. Spectra of M giant stars were obtained for comparison. Most of the stars were observed in 1984–6 with the 400 g mm<sup>−1</sup> grating (resolution 5 Å, range 5200–7700 Å), which was more efficient as it was blazed for 7500 Å, nearer the peak emission intensity of these stars than the 5000 Å of the lower-resolution grating. These spectra were classified using a set of M and S spectra obtained by Lloyd Evans & Catchpole (1989), with the addition of a few carbon stars. The S and C spectra could also be used in classifying the lower-resolution spectra. The results are entered in Tables 1 and 2.

The spectra obtained differ from those of Miras in the solar neighbourhood in two respects: the proportion of stars with modified composition (S or C) is large, and the M and S stars have quite early spectra on average. The spectral types found for R110, R120, R123, C5, C38 and C48 are all very early by general galactic standards. The observations necessarily avoid phases near minimum light but do not generally represent the spectrum at maximum. These features have been reported in other studies: the high proportion of carbon-rich compared with O-rich stars among red giants in the LMC is especially well known and is thought to result from the greater ease of overturning the C/O ratio by the dredge-up of carbon produced by the triple-alpha process when the initial O/H ratio is low. Ti is also underabundant as part of the general metal weakness, so that TiO is, in effect, doubly weakened by comparison with typ-

ical galactic stars. A further consequence of this is that visual light amplitudes, which are greatly enhanced by the variation of V-band blanketing round the light cycle, are likely to be smaller than those of galactic Miras of similar period. The division between semiregular and Mira variables, which is conventionally defined by light amplitude at a given period, is likely to occur at a smaller amplitude in the LMC.

## 3 THE MACHO DATA

### 3.1 Positions

Accurate positions for the Miras on the original list of Glass & Lloyd Evans (1981) were obtained from the Digitized Sky Survey, for which the output charts are furnished with World Coordinate System positional calibrations in their FITS headers. The original finding charts were used to identify the stars on ‘second-generation’ red charts. The MACHO data base contains astrometric positions derived for each field from the Guide Star Catalog. The newly derived positions of the Glass & Lloyd Evans stars are given in Table 2. The WBP and GR positions were generally found to be satisfactory (see the next section).

### 3.2 Light curves

The MACHO data base was searched for Mira-like variables within 5-arcsec radius of the available positions. The light curves of the nearest stars to the nominal positions were displayed one-by-one using the MACHO on-line facility. In general, the counterparts, usually easy to identify by their large amplitudes, were found within 2 arcsec of the nominal positions. In 13 cases there was no MACHO overlap or the star was too bright on the MACHO template and could not readily be extracted. The MACHO identifications are given in Table 1. The light-curve information was downloaded for further processing. The  $r$ -band data are shown in Figs 1 and 2. The  $b$ -band data were generally not used.

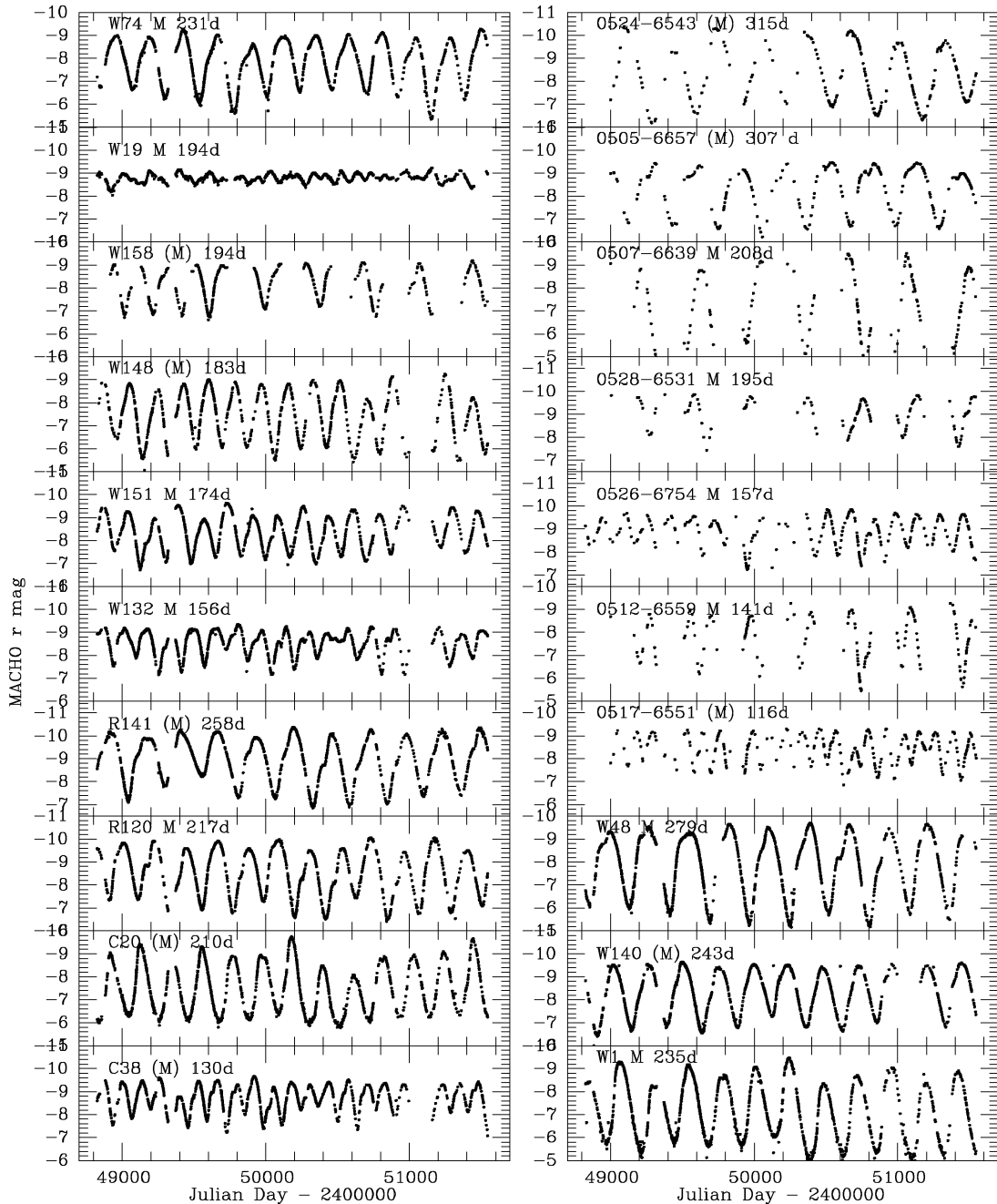
There were some surprises: three of the WBP stars were found to be semiregular variables rather than Miras. Given the small  $I$  amplitudes listed by Wood et al. (1985) for the S star W19 and the carbon star W46, this might have been expected. The carbon star W30 has a much shorter period and a smaller MACHO  $r$ -amplitude than listed by WBP. However, its identification is uncertain, since the WBP coordinates refer to a point about 16 arcsec away from the star marked on the chart (measured in the infrared work and assumed to be the correct one).

The time-span of the MACHO observations is relatively short but even so it is possible to discern a pattern in the light curves that is known from galactic Mira variables. The stars of M type show variation in the brightness of individual maxima and minima, such that the amplitude of individual cycles may differ, but there is little sign of overall changes in light level where the whole curve is shifted up or down. The carbon stars, in contrast, show several possible examples of changing mean light. This is seen to a greater or lesser degree in 0534–6531, W103, R153, 0537–6740 and 0541–6631. Feast et al. (1984) attributed this phenomenon, in galactic carbon stars, to the production of obscuring dust by the star. Lloyd Evans (1997) used AAVSO light curves covering many decades to show that fading in the light curves of carbon Miras is well correlated with red  $J - K$ , which implies that they are the stars with most circumstellar dust. Four of the five stars noted above are amongst

**Table 2.** Positions of TLE stars in 2000 coordinates from MACHO and DSS retrievals.

TLE name	R.A.	Dec.	Type
C2	05 20 46.725	−69 01 23.98	CSe
C7	05 20 28.173	−69 16 40.83	Ce
C11	05 18 16.23	−69 21 29	M6 (est. no MACHO)
C20	05 16 26.441	−69 09 59.87	M4
C38	05 19 42.412	−69 25 22.2	M2
C48	05 15 18.960	−69 14 17.97	M0, M4–5S
R49	05 16 37.522	−69 15 59.83	S6/2.5
R120	05 15 39.869	−69 07 32.38	K5e
R141	05 15 26.061	−69 15 06.58	M1e
R110	05 18 19.6	−69 03 30	M2, M5 (est. no MACHO)
R153	05 19 58.161	−69 14 16.86	C, 4e
R105	05 18 36.3	−69 19 02	M6Se (est. no MACHO)

Note: C48 and R49, although used in the work of Glass & Lloyd Evans (1981), were omitted from the FGWC paper because of suspected contamination of the  $K$  photometry by nearby stars or stars in the reference fields. The MACHO identifications of these stars are 79.5742.23 and 79.6863.18 and their MACHO periods are 176.2 and 373.4 d, respectively; for comparison, the discovery periods given by Glass & Lloyd Evans (1981) were 170 and 390 d.

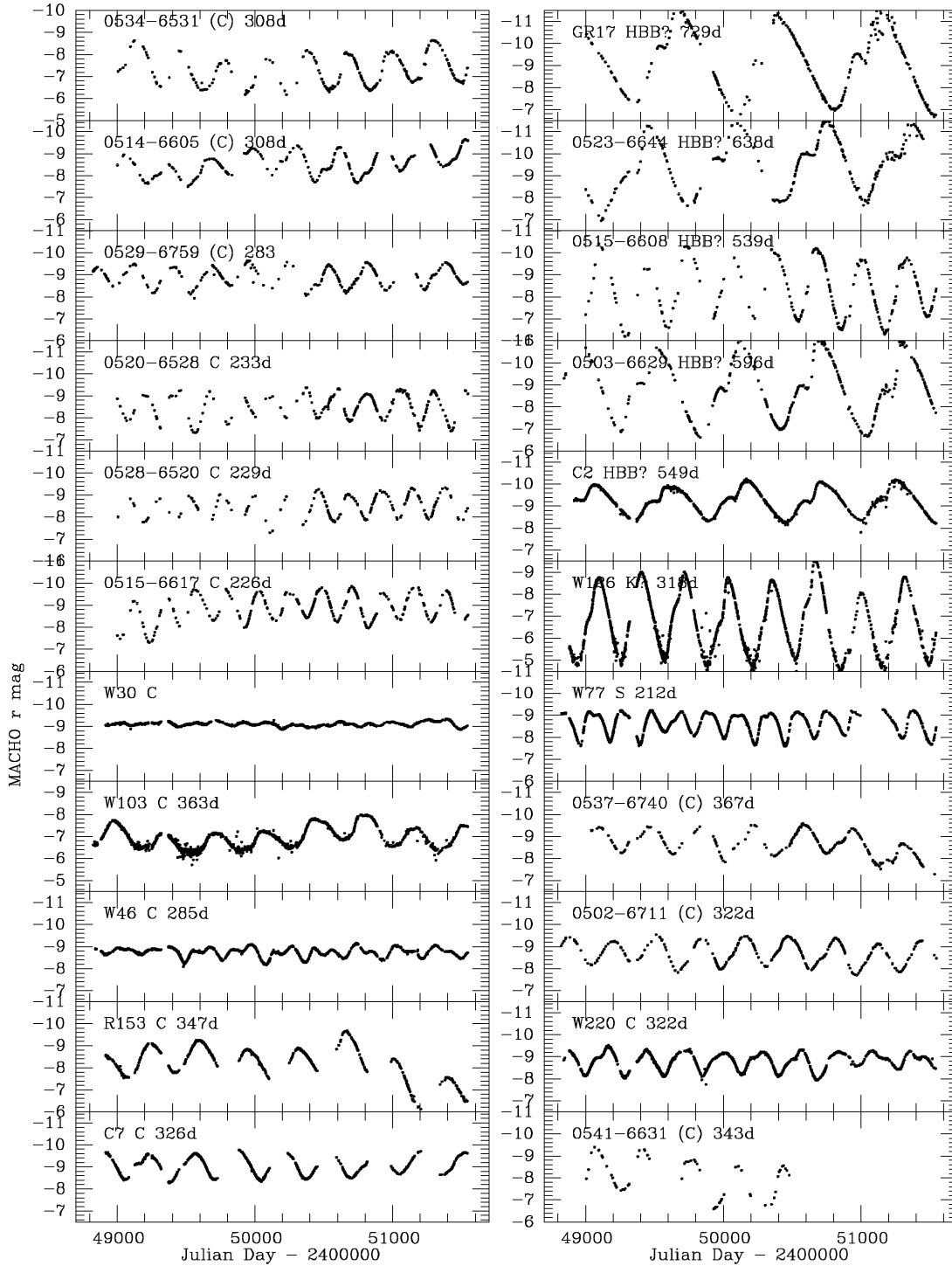


**Figure 1.** MACHO light curves for M and presumed M stars in the defining sample. W19 is evidently not a Mira.

the reddest of those for which FGWC provide *JHK* photometry, with  $J - K$  greater than 2.60, so that the same situation pertains in the LMC.

The sample of FGWC contains six stars with periods greater than 420 d that are too bright to fit the period–luminosity relation for stars with periods shorter than 420 d. One of these, 0523–6644 = RGC69 (Reid et al. 1988), is an S star with enhanced lithium (Smith et al. 1995). C2 is a CS star and also has a very strong lithium line (this paper). Wood, Bessell & Fox (1983) suggested that stars on the upper asymptotic giant branch (AGB) might be undergoing hot-bottom burning (HBB) and that is the explanation given by Smith et al. (1995) for the high frequency of lithium enhancement in stars, mostly of spectral type S, near the upper luminosity limit

of AGB stars. The predominance of S stars and the relative lack of carbon stars at higher luminosity also results from the operation of HBB. The presence of a CS star, as a star which has just crossed the boundary from S to C or is in the process of changing back from C to S, is consistent with this picture. Smith et al. (1995) find that well-developed carbon stars, with or without enhanced lithium, only occur at lower luminosity. Whitelock & Feast (2000) suggest that all the Miras with luminosities above the  $P-L$  relation are there because of HBB. The five suspected HBB stars recovered here have distinctive light curves, characterized by well-developed bumps on the rising branch. Less developed bumps of this kind were noted in about one-third of the O-type Miras and in only two of the C-type Miras.



**Figure 2.** MACHO light curves for C, presumed C, HBB and miscellaneous stars in the defining sample.

### 3.3 Periods

Periods were searched for using a least-squares Fourier fitting program. The periods are listed in Table 1, together with those used for the FGWC period–magnitude determinations. The periodograms were printed out for each star. The period with the largest amplitude was used to produce a folded light curve in each case.

The results indicate that the original periods were usually close to being correct. In addition, it is clear that they have remained stable over two to three decades. Exceptions to this are W30, which has turned out to be an irregular variable of small amplitude, and GR0537-6740, which nevertheless is not conspicuously irregular in the MACHO data. The period of GR17, which falls among the ‘HBB’, also disagrees. It is so long that only two or three cycles were covered by the discovery observations.



Four periodograms (for C7, R153, W19 and W46) showed subsidiary peaks at periods just greater than half (0.51–0.54 times) the main peak. The latter two are semiregular variables. R153 seems to be a normal carbon Mira, but C7 is a carbon Mira with a rather small amplitude.

Two carbon stars show additional frequencies slightly higher than the main ones. These are the semiregular W19 and the Mira GR0520–6528.

#### 4 AMPLITUDES

The amplitudes (see Table 1) were extracted graphically from the folded light curves. Two parameters were measured because almost all the light curves show a shift in zero-point from cycle to cycle. The first parameter (tabulated as ' $r$  amp') is the overall peak-to-peak amplitude, omitting obviously discrepant points (due to noise). The second parameter (tabulated as ' $\Delta r$  amp') is the average of the scatter at minimum and maximum (see Fig. 3 for an explanation). Since the mean magnitudes used in computing the  $K$ ,  $\log P$  relations are simply the mean of the maximum and minimum observed magnitude (see FGWC for more details),  $\Delta K$ , the equivalent quantity at  $K$ , is representative of the variability of the mean to be expected at that wavelength. The  $K$ -band infrared amplitude from the original data (given in Glass et al. 1990) is also listed.

The average  $r$  amplitude for 19 M or (M) stars is 3.4 mag (standard deviation, s.d. = 0.6) and their average  $\Delta r$  amp is 0.8 (s.d. = 0.3). The corresponding figures from 20 MACHO light curves with sufficient phase coverage from the Baade's window Sgr I field (see Glass et al. 1995) in the Milky Way galaxy, where all the Miras are believed to be of M type, is, for the  $r$  amplitude, 3.7 (s.d. 0.6) and for the  $\Delta r$  amp is 0.7 (s.d. = 0.2). The LMC and Sgr I numbers are probably not significantly different for periods in excess of about 200 d (see Fig. 4). However, it is clear that the LMC sample contains a number of short-period 'Miras' of lower amplitude, that do not seem to have counterparts in the galactic Sgr I field.

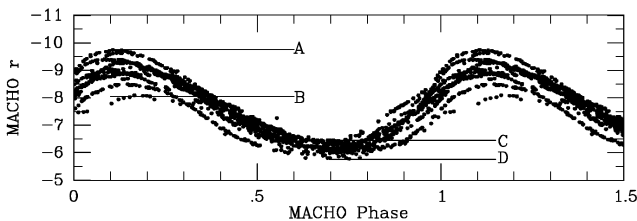
Cioni et al. (2001) found that there is not a clear division in amplitude between Miras and semiregular variables in their EROS-based sample of stars in the LMC, although stars with  $R_E$  amp < 1.0 are much more numerous than those with higher amplitudes. This question is being investigated in more detail by Glass, Cioni & Schultheis (in preparation).

For 13 C and (C) stars, the average  $r$  amplitude is 2.0 mag (s.d. = 0.6) and  $\Delta r$  amp = 0.7 (s.d. = 0.5).

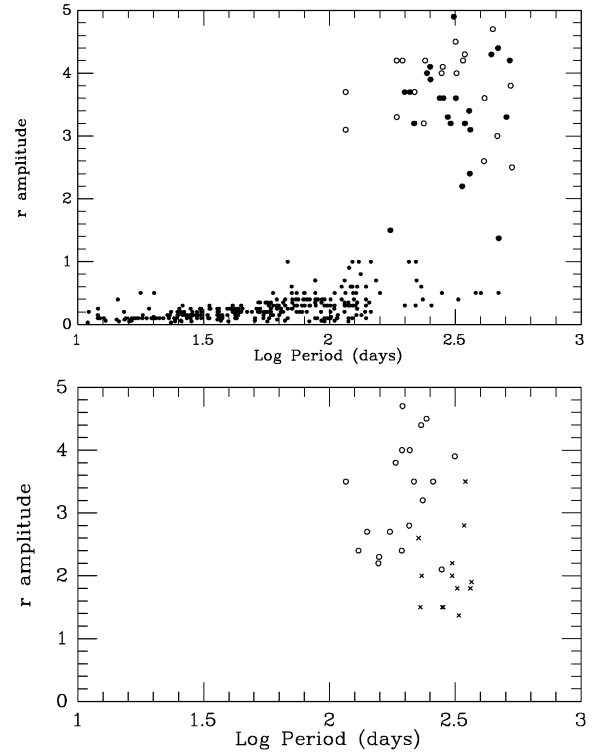
The solitary Wood et al. (1985) K? star has a rather large amplitude of 5.1 mag,  $\Delta r$  amp = 1.2.

The Wood et al. S star has  $r$  amplitude = 1.6 and  $\Delta r$  amp = 0.5.

It is thought that the amplitude of variation in Miras is affected by the chemical composition of the atmosphere. In particular, the



**Figure 3.** Phased light curve of C20, period 209.9 d. The  $r$  amplitude is given by A–D and the quantity ' $\Delta r$  amp', an indication of the amount by which the amplitude varies from cycle to cycle, is given by the average of (A–B) and (C–D) (see Table 1).

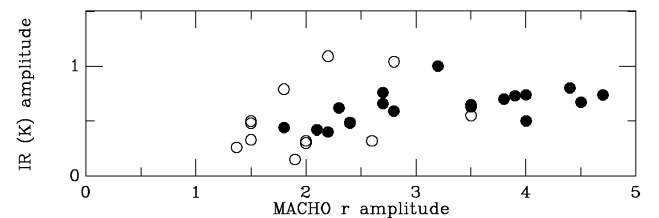


**Figure 4.** Comparison of amplitude versus period for M-giant stars in Baade's windows and the LMC. Top: Baade's windows: the Miras from the 1200 arcmin<sup>2</sup> Lloyd Evans (1976) Sgr I field are shown with the Alard et al. (2001) SRVs from two equal areas totalling 450 arcmin<sup>2</sup> in Sgr I and NGC 6522. It is probable that all of these stars are O-rich. A clear gap exists between 1 and 2 mag amplitude. Bottom: O-rich Miras (open circles) and C-rich Miras (heavy dots) from the LMC S Dor field of Lloyd Evans (1972). Most C-rich Miras and some shorter-period O-rich Miras (<200 d) have amplitudes in the 1–2 mag range not occupied in the Galaxy. The amplitudes given here are determined from second-order least-squares Fourier fits to the data to be compatible with the eye estimates used in Alard et al.

condensation of TiO in M Miras causes their visual amplitudes to be more than expected from temperature changes alone (Smak 1966; Reid & Goldston 2002). Certainly, the evidence here suggests that the M-type Miras have larger amplitudes than the C-type ones.

#### 4.1 Comparison of $r$ - and $K$ -band amplitudes

In Fig. 5 the  $K$ -band amplitudes of the sample are plotted against those in the  $r$  band. The  $r$ -band amplitudes seen in the LMC include many below 2 mag. The O-rich Miras have larger visual amplitudes than the C-rich ones.



**Figure 5.**  $K$  amplitudes versus MACHO  $r$ -band amplitudes. Known and likely carbon-rich Miras are shown with open circles; M Miras with solid points.

The parameter ‘ $\Delta r$  amp’ is an indication of how much the average  $r$  mag can vary over the long term. It is typically 1 mag or less. The equivalent quantity for  $K$  magnitudes,  $\Delta K$  amp, is difficult to estimate because long-term detailed multicycle observations are not available for ordinary Miras. However, the solar neighbourhood data presented by Whitelock, Marang & Feast (2000) give an indication. Some of their objects have been monitored, though not with very frequent sampling, over many cycles and folded light curves are given. The mean peak-to-peak amplitude of 18 of their O-rich Miras with large numbers of observations, determined in the same way as in this work, is 0.83 (s.d. = 0.26). The mean  $\Delta K$  amp is 0.23 ( $\sigma$  = 0.07).

For O-rich Miras it appears that a long-term ratio of  $\Delta K$  amp to  $K$  amplitude of about 0.2–0.3 is appropriate.

$\Delta K$  amp can be thought of as the cycle-to-cycle variation in average  $K$  magnitudes. The scatter in the period–luminosity relation for observations that encompass only one or two cycles is thus expected to be about  $0.23 \pm 0.07$  mag peak to peak, equivalent to a standard deviation of about half this amount. The observed scatter in the  $K$ – $\log P$  relation is  $\pm 0.13$  mag. The similarity of these numbers is probably fortuitous, but a long-term series of detailed observations will undoubtedly lead to a better-defined mean  $K$  magnitude for a Mira and it may be that the scatter in the  $\log P$ ,  $K$  relation can then be reduced. The data available at the moment for the LMC Miras are not sufficient to test this possibility.

## 5 PERIOD–MAGNITUDE RELATIONS

### 5.1 M stars

In what follows, the representative  $K$  magnitudes have been taken from table 1 of FGWC.

Table 3 is a list of the Mira-like M stars in the sample. Stars without MACHO light curves are marked with asterisks.

The period–luminosity relation derived from all the stars in Table 3, assuming all the errors are in the  $K$  magnitudes and that the periods are perfectly determined, is

$$K = (-3.52 \pm 0.21) \log P + (19.64 \pm 0.49) (N = 26) \sigma = 0.13.$$

This is not significantly different from the relation given for O-rich Miras in FGWC, indicating that the errors in the periods have had a negligible effect. Eliminating the stars without MACHO light curves does not change the  $K$ ,  $\log P$  relation but increases the probable errors on the slope and intercept terms:

$$K = (-3.53 \pm 0.26) \log P + (19.64 \pm 0.61) (N = 20) \sigma = 0.13.$$

It is evident that the periods are very well determined and that the scatter in the fit, if not due to possible mass or metallicity effects as mentioned in the introduction, must arise from the  $K$  magnitudes of the stars concerned. The  $K$  data for the stars with the highest residuals (exp. – obs.) have been examined in detail, but no obvious reason for rejecting any of these points can be found. The data and fits are shown in Fig. 6.

To improve the  $K$ -band data requires even better phase coverage for more cycles. The residuals can arise in the following ways.

(i) Intrinsic variability of the stars. Long-term observations of M-type Miras in the IR with detail comparable to the MACHO curves are rare. Maran et al. (1977) report 2.7- $\mu$ m monitoring of the Mira variables S Ori (414 d), S CMi (330 d), Mira itself (330 d), R Aql (284 d) and R Leo (309 d).

**Table 3.** M stars used in defining the period–luminosity relations.

Name	$\log P$	$K$	exp. – obs.
C38	2.115	12.12	0.065
C11*	2.305	11.51	0.006
C20	2.322	11.54	–0.084
R120	2.336	11.38	0.027
R141	2.412	10.99	0.149
R110*	2.417	11.29	–0.169
R105*	2.623	10.29	0.106
W132	2.194	11.67	0.237
W151	2.240	11.74	0.005
W148	2.263	11.82	–0.156
W158	2.287	11.77	–0.191
W94*	2.342	11.28	0.105
W74	2.364	11.49	–0.182
W1	2.371	11.48	–0.197
W140	2.385	11.19	0.044
W48	2.445	10.99	0.033
0517–6551	2.065	12.25	0.111
0512–6559	2.148	12.13	–0.061
0526–6754	2.197	11.79	0.106
0528–6531	2.289	11.48	0.092
GR13*	2.305	11.59	–0.074
0507–6639	2.317	11.57	–0.097
0533–6807*	2.393	11.38	–0.174
0537–6607*	2.453	11.02	–0.026
0505–6657	2.488	10.67	0.201
0524–6543	2.498	10.71	0.126

Note: stars with asterisks were not found in the MACHO data base. The periods and  $K$  magnitudes of FGWC are given for these cases.

(ii) Incomplete light curves.

(iii) The possibility that the photometry was affected by the presence of extra stars in the observing or the reference aperture, a problem encountered in aperture photometry, or that random errors have occurred.

### 5.2 $\log P$ – $m_{\text{bol}}$ relation

The period–luminosity relation derived from all the stars in Table 3, assuming all the errors are in the  $m_{\text{bol}}$  magnitudes, is

$$m_{\text{bol}} = (-3.06 \pm 0.26) \log P + (21.50 \pm 0.61) (N = 26)$$

$$\sigma = 0.16.$$

Again, the  $M_{\text{bol}}$  values have been taken from FGWC.

The equivalent expression from FGWC is

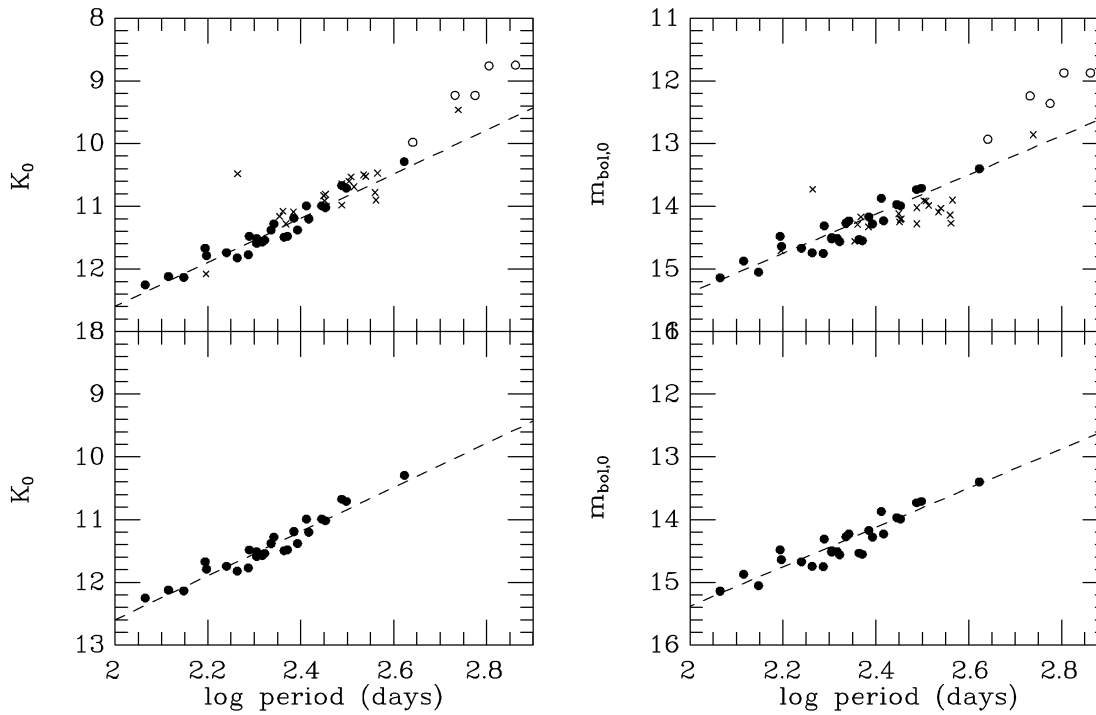
$$m_{\text{bol}} = (-3.00 \pm 0.24) \log P + (21.35 \pm 0.657) (N = 29)$$

$$\sigma = 0.16,$$

which is not significantly different.

## ACKNOWLEDGMENTS

The STAR programs written by Dr L. A. Balona (SAAO) were used in the Fourier analysis of the light curves. This paper utilizes public domain data originally obtained by the MACHO Project, whose work was performed under the joint auspices of the US Department of Energy, National Nuclear Security Administration by the University of California, Lawrence Livermore National Laboratory under contract no W-7405-Eng-48, the National Science Foundation through



**Figure 6.** Period–luminosity relations using the new periods from MACHO, when available; otherwise the periods given by FGWC. The lower panel in each case is for M stars only. The upper panel contains the carbon stars (×) and the presumed HBB stars (o). The M stars are those of Table 3.

the Center for Particle Astrophysics of the University of California under cooperative agreement AST-8809616, and the Mount Stromlo and Siding Spring Observatory, part of the Australian National University. We acknowledge use of the Digitized Sky Survey produced at the Space Telescope Science Institute under US Government grant no NAG W-2166, based on material taken at the UK Schmidt Telescope, operated by the Royal Observatory Edinburgh with funding from the UK Science and Engineering Research Council and later by the Anglo-Australian Observatory.

## REFERENCES

- Alard C. et al., 2001, *ApJ*, 552, 289  
 Alcock C. et al., 2000, *AJ*, 119, 2194  
 Cioni M.-R., Marquette J.-B., Loup C., Azzopardi M., Habing H.J., Lasserre T., Lesquoy E., 2001, *A&A*, 377, 945  
 Feast M.W., Whitelock P.A., Catchpole R.M., Roberts G., Overbeek M.D., 1984, *MNRAS*, 211, 331  
 Feast M.W., Glass I.S., Whitelock P.A., Catchpole R.M., 1989, *MNRAS*, 241, 375 (FGWC)  
 Glass I.S., Lloyd Evans T., 1981, *Nat*, 291, 303  
 Glass I.S., Reid N., 1985, *MNRAS*, 214, 405 (GR)  
 Glass I.S., Catchpole R.M., Feast M.W., Whitelock P.A., Reid I.N., 1987, in Kwok S., Pottasch S.R., eds, *Late Stages of Stellar Evolution*. Reidel, Dordrecht, p. 51  
 Glass I.S., Whitelock P.A., Catchpole R.M., Feast M.W., Laney C.D., 1990, *SAAO Circulars* No 14, p. 63  
 Glass I.S., Whitelock P.A., Catchpole R.M., Feast M.W., 1995, *MNRAS*, 273, 383  
 Hughes S.M.G., Wood P.R., 1990, *AJ*, 99, 784  
 Jorden A.R., Read P.D., van Breda I.G., 1982, *Instrumentation in Astronomy IV*, SPIE, 331, 368  
 Lloyd Evans T., 1972, in Muller A.B., ed., *The Magellanic Clouds*. Reidel, Dordrecht, p. 74  
 Lloyd Evans T., 1976, *MNRAS*, 174, 169  
 Lloyd Evans T., 1992, in Warner B., ed., *ASP Conf. Ser. Vol. 30, Variable Stars and Galaxies*. Astron. Soc. Pac., San Francisco, p. 169  
 Lloyd Evans T., 1997, *MNRAS*, 286, 839  
 Lloyd Evans T., Catchpole R.M., 1989, *MNRAS*, 237, 219  
 Maran S.P. et al., 1977, *Infrared Phys.*, 17, 565  
 Reid M.J., Goldston J.E., 2002, *ApJ*, 568, 931 (errata, *ApJ*, 572, 694)  
 Reid N., Glass I.S., Catchpole R.M., 1988, *MNRAS*, 232, 53  
 Smak J.I., 1966, *ARA&A*, 4, 19  
 Smith V.V., Plez B., Lambert D.L., Lubowich D.A., 1995, *ApJ*, 441, 735  
 Whitelock P.A., Feast M.W., 2000, *Mem. Soc. Astr. Ital.*, 71, 601  
 Whitelock P.A., Marang F., Feast M.W., 2000, *MNRAS*, 319, 728  
 Wood P.R., 1990, in Mennessier M.O., Omont A., eds, *From Miras to Planetary Nebulae*. Editions Frontières, Paris, p. 67  
 Wood P.R., Bessell M.S., Fox M.W., 1983, *ApJ*, 272, 99  
 Wood P.R., Bessell M.S., Paltoglou G., 1985, *ApJ*, 290, 477 (WBP)

This paper has been typeset from a  $\text{\LaTeX}$  file prepared by the author.