Emmy Noether

From Wikipedia, the free encyclopedia

Amalie Emmy Noether (German: [ˈnøːtɐ]; 23 March 1882 – 14 April 1935) was an influential German mathematician known for her groundbreaking contributions to abstract algebra and theoretical physics. Described by David Hilbert, Albert Einstein and others as the most important woman in the history of mathematics,[1][2] she revolutionized the theories of rings, fields, and algebras. In physics, Noether's theorem explains the fundamental connection between symmetry and conservation laws.[3]

She was born to a Jewish family in the Bavarian town of Erlangen; her father was the mathematician Max Noether. Emmy originally planned to teach French and English after passing the required examinations, but instead studied mathematics at the University of Erlangen, where her father lectured. After completing her dissertation in 1907 under the supervision of Paul Gordan, she worked at the Mathematical Institute of Erlangen without pay for seven years. In 1915, she was invited by David Hilbert and Felix Klein to join the mathematics department at the University of Göttingen, a world-renowned center of mathematical research. The philosophical faculty objected, however, and she spent four years lecturing under Hilbert's name. Her *habilitation* was approved in 1919, allowing her to obtain the rank of *Privatdozent*.

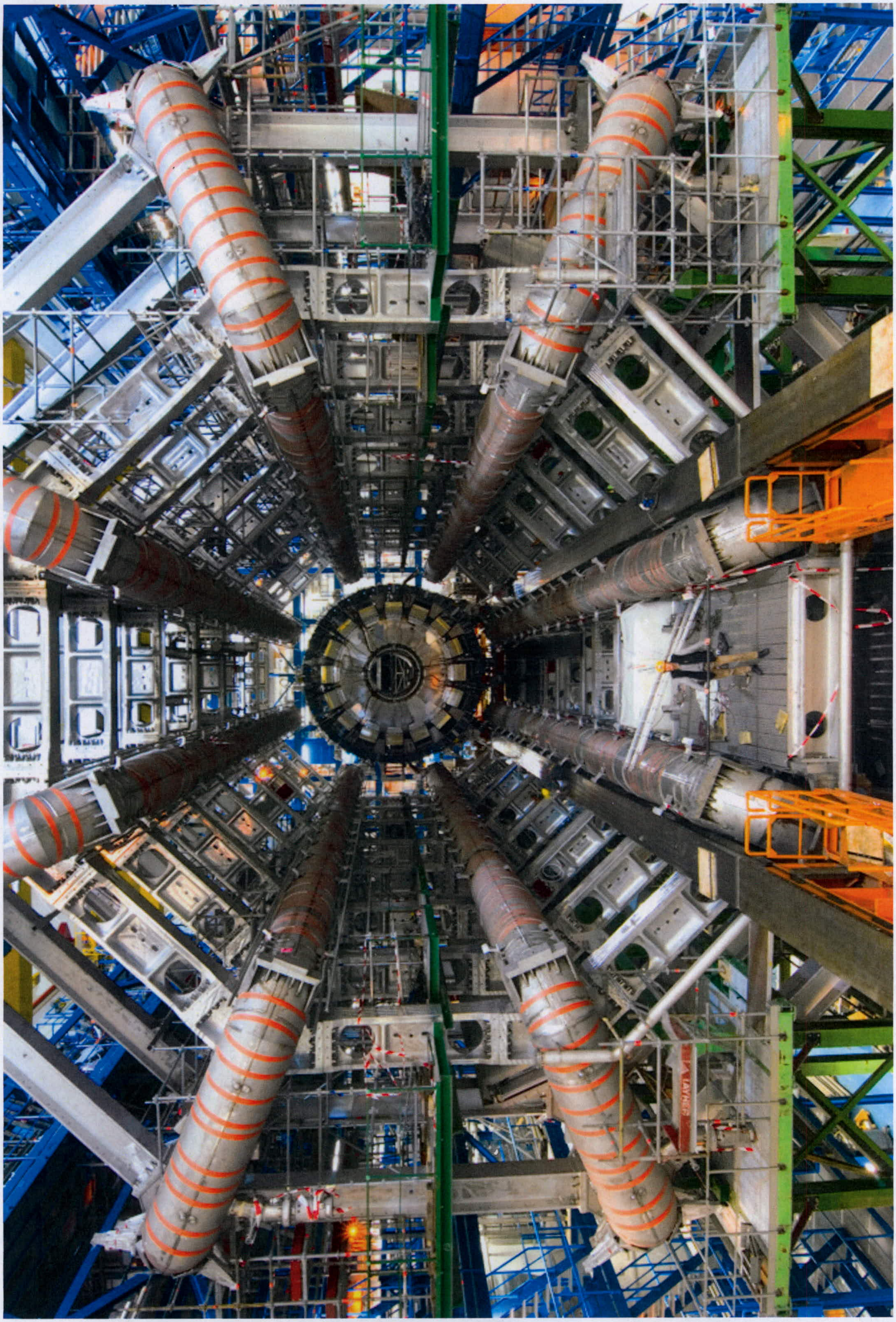
Noether remained a leading member of the Göttingen mathematics department until 1933; her students were sometimes called the "Noether boys". In 1924, Dutch mathematician B. L. van der Waerden joined her circle and soon became the leading expositor of Noether's ideas: her work was the foundation for the second volume of his influential 1931 textbook, *Moderne Algebra*. By the time of her plenary address at the 1932 International Congress of Mathematicians in Zürich, her algebraic acumen was recognized around the world. The following year, Germany's Nazi government dismissed Jews from university positions, and Noether moved to the United States to take up a position at Bryn Mawr College in Pennsylvania. In 1935 she underwent surgery for an ovarian cyst and, despite signs of a recovery, died four days later at the age of 53.

Noether's mathematical work has been divided into three "epochs".[4] In the first (1908–1919), she made significant contributions to the theories of algebraic invariants and

Emmy Noether

Amalie Emmy Noether

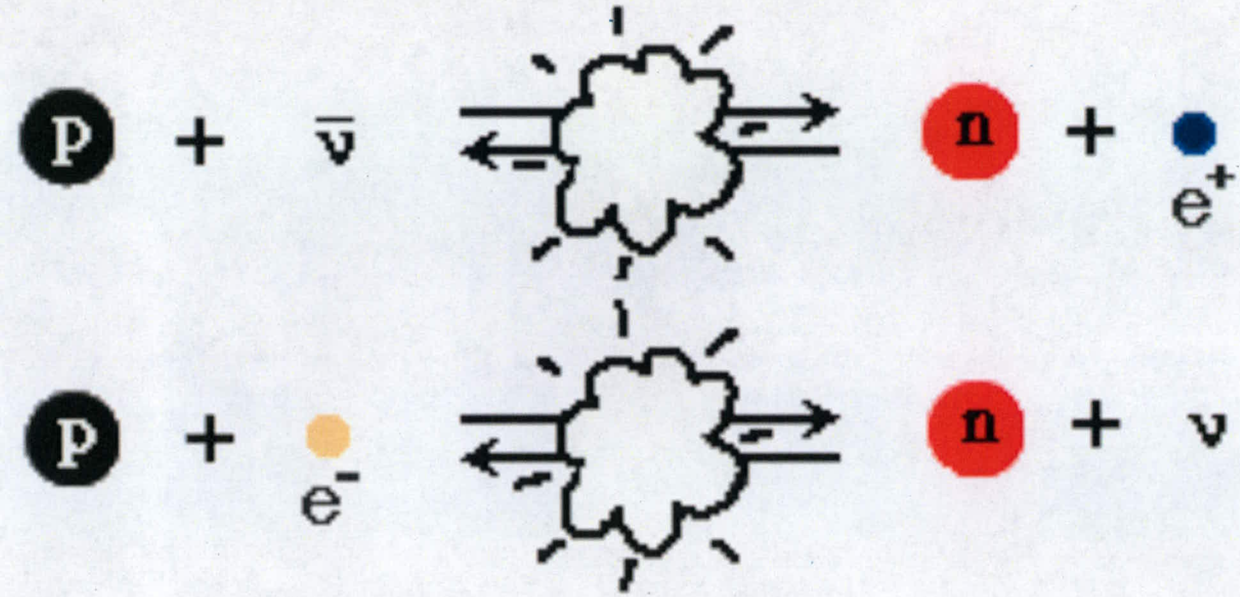
Born	23 March 1882 Erlangen, Bavaria, Germany
Died	14 April 1935 (aged 53) Bryn Mawr, Pennsylvania, USA
Citizenship	Germany
Fields	Mathematics and Physics
Institutions	University of Göttingen Bryn Mawr College
Alma mater	University of Erlangen
Doctoral advisor	Paul Gordan



proton/neutron conversions

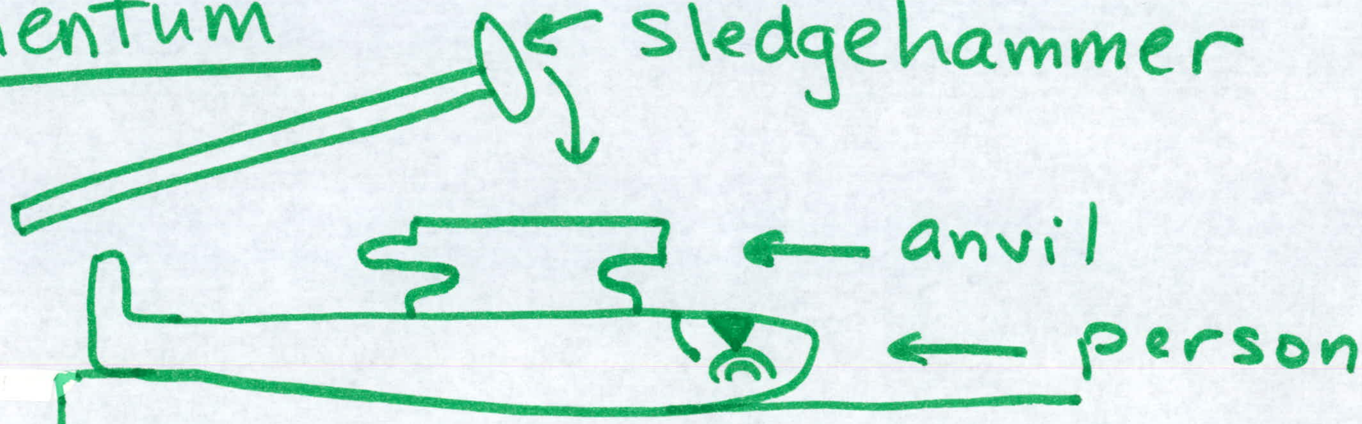
Reaction #1:

Reaction #2:



(The double arrows indicate these reactions go both ways.)

Momentum



The anvil shields the horizontal person from most of the sledge-hammer's

(a.) momentum

(b.) kinetic energy

(c.) both momentum and kinetic energy

(d.) neither momentum nor kinetic energy.

Astronomy Picture of the Day

Discover the cosmos! Each day a different image or photograph of our fascinating universe is featured, along with a brief explanation written by a professional astronomer.

2011 May 3

Globular Cluster M15 from Hubble
Credit: ESA, Hubble, NASA

Explanation: Stars, like bees, swarm around the center of bright globular cluster M15. This ball of over 100,000 stars is a relic from the early years of our Galaxy, and continues to orbit the Milky Way's center. M15, one of about 150 globular clusters remaining, is noted for being easily visible with only binoculars, having at its center one of the densest concentrations of stars known, and containing a high abundance of variable stars and pulsars. This sharp image, taken by the Earth-orbiting Hubble Space Telescope, spans about 120 light years. It shows the dramatic increase in density of stars toward the cluster's center. M15 lies about 35,000 light years away toward the constellation of the Winged Horse (Pegasus). Recent evidence indicates that a massive black hole might reside as the center of M15.

Tomorrow's picture: celestial trails

Momentum

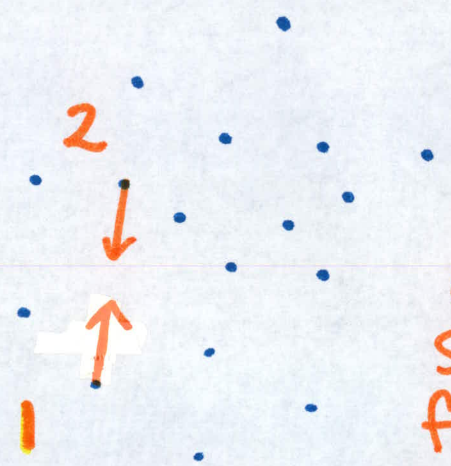
$\vec{p} = m\vec{v}$ kg·m/sec

$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$

(constant m)

$\vec{F}_{NET\ on\ Star\ 1} + \vec{F}_{NET\ on\ Star\ 2} + \vec{F}_{NET\ on\ Star\ 3} + \ldots = ?$

Globular Cluster



1 exerts a force on 2; 2 exerts the same magnitude force on 1 (gravity)

Each star attracts every other star in cluster.

Each star experiences a net force due to all the other stars.

Call the forces each star exerts on every other star an <u>internal</u> force.

If there are no other forces from outside the globular cluster exerted on any star that is part of the cluster then the net force on the cluster as a whole is <u>zero.</u>

Then the total momentum of the cluster ($\vec{p}_{star\,1} + \vec{p}_{star\,2} + \ldots$) is <u>constant</u>

→ Conservation of Momentum

Individual stars in the globular cluster are moving and accelerating (meaning $\vec{a} \neq 0$ and not simply it's speeding up) but is there any way we can think of the cluster (as one entity, like a block or the particles that make up a gas) as being at rest? In other words, where I don't see the cluster getting closer, nor farther away from me? → Zero Momentum Frame!

When a particle experiences a net force:

(a.) It accelerates ($\vec{a} \neq 0$).

(b.) It's velocity changes ($d\vec{v}/dt \neq 0$):

- Speed changes.
- direction of motion changes.
- both change.

(c) It's momentum changes.

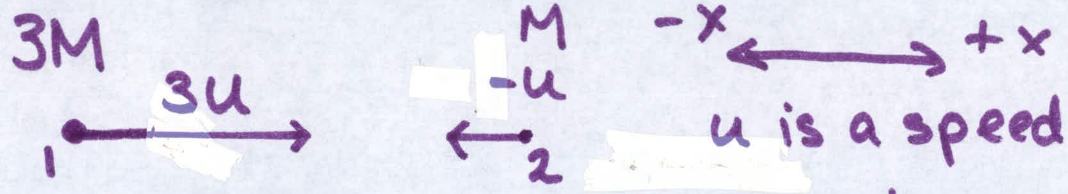
- $\vec{p} = m\vec{v}$ ($= m v_x \hat{i} + m v_y \hat{j}$)
- $d\vec{p}/dt = m\, d\vec{v}/dt = m\vec{a} = \vec{F}_{net}$

What might happen?

(d.) Kinetic energy changes (because speed changes)

(e.) Force does work on particle.

Collision in 1 Dimension



u is a speed.

Particle moving in +x direction has 3 times the mass and 3 times the speed of particle moving in -x direction.

Before Collision Particles exert no force on each other.

During the Collision they do exert forces on each other.

Express momentum in symbols:

$$\left.\begin{aligned} P_1 &= 3M\,3u = +9Mu \\ P_2 &= M(-u) = -Mu \end{aligned}\right\} P_1 + P_2 = +8Mu$$

← Net momentum in +x direction. (vector)

Kinetic Energy:

$$\left.\begin{aligned} K_1 &= \tfrac{1}{2}3M(3u)^2 = \tfrac{27}{2}Mu^2 \\ K_2 &= \tfrac{1}{2}M(-u)^2 = \tfrac{1}{2}Mu^2 \end{aligned}\right\} K_1 + K_2 = 14Mu^2$$

Kinetic Energy has no direction (scalar).

$$+8Mu = P_1' + P_2'$$

$$= 3Mv_1 + MV_2$$

$$14Mu^2 = \frac{1}{2}3Mv_1^2 + \frac{1}{2}MV_2^2$$

$$0 = P_1' + P_2' = 3Mv_1 + MV_2$$

$$0 = 3V_1 + V_2 \rightarrow V_2 = -3V_1$$

Collision in 1 Dimension (continued)

Is there a way I can imagine this system of colliding particles as being at rest (like seeing the globular cluster as a whole at rest?)

→ Zero Momentum Frame

OR Center of Mass Frame

$3M \longrightarrow 3u$ as seen by Frank 

Frank sees Liz moving in the same direction as the particle of mass 3M, 1 meter behind it.

Frank says Liz is not catching up to the particle, nor falling further behind it.

Question: How fast is Liz moving according to Frank? <u>3u</u>

Question: How fast is particle moving according to Liz (or we say "with respect to Liz's frame of reference?)

<u>0</u>, It's not moving.

What about the momentum according to Frank and Liz? Do they agree on the particle's momentum?

Frank says particles' momentum = $\underline{3M \times 3u}$

Liz says particles momentum = $\underline{3M \times 0}$

But what about conservation of momentum? Is it violated? And k?

Frank says particle's k = $\underline{\frac{1}{2} 3M (3u)^2} = \frac{27}{2} M u^2$

Liz says particles k = $\underline{\frac{1}{2} 3M (0)^2} = 0$

Emmy walks in the same direction as Liz, but behind her.

Frank says Emmy moves at $2u$, Liz moves at $3u$, and the particle moves at $3u$.

Question: How fast does Emmy say Liz and the particle are moving in her (Emmy's) reference frame? $\underline{+u}$

Question: How fast Liz say Emmy is moving in her (Liz's) reference frame? $\underline{-u}$

What is particle's p and K according to Emmy?

$P = 3Mu$, $K = \frac{1}{2} 3Mu^2 = \frac{3}{2} Mu^2$

Emmy is in the particle's zero momentum frame.

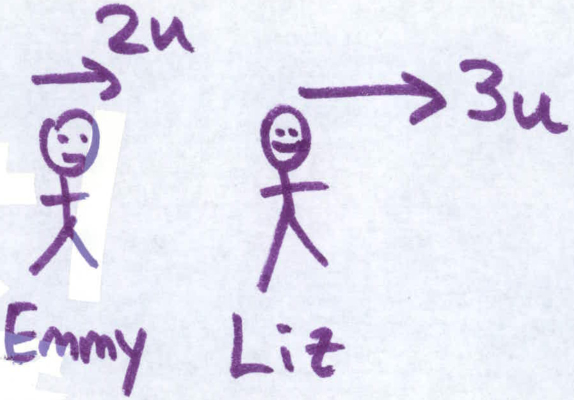
What's the zero momentum frame of our colliding particles?

Recall: $\overset{3M}{\underset{1}{\bullet}} \xrightarrow{3u}$ $\quad \overset{M,\,-u}{\underset{2}{\longleftarrow\bullet}}$

$$P_T = P_1 + P_2 = 9Mu - Mu = +8Mu$$

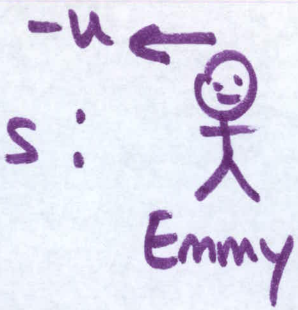
Liz, nor Frank, are in the system of two particle's zero-momentum frame (also called the center of mass frame.)

Frank sees:



$$\underset{1}{3M} \longrightarrow 3u \qquad \underset{2}{M} \longleftarrow -u$$

Liz sees:



$$\underset{1}{3M} \bullet \qquad \underset{2}{M} \longleftarrow$$

Speed of $2 = 3u + u$
$= 4u$

$$P_T = P_1 + P_2 = 3M \times 0 + M\,4u \times (-1)$$
$$= -4\,Mu \leftarrow \text{Not zero-momentum frame.}$$

Emmy sees:

$$\underset{1}{3M} \longrightarrow 3u - 2u = u \qquad \underset{2}{M} \longleftarrow -u - 2u = -3u$$

$$P_T = P_1 + P_2 = 3Mu - 3Mu = 0 \leftarrow \text{zero momentum Frame!}$$

Zero-Momentum or Center-of-Mass (CM) is found by doing

$$P_T = P_1 + P_2 = M_T V_{cm}$$

where $M_T = M_1 + M_2 = 3M + M = 4M$

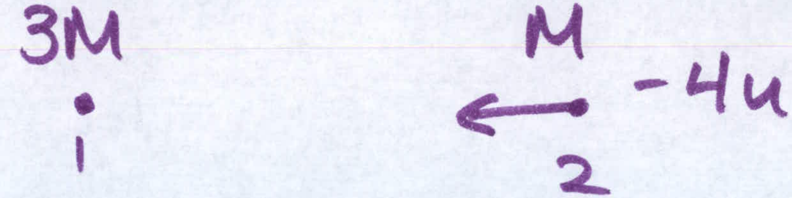
So $$V_{cm} = \frac{P_1 + P_2}{M_T} = \frac{M_1 V_1 + M_2 V_2}{M_1 + M_2}$$

Frank: $$V_{cm} = \frac{3M \times 3u + M \times (-u)}{3M + M} = \frac{8Mu}{4M} = 2u$$

In CM Frame:

$$3M \;\; \underset{1}{\bullet}\xrightarrow{3u-2u=u} \qquad \xleftarrow{-u-2u=-3u}\underset{2}{\bullet} \;\; M$$

What does Liz find for the velocity of the center of mass frame?



$$M_T v_{cm} = 3M \times 0 + M(-4u) = -4Mu$$

$$(3M+M)v_{cm} = -4Mu$$

$$v_{cm} = \frac{-4Mu}{4M} = -u$$

Center-of-Mass

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Looks like this is the derivative of three position variables

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

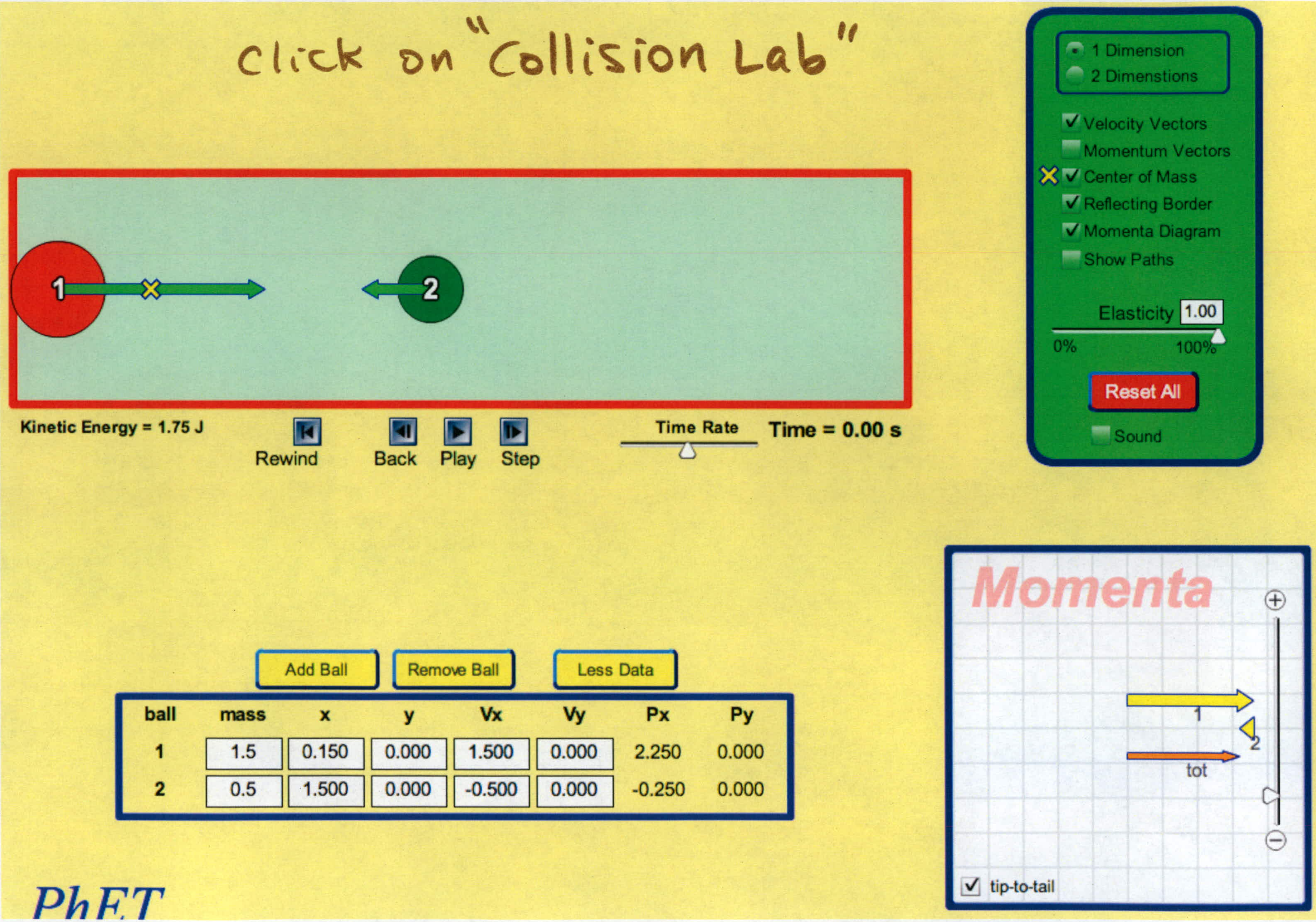
r_1, r_2 position vectors of particles 1 and 2.

r_{cm} = position vector of the center of mass point of the system of two particles.

4Be

phet.colorado.edu

Click on "Collision Lab"



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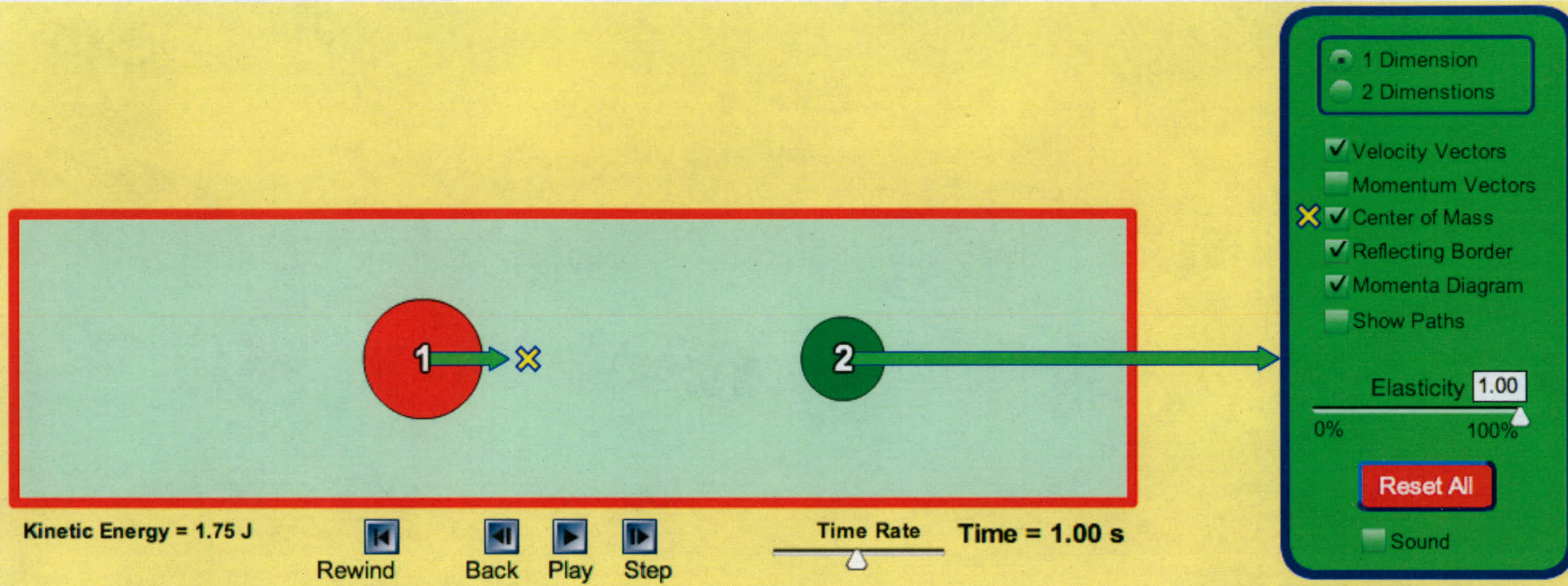
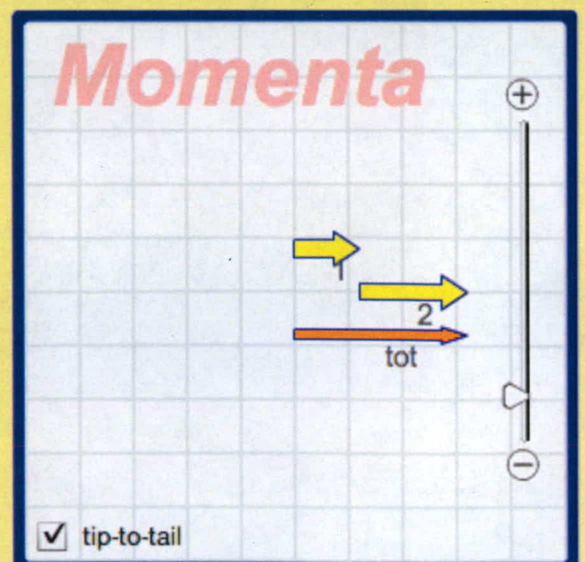
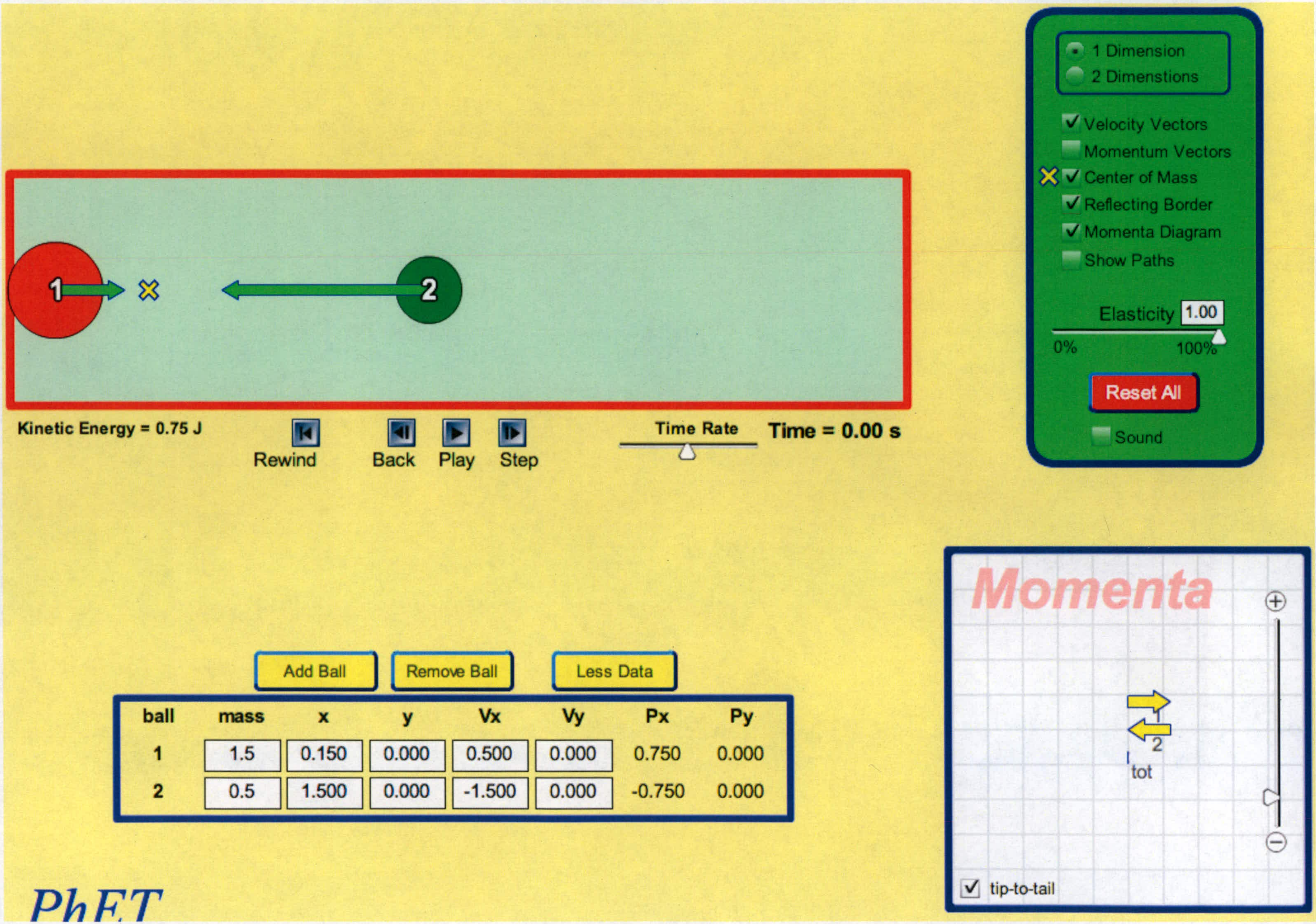


Add Ball | Remove Ball | Less Data

ball	mass	x	y	Vx	Vy	Px	Py
1	1.5	1.173	0.000	0.500	0.000	0.750	0.000
2	0.5	2.387	0.000	2.500	0.000	1.250	0.000

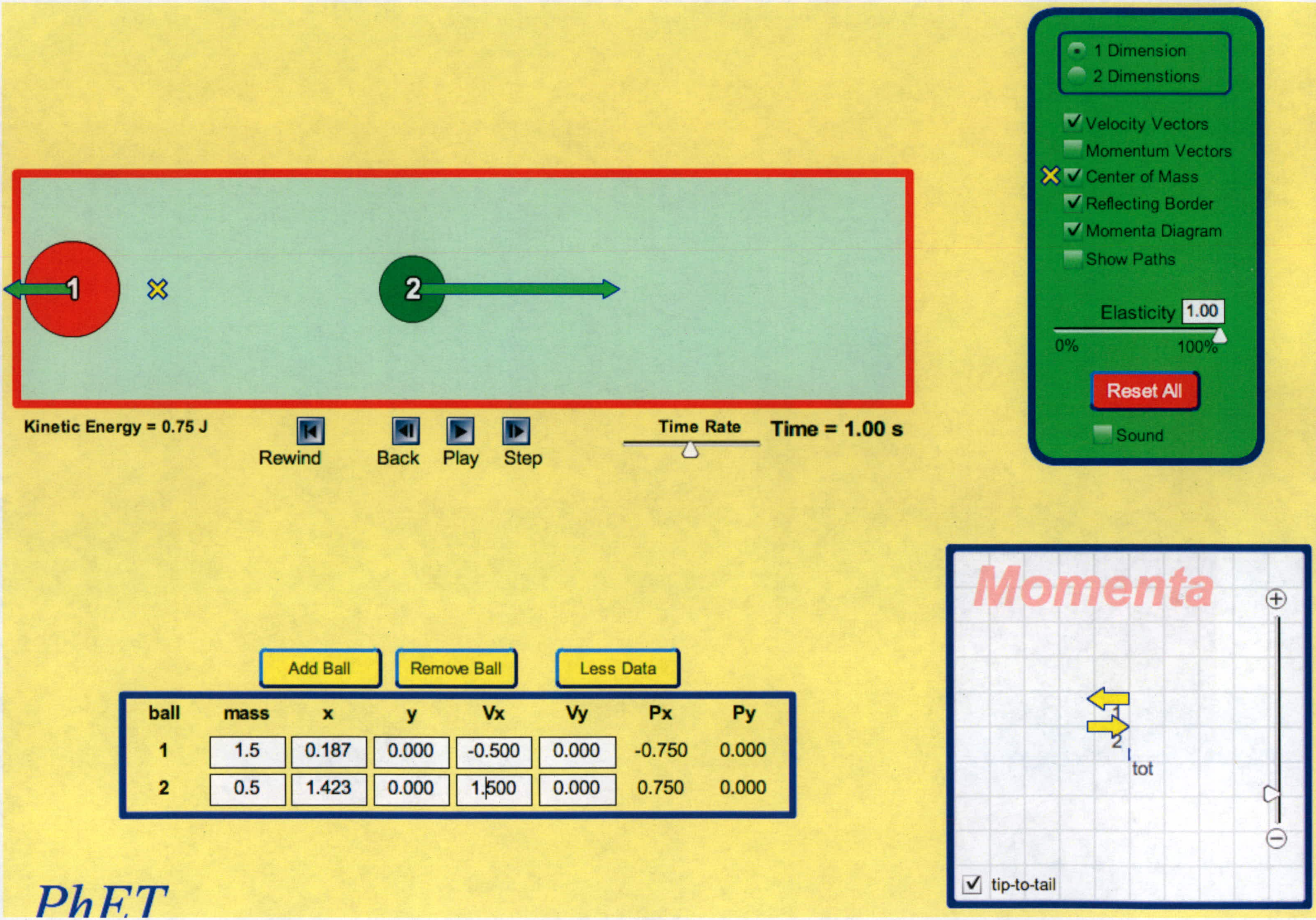


5Be



ball	mass	x	y	Vx	Vy	Px	Py
1	1.5	0.150	0.000	0.500	0.000	0.750	0.000
2	0.5	1.500	0.000	-1.500	0.000	-0.750	0.000

5Af


1 Dimension
2 Dimenstions
Velocity Vectors
Momentum Vectors
Center of Mass
Reflecting Border
Momenta Diagram
Show Paths
Elasticity 1.00
0%
100%
Reset All
Sound
1
2
Kinetic Energy = 0.75 J
Rewind
Back
Play
Step
Time Rate
Time = 1.00 s
Add Ball
Remove Ball
Less Data
ball mass x y Vx Vy Px Py
1 1.5 0.187 0.000 -0.500 0.000 -0.750 0.000
2 0.5 1.423 0.000 1.500 0.000 0.750 0.000
Momenta
1
2
tot
tip-to-tail
PhET

Two Particles, 1-dimension, before collision

Situation as posed:

3M (1) $\xrightarrow{3u}$ M (2) $\xleftarrow{-u}$

$$v_{cm} = \frac{3M\,3u + M(-u)}{3M+M} = 2u$$

$P_1 = 9Mu$ $P_2 = -Mu$

In Center of mass frame:
($V_{cm} = 0$ in this frame)

3M (1) $\longrightarrow$ $3u - 2u = u$ M (2) $\longleftarrow$ $-u - 2u = -3u$

$P_1 = 3Mu$ $P_2 = M(-3u) = -3Mu$

$(P_1 + P_2 = 0)$

$$K_1 = \frac{1}{2} 3M u^2 = \frac{3}{2} M u^2$$

$$K_2 = \frac{1}{2} M (-3u)^2 = \frac{9}{2} M u^2$$

$$K_1 + K_2 = \frac{3}{2} M u^2 + \frac{9}{2} M u^2 = 6 M u^2$$

Two Particles, 1 dimension, after collision

In center of mass frame:

Collision does not change total momentum, so $P_1 + P_2 = 0$ (conservation of momentum) (No external forces)

Let v_1 = velocity of particle 1 after collision

v_2 = velocity of particle 2 after collision

$$P_1 + P_2 = 3Mv_1 + Mv_2 = 0 \Rightarrow 3v_1 + v_2 = 0$$

$$\rightarrow v_1 = -v_2/3$$

Elastic Collision → Kinetic Energy unchanged by collision, so it's still $6Mu^2$

Two Particles, 1-dimension, after collision

$K_1 = \frac{1}{2} 3 M v_1^2 \rightarrow K_1 + K_2 = 6Mu^2 \rightarrow \frac{3}{2} M v_1^2 + \frac{1}{2} M v_2^2 = 6Mu^2$

$K_2 = \frac{1}{2} M v_2^2$

AND $v_1 = -\frac{v_2}{3}$, so

gives $\frac{3}{2} M (-v_2/3)^2 + \frac{1}{2} M v_2^2 = 6Mu^2$

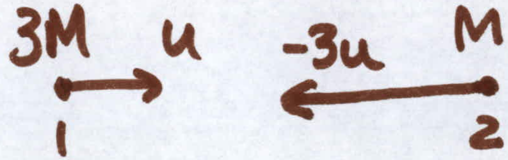
$$\frac{Mv_2^2}{6} + \frac{Mv_2^2}{2} = 6Mu^2$$

$$\frac{2}{3} M v_2^2 = 6Mu^2 \rightarrow v_2^2 = 9u^2 \text{ or } \boxed{v_2 = 3u}$$

velocities after collision $\rightarrow$ $\boxed{v_1 = -\frac{v_2}{3} = -u}$

Two Particles, 1-d, Before & After CM Frame

Before :



After :

Particles reverse direction with no change in speed in CM Frame.

What does after collision look like in the reference frame problem was originally stated?

Two Particles, 1d, After Collision, Original Frame

In original frame we found

$$v_{cm} = \frac{3M \times 3u + M(-u)}{3M + M} = 2u$$

and transformed $\underset{1}{\bullet}\xrightarrow{3M \quad 3u}$ $\xleftarrow{-u \quad M}\underset{2}{\bullet}$

into $\underset{1}{\bullet}\xrightarrow{3M \quad 3u-2u=u}$ $\xleftarrow{-u-2u=-3u}\underset{2}{\bullet}$ M

After collision $\xleftarrow{3M \quad -u}\underset{1}{\bullet}$ $\underset{2}{\bullet}\xrightarrow{M \quad 3u}$

CM Frame

After Collision, original frame $\underset{1}{\bullet}\xrightarrow{3M \quad -u+2u=u}$ $\underset{2}{\bullet}\xrightarrow{M \quad 3u+2u=5u}$

Particles move in same direction.

Two Particles, 1D, Kinetic Energy

What is kinetic energy, K, after and before collision in original frame?

Before: $K_1 = \frac{1}{2} 3M (3u)^2 = \frac{27}{2} M u^2$

$K_2 = \frac{1}{2} M (-u)^2 = \frac{1}{2} M u^2$

$K_1 + K_2 = {}^{28}/_{2} M u^2 = 14 M u^2$

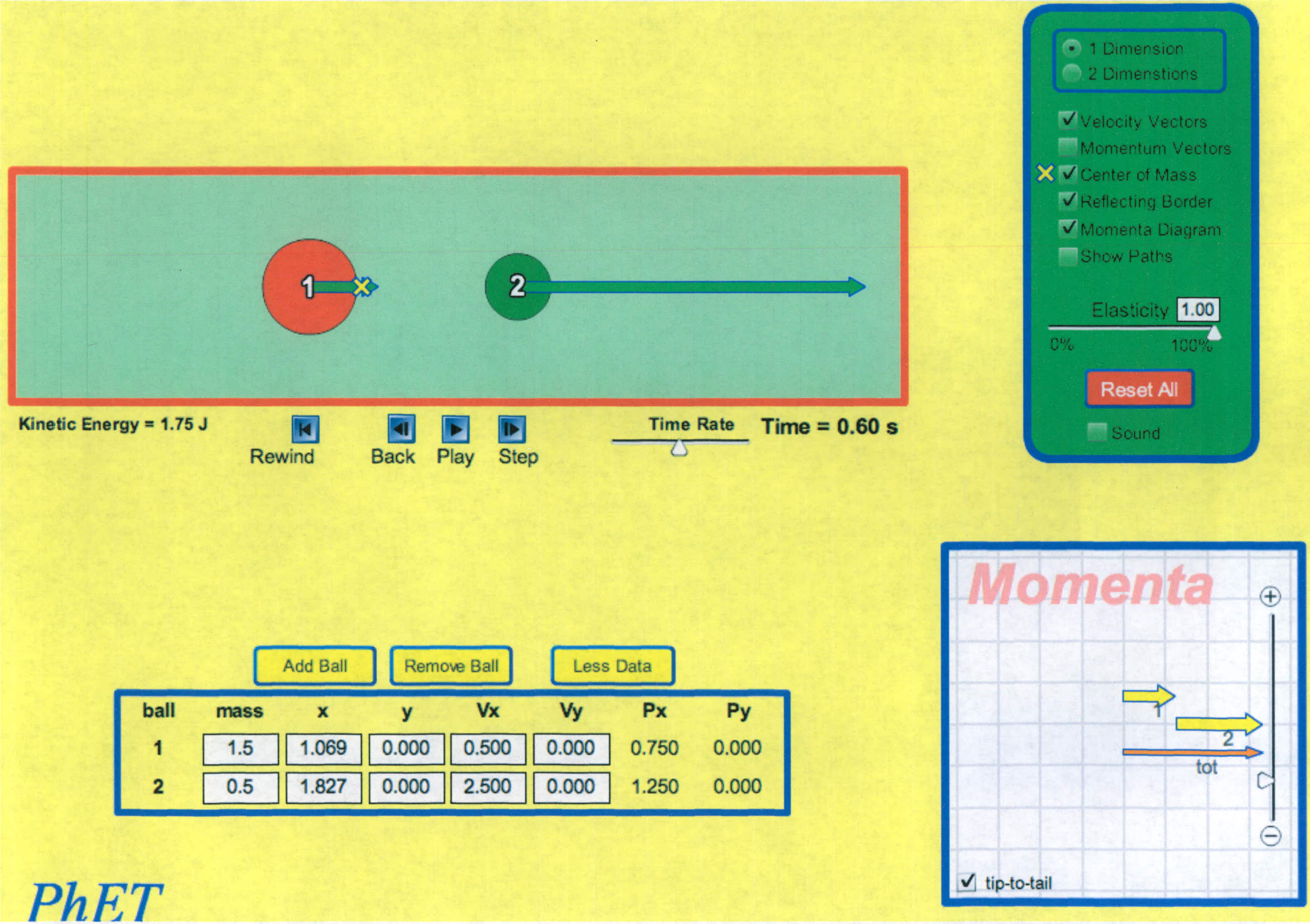
After: $K_1 = \frac{1}{2} 3M u^2$

$K_2 = \frac{1}{2} M (5u^2) = \frac{25}{2} M u^2$

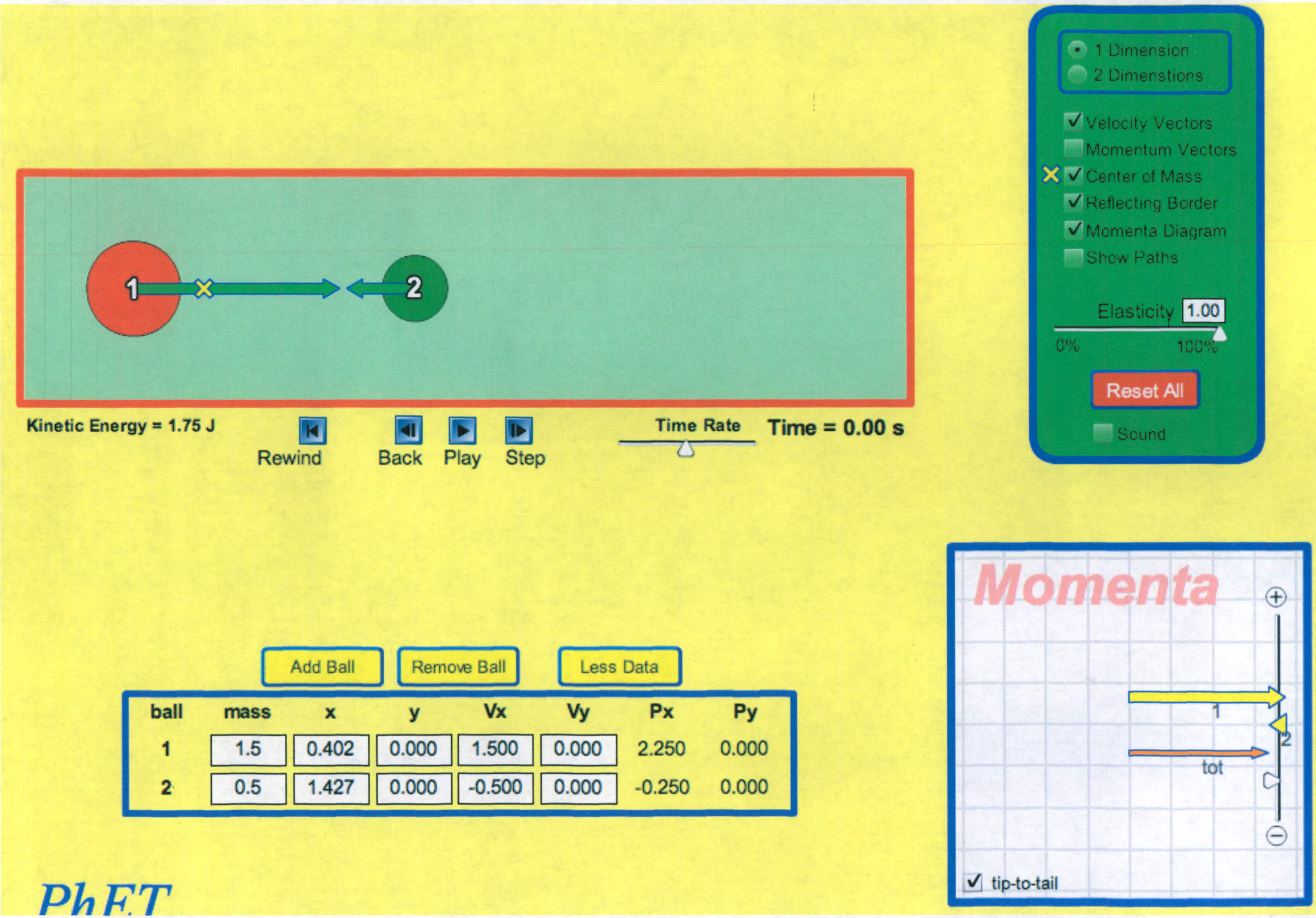
$K_1 + K_2 = \frac{28}{2} M u^2 = 14 M u^2$

Same

Elastic Collision K conserved in each frame.



ball	mass	x	y	Vx	Vy	Px	Py
1	1.5	1.069	0.000	0.500	0.000	0.750	0.000
2	0.5	1.827	0.000	2.500	0.000	1.250	0.000



ball	mass	x	y	Vx	Vy	Px	Py
1	1.5	0.402	0.000	1.500	0.000	2.250	0.000
2	0.5	1.427	0.000	-0.500	0.000	-0.250	0.000