Investigating Monte Carlo estimates for Determinants of Large, Sparse Matrices

David Young
Supervisor: Mike Peardon

INTRO

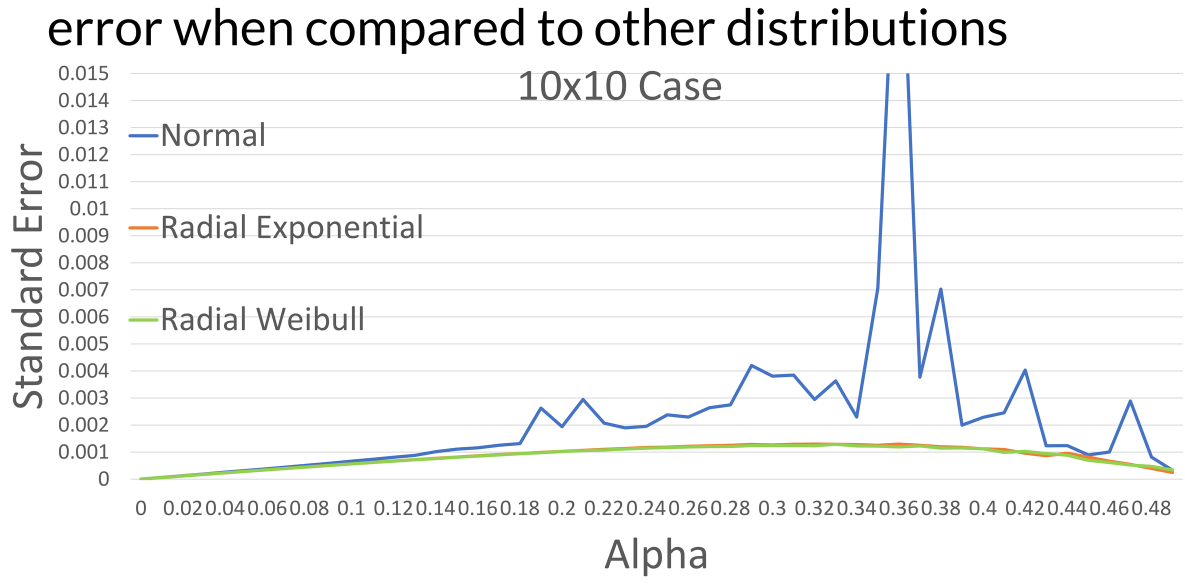
- One existing formula for calculating Matrix determinants is obtained by applying M as a linear transform on the Gaussian, and is evaluated using Monte Carlo sampling
- I investigated the difference in the results when Monte Carlo sampling was used on integral representations obtained by applying the same method to different distributions

METHODS

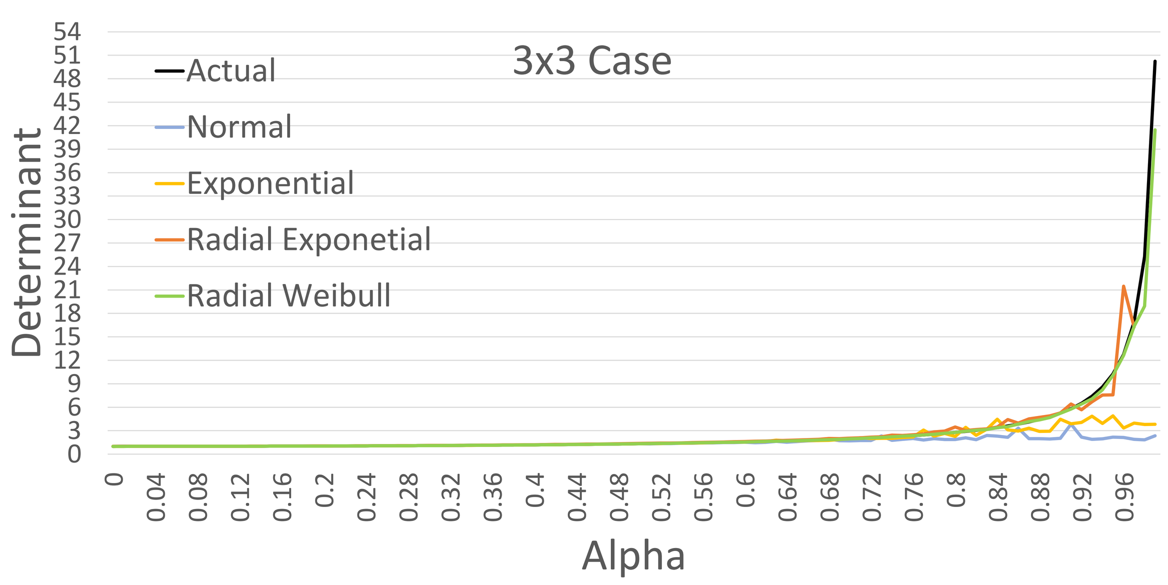
1. Used above method on suitable distributions to obtain different representations
2. For each integral representation, wrote python program which used 1,000,000 samples to find biased and unbiased estimator for det M for different test M
3. Compared results to analytically computed determinants, and compared standard error produced by different distributions

RESULTS

- Variance of Gaussian method does not exist for certain matrices and produces large and erratic standard error when compared to other distributions



- Radial Weibull and Radial Exponential produced an estimator significantly closer than Gaussian in certain cases



Monte Carlo estimates for Matrix Determinants can be **significantly improved** by selecting a **sampling distribution** that suits the Matrix

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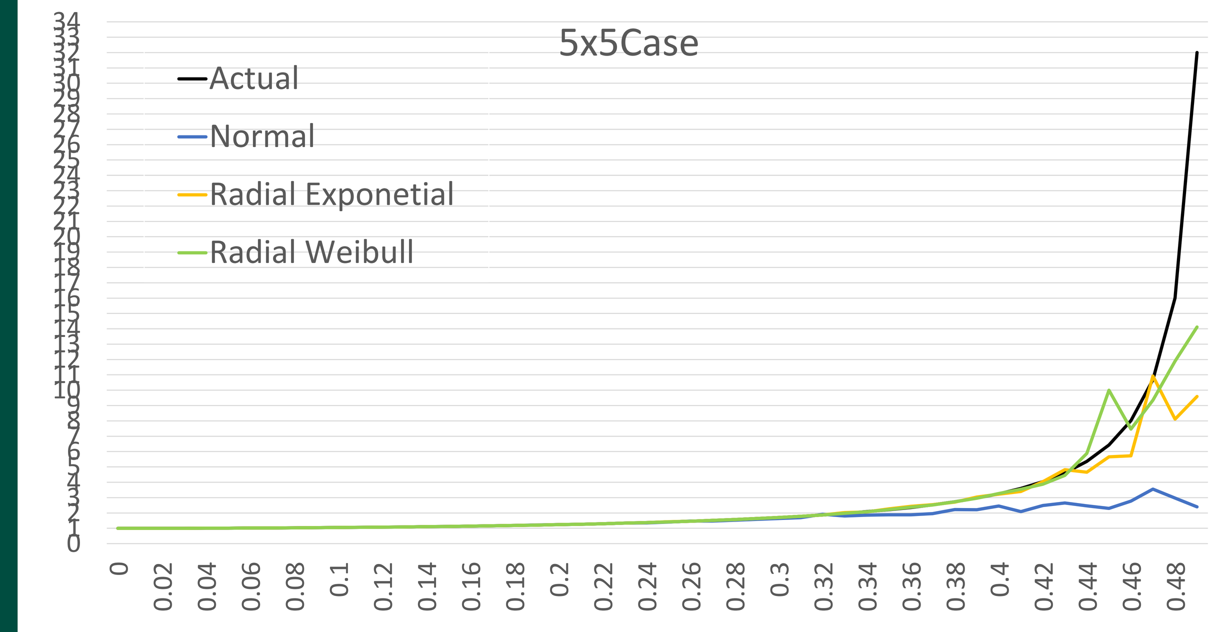

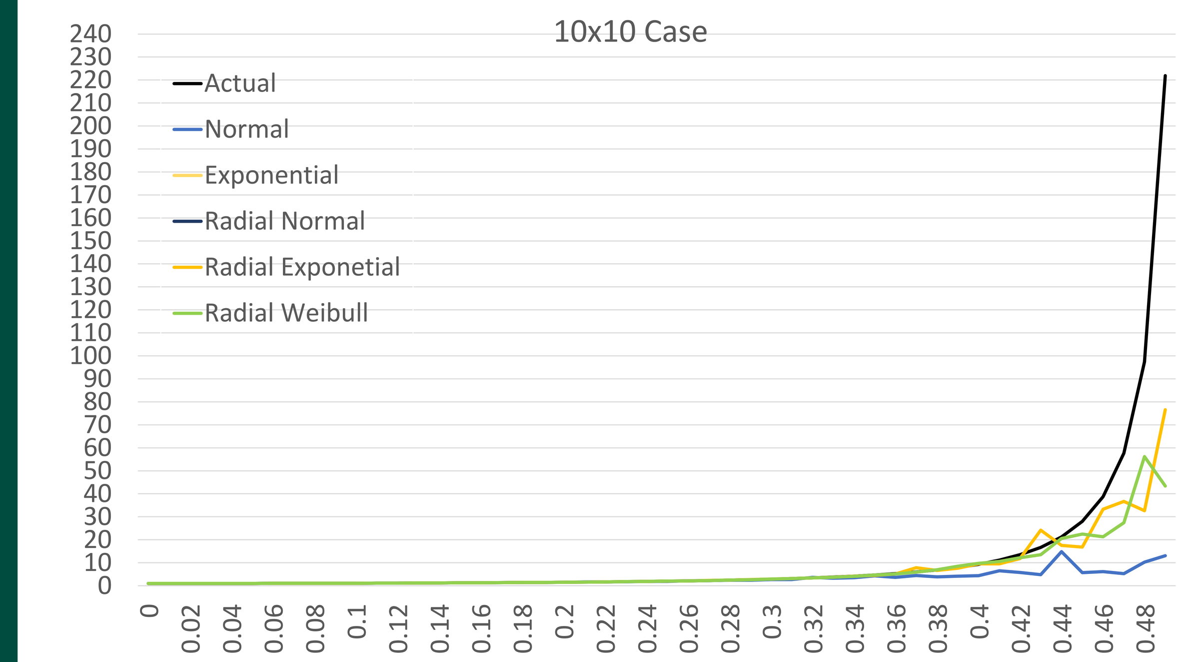

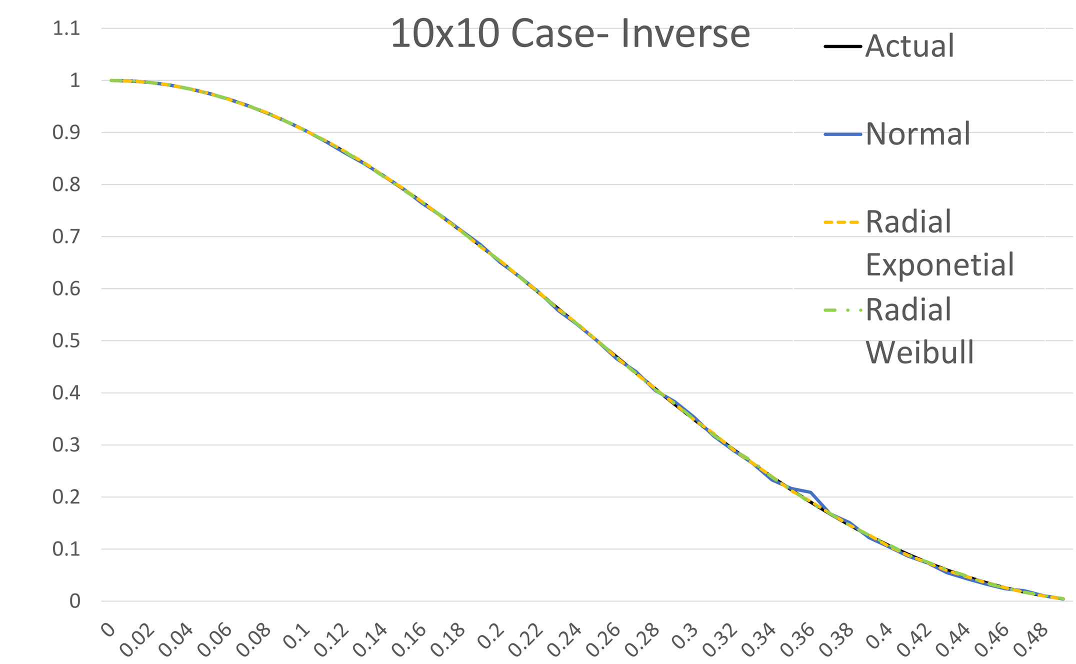

$$\det M = \int_{\mathbb{R}^n} \frac{e^{-\frac{1}{2}x^t (M^2)^{-1} x}}{(\sqrt{2\pi})^n} \prod_{i=0}^{n} (dx_i)$$

$$\det M = \int_{\mathbb{R}^n} \frac{\Gamma(\frac{n+1}{2})}{2\pi^{\frac{n+1}{2}}} \frac{e^{-|M^{-1}x|}}{|M^{-1}x|^{n-1}} \prod_{i=0}^{n} (dx_i)$$

$$\det M = \int_{\mathbb{R}^n} \frac{\Gamma(\frac{n+1}{2})}{4\pi^{\frac{n+1}{2}}} \frac{e^{-|M^{-1}x|^{\frac{1}{2}}}}{|M^{-1}x|^{(n-\frac{1}{2})}} \prod_{i=0}^{n} (dx_i)$$

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin