

Declining Labor Turnover and the Importance of Intensive Margin Adjustment*

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Abstract

The contribution of intensive margin adjustments to the cyclical fluctuations in total hours worked has increased in the US since the 1980s. I document that the job tenure length has increased during this period and labor hours adjustments in recessions are more prominent in economies with higher job tenure lengths. I build a search-and-matching model with part-time workers and job-specific human capital accumulation. With the model, I claim that the improvement in initial match quality can account for the increased use of intensive margin adjustments along the business cycle. A policy simulation shows that subsidizing intensive margin adjustments via Short-Time Compensation (STC) policy is more effective in reducing unemployment volatility when the initial match productivities are higher and job separations are lower.

Keywords: Intensive margin adjustments, endogenous separation, human capital, business cycle

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1 Introduction

The total hours worked show cyclical movements along the business cycle but it has been increasingly the hours per worker that contribute more to the cyclicalities in the last four decades. For workers and firms, there are two different ways to decrease their labor input for production in times when the aggregate productivities are low. One is by firing workers which is the extensive margin of labor adjustment. The other is by reducing the hours per worker which is the intensive margin adjustment. Since the 1980s, there has been a steady decline in the unemployment inflow rates. ([Fujita \(2018\)](#) and [Faberman \(2017\)](#)). Between 1976 and 2000, the monthly unemployment inflow rates were on average 2.1 percent which has decreased to 1.6 percent post-2000. On the other hand, the labor market flows between full-time (FT) to part-time (PT) workers are steady at 4 percent during the same period. It implies that intensive margin adjustments have become relatively more important as a predominant method to adjust labor input compared to firing workers.

One of the major differences between extensive and intensive margin adjustments is that their implications on job-specific human capital. Once a job match is dissolved, the worker has to find another employer making the human capital that is specific to one's previous job obsolete. In contrast, reducing the hours worked allow workers and firms to preserve their job-specific human capital by adjusting the labor input with hours worked without ending their relationship. After the aggregate productivity goes back up to the normal level, workers with reduced hours can again work for their original hours with their preserved human capital.

In this paper, I consider the improved initial match quality as a driving force behind the decline in labor turnover. [Mercan \(2017\)](#) and [Pries and Rogerson \(2022\)](#) both consider the improvement in initial signal about the quality of job matches as a source of reduced labor market fluidity. As a result of less noise, the quality of initial matches improve as firms and workers become more selective in forming matches. Here, I directly shift initial

match productivity distribution instead of modelling the signalling process in order to keep the model simple.

I start my analysis with decomposing the change in total hours worked into changes in employment and hours per worker from NBER peak before NBER dated recessions. Using the aggregate data from Labor Productivity and Costs (LPC), I find that a larger share of the decrease in total hours worked has been coming from reductions in working hours of incumbent workers rather than dissolving their match in more recent recessions. Among OECD countries, the United States is known to be less reliant on intensive margin adjustments compared to other countries (Ohanian and Raffo (2012)). However, little has been explored how the importance of intensive margin adjustments has changed over time.

The steady decline in the rate of job separations has resulted in the right shift in job tenure distribution. Hyatt and Spletzer (2016) record that the fraction of workers who stayed in their job for less than a year has been constantly decreasing for last four decades. Considering that workers are more likely to possess higher amount of job-specific human capital when they stay in their job for longer, workers and firms would be increasingly less willing to forgo the current match in times of economic downturns. Using within state variations in job tenure distribution, I find that in economies with high job tenure, they are more likely to use intensive margin than extensive margin adjustments in recessions.

One of the two main goals of my theoretical exercise is to quantitatively assess the role of improved initial match productivities in determining the relative importance of intensive margin adjustments. I add three main features to the standard search-and-matching model. There are part-time workers, on-the-job search, and job-specific human capital to incorporate the different consequences in human capital between extensive margin and intensive margin adjustments. After calibrating the model to the 1980-2000 period data, I measure the size of change in initial match productivities that is needed to replicate the decline in labor turnover in post-2000 economy. 12% increase in the initial match productivities can replicate the decline in job separation rates from 2.1% in pre-2000 period to 1.6% in post-2000 period.

The second goal is to quantify the effects of Short-Time Compensation (STC) in reducing unemployment volatility in pre-2000 and post-2000 economies. The short-time compensation is a labor subsidy scheme that is designed to incentivize firms and workers to reduce hours worked temporarily instead of firing them. The reduced wage due to hours reduction is subsidized in states where STC program exist. California was the very first state where it started STC in 1978. Now, 27 states implement the policy but the take-up rates have been steadily low that the amount of STC paid out is less than 1 percent of Unemployment Insurance (Faberman (2017)). Even though STC policy utilization momentarily increased during COVID-19 pandemic, the share of Unemployment Insurance (UI) initial claims that were STC has been still very small at 1 percent at its highest(Krolikowski and Weixel (2020)). In contrast, European countries such as Germany and France have successfully managed to reduce unemployment fluctuations using STC along the business cycle (Tilly and Niedermayer (2017) and Giupponi and Landaïs (2022)). Therefore, it is worthwhile to examine how much successfully promoting STC in the US can stabilize the labor market in recessions and if it is still valuable in the current low labor turnover economy.

2 Related Literature

There are three different strands of literature that are closely related to this paper. First of all, this paper contributes to the literature of declining labor turnover. There have been several attempts to uncover the reason behind the diminishing fluidity in the US labor market. Fujita (2018) explores the increasing risk of skill loss during unemployment as a reason behind the reduced turnover. On the other hand, Mercan (2017) and Pries and Rogerson (2022) emphasizes the improved signal on new matches as the driving force behind the lessend fluidity¹. I contribute to this literature by analyzing effects of improvements in initial match quality on the use of intensive margin adjustments where the improvement is

¹The major difference between these two is that the former investigates job-to-job transitions and the latter job destruction rates

calibrated to replicate the decline in job separation rates.

The second strand of literature that this paper is related to is measuring the importance of intensive margin adjustments. [Ohanian and Raffo \(2012\)](#) and [Cacciatore et al. \(2020\)](#) have confirmed that hours per worker accounts for one-third of the unconditional volatility of aggregate hours in the United States which is in line with previous research including [Cho and Cooley \(1994\)](#)). However, little has been explored how the importance of intensive margin adjustments has changed over time in the United States. I contribute to the literature by finding that the cyclical fluctuations in total hours worked has been increasingly due to hours per worker fluctuations in recessions.

Some of the recent literature in intensive margin adjustments has focused on fluctuations in part-time workers when it comes to analyzing change in hours per worker. [Borowczyk-Martins and Lalé \(2019\)](#) find that cyclical variation in hours per worker is largely driven by fluctuations in the share of part-time workers and it is especially true in recessions. [Gomis-Porqueras and Griffy \(2020\)](#) develop a random search model that incorporates part-time workers by introducing different acyclical match maintenance costs to full and part-time matches. I add on-the-job search and job-specific human capital accumulation to fully incorporate the virtue of preserving human capital in intensive margin adjustments. The random search model is tractable because I assume Bertrand competition between poaching and incumbent firms following [Lise and Robin \(2017\)](#).

Lastly, this paper is related to the literature of Short-Time Compensation (STC). Most of the papers in this literature have mainly analyzed STC in European countries ([Tilly and Niedermayer \(2017\)](#) and [Giupponi and Landais \(2022\)](#)). It is mainly because Germany has been the most successful countries in introducing STC policy, which is called Kurzarbeit. Other European countries also extensively use STC to reduce unemployment fluctuations in recessions. In contrast, STC policy has gained little traction in the US even though it exists ([Krolkowski and Weixel \(2020\)](#)). Therefore, I examine the effects of STC policy once the policy is successfully promoted and widely used. Specifically, I compare its effectiveness on

reducing unemployment volatility in low and high labor turnover economies.

3 Empirical Analysis

3.1 Labor Turnover and Intensive Margin Adjustments

To find out how the importance of intensive margin adjustments has changed over time in recessions, I use hours per worker and employment quarterly data from Labor Productivity and Costs (LPC). Then, I can decompose the total hours change into hours per worker change and employment change.

$$\Delta \log(\text{Total Hours}) = \Delta \log(\# \text{ of Employees}) + \Delta \log(\text{Hours per Worker})$$

Figure 1 records the change in total hours from the quarter before the NBER dated recessions to the trough in four different recession periods post-1980. Both total hours and hours per worker decreases during the recession periods. As shown in Table 1, the cyclical component of total hours worked decreased 7.24 percent during the 1980 twin recessions from the peak to trough. The decrease in hours per worker contributed only 11.68 percent of the decrease in total hours. Compared to the 1980s, the share of hours per worker more than doubled in its contribution to the peak-to-trough decline in total hours worked during the Great Recession reaching 25.08 percent. The increasing share of hours per worker is consistent across using different filtering parameters and methods²

3.2 Job Tenure and Full-Time/Part-Time Transitions

Using the Current Population Survey (CPS) monthly survey, I measure monthly unemployment inflow rates over time utilizing its rotating panel structure. It repeats the findings from previous research of Fujita (2018) and Molloy et al. (2016) that the unemployment inflow

²The filtering methods include Hodrick-Prescott, Baxter-King, and Hamilton filters

Table 1: Peak-to-Trough Changes in Hours

	Total Hours	Hours per worker	Hour share(HP)	Hour share(BK)
1980	7.24%	0.85%	11.68%	8.88%
1990	1.70%	0.20%	11.53%	14.34%
2001	3.11%	0.64%	20.46%	19.94%
2007	7.39%	1.83%	25.08%	26.36%

Peak-to-trough changes are HP-filtered log deviations from NBER peak quarter to trough quarter for each recession. Total hours and hours per worker series are logged and calculated after HP-filtering the data with a smoothing parameter of 1600. Total hours is defined as an employment level multiplied by average weekly hours worked per worker.

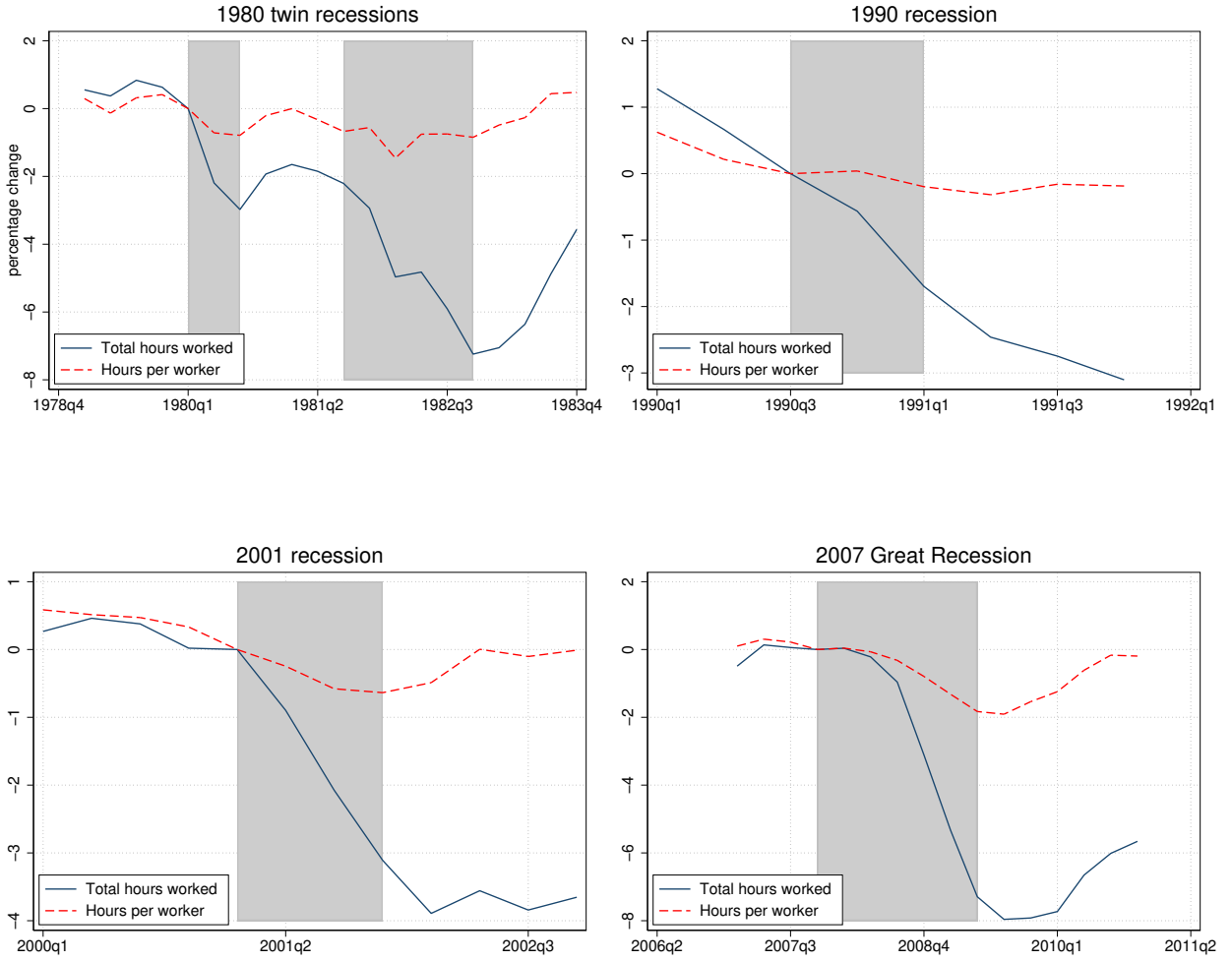
rates and job destruction rates have been on a secular decline since 1980s. Before 2000, the monthly unemployment inflow rates have been on average 2.1% but it has decreased to 1.6% after 2000.

Another dimension of the secular change in labor market is the decline in short-duration job. Naturally, workers would stay in their job for a longer period of time if they are less likely to be separated from it. Therefore, the right shift in job tenure distribution is a mirror image of declining job separation rates. Figure 2 shows that unemployment inflow rates have been steadily declining. This is especially true because the decline in unemployment inflow rates has been concentrated in low-tenure workers (Pries and Rogerson (2022)). Figure 3 confirms this trend suggesting that the job tenure distribution has been on a secular right shift. Moreover, the decline in share of workers with less than two years of job tenure has been driving this trend. Over the years, the labor market has been filled with workers with more job-specific human capital.

State-level data also repeat the pattern of the decrease in job separation rates and the right shift in tenure distribution across time. Figure 4 shows the negative relationship between average unemployment inflow rates in each state and its share of worker with more than 3 years of job tenure. In states with higher labor turnover, there are less workers staying in the same job for a given period³.

³This relationship holds true with alternative measure of job tenure such as the share of workers with more than 2 years of job tenure or the average length of tenure

Figure 1: Peak-to-Trough Changes in Hours



One caveat with the CPS job tenure supplement is that it started in 1996, which does not give the full picture of change in job tenure distribution in 1980s⁴. Alternatively, I exploit within-state variations in job tenure distribution to measure the relationship between job tenure and labor adjustments. Here, I run the following regression:

$$\log y_{it} = \beta_1 \log U_{it} \cdot \log T_{it} + \beta_2 \log U_{it} + \beta_3 \log T_{it} \\ + \beta_4 X_{it} + \lambda_i + \lambda_t + \varepsilon_{it}$$

⁴1983 and 1987 job tenure supplement exist but it was fielded as part of ob Tenure/Occupation Mobility and Training Supplement and under a different sample universe

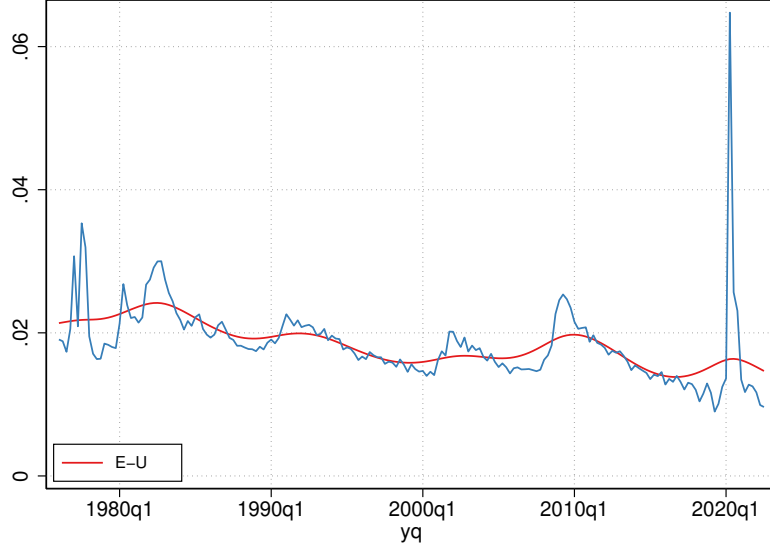


Figure 2: Employment to unemployment

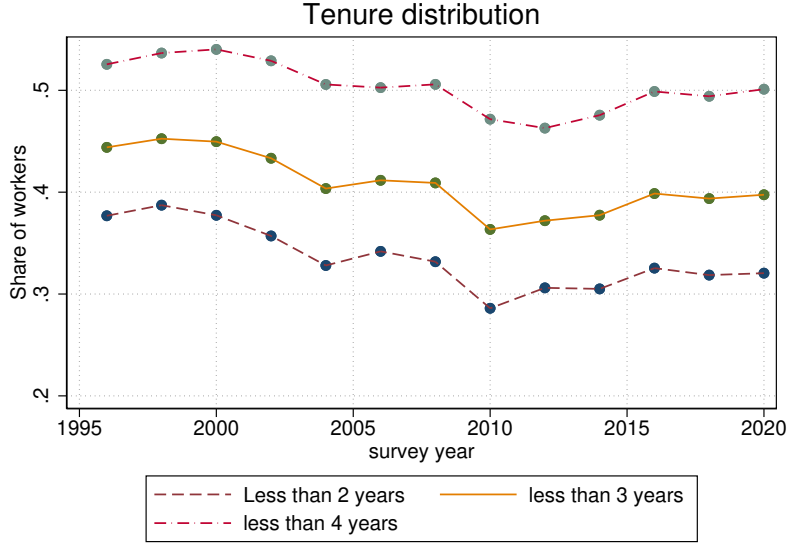


Figure 3: Job Tenure Distribution

where y_{it} is the yearly average of monthly transition probabilities of each state for years when the job tenure data is available ⁵. State-level unemployment rates are denoted as U_{it} and the share of workers with more than 3 years of tenure within state is denoted as T_{it} . The main coefficient of interest is β_1 which measures how differently business cycle (U_{it}) affects

⁵Job tenure data is available biannually starting from 1996. CPS Job Tenure Supplement survey is conducted in February until 2000 and January starting from 2002

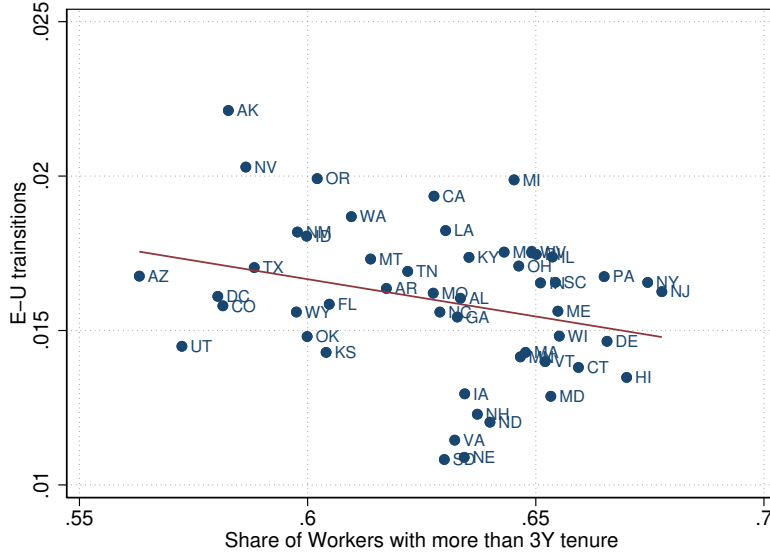


Figure 4: State-level E-U rates and share of workers with more than 3 years

labor market flows y_{it} depending on the state-level job tenure distribution (T_{it}). I control for state and year fixed effects. I also include control (X_{it}) for average age, share of female workers, and year of education.

I consider two different dependent variables that are full-time to part-time (F-P) and employment to unemployment transitions (E-U). The former represents intensive margin adjustments while the latter represents extensive margin adjustments⁶. β_2 measures the elasticity of labor flows with respect to state unemployment rates U_{it} . For both F-P and E-U flows, this elasticity is positive which means that they go up in recessions. The estimated signs of the main coefficient β_1 are the opposite between the dependent variable F-P and E-U transitions meaning the job tenure distribution has contrasting effects to the above mentioned elasticities. The negative β_1 for column (1) and (2) implies that 1 percent increase in the share of high tenure workers within states is associated with 0.3 percent increased elasticity of F-P flows with respect to unemployment rates. Conversely, 1 percent increase in the share of high tenure workers is associated with 0.8 percent decreased elasticity of E-U flows with

⁶Borowczyk-Martins and Lalé (2019) have shown that change in the number of part-time workers account for large part of hours per worker change and more than half of the change comes from transitions between full-time and part-time positions. Over 90 percent of these transitions happen within the same employer

Table 2: Job Tenure and Labor Market Flows across States

	(1)	(2)	(3)	(4)
	F-P	F-P	E-U	E-U
Unem. x more than 3y	0.357** (0.163)	0.327* (0.175)	-0.836*** (0.286)	-0.344 (0.229)
Unem.	0.400*** (0.0695)	0.380*** (0.0784)	0.434*** (0.126)	0.536*** (0.109)
more than 3y	1.017* (0.515)	1.108* (0.556)	-3.890*** (0.858)	-1.338* (0.684)
Controls	Y	Y	Y	Y
State FE	Y	Y	Y	Y
Time FE	N	Y	N	Y
N	663	663	663	663
adj. R^2	0.485	0.523	0.661	0.868

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

respect to unemployment rates according to column (3) and 0.3 percent according to column (4). ⁷

This state-level evidence shows that the increase in job tenure is related to increased use of the FT to PT transitions in recessions. Conversely, the increase in job tenure is related to the decreased use of job separation rates.

4 Search and Matching Model with Part-Time Workers

I have shown two empirical facts. Firstly, I found that hours per worker decrease contributed more to the total hours decrease in post-2000 recessions than in recessions before 2000 from the aggregate data. One of the most pronounced change in labor market in the last four decades were the steady decline in unemployment inflow rates. Because of this fact, there

⁷Using job destruction rates from Quarterly Workforce Indicator (QWI) instead of job separation rates also similarly show negative β_1 coefficients

have been an increase in the amount of job-specific human capital which is observed as a right shift in job tenure distribution. Using the within state variation, I found that this right shift is related to increased use of intensive margin adjustments in recessions.

In this section, I theoretically examine the main hypothesis that the improvement in initial match productivities led to the increased use of intensive margin adjustments in recessions. I build a search and matching model with part-time workers to add intensive margin adjustments to the standard Diamond-Mortensen-Pissarides (DMP) model ([Mortensen and Pissarides \(1994\)](#)). Moreover, I add job-specific human capital accumulation and on-the-job search to consider that intensive margin adjustments preserve job-specific human capital unlike extensive margin adjustments. Then, I calibrate the model in pre-2000 economy and estimate the size of the initial match productivity improvements to replicate the decline in job separation rates in the data. After that, I compare the response of pre-2000 and post-2000 economies to aggregate productivity shocks.

4.1 Environment

I build a random search model with counter-cyclical part-time rates where workers accumulate job-specific human capital and on-the-job search. Workers either have low or high human capital ($h^h > 1$ or $h^l = 1$). Every match starts from low human capital level and can exogenously attain high human capital with a fixed probability (ϵ) each month. High human capital workers lose their job-specific human capital once they lose their job or move to another job.

Each worker draws match-specific productivity (x_t) that follows an AR(1) process from its stationary distribution. As [Gomis-Porqueras and Griffy \(2020\)](#), There is a constant overhead cost for each match and full-time positions pay higher amount compared to part-time positions ($T_p < T_f$). Figure [A.2](#) is a suggestive evidence that there exists such cost but the yearly frequency of observation does not allow to investigate the overhead cost change related to hours worked change. At the same time, part-time workers produce less amount of

goods than full-time counterparts ($H^{pt} < H^{ft}$). It leads to sorting low match productivity workers into part-time positions. Workers get an opportunity to draw the match-specific productivity again once they contact a poaching firm and moves if their expected surplus is higher than their current one. Workers can be separated either exogenously or endogenously. The endogenous separation thresholds move counter-cyclically and the aggregate productivity (z_t) also follows an AR(1) process.

There are a few assumptions that make this model tractable. Firstly, match formation and destruction are efficient. Workers leave their match once they find a match that gives them higher surplus. Also, firms are assumed to make state contingent offers and counter offers to workers. Lastly, the incumbent firm and poaching firm engage in Bertrand competition following [Lise and Robin \(2017\)](#) to ensure tractability.

At the start of the period, human capital accumulates exogenously. Then, the aggregate Z_t and match-specific productivities X_t are realized. After that, exogenous separations happens with probability δ and remaining workers are subject to endogenous separations depending on their realized productivity. Among those who survived the separations, workers below the part-time threshold work only part-time. After observing the allocation of workers of that period, firms post their vacancies and search and matching take place. After unemployed and employed workers find their new job, they produce.

4.2 Value Functions

4.2.1 Unemployed Value Function

$$\begin{aligned} U_t &= b + \beta \mathbb{E}[(1 - \lambda_{t+1})U_{t+1} + \lambda_{t+1} \int \max\{U_{t+1}, W_{t+1}(x_{t+1})\} d\Gamma(x) | z_t] \\ &= b + \beta \mathbb{E}[U_{t+1} | z_t] \end{aligned}$$

The flow value of unemployment is b and time-invariant. The probability of unemployed

worker finding a vacancy at period $t + 1$ is denoted as λ_{t+1} . Conditional on finding a job offer, an unemployed worker takes the job if it give higher value than staying unemployed. Let $W_{t+1}(x_{t+1})$ be the value offered to unemployed worker by match of productivity x_{t+1} . $U_{t+1} = W_{t+1}(x_{t+1})$ holds because it is assumed that firms have all the bargaining power. Firms extract all the rents from unemployed workers making them indifferent between employment and unemployment. $\Gamma(x)$ is a stationary distribution of x_t .

4.2.2 Joint Value Function

$$J_t^j(x_t) = \max\{J_t^{ft,j}(x_t), J_t^{pt,j}(x_t)\}$$

where $\eta \in \{ft, pt\}$ and $j \in \{l, h\}$

Joint value function is a sum of the job's value for the worker and the firm. Therefore, the wage does not appear in the equation. The joint value function takes the maximum value between the match's value in full-time and part-time position. The value depends on the aggregate productivity z_t , match specific productivity x_t , and human capital $j \in \{l, h\}$.

$$\begin{aligned} J_t^{\eta,j}(x_t) = & z_t \cdot H^\eta \cdot x_t \cdot h^j - T^\eta \\ & + \beta \sum_{j'=l,h} p^{jj'} \cdot \mathbb{E} \left[(1 - (1 - \delta) \mathbf{1}\{J_{t+1}^{j'}(x') \geq U_{t+1}\}) \cdot U_{t+1} \right. \\ & + (1 - \delta) \mathbf{1}\{J_{t+1}^{j'}(x') \geq U_{t+1}\} \left((1 - s\lambda_{t+1}) J_{t+1}^{j'}(x') \right. \\ & \left. \left. + s\lambda_{t+1} \int \max\{J_{t+1}^{j'}(x'), W_{t+1}(x', y)\} d\Gamma(y) \right) \right] \Big| z_t, x_t \end{aligned}$$

Each match produces product of aggregate productivity z_t , hours worked H^η , match productivity x_t , and human capital h^j minus match maintenance cost T_η at period t . In the next period, the match either 1. dissolve due to an exogenous or endogenous separation, 2. fail to find new employer, 3. or succeed on-the-job search.

For workers who succeed on the job search, if $J_{t+1}^l(y) \geq J_{t+1}^{j'}(x')$, because of the Bertrand

competition assumption, the poaching firm does not offer higher value than the incumbent firm. Thus, $W_{t+1}(x', y) = J_{t+1}^{j'}(x')$ holds and worker moves to firm with productivity y . If $J_{t+1}^l(y) < J_{t+1}^{j'}(x')$, the worker does not move and keep the current value $J_{t+1}^{j'}(x')$. So, $\max\{J_{t+1}^{j'}(x'), W_{t+1}(x', y)\} = J_{t+1}^{j'}(x')$ for all y holds. Therefore, it is simplified as below.

$$\begin{aligned} J_t^j(x_t) &= z_t \cdot H^n \cdot x_t \cdot h^j - T^n \\ &+ \beta \sum_{j'=l,h} p^{jj'} \cdot \mathbb{E} \left[(1 - (1 - \delta) \mathbf{1}\{J_{t+1}^{j'}(x') \geq U_{t+1}\}) \cdot U_{t+1} \right. \\ &\quad \left. + (1 - \delta) \mathbf{1}\{J_{t+1}^{j'}(x') \geq U_{t+1}\} J_{t+1}^{j'}(x') \middle| z_t, x_t \right] \end{aligned}$$

The human capital accumulation $p^{jj'}$ follows the process below. Each period, an employed worker with low human capital h^l become a worker with high human capital h^h with exogenous probability ϵ conditional on staying at the same employer. Once the human capital is gained, employee does not lose the human capital and it only disappears once the worker moves to another employer or lose the job.

$$p^{jj'} = \begin{cases} \epsilon & \text{if } j = l \text{ and } j' = h \\ 1 - \epsilon & \text{if } j = l \text{ and } j' = l \\ 1 & \text{if } j = h \text{ and } j' = h \\ 0 & \text{if } j = h \text{ and } j' = l \end{cases}$$

4.2.3 Surplus Function

Surplus function is a joint value subtracted by unemployed value which characterizes the mobility and hours worked decisions of worker-firm matches. After some algebra, the surplus

functions are simplified as below.

$$\begin{aligned}
S_t^{\eta,l}(x_t) &= z_t \cdot H^\eta \cdot x_t \cdot h^l - T^\eta - b \\
&\quad + \beta(1 - \epsilon)(1 - \delta) \cdot \mathbb{E}[\max\{S_{t+1}^l(x'), 0\} | z_t, x_t] \\
&\quad + \beta\epsilon(1 - \delta) \cdot \mathbb{E}[\max\{S_{t+1}^h(x'), 0\} | z_t, x_t] \\
S_t^{\eta,h}(x_t) &= z_t \cdot H^\eta \cdot x_t \cdot h^h - T^\eta - b \\
&\quad + \beta(1 - \delta) \cdot \mathbb{E}[\max\{S_{t+1}^h(x'), 0\} | z_t, x_t]
\end{aligned}$$

Then, endogenous separation and part-time thresholds are given by

$$\begin{aligned}
S_t^{ft,l}(\hat{x}^l) &= S_t^{pt,l}(\hat{x}^l) \\
S_t^{pt,l}(\hat{x}^l) &= 0 \\
S_t^{ft,h}(\hat{x}^h) &= S_t^{pt,h}(\hat{x}^h) \\
S_t^{pt,h}(\hat{x}^h) &= 0
\end{aligned}$$

The surplus from a match at time t depends on time only through the TFP z_t and match-specific productivities x_t . It does not depend on the distributions of human capital, unemployed workers, or employed workers. For low productivity matches under threshold \hat{x}^l (\hat{x}^l for high human capital workers), it is more beneficial to work part-time and save overhead costs than to work full-time and pay high overhead costs. The range of productivities that gives higher value in part-time positions are wider when the aggregate productivity is low.

4.2.4 Vacancy Posting

$$V_t = -\kappa + q_t \left[P(u_{t+}) \int \max\{S_t^l(x_t), 0\} d\Gamma(x_t) \right. \\ \left. + P(e_{t+}) \sum_{j=l,h} \int \int_{\hat{x}^j} \max\{S_t^l(y) - S_t^j(x_t), 0\} L_{t+}(x, j) dx d\Gamma(y) \Big| z_t \right]$$

Vacant firms pay vacancy post κ . Vacancy is filled with probability q_t . New matches always possess low human capital including those who job-to-job transitioned to new positions. $\Gamma(\cdot)$ is stationary distribution of match-specific productivity, $L_t(x, j)$ is share of workers with productivity x and human capital j at period t . Period $t+$ denotes a time in period t after endogenous job separations and full-time part-time transitions happened and before search and matching take place.

$$P(u_{t+}) = \frac{u_{t+}}{s} \\ P(e_{t+}) = \frac{s \cdot \sum_{j=l,h} \int_{\hat{x}} L_{t+}(x, j) dx}{s}$$

$P(u_{t+})$ denotes a probability that the vacancy is filled with an unemployed job searcher and $P(e_{t+})$ denotes a probability that the vacancy is filled with an employed job searcher.

$$u_{t+} = u_t + \sum_{j=l,h} \int \left[\mathbf{1} \{S_t^j(x) < 0\} + \delta \mathbf{1} \{S_t^j(x) \geq 0\} \right] L_t(x, j) dx \\ L_{t+}(x, l) = (1 - \delta) \mathbf{1} \{S_t(x) \geq 0\} L_t(x, l) \\ L_{t+}(x, h) = (1 - \delta) \mathbf{1} \{S_t(x) \geq 0\} L_t(x, h)$$

The unemployed search pool consists of unemployed workers at the start of the period t and workers who are newly separated at the start of period t . The employed search pool consists

of employed workers who survived exogenous and endogenous job separations at period t .

4.2.5 Matching Function

While unemployed workers' relative contact probability with a poaching firm is normalized to 1, part-time and full-time workers meet a poaching firm with probability of $s < 1$. The market tightness is defined as

$$\theta_t \equiv \frac{v_t}{s_t}$$

where

$$s_t = u_{t+} + s \sum_{j=l,h} \int_{\hat{x}} L_{t+}(x, j)$$

Then, I assume a standard matching function with match efficiency normalized to 1 and can define job-finding rates and vacancy filling rates as

$$\begin{aligned} M_t &= s_t^\nu v_t^{(1-\nu)} \\ \lambda_t &= \frac{M_t}{s_t} \\ q_t &= \frac{M_t}{v_t} \end{aligned}$$

4.2.6 Law of Motion

$$\begin{aligned}
u_{t+1} &= u_{t+} \left[1 - \int \lambda_t \mathbf{1}\{S_t^l(x) \geq 0\} d\Gamma(x) \right] \\
L_{t+1}(x', l) &= \pi(x'|x) \int \left[(1 - \epsilon) L_{t+}(x, l) \left[1 - s \int \lambda_t \mathbf{1}\{S_t^l(y) > S_t^l(x)\} d\Gamma(y) \right] \right. \\
&\quad \left. + \sum_{j=l, h} \int s \lambda_t \mathbf{1}\{S_t^l(x) > S_t^j(y)\} L_{t+}(y, j) dy \right] dx \\
&\quad + u_{t+} \lambda_t \Gamma(x) \\
L_{t+1}(x', h) &= \pi(x'|x) \int \left[L_{t+}(x, h) \left[1 - s \int \lambda_t \mathbf{1}\{S_t^l(y) > S_t^h(x)\} d\Gamma(y) \right] \right. \\
&\quad \left. + \epsilon L_{t+}(x, l) \left[1 - s \int \lambda_t \mathbf{1}\{S_t^l(y) > S_t^h(x)\} d\Gamma(y) \right] \right] dx \\
p_{t+1} &= \sum_{j=l, h} \int_{\hat{x}}^{\hat{x}} L_{t+1}(x, j) dx \\
f_{t+1} &= \sum_{j=l, h} \int_{\hat{x}}^{\hat{x}} L_{t+1}(x, j) dx
\end{aligned}$$

When unemployed searchers fail job search, they start as an unemployed worker in the next period. If they succeed search, they start as a low human capital worker. When a low human capital worker stays in the same job, they accumulate human capital with exogenous probability ϵ . When a worker succeed in on-the-job search, they always start again as a low human capital worker.

4.2.7 Stochastic Equilibrium

1. For given $\{b, \beta, \delta, h^l, h^h, \epsilon, H^{pt}, H^{ft}, T^{pt}, T^{ft}\}$ and stochastic process of $\{z_t, x_t\}$, the surplus function $S^{\eta, j}(x)$ is sufficient to determine all decisions regarding worker mobility and work hours
2. For a given distribution $\Gamma(x)$ where match-specific productivity x is drawn from, va-

cancy cost κ , and meeting technology $M(s, V)$; and for any given initial share of unemployed workers u_o and match productivity distribution $L_0(x, j)$, a sequence of market tightness, unemployed workers, and worker-firm matches $\{\theta_t, u_{t+1}, L_{t+1}\}_{t=0}^T$ can be calculated by using the surplus function, given sequence of $\{z_t\}_{t=0}^T$ and match-specific process for each match

4.2.8 Calibration Procedure

To start with, I externally calibrate the flow value of unemployment b as 0.7 following [Mortensen and Éva Nagypál \(2007\)](#). The model is estimated in monthly frequency and the matching elasticity ν is set at 0.5 which is from micro studies. The TFP process is AR(1) in log and the persistence of the process ρ_z and the standard deviation of the shock ϵ_z follow [Hagedorn and Manovskii \(2008\)](#). The probability of exogenous separation δ is fixed at 2%. The hours worked of part-time workers H^{pt} is normalized at 1 and that of full-time workers H^{ft} is estimated using CPS actual weekly hours worked. The human capital accumulation takes on average 2 years conditional on staying in the same employer. The relative productivity of high human capital workers h^h are from the estimated return to 2 years of job tenure estimated by [Buchinsky et al. \(2010\)](#) where the low-type productivity h^l as normalized as 1.

Then, I have 6 different parameters left to estimate and use 6 first moments from data to pin them down. The internal calibration is conducted in two different levels of loops.

1. For each iteration, on-the-job search intensity s and vacancy posting cost κ are estimated to match job-to-job transition rates and job-finding rates. It is possible to separate this step because the value function is solved without these two parameters.
2. Then, overhead costs (T_f and T_p) and match productivity process parameters (ρ_x and ϵ_x) are jointly estimated to match average part-time rates, job separation rates, full-time to part-time transition rates, and part-time to full-time transition rates in 1980-2000 period

Table 3: External Calibration

External calibration		
Parameter	Value (and source)	Description
b	0.7 (Mortensen and Nagypál 2007)	Flow value of unem.
β	0.9966 (Monthly frequency)	Discount factor
ν	0.5	Matching elasticity
ρ_z	0.94 (Hagedorn and Manovskii 2008)	TFP persistence
ε_z	0.0034 (Hagedorn and Manovskii 2008)	TFP S.D.
δ	0.02	Exo. sep.
H^{ft}	2.067 (Current Population Survey)	Full-time hours
H^{pt}	1 (Normalized)	Part-time hours
ϵ	$\frac{1}{24}$ (2 years to accumulate human capital)	Upgrade prob.
h^h	1.134 (Buchinsky et al. 2010)	High-type productivity
h^l	1 (Normalized)	Low-type productivity

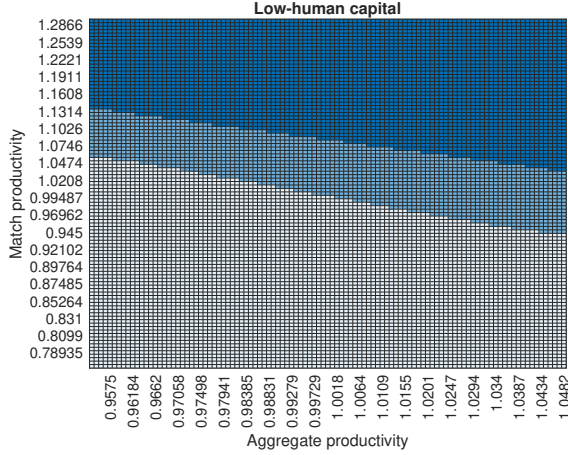


Figure 5: Low-type

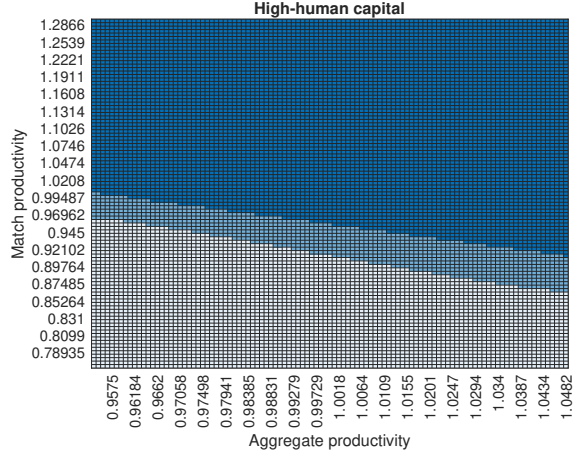


Figure 6: High-type

The resulting policy functions are shown in fig. 5 and fig. 6. Low-type workers face higher intensive- and extensive- margin thresholds than high type workers. It implies that once human-capital is accumulated, workers are less likely to separate. Instead, low productivity high-types tend to be kept as part-time workers once faced with negative shocks.

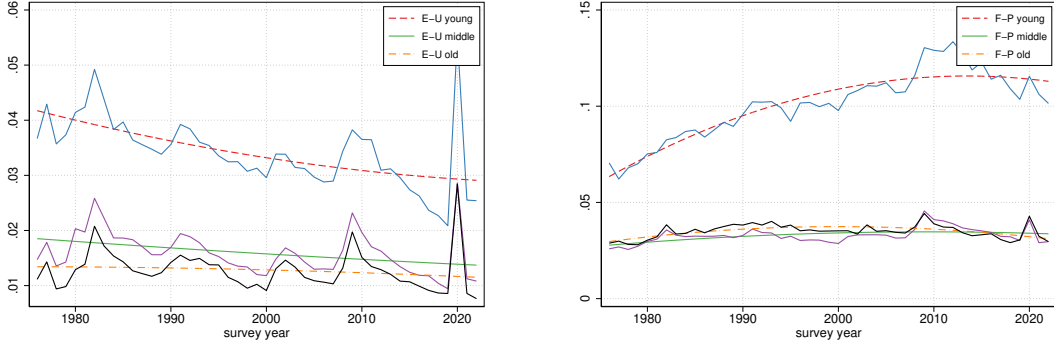
4.3 Improvements in Initial Match Productivities

When it comes to the size of decrease in unemployment inflow rates, young workers show the steepest decline over the years compared to prime-aged and old workers (-17% vs. -12%

Table 4: Internal Calibration

Internal calibration		
Parameter	Value	Description
T_f	1.67	Full-time overhead cost
T_p	0.51	Part-time overhead cost
ρ_x	0.974	Match productivity persistence
ε_x	0.029	Match productivity S.D.
s	0.375	OJS intensity
κ	0.5273	Vacancy cost
Model moments		Data moments
mean(PR)	0.174	0.170
mean(E-U)	0.0212	0.0213
mean(P-F)	0.2128	0.2204
mean(F-P)	0.0407	0.0430
mean(J-J)	0.0263	0.028
mean(U-E)	0.3705	0.3698

Figure 7: Trends in labor turnover by age group



vs. -5%) as shown in fig. 7. In contrast, young workers' full-time to part-time rates has increased the most (30% vs. 12% vs. 1%). This implies that young workers who are new to the labor market are more likely to find matches with a better fit than in the past. In the following section, I estimate the size of the shift in initial match productivity distribution that replicates the size of decline in job separation rates in the data.

4.3.1 Estimating the change in initial match productivities

The baseline model is calibrated to the pre-2000 data moments. Before 2000, monthly job separations were on average 2.1%. In this section, I estimate the size of the improvement in

Table 5: Calibration to Post-2000 Economy

Internal calibration		
Parameter	Value	Description
μ	12	Improvement in initial match prod. (%)
T_p	0.4886	Part-time overhead cost
s	0.25	OJS intensity
κ	2.8906	Vacancy cost
Model moments		Data moments
mean(PR)	0.1707	0.170
mean(E-U)	0.0169	0.0167
mean(J-J)	0.0271	0.028
mean(U-E)	0.3637	0.3698

Table 6: Correlation between total hours and hours per worker

σ	Pre-2000	Post-2000
Model	0.861	0.973
Data	0.493	0.643

initial match productivities needed to replicate the decline in job separation rates to 1.6%. For the baseline model, the initial match productivity distribution $\Gamma(x)$ was the stationary distribution of the match productivity process. Here, I shift the distribution $\mu\%$ to the right without changing the variance of the distribution. Since any change in $\Gamma(x)$ does not affect the policy function, the right shift results in less amount of part-time workers. Considering the fact that the part-time rates have been stable across the periods, I recalibrate the overhead cost T_p to keep the part-time rate at 17% as in the data⁸.

The estimation result in table 5 shows that a 12% increase in initial match productivities can replicate the decrease in job separation rates. Since the improvement in initial match productivities make posting vacancy more attractive, Vacancy posting cost went up to hold the hiring rates at a steady-level at 36%. On the other hand, the estimated on-the-job search intensity went down to keep the job-to-job transition rates at 2.7% level since new jobs are now more attractive to incumbent workers. Across the simulated periods, the correlation

⁸Without recalibrating the part-time overhead cost, the part-time rate goes down to below 10%. The main finding that the relative importance of intensive margin adjustments has increased still holds without recalibrating T_p

between logged hours per worker and total hours variables is higher in the model calibrated to post-2000 economy than that to pre-2000 economy as shown in table 6. In logged and then HP-filtered aggregate data, the correlation went up by 15%p. In the model counterpart, it went up by 12%p.

4.3.2 Impulse Response Functions

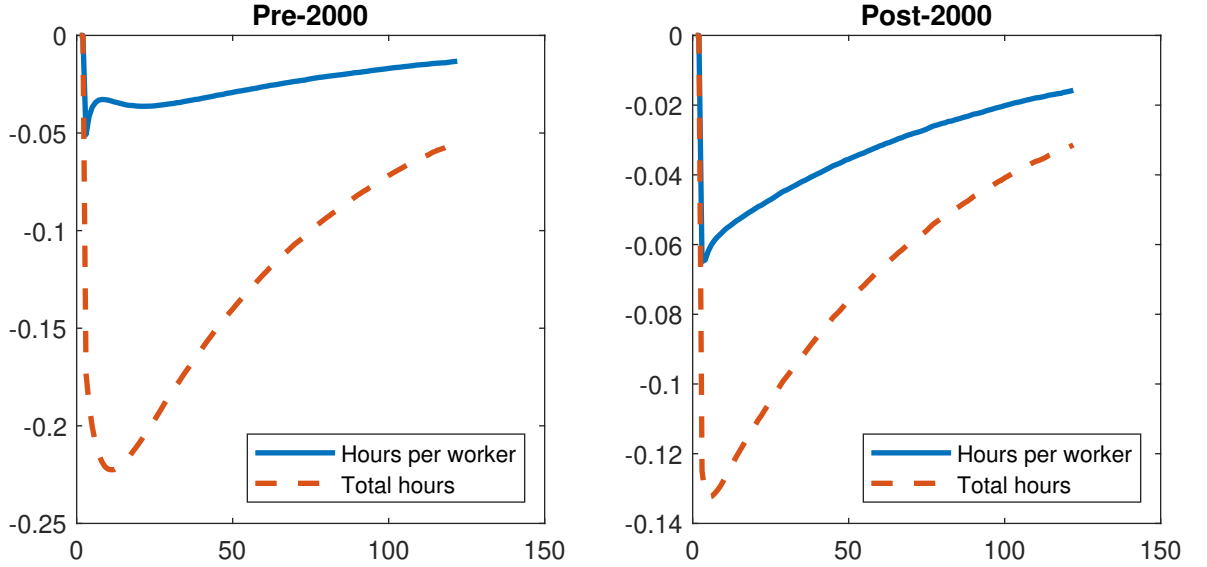
In this section, I compare the response of economies with different rates of labor turnover with regards to a two standard deviation negative aggregate productivity shock. Specifically, I compare the importance of intensive margin adjustments in economies with average job separation rate of 2.1% (pre-2000) with those with 1.6% (post-2000) in fig. 8. I simulate the response for 120 month periods for each economy.

In the high job separation economy, the total hours worked decrease more than 20% with a negative productivity shock. The hours per worker decrease about 5%. In contrast, the total hours worked decrease only 13% in the the low jobseparation economy but the hours per worker decrease more than 6% with regard to the same size of negative productivity shock. This implies that the improvment in initial match productivities led the economy to rely more on hours per worker adjustments in economic downturns.

Additionally, I retriive the process of the aggregate productivities that replicates unemployment rates from the data to compare the labor market responses in order to an identical series of aggregate productivity shocks in two differnt economies. I first find a series of aggregate productivities z_t that replicates the time series of unemployment rates in the post-2000 period with the post-2000 calibration. Using the same series of aggregate productivities, I simulate the unemployment rates with the pre-2000 calibration.

In Figure 9, the unemployment rates in post-2000 economy closely follow the actual unemployment rates from the data since I targeted the data directly when retrieving the series of z_t . The solid line is simulated unemployment rates with pre-2000 calibration using the same series of z_t . The major difference is that the unemployment rates are much more volatile

Figure 8: Impulse response function of hours per worker and total hours

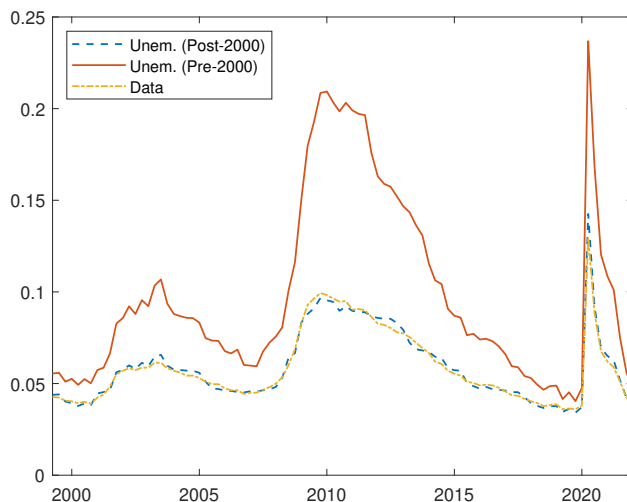


with pre-2000 calibration. Even though we cannot observe how the low labor turnover economy would have behaved post-2000 directly, feeding in the retrieved aggregate productivity process into it shows drastically different unemployment dynamics compared to the high labor turnover economy. The unemployment volatility is much higher in low turnover economy and unemployment rates also soar higher in the trough of the Great Recession.

I continue the exercise with the same set of productivity process in fig. 9. In contrast to the result in unemployment rates, the responses of hours per worker were more volatile for post-2000 economy compared to pre-2000 economy for the same series of the aggregate productivity process. The biggest difference between these two series of data is coming from the period around the trough of the Great Recession. It implies that low labor turnover economy rely more on intensive margin adjustments than high labor turnover economy in recessionary episodes.

In fig. 11, I show the average job separation rates in 10 deciles of realized aggregate productivities for two different calibrations. The slope of job separation rates is steeper for pre-2000 calibration with regards to the aggregate productivity which means that the

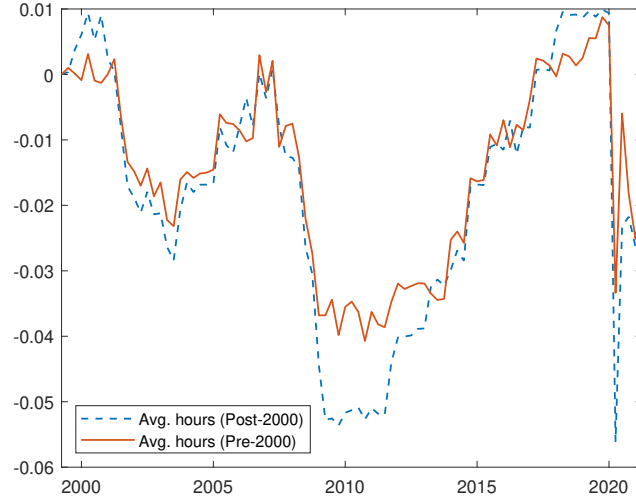
Figure 9: Simulated unemployment rates



counter-cyclicality of job separation rates are higher. It confirms that the use of job separation is more cyclical in the pre-2000 (low job separation) economy compared to the post-2000 (high job separation) economy. For the full-time to part-time transition rates shown in fig. 12, however, the post-2000 calibration shows higher counter-cyclicality. For post-2000 economy, FT to PT transitions play a bigger role in recessions.

In fig. 13, I calculate the average share of high-human capital workers in the simulated economy in 10 deciles of realized aggregate productivities. With pre-2000 calibration, the amount of human capital fluctuates cyclically because high-type workers are more likely to be separated when aggregate productivities are low. In contrast, the cyclicality of human capital is significantly lower with post-2000 calibration meaning that the increased use of intensive margin adjustments serves to preserve human capital in recessions.

Figure 10: Simulated hours per worker



5 Effects of Short-Time Compensation on Unemployment Volatility

Short-Time Compensation (STC) is a labor market policy that encourages retaining full-time workers with reducing hours instead of separating them when firms and workers want to adjust their labor input. With STC, employers reduce hours and pay of workers. The government makes up for all or part of the lost wages due to the hours reduction. Since STC helps firms hoard their existing labor, firms would apply for STC if workers are too costly to lose either because they have human capital or high match productivities. In the US, however, the STC take-up rates has been very small and only 1% of the initial UI claims were STC benefits. Therefore, I use my model to simulate the successful promotion of the STC and evaluate how effective STC is at stabilizing unemployment volatility. By comparing cost equivalent policies of STC in the model, I find that STC gains its relative strength in economies where the labor turnover is less prevalent since they have a higher share of workers

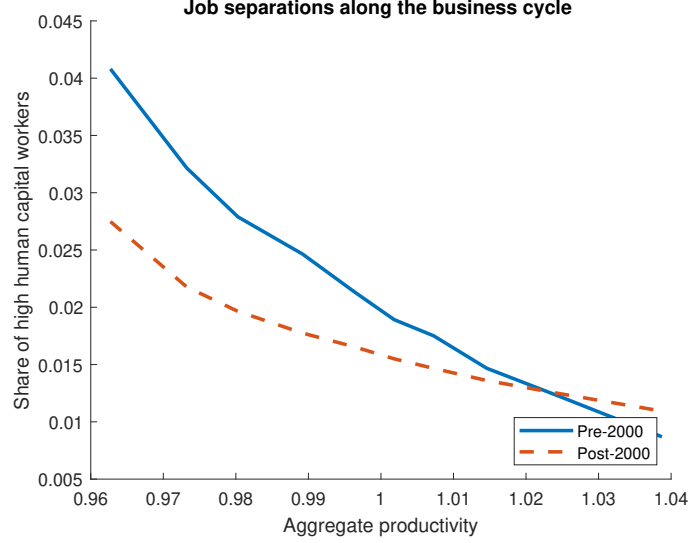


Figure 11: Separation rates

with accumulated human capital that benefits from STC in economic downturns.

$$\begin{aligned}
S_t^{p,h}(x_t, 1) = & z_t \cdot H^{pt} \cdot x_t \cdot h^h - T^{pt} - b + Subsidy \\
& + \frac{5}{6}\beta(1 - \delta) \cdot \mathbb{E}[\max\{S_{t+1}^h(x', 1)\}|z_t, x_t] \\
& + \frac{1}{6}\beta(1 - \delta) \cdot \mathbb{E}[\max\{S_{t+1}^h(x', 0)\}|z_t, x_t]
\end{aligned}$$

Once the worker works for a full-time schedule, the worker is eligible for STC and gets subsidy once the worker is moved to part-time positions. Subsidized workers become ineligible with exogenous probability of $1/6$ each month reflecting the fact that the eligibility expire in 26 weeks in states with STC policy. Workers who are just hired in part-time positions and job-to-job transitioned to part-time positions are not eligible for STC. STC subsidizes within-firm F-P transitions to preserve job-specific human capital.

The volatility of unemployment is defined as a standard deviation of unemployment rates in log. The amount of subsidy is set to be cost-equivalent in both economies. STC reduces the level and volatility of unemployment rates as shown in the first column for both cases. In the low separation economy (post-2000), it becomes more effective in decreasing unemployment

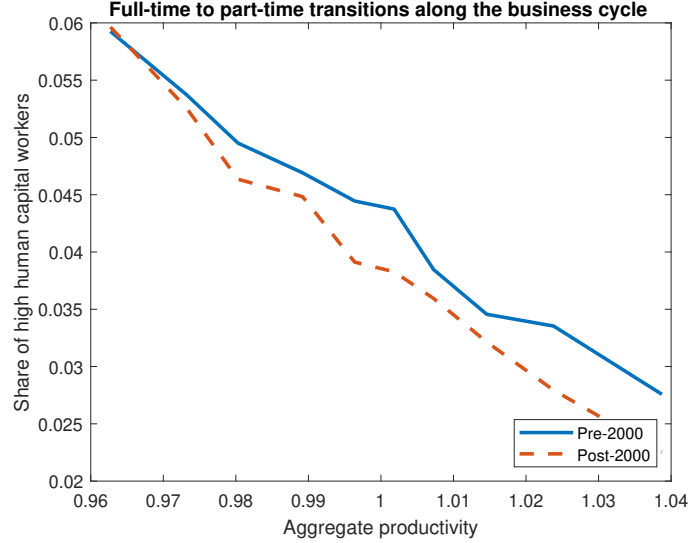


Figure 12: Full-time to part-time transition rates

	U	$\sigma(U)$
Pre-2000		
Baseline	0.0729	0.0639
STC	0.0487 (-33.2%)	0.0419 (-34.46%)
Post-2000		
Baseline	0.0527	0.0351
STC	0.0389 (-26.2%)	0.0207 (-42.1%)

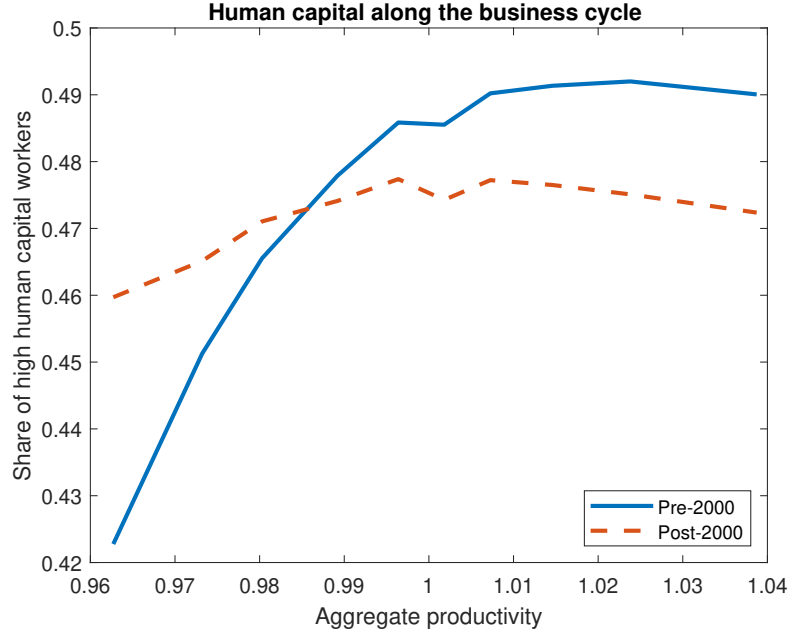
Table 7: Level and volatility change in unemployment rates due to STC

volatility.

6 Conclusion

There is a growing number of research showing that the US economy is becoming less dynamic over time. In the labor market, a secular decline in job separation rates for the last four decades is one of the most prominent phenomena of the reduced dynamism. A recent strand of literature focuses on a possibility that the development in information technology has led workers and firms to form better matches and caused the secular decline in unemployment inflow rates. In this paper, I have claimed that the improvement in the match quality caused firms and workers to use more intensive margin adjustments alternatively in recessions. The

Figure 13: Human capital fluctuations



main mechanism behind it is that a better job match accrues job-specific human capital with its longer job tenure. In order to avoid losing the human capital, firms and matches use relatively more intensive margin adjustments instead of separating workers in economic downturns. This suggests that the reduced dynamism has led intensive margin adjustments to play a larger role in recessions than before.

I add three model features of part-time work, on-the-job search, and human capital accumulation to the standard DMP model with match-specific productivity and endogenous separations. This model allows me to simulate the policy experiment of successfully implementing Short-Time Compensation (STC) scheme in the US economy and take human capital preserving nature of the policy into account. The policy is more effective in reducing unemployment volatility in a high initial match productivity and low job separation economy than a high job separation economy. This implies that STC policy stabilizes labor market against negative productivity shocks especially well in economy with low labor turnover rates and keep its job-specific human capital.

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A Appendix

I compare the incidence of FT (PT) to unemployment transitions and FT-PT transitions using the individual level job tenure data. In Table A.1, I run logit regressions using individual-level transition incidence as dependent variables and confirm that high tenure workers are less likely to go through E-U or F-P transitions. Furthermore, the decrease in hazard rate along the tenure is much steeper for F-P flows.

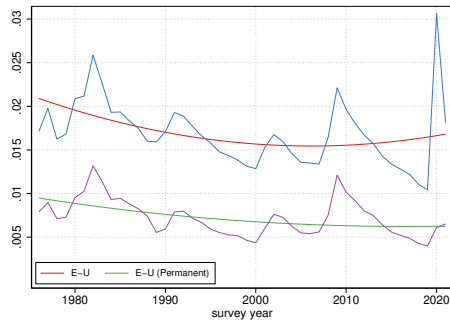
Table A.1: Job Tenure and Labor Market Flows

	E to U	F to P	P to F
tenure	-0.100*** (0.003)	-0.031*** (0.001)	0.015*** (0.001)
age	-0.049*** (0.006)	-0.191*** (0.004)	0.174*** (0.004)
agesq	0.001*** (0.000)	0.002*** (0.000)	-0.002*** (0.000)
education	-0.016*** (0.001)	-0.014*** (0.000)	-0.004*** (0.000)
female	-0.295*** (0.026)	0.592*** (0.016)	-0.696*** (0.018)
R-squared	0.052	0.046	0.04
N	490538	403637	86901

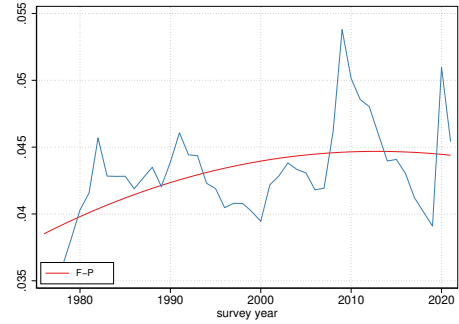
* p<0.05, ** p<0.01, ***p<0.001

Figure A.1 shows the trend in monthly probabilities of employment/unemployment and full-time/part-time transitions. Panel (a) shows that there has been a decline in unemployment inflow rates (E-U) while panel (b) shows an increase in full-time to part-time (F-P) probabilities.

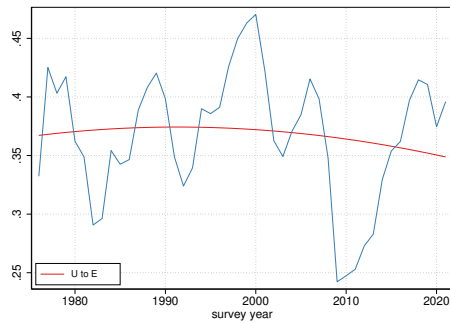
Figure A.1: Trend in Labor Market Flows



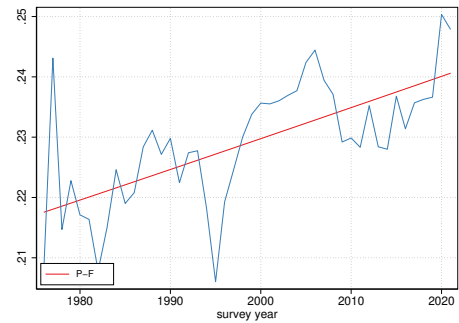
(a) Employment to unemployment



(b) Full-time to part-time

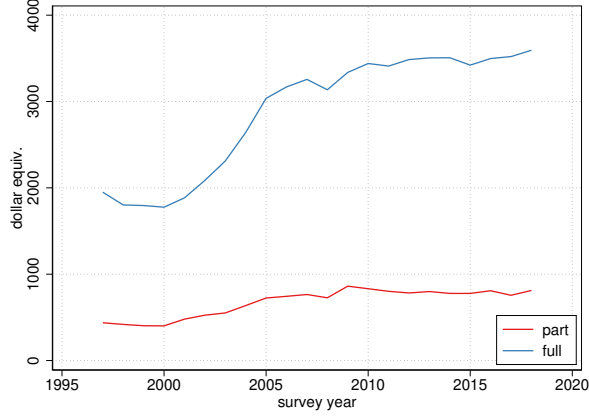


(c) Unemployment to employment



(d) Part-time to Full-time

Figure A.2: Acyclical Cost of Match



Dollar values of employer contribution to health insurance from ASEC march supplement.

B Derivation of Surplus Functions

$$S_t^j(x) = \max\{S_t^{ft,j}(x), S_t^{pt,j}(x)\}$$

$$S_t^{\eta,j}(x) = J_t^{\eta,j}(x) - U_t$$

$$= z_t \cdot H^\eta \cdot x_t \cdot H^j - T^\eta - b$$

$$+ \beta \sum_{j'=l,h} p^{jj'} \cdot \mathbb{E} \left[(1 - (1 - \delta) \mathbf{1}\{J_{t+1}^{j'}(x') \geq U_{t+1}\}) \cdot U_{t+1} \right.$$

$$\left. + (1 - \delta) \mathbf{1}\{J_{t+1}^{j'}(x') \geq U_{t+1}\} J_{t+1}^{j'}(x') - U_{t+1} \middle| z_t, x_t \right]$$

$$= z_t \cdot H^\eta \cdot x_t \cdot H^j - T^\eta - b$$

$$+ \beta(1 - \delta) \sum_{j'=l,h} p^{jj'} \cdot \mathbb{E} \left[\mathbf{1}\{J_{t+1}^{j'}(x') \geq U_{t+1}\} (J_{t+1}^{j'}(x') - U_{t+1}) \middle| z_t, x_t \right]$$

$$= z_t \cdot H^\eta \cdot x_t \cdot H^j - T^\eta - b$$

$$+ \beta(1 - \delta) \sum_{j'=l,h} p^{jj'} \cdot \mathbb{E} [\max\{S_{t+1}^{j'}(x'), 0\} | z_t, x_t]$$