TRAVIOLI: A Dynamic Analysis for Detecting Data-Structure Traversals

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Abstract—Traversal is one of the most fundamental operations on data-structures, in which an algorithm systematically visits some or all of the data items of a data-structure. We propose a dynamic analysis technique, called TRAVIOLI, for detecting data-structure traversals. We introduce the concept of acyclic execution contexts that enables precise detection of traversals of arrays and linked data-structures such as lists and trees in the presence of loops and recursion. We describe how information about program functions containing data-structure traversals can be used for manually generating performance regression-tests, for discovering performance bugs caused by redundant traversals, and for traversal comprehension. We evaluate TRAVIOLI on five real-world JavaScript programs. In our experiments, TRAVIOLI produced fewer than 4% false positives. We were able to construct performance tests for 93.75% of the reported true traversals. TRAVIOLI also found two asymptotic performance bugs in widely used JavaScript frameworks D3 and express.

I. INTRODUCTION

Data-structures form the building blocks of almost all programs. As programs grow large, the implementations of their data-structures and of the code modules that use these data-structures also becomes complex. Researchers have developed several tools to identify, to visualize, and to reason about data-structures using static or dynamic program analysis techniques [11, 17, 24, 27, 31, 35]. Most of these tools focus on discovering a *concise representation* or *abstraction* of data-structures in program memory at one or more program-locations during program execution.

We focus on understanding where and how data-structures are *traversed* in a program. Traversal is perhaps one of the most fundamental operations on data-structures, in which an algorithm systematically visits some or all of the data items of a data-structure [32]. The running time of program functions that perform traversals usually increases with the increase in the size of the input data-structures. Thus, a proper understanding of how a program traverses its data-structures is crucial for characterizing the program's performance.

We propose a dynamic analysis technique, called TRAVIOLI, to detect data-structure traversals in a program. The technique works by analyzing read-events generated by an execution of the program. TRAVIOLI reports a data-structure traversal if it finds that 1) the program reads two or more different memory-locations at a program-location, and 2) those memory-locations either belong to the same object, or belong to different objects connected by a series of pointers. A key

contribution of this work is the notion of *acyclic execution contexts* (AECs), which enable precise detection of traversals in the presence of loops and recursion with very few false positives.

We describe three applications of TRAVIOLI and AECs. In one application, we show how we can use TRAVIOLI to help write performance regression-tests. In a second application, we show how TRAVIOLI can be used to detect redundant traversals bugs. In the final application, we show how we can construct an *access graph* to help visualize data-structure traversals found in a program.

A key advantage of TRAVIOLI is that it can detect a traversal even if a program is executed on a small unit test—the program does not need to execute a program-location many times to detect a traversal. Another key advantage of TRAVIOLI is that it can detect a traversal even if the traversal involves recursive function calls and loop iterations.

We have implemented TRAVIOLI for JavaScript and made it publicly available at https://github.com/rohanpadhye/travioli. We applied TRAVIOLI to 5 popular JavaScript projects. In our experimental evaluation we found that the false postive rate of TRAVIOLI's traversal detection is 4%. In 93.75% of reported true traversals, we managed to create a performance regression-test. TRAVIOLI discovered two previously unknown performance bugs due to redundant traversal in widely used JavaScript frameworks—D3 and express, which have been confirmed by the respective project's developers. We also discuss a case-study where we used *access graphs* constructed by TRAVIOLI to understand a complex tree traversal in angular.js.

II. OVERVIEW

A. Traversing Functions

We propose a technique to identify functions that traverse input data-structures and whose running time could be arbitrarily increased by increasing the size of the input data-structure. We call such functions *traversing functions*. In the rest of the section, we define *traversing functions* through a series of examples written in JavaScript and informally describe an algorithm to detect such functions using dynamic program analysis.

Consider the function sum in Figure 1. The function iterates over an input array of objects, arr, and computes the sum of the val field of the objects it contains. The function is an

```
1  /* Sum values in records in array */
2  function sum(arr) {
3    var result = 0, record, i;
4    for(i = 0; i < arr.length; i++){
5      record = arr[i];
6      result += record.val;
7    }
8    return result;
9  }
```

Fig. 1. A function that traverses an array.

```
1  /* Compute the length of linked-list  */
2  function len(list) {
3    var count = 0;
4    while (list != null) {
5      count++;
6      list = list.next;
7    }
8    return count;
9  }
```

Fig. 2. A function that traverses a linked-list.

```
1  /* Add values from a pair of array elements. */
2  function addPair(arr) {
3    var rx = arr[0];
4    var ry = arr[1];
5    return rx.val
6         + ry.val;
7  }
```

Fig. 3. A non-traversing function.

example of a simple data-structure traversing function. The running time of the function can be increased by increasing the size of the input array.

Although in this example we could easily identify the input to the function (i.e. the array `arr`), this may be non-trivial for complex functions where inputs could be passed via global or static variables. We define *read-footprint* to precisely capture the set of inputs to a function. A *memory-location* is the address of a piece of memory that stores a program value that can be read by a program. A memory-location is often denoted in a program by a variable, an element of an array, or a field of an object. The *read-footprint* of a function consists of all memory-locations that are read by an execution of the function without any prior write to them by the execution. Such memory-locations could be treated as the input to the function. For example, the read-footprint of the `sum` function consists of the array `arr`, all its elements (accessed via `arr[i]`), the `length` field of the array (accessed via `arr.length`), and the field `val` of the objects stored in the array (accessed via `record.val`). In constrast, the memory-locations denoted by the variables `i` and `record` are not part of the read-footprint because in any execution of `sum`, `sum` first writes them before reading them.

Given the definition of a read-footprint, we can define a traversing function as follows: We say that a function is a *traversing function* if the size of its read-footprint can be arbitrarily increased by providing suitable inputs, possibly of larger size. If a function is a traversing function, then we say that the function contains a *traversal*.

The function `sum` in Figure 1 contains a traversal because the size of the read-footprint increases if the size of the input array `arr` is increased.

The function `len` in Figure 2 is another example of a traversing function. The read-footprint of the `len` function consists of the memory-location denoted by `list` and the

memory-locations denoted by the `next` field of all objects reachable from `list` by following the `next` field zero or more times. The read-footprint of this function can be increased by increasing the size of the list passed as an argument.

In constrast, the function `addPair` in Figure 3 is not a traversing function. The function `addPair` adds the values of the first two elements of the input array. While this function also reads multiple elements of `arr`, it is not a traversing function because the size of its read-footprint is always bounded regardless of the size of the input array or the values it contains.

B. Detecting Traversing Functions

The problem of determining if a function contains a traversal is undecidable in general. However, in many cases, one can determine whether a function has a traversal either by analyzing the source code or by analyzing an execution of the function. We propose a dynamic analysis technique, called TRAVIOLI, to determine if a function contains a traversal. TRAVIOLI works by checking a set of conditions on an execution of the function—if the conditions are satisfied then we say that the function contains a *possible* traversal. Our technique is approximate in the sense that it can give both false positives and negatives. However, we have identified a set of conditions which if satisfied often accurately indicate the presence of a traversal. A key feature of TRAVIOLI is that we do not need to invoke the function on an input having a large read-footprint—TRAVIOLI can detect a traversal by analyzing the execution of the function on a small test input. We next introduce these conditions using a series of motivating examples.

TRAVIOLI uses program instrumentation to generate a trace of *events* corresponding to reads and writes of memory-locations. In the following discussion, whenever an execution of a function reads a memory-location that the function execution has not written before, we call it a *input read-event*. An input read-event contains the address of the memory-location being read, the value being read, and the program-location where the read is performed by the function. TRAVIOLI determines the input read-events during each function execution and analyzes them to determine if the function has a traversal.

From executions of `sum` and `len` in Figures 1 and 2, respectively, one can observe that different memory-locations are read at the same program-location: `sum` reads the elements of the array `arr` at line 5 and `len` reads the `next` field of the `list` objects at line 6. This observation suggests that a traversal within a function should satisfy the following two conditions:

```
1  /* Get the third element of a linked list */
2  function third(list) {
3    var node = n(list);
4    node =  n(second);
5    return node.data;
6  }
7  function n(node) {
8    return node.next;
9  }
```

Fig. 4.   Another example of a non-traversing function.

```
1  /* Check if a linked-list contains a value */
2  function contains(list, x) {
3    if (list === null) {
4      return false;
5    } else if (list.data === x) {
6      return true;
7    } else {
8      var tail = list.next;
9      return contains(tail, x);
10   }
11 }
```

Fig. 5.   A recursive function containing a traversal.

C1. At least two input read-events at some program-location ℓ access different memory-locations, and

C2. the memory-locations involved in the input read-events either belong to the same object, or belong to different objects connected by a series of pointers.

Note that `addPair` in Figure 3 does not satisfy the first condition because the two elements of the array are read at different program-locations—lines 3 and 4, respectively.

The above two conditions result in a false positive for the function `third` in Figure 4. The function `third` calls `n` twice, and line 8 accesses `next` field of objects connected by a pointer. Thus both conditions are satisfied. However, `third` does not contain a traversal, since the read-footprint of `third` is bounded to at most two linked-list nodes. The imprecision arises because the first condition requires two input read-events to occur at similar execution points, where two execution points are deemed similar if they have same program-locations. This notion of similarity of two execution points is too coarse-grained. We can alleviate this problem if we say two execution points are similar if they have similar calling contexts. We use a variant of calling contexts, called execution contexts, to decide similarity of two execution points.

Definition 1. The *execution context* of an event with respect to an execution of a function f is a sequence $(f_1 : \ell_1)(f_2 : \ell_2) \ldots (f_n : \ell_n)$, where

- f_1 is the function f,
- for each i such that $1 \leq i < n$, ℓ_i is the program-location within function f_i where f_{i+1} is invoked in the current execution, and
- the function f_n is currently executing the program-location ℓ_n to generate the input read-event.

For example, in an execution of the function `third` in Figure 4, the two input read-events at line 8 have the execution contexts: `(third:3)(n:8)` and `(third:4)(n:8)`, respectively with respect to the execution of the function `third`. Unless otherwise specified, we always refer to execution contexts with respect to the execution of the function being analyzed for traversals. In order to remove the false positive for `third`, we refine the first condition for traversal as follows:

C1. At least two input read-events at some execution context access different memory-locations.

The revised condition gives no false positive for any of the previous examples. Unfortunately, this revision, which uses

```
1  /* Alternately add and subtract elements. */
2  function alt(list) {
3    return p(list, true, 0);
4  }
5  function p(node, flag, total) {
6    if (node != null) {
7      var value = node.data;
8      return flag ? q(node, flag, total + value)
9                  : q(node, flag, total - value);
10   } else {
11     return total;
12   }
13 }
14 function q(node, flag, total) {
15   var tail = n(node);
16   return p(tail, !flag, total);
17 }
18 function n(node) {
19   return node.next;
20 }
```

Fig. 6.   Mutually recursive functions containing a traversal.

a fine-grained notion of similarity of execution points, introduces false negatives—it fails to detect data-structure traversals via recursive functions, such as the function `contains` defined in Figure 5.

In the function `contains`, a recursive traversal occurs at line 8, but its execution does not meet condition C1 because the execution contexts of the input read-events at this program-location are different. In particular, the execution context is `(contains:8)` for the first input read-event, `(contains:9)(contains:8)` for the second input read-event, `(contains:9)(contains:9)(contains:8)` for the third input read-event, and so on. Such execution contexts become more complicated for more complex functions involving mutual recursion, such as the function `alt` in Figure 6.

The function `alt` traverses the input `list` and alternately adds and subtracts values of its nodes to the total. The boolean `flag` passed to function `p` at line 8 decides which operation to perform, and this flag is toggled by the function `q` at line 16. Here, `p` and `q` are mutually recursive, and the traversal of the linked list occurs at line 19 after `q` calls `n` at line 15. The first time program control reaches line 19, the execution context is `(alt:3)(p:8)(q:15)(n:19)`; the second-time a different branch is taken in `p`, and thus the context is `(alt:3)(p:8)(q:16)(p:9)(q:15)(n:19)`, and so on.
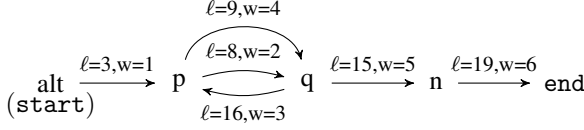
Fig. 7. Execution context graph for the execution context `(alt:3)(p:8)(q:16)(p:9)(q:15)(n:19)`.

Example	Execution contexts	AEC
Fig. 4,	(third:3)(n:8)	(third:3)(n:8)
Line 8	(third:4)(n:8)	(third:4)(n:8)
Fig. 2,	(len:6)	**(len:6)**
Line 6	(len:6)	**(len:6)**
Fig. 5,	(contains:8)	**(contains:8)**
Line 8	(contains:9) (contains:8)	**(contains:8)**
Fig. 6,	(alt:3)(p:8)(q:15)(n:19)	**(alt:3)(p:8)(q:15)(n:19)**
Line 19	(alt:3)(p:8)(q:16)(p:9)(q:15)(n:19)	**(alt:3)(p:8)(q:15)(n:19)**

TABLE I
EXECUTION CONTEXTS AND AECS FOR FIRST TWO READ-EVENTS.

In TRAVIOLI, a key observation we make is that, despite the differences in the execution contexts of the input read-events involved in a traversal, the contexts are equivalent *modulo recursion* (i.e. after removing any cycles). Such reduced execution contexts, which we define next, are called *acyclic execution contexts* (AEC) and they are constructed as follows.

For an execution context $(f_1 : \ell_1)(f_2 : \ell_2) \ldots (f_n : \ell_n)$, we first construct an *execution context graph* consisting of a node for each unique function f_i and a special node `end`. Moreover, let `start` denote the node corresponding to f_1. For every consecutive pair $(f_i : \ell_i)(f_{i+1} : \ell_{i+1})$ in the execution context, we add a directed edge from f_i to f_{i+1} with label ℓ_i and weight i. Additionally, we add an edge from f_n to `end` with label ℓ_n and weight n. For the example in Figure 6, the execution context graph for the second input read-event at line 19 is shown in Figure 7, where the edges are labeled by the program-locations ℓ and weights w.

Definition 2. The **acyclic execution context** (AEC) of an execution context is the sequence $(f_1 : \ell_1)(f_2 : \ell_2) \ldots (f_k : \ell_k)$ such that $f_1 \xrightarrow{\ell_1} f_2 \ldots f_k \xrightarrow{\ell_k} f_{k+1}$ is the shortest weighted-path from `start` to `end` in the execution context graph of the execution context.

For the graph in Figure 7, the acyclic execution context is `(alt:3)(p:8)(q:15)(n:19)`. As the edge weights correspond to the position of the edge in the execution context, multiple edges between two nodes are disambiguated by choosing the edge corresponding to the least recent function invocation.

Two distinct execution contexts that have the same AEC are the recursive analog of distinct iterations of a single loop. Unlike execution contexts that can grow unboundedly, AECs are bounded because the number of permutations of distinct functions in a program is finite. We found that AECs are a useful abstraction for clustering execution contexts of input read-events involved in a traversal—such an abstraction helps us to merge execution points involved in a traversal in a precise way irrespective of whether the traversal involves recursive calls or loop iterations.

Table I lists, for some example functions and program-locations (column 1), the execution contexts (column 2) and corresponding AECs (column 3) for the first two input read-events, when the functions are provided an input linked-list containing at least two nodes. The first row shows that the AECs for input read-events at line 8 in the function `third` are distinct; therefore, `third` is not a traversing function. The last three rows show that for the functions `len`, `contains` and

`alt`, multiple input read-events at the given locations have a common AEC. Therefore, each of these functions contain a traversal.

We can now refine the conditions that a traversing function should satisfy in terms of AECs as follows:

C1. At least two input read-events having same AECs access different memory-locations, and

C2. the memory-locations involved in the input read-events either belong to the same object, or belong to different objects connected by a series of pointers.

We call the AEC of the input read-events as a *traversal point*. In general, a traversing function may contain more than one traversal point.

III. FORMAL DESCRIPTION

TRAVIOLI identifies the traversing functions in a program by analyzing an execution of the program. TRAVIOLI first instruments the program under analysis to generate runtime events. The instrumented program is executed with a suitable set of inputs to generate a trace of runtime events. From the generated trace, TRAVIOLI determines the input read-events for every function execution. TRAVIOLI then analyzes the sequence of input read-events during every function execution to detect traversals. We next describe each of these steps formally.

A. Events and Traces

TRAVIOLI tracks reads and writes of every memory-location during an execution of a program. In a program, a memory-location can be denoted by a local variable, a global variable, a field of an object, or an element of an array. A memory-location is represented by a pair (obj, fld), where

- if the memory-location is a field of an object, then obj is the address of the object and fld is the name of the field,
- if the memory-location is an element of an array, then obj is the address of the array and fld is the index of the element,
- if the memory-location is a local variable, then obj is the address of the activation record object containing the variable and fld is the name of the variable, and
- if the memory-location is a global variable, then obj is the address of a singleton global object denoting the global scope and fld is the name of the variable.

TRAVIOLI instruments a program to generate the following four kinds of events:

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1) READ$\langle \ell, obj, fld, val \rangle$ denotes the read of a memory-location (obj, fld) at program-location ℓ. The result of the read, val, can be a scalar or the address of another object.

2) WRITE$\langle \ell, obj, fld, val \rangle$ denotes the write of a memory-location (obj, fld) at program-location ℓ. Here, val is the new value that is assigned to the memory-location. At function calls, write-events are generated for each argument passed to the function, where each formal argument is treated as a local variable.

3) CALL$\langle \ell, f, a \rangle$ is an event corresponding to the invocation of the function f at the program-location (i.e. call-site) ℓ. a is a freshly generated unique identifier for the newly created activation record object for the function invocation.

4) RET$\langle \ell, a \rangle$ is an event corresponding to a function returning to its caller. Here, ℓ is the program-location of the return instruction and a is the identifier of the current activation record, which is about to be destroyed. Note that each unique value of a appears in exactly one call and one return event in the program execution.

The execution of an instrumented program generates a trace of events. We identify the *execution of a function* started by the event CALL$\langle \ell, f, a \rangle$ by the activation record identifier a. For a function execution denoted by a, we use TRACE(a) to denote the sequence of events generated by the function execution including the call and return events that start and end the execution of the function, respectively. Note that if a function f' is invoked during the execution of a function f with activation record a, and if this invocation creates an activation record a', then TRACE(a') is a subsequence of TRACE(a).

B. Read-Traces and Read-Footprints

To compute the read-footprint of a function execution, we need to determine the set of memory-locations that are read before being written during the execution. We define the *read-trace* of a function execution a, denoted by RTRACE(a), as the largest set of events e_i such that:

- $e_i = $ READ$\langle *, obj, fld, * \rangle$
- $e_i \in $ TRACE(a)
- $\forall j \colon (e_j = $ WRITE$\langle *, obj, fld, * \rangle) \in $ TRACE$(a) \Rightarrow j > i$

The third condition ensures that if there is a write to the memory-location (obj, fld) in the trace, then it must occur *after* e_i. Then, the *read-footprint* of a function execution a, denoted by FP(a), is computed as:

$$\text{FP}(a) = \{(obj, fld, val) \mid \text{READ}\langle *, obj, fld, val \rangle \in \text{RTRACE}(a)\}$$

C. Detecting Traversals

For every function execution a and for each event e in TRACE(a), we compute the execution context of e with respect to a, denoted by EC(a, e) as follows:

- If e is the first event of TRACE(a) and is of the form CALL$\langle \ell, f, a \rangle$, then EC$(a, e) = \epsilon$, i.e. the empty sequence.

- If e is not the first event of TRACE(a) and is generated at program-location ℓ, and if the latest call-event before e without a matching return event before e is $e' = $ CALL$\langle \ell', f', a' \rangle$, then EC$(a, e) = $ EC$(a, e').(f', \ell)$, where $s.(f, \ell)$ is the sequence obtained by appending the pair (f, ℓ) to the sequence s.

This is a formal version of Definition 1 given in Section II-B.

Once we have computed the execution context of an event with respect to a function execution, we determine its acyclic execution context as per Definition 2. Let us denote the acyclic execution context of an event e with respect to a function execution a by AEC(a, e).

Next, we define a connectivity relation between objects accessed in function execution a. We define the binary relation $\overset{a}{\leadsto}$ such that $o_1 \overset{a}{\leadsto} o_n$ holds if and only if there exists a sequence $(o_1, f_1, o_2), (o_2, f_2, o_3), \ldots (o_n, f_n, val)$, such that each element of the sequence is in the read-footprint FP(a). This relation is reflexive and transitive.

We can now formally describe the conditions we check to detect if a function execution contains a traversal: If there exist two input-read events $e_i = $ READ$\langle \ell_i, obj_i, fld_i, val_i \rangle$ and $e_j = $ READ$\langle \ell_j, obj_j, fld_j, val_j \rangle$ such that:

- $e_i, e_j \in $ RTRACE(a)
- $(obj_i, fld_i) \neq (obj_j, fld_j)$
- AEC$(a, e_i) = $ AEC$(a, e_j) = \alpha$
- $obj_i \overset{a}{\leadsto} obj_j$ or $obj_j \overset{a}{\leadsto} obj_i$

then we mark the function associated with a as a traversing function and α as a traversal point. There may be more than one acyclic execution context marked as a traversal point for a function f across one or more of the function's executions.

IV. APPLICATIONS

We describe three applications of our traversal detection algorithm and AECs.

A. Performance Test Generation

Major software projects such as the Chrome browser use sophisticated frameworks to continuously perform performance regression testing [3], where the application is benchmarked at different versions in the development history; performance bugs are discovered by identifying code changes that cause statistically significant deviations in the measurements. Unfortunately, performance regression testing is not as widely used as functional testing. While there exists several code coverage tools for measuring completeness of functional tests, there is a lack of tool support to identify code modules that should be the focus of performance tests.

Previous research suggests that hard-to-detect performance bugs are often exposed when applications are executed with large-scale inputs and/or with special input values [18]. Our technique can be used to identify the former case. If functional unit tests for an application are available, we can use TRAVIOLI to find functions that traverse input data-structures and assist developers in constructing performance unit tests that forces long traversals of the input data-structures.

```
10  /* Creates a linked-list with numbers 1..N */
11  function makeRange(N) {
12    var node, rangeList = null;
13    for (var i = N-1; i >= 0; i--) {
14      node = {data: i, next: rangeList};
15      rangeList = node;
16    }
17    return rangeList;
18  }
19  /* Test the len() function */
20  function lenTest() {
21    var myList = makeRange(3);
22    var myTail = myList.next;
23    assert(len(myList) === 3);
24    assert(len(myTail) === 2);
25  }
```

Fig. 8.   Unit test for the `len` function from Fig. 2.

```
1  /* Execute 'len' over a large linked-list. */
2  var benchmark = new Benchmark.Suite;
3  var bigList = makeRange(100000);
4  benchmark.add('len-perf', function () {
5    return len(bigList);
6  }).run();
```

Fig. 9.   Performance test for the `len` function from Fig. 2.

```
1  /* Does 'list' contain everything in 'arr'? */
2  function containsAll(list, arr) {
3    for (var i = 0; i < arr.length; i++) {
4      var item = arr[i];
5      if (contains(list, item) == false) {
6        return false;
7      }
8    }
9    return true;
10 }
```

Fig. 10.   A function that redundantly traverses a list.

We next illustrate the process of creating a performance unit test from a functional unit test using TRAVIOLI. Consider the function `len` in Figure 2 and a unit test in Figure 8. The unit test, `lenTest`, first invokes the function `makeRange` at line 21 to create a sample list of the first three natural numbers. Lines 23 and 24 contain calls to `len` and assertions to ensure that the result matches the expected length of the list. If this unit test is provided to TRAVIOLI, the following report is generated:

```
Data-structure 'list' in function 'len':
- Traversal point: (len:6) up to 3 times
  - Reached from the following contexts:
    1. (lenTest:23)(len:6)
    1. (lenTest:24)(len:6)
  - Values last-written at following contexts:
    1. (lenTest:21)(makeRange:14)
```

TRAVIOLI detects one traversal point, which traverses the data structure `list` in the function `len`. Note that the traversal is identified when analyzing the executions of the function `len` and not `lenTest`, as the linked-list is not an external input to the latter. The traversal point (len:6) is with respect to the function `len`. We collect three types of information for each is a traversal point α in a function f.

1) We associate a traversal point counter that counts the maximum number of times f has executed α. For example, when `len` is invoked at line 23, TRAVIOLI finds three events at the traversal point. For the second invocation, from line 24, the traversal point is seen twice. The maximum count is three; therefore the report mentions "up to 3 times".

2) For all events at AEC α in the trace of an execution of f, TRAVIOLI collects AECs with respect to the program's entry function and lists them under the header "Reached from...". Assuming that the entry function in the example is `lenTest`, the events at the traversal point have the following two AECs with respect to the entry function: (lenTest:23)(len:6) and (lenTest:24)(len:6). This information helps us to understand how the traversing function was invoked in the functional unit test.

3) For all memory-locations m read at the traversal point α in an execution of f, we track the event at which m was last written before the read at the traversal point α. TRAVIOLI collects the AECs, with respect to the entry function, for all such write events and lists them under the header "Values last-written at...". In the running example, the last write for all memory-locations read at the traversal point occurs at the AEC (lenTest:21)(makeRange:14). This information helps to understand how a input data-structure gets created.

In our evaluation, we found that this report provides useful information to track where and how input data-structures were constructed and provided as inputs to traversing functions. If the goal of a developer is to write a performance unit test exercising a traversal point, she/he can use this information to write a test that will increase the value of the traversal point counter by several orders of magnitude. Figure 9 shows a possible performance test for the running example, using the API of Benchmark.js [1], a performance test framework. To verify that this test does indeed have a large read-footprint, we can run it through TRAVIOLI to generate a report which shows an increased value of the counter for the traversal point:

```
Data-structure 'list' in function 'len':
- Traversal point: (len:6) up to 100000 times
  ...
```

B. Detecting Redundant Traversal Bugs

TRAVIOLI can also be used to detect *redundant traversals*, such as the traversal in the function `containsAll` shown in Figure 10. The function `containsAll` takes as input a linked list `list` and an array `arr` and returns `true` if an only if all items in the array are also present in the list, by repeatedly invoking the `contains` function defined in Figure 5. The list is traversed multiple times without any change to its data—this is a case of redundant traversal. If the list contains n elements
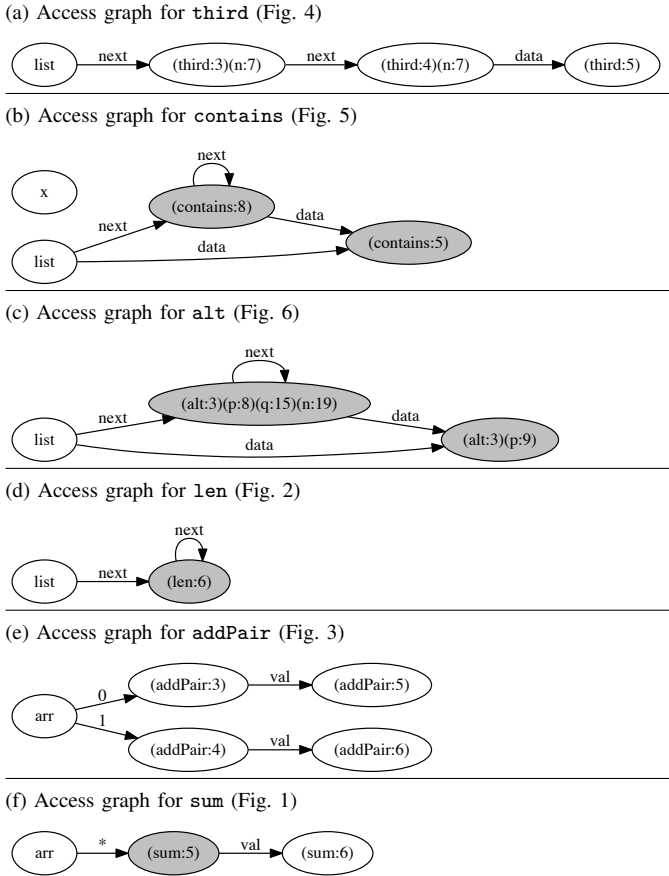
(a) Access graph for `third` (Fig. 4)

(b) Access graph for `contains` (Fig. 5)

(c) Access graph for `alt` (Fig. 6)

(d) Access graph for `len` (Fig. 2)

(e) Access graph for `addPair` (Fig. 3)

(f) Access graph for `sum` (Fig. 1)

Fig. 11. Access graphs for various examples presented in the paper.

and the array is of length m, then the worst-case complexity of `containsAll` is $\mathcal{O}(mn)$. Such instances are often indicative of performance bugs and can be fixed by using different data-structures (such as hashed sets) or caching. TRAVIOLI found two such bugs in popular JavaScript projects and the developers of the projects acknowledges them as performance bugs.

In order to determine if a traversal in a function is redundant, we need to analyze the sequence of concrete memory-locations (i.e. actual memory addresses) read at a traversal point of the function. If the sequence contains repeated contiguous subsequences, then we know that the memory-locations in these contiguous subsequences are traversed repeatedly. We then say the function has a redundant traversal. Formally, if the sequence of memory-locations read at a traversal point can be partitioned into the contiguous subsequences $\beta_1, \beta_2, \ldots, \beta_k$ where $k \geq 2$ and for each $1 \leq i, j \leq k$, either β_i is a prefix of β_j or β_j is a prefix of β_i, then the sequence of memory-locations indicate a redundant traversal.

For example, if a, b and c are concrete memory-locations (i.e. actual memory addresses as observed in a function execution), then the sequence of reads $abcaab$ can be partitioned into repeating contiguous subsequences $(abc)(a)(ab)$ indicating redundant traversals. On the other hand, the sequence $abcacab$ if partitioned as $(abc)(ac)(ab)$ does not indicate a redundant traversal because ac is not a prefix of ab and vice versa.

In general, a redundant traversal can be detected by a memory-location sequence as short as aba or aab; therefore, TRAVIOLI can detect redundant traversals from functional unit tests alone. Moreover, TRAVIOLI can detect redundant traversals in functions that use recursion, such as the example in Figure 10, which cannot be detected with existing approaches [21, 22].

C. Traversal Comprehension

Recall that the read-footprint of a function is the set of all memory-locations that are read by a function execution without any prior write to them by the execution. A memory-location in a read-footprint, which we call an input memory-location, can be reached from a program variable via a series of one or more fields or array indices. An input memory-location can be described using an *access path*. An *access path* α in a function execution is a finite non-empty sequence of the form $v.k_1.k_2 \cdots k_n$, where $n \geq 0$, v is a variable name, and each k_i is either a field name or an array index. The recursive definition of an access path is as follows: The access path v represents the value of the variable v before the function execution starts, and the access path $\pi.k$ represents the value stored in the field or array index k of the object whose access path is π. For example, the set of input memory-locations read by the function `third` in Figure 4 can be represented by the access paths `list`, `list.next`, `list.next.next`, and `list.next.next.data`. Note that more than one access path may refer to the same memory-location.

While analyzing the traversals reported by TRAVIOLI in our experimental evaluation, we found it useful to represent access paths representing the various input memory-locations as a finite graph, called an *access graph*. An access graph has a node for each program variable and for each AEC where an input memory-location is read. Nodes represent a set of values, which may be scalars or addresses of objects. A variable node corresponding to a variable v represents the value stored in v at the beginning of the function execution. An AEC node corresponding to an AEC α represents the values read by an input read-event at α. There is an edge with label k from any node n to an AEC node α if the field k of an object denoted by the n-node is read in an input read-event at the AEC α. If more than one field of objects represented by node n are read at the AEC α, then the edge from the n node to the α node is labeled with $*$. This happens when multiple elements of an array or multiple fields of an object are read at the AEC α. According to this definition, variable nodes do not have incoming edges. Moreover, all AEC nodes are reachable from at least one variable node. An AEC node is colored grey if the corresponding AEC is a traversal point.

An access graph concisely captures the access paths of all input memory-locations read at each AEC. In particular, a path in the graph from a variable node to an AEC node α represents an access path of the input memory-location read at α. For such a path in the graph, the corresponding access path begins with the variable that is denoted by the first node in the path, and is followed by all fields that are
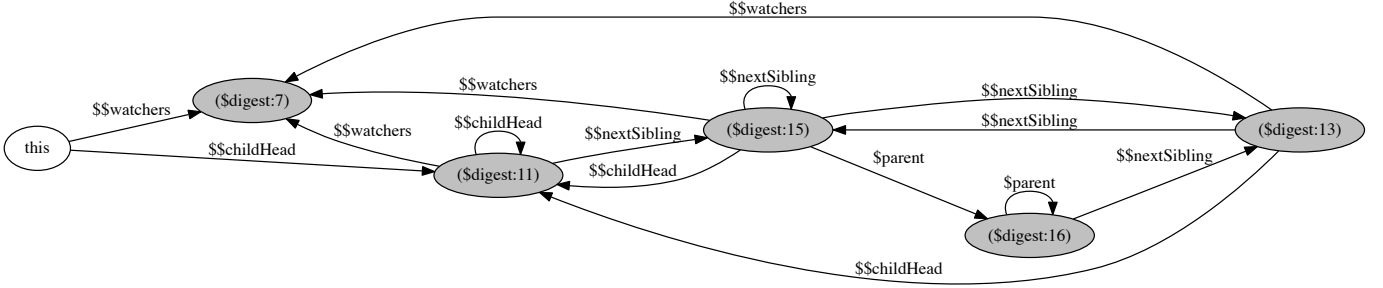
Fig. 12.   Access graph for the code in Figure 13.

```
1  Scope.prototype.$digest = function() {
2    var target, current, watchers /*, ... */;
3    target = this;
4    current = target;
5    // ...
6    do {
7      if (current.$$watchers) { /* ... */ }
8
9      // Insanity Warning: ...
10     // yes, this code is a bit crazy, but...
11     if (!(next = ((current.$$childHead)
12       (current !== target &&
13         current.$$nextSibling)))) {
14       while (current !== target &&
15         !(next = current.$$nextSibling)) {
16         current = current.$parent;
17       }
18     }
19   } while ((current = next));
20 }
```

Fig. 13.   An excerpt of a tree-traversal in Angular.js.

labels of edges on the path to the AEC node α. For example, in Fig. 11a, the access path of an input memory-location read at AEC (third:5) is list.next.next.data. In Fig. 11b, the access graph contains a cycle. Therefore, the access paths of the input memory-locations read at AEC (contains:5) are list.data, list.next.data, list.next.next.data, and so on. Thus the access graph provides a bounded representation of an unbounded number of access paths.

Figures 11b, 11c and 11d represent access graphs of three functions that traverse linked-lists in different ways, but the access graphs provide similar abstractions because in each case the input lists are traversed via the next field at a single AEC. Figures 11e and 11f depict access graphs for functions that read array elements. In addPair, array elements are read at two distinct AECs; therefore, the graph contains two branches starting from arr. On the other hand, sum traverses the array and this is captured by the wild-card $*$ that labels the edge from arr to the AEC (sum:5). The access paths that reach this AEC are arr.*, which indicate that more than one field (or in this case, more than one array index) of the variable arr is read at the AEC (sum:5). Similarly, the access paths that are read at AEC (sum:6) are arr.*.val, which represent the val fields of the elements contained in the array arr.

Note that an access graph does not describe the order of traversal, its stopping criteria, or any other program invariants. They are primarily an aid for understanding how functions follow a series of pointers to traverse a data-structure, and to map these events to their acyclic execution contexts. Moreover, access graphs are approximate because they are constructed via dynamic analysis; therefore, an access graph can only represent reads that are observed during an execution.

We can use access graphs to determine access paths that identify the data-structure being traversed. We call such an access path the *root* of the data-structure. The roots are determined by identifying the shortest access path corresponding to access graph nodes n that are predecessors of AEC nodes marked as traversal-points, such that the corresponding path from a variable node to node n does not pass through any AEC marked as a traversal point. For example, the root of the data structure traversed in Fig. 11b is simply list.

Construction: We construct one access graph per function f in the program. We add nodes and edges during every execution a of f that is analyzed for traversals. We ensure there exists one node corresponding to every variable that is an input memory-location and which is read at least once during a. We also ensure the existence of a node for every unique AEC encountered when analyzing input read-events in RTRACE(a). We maintain a mutable mapping called *last* that maps an object address to an access graph node. We initialize *last* such that $last(obj) = v$ if the input variable v contains the address of obj. We then analyze RTRACE(a) in sequence as follows:

1) Let the next event be $e = \text{READ}\langle \ell, obj, fld, val \rangle$ having AEC$(a, e) = \alpha$.
2) If $last(obj)$ is defined as node n, then:
   - If there is no existing edge in the access graph from n to α, then we add such an edge with label fld.
   - If there already exists an edge in the access graph from n to α with label $k' \neq fld$, then we update the edge label to the wild-card symbol '$*$'.
3) We update the mapping $last(obj) = \alpha$ and repeat from step 1 if there are more events remaining.

Case Study: We discuss how we used access graphs to understand a complex and crucial traversal in the source code of the widely used Angular.js framework [2] (see code in Figure 13). A object of type Scope denotes a node in a tree data-structure. $digest is a method of Scope. Each Scope object has a field $parent pointing to its parent node, a field $$childHead pointing to its first child, and a field $$nextSibling pointing to its right-sibling. The $digest

method, when invoked on a scope object s, traverses the entire sub-tree rooted at s in a depth-first order using a pair of nested loops at lines 6–19. The variable `current` points to a different node in each iteration of the outer loop and processes its `$$watchers` field at line 7—we omit this code for brevity. This tree traversal requires $\mathcal{O}(1)$ memory, unlike conventional traversals that use recursive functions. The traversal code is quite difficult to understand, as noted by the developers' own comments at lines 9–10.

To visualize the depth-first traversal, we use TRAVIOLI to analyze the execution of this code using a sample input of scope-tree with ten nodes. The access graph is shown in Figure 12. The AEC node (`$digest:7`) corresponds to all values read at line 7. The traversal can be understood by tracing all paths to the AEC node (`$digest:7`) representing the read of the field `$$watchers` of the variable `current` at line 7. We are interested in understanding what `current` represents at that AEC. In the simplest case, there is an edge from variable `this` to (`$digest:7`), representing the access path `this.$$watchers`. Other ways of reaching (`$digest:7`) are from the AEC nodes (`$digest:11`), (`$digest:15`), and (`$digest:13`). The AEC node (`$digest:11`) can be reached in several ways, such as by starting at `this` and by following one or more `$$childHead` fields to traverse the first-childs of sub-trees. The `$$nextSibling` field of the objects retrieved at this AEC can be read at AEC (`$digest:15`) to access the next right sibling. We can traverse a series of right siblings at this AEC by following the self-loop. Alternatively, the sub-trees rooted at these siblings can be traversed by following the edge labeled `$$childHead` back to AEC (`$digest:11`) and thus repeating the traversal process. Yet another path from the node (`$digest:15`), representing the sibling objects, follows the `$parent` field to AEC (`$digest:16`). From this parent object, we can follow its parent via the self loop, or follow the edge labeled `$$nextSibling` to node (`$digest:13`), which captures the action of moving to the parent's sibling. We can reason about the node (`$digest:13`) in a similar fashion. An interesting observation is that there is no path with consecutive edges `$$parent.$$childHead`, because this would result in a cycle. Similarly, there is no edge from AEC (`$digest:16`), where `$$parent` fields are traversed, to (`$digest:7`), where the `$$watcher` field is processed, because parent nodes are processed before their children. In our view, such graphical representations provide useful insights into understanding how and where different parts of a data-structure are traversed.

V. EVALUATION

We have implemented TRAVIOLI in the Jalangi framework [28]. We use DOT [16] format for access graphs. We evaluate TRAVIOLI on a set of five open-source JavaScript projects. The projects were chosen because they are widely used, they have comprehensive unit tests that can be launched from command-line using Node.js [10], and they represent a variety of scenarios where data-structure performance may be important. The projects include `d3-collection` [5], a data-structure library used in the popular D3 [4] visualization toolkit, `immutable-js` [8], an immutable data-structure library developed by Facebook, `d3-hierarchy` [6], which provides algorithms for visualizing hierarchical data-sets, `express` [7], a server-side web framework, and `mathjs` [9], an extensive math library. We analyze the `matrix` module of `mathjs`. The source code of TRAVIOLI has been made publicly available at https://github.com/rohanpadhye/travioli, along with the scripts to reproduce the experiments described in this section.

Table II provides an overview of experiments performed on a MacBook Pro with an Intel Core i7-4770HQ processor and 16GB RAM running OS X 10.10 and Node.js v4.4.0. All listed runtimes are in seconds. Column 1 lists the candidate projects, column 2 lists the number of unit tests in their test suites, column 3 reports the running time of the corresponding test suites, and column 4 reports the running time of the instrumented test suites, including the time to instrument the source files (project + dependencies) and the time to generate events. Column 5 lists the number of events that are generated and subsequently analyzed. Columns 6–8 report the time required to analyze these events, the number of function executions analyzed for traversals, and the number of unique functions for which access graphs are generated. Although we compute the read-trace for all function executions, we exclude analysis of functions from the project's dependencies or test suites. Columns 9–11 report the results of traversal detection: the number of traversing functions, the number of distinct access paths identified as roots of data-structures (cf. Section IV-C), and the number of distinct AECs marked as traversal points. Columns 12–14 repeat this information for redundant traversals (cf. Section IV-B).

For each candidate project, the instrumented tests as well as the analysis of traces completed within few minutes. We evaluate the quality of the traversals reported by answering four research questions:

RQ1. *Do the traversals reported by* TRAVIOLI *contain false positives?*

RQ2. *Can we generate peformance tests for the traversals reported by* TRAVIOLI*?*

RQ3. *Do the redundant traversals reported by* TRAVIOLI *contain false positives?*

RQ4. *Do the redundant traversals reported by* TRAVIOLI *correspond to performance issues?*

We answer RQ1 and RQ2 by manually evaluating a subset of the traversals reported by TRAVIOLI. For each candidate project, we randomly sample up to 10 access paths reported as roots of data-structures being traversed, and randomly pick one corresponding traversal point. Similarly, we answer RQ3 and RQ4 by manually evaluating a random subset of the traversal points that are reported as *redundant* by TRAVIOLI.

RQ1: Of the 50 traversal points that were randomly sampled across all candidate projects, we found only two false positives: one in `immutable-js` and another in `express`. In `immutable-js`, an array data-structure was incorrectly reported to be traversed. The false positive resulted from a

Application	Test Suite		Instrumented Tests		Analysis			All Traversals			Redundant Traversals		
	Test Cases	Run. Time	Run. Time	Events Logged	Run. Time	Function Invocations	Unique Func.	Unique Func.	Unique Roots	Unique AECs	Unique Func.	Unique Roots	Unique AECs
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
d3-collection	233	0.21	7.95	593,752	3.88	1,340	37	13	15	36	0	0	0
immutable-js	418	0.65	81.12	11,677,694	149.69	260,642	513	239	460	2,859	45	62	106
d3-hierarchy	49	0.18	10.14	1,021,083	6.40	5,523	50	20	23	88	1	1	1
mathjs:matrix	357	0.59	43.80	4,341,074	44.33	26,931	282	128	226	2,261	31	13	1,444
express	696	2.12	81.09	7,454,087	91.52	53,382	158	50	81	1,847	2	2	2

TABLE II

OVERVIEW OF EXPERIMENTS CONDUCTED TO EVALUATE TRAVIOLI. ALL TIMES ARE IN SECONDS.

Application	False Positives	Restricted Traversals	Perf. Tests Generated	Total
d3-collection	0	0	10	**10**
immutable-js	1	3	6	**10**
d3-hierarchy	0	0	10	**10**
mathjs:matrix	0	0	10	**10**
express	1	0	9	**10**
Total	**2**	**3**	**45**	**50**

TABLE III

EVALUATION OF SAMPLED TRAVERSAL POINTS.

Application	False Positives	Restricted Traversals	Necessary Redundancies	Perf. Bugs	Total
d3-collection	0	0	0	0	**0**
immutable-js	4	6	0	0	**10**
d3-hierarchy	0	0	0	1	**1**
mathjs:matrix	0	3	7	0	**10**
express	0	1	0	1	**2**
Total	**4**	**10**	**7**	**2**	**23**

TABLE IV

EVALUATION OF SAMPLED REDUNDANT TRAVERSAL POINTS.

related traversal of a hashmap that mapped strings to integer values; the resulting integers were used as indices to access a single element of different arrays. The array accesses occured within the same loop that traversed the hashmap, and in at least two iterations a common array was accessed at the same AEC; therefore, the conditions that TRAVIOLI checks for detecting traversals were satisfied. In express, one traversal point was in the test suite itself, in a function that was supplied as a callback parameter to express. Since the traversal was not really part of express we marked this as a false positive. The remaining 48 traversal points were true traversals.

RQ2: We attempted to manually construct performance tests for the 48 true traversal points using the method described in Section IV-A. If we were able to construct a test that increased the counter of the AEC corresponding to the sampled traversal point by a factor of 100, then we report this sample as a case of successful performance test generation.

We found three instances where we were not able to construct performance tests, because the traversing function corresponding to the sampled traversal point was private, and because the function could only be invoked by the project's own internal modules with an input of a bounded size. We consider such traversal points as *restricted* traversals. In our experiments, all three restricted traversals were found in immutable-js. The corresponding traversing functions perform traversals of input arrays. However, these functions are only used to traverse arrays contained in nodes of a bit-partitioned vector trie. Each node in such a trie can have up to a maximum of 32 child nodes. These child nodes are stored in an array, whose traversal was reported by TRAVIOLI. The array traversal counter can never exceed 32.

We summarize the results of the evaluation of RQ1 and RQ2 in Table III. The false positive rate in our evaluation was 4%. For the true positives, we could construct performance tests in 93.75% of the cases.

RQ3: From the reported redundant traversals, we sampled 10 data-structures and one corresponding traversal point from both immutablejs and mathjs. d3-hierarchy and express

contained fewer than 10 reports of redundant traversals and we analyzed all those cases. No redundant traversal was reported for d3-collection. We manually analyzed a total of 23 redundant traversals.

If a reported redundant traversal point does not really perform redundant computations, we mark it as a false positive. We found four such false positives in our evaluation. All false positives were in immutable-js. The sequence of memory-locations read at the reported traversal points did contain repeated contiguous subsequences, but this was specific to the particular inputs in the test suites. The corresponding traversing functions do not perform redundant computations in general.

RQ4: Of the 19 sampled redundant traversals that were true positives, we manually determined if they represented real performance issues. Some of these traversal points were *restricted* traversals, because the corresponding traversing functions can only be provided inputs of a bounded size due to certain program invariants. For example, in immutable-js, several restricted redundant traversals of ArrayMap nodes were observed. These maps are implemented as an array of key-value pairs; retrieving an entry from the map requires traversing the array until a matching key is found. The public Map API internally uses ArrayMap nodes only for maps containing fewer than 8 elements, as it trades-off the slightly longer lookup time to avoid allocating extra memory for hashtables. As more elements are added to the map, the implementation automatically switches to a different representation with more efficient lookup times. In express, one reported redundant traversal was restricted because the traversing function is private and is only ever invoked internally with an input list of distinct HTTP methods (e.g. GET, POST). As express supports only 26 HTTP methods, this never becomes a bottleneck.

For the remaining samples, we could construct tests to exercise the redundant traversals of very large inputs. However, in mathjs, several redundant traversals were observed that were not really performance bugs. For example, an algorithm

to multiply two matrices requires traversing at least one matrix multiple times. We marked such samples as *necessarily redundant*.

Two of the reported redundant traversals were real performance bugs—they were confirmed by the developers. In `d3-hierarchy`, TRAVIOLI found a bug in the implementation of binary treemaps, which are a visualization of hierarchical data as rectangles that are repeatedly partitioned into two sets. The implementation partitions an array of numbers by computing an index such that the sums of the left and right sub-arrays are approximately equal. This process is recursively repeated for each partition, resulting in a binary tree. We detected, from a simple unit test, that the algorithm to find the index to partition the array performed redundant traversals at each step to compute the sums of the sub-arrays. We were able to show that in the worst-case the implementation had complexity $\mathcal{O}(n^2)$. We reported and fixed this bug (see https://github.com/d3/d3-hierarchy/issues/44), by computing the sums of all prefixes of the input array ahead-of-time, and using a binary search to find the partition index at each step. The fixed implementation is $\mathcal{O}(n \log n)$ in the worst-case, and provides about a $30\times$ speed-up for a binary treemap with 1,000 nodes.

The second bug was found in `express`. When an `express` application is configured to support m URL patterns with n handlers using a particular API, the list of URL patterns is redundantly traversed once per handler to construct a regular expression that combines all patterns. As regular expression compilation is relatively expensive, this implementation may lead to longer start-up times for some applications. We reported this as a performance issue—the developers acknowledged the issue (see https://github.com/expressjs/express/issues/3065).

Table IV summarizes the evaluation of redundant traversals. 17.4% of the reports were false positives. 52.6% of the true positives were restricted, and 36.8% were benign. TRAVIOLI found two real performance bugs that have been confirmed by the developers.

TRAVIOLI uses dynamic analysis; therefore, it can only detect and report traversals if they occur during program execution. We cannot precisely evaluate TRAVIOLI for *false negatives*, because it is not possible to statically determine all traversal points, and it is not feasible to manually evaluate all candidate acyclic execution contexts.

VI. RELATED WORK

1) Redundant computation bugs: Clarity [22] uses static analysis to detect program functions in which an $\mathcal{O}(n)$ traversal occurs $\mathcal{O}(m)$ times redundantly. As our analysis is dynamic, we can determine if repeated traversals are redundant at a finer level of granularity. For example, if a binary-search-tree is repeatedly queried for different values, we do not report a redundancy if at least two traversals follow different paths in the tree. Clarity conservatively assumes all conditional branches to be equally likely, and thus cannot make such fine-grained distinctions automatically. Clarity therefore uses source-level annotations to recognize operations on standard Java collections that have sub-linear average time complexity. On the other hand, Clarity's static analysis is a sound over-approximation, while our dynamic analysis is subject to false negatives. Toddler [21] uses dynamic analysis to detect similar memory access patterns at the same execution context. It detects redundancies by analyzing the execution of long-running performance tests and extracting similarities in memory accesses across loop iterations. Our use of AECs and object connectivity allow us to detect traversals from as little as two iterations, and therefore we can detect redundant traversals using unit tests. Our use of acyclic execution contexts allows detecting recursive data-structure traversals, which is not supported by either of these tools.

A related tool—MemoizeIt [15]—uses dynamic analysis to detect functions whose computation can be memoized. Such an approach can possibly detect bugs like the one we found in `express`. However, the bug that we found in `d3-hierarchy` belongs to a different class, because an input array is redundantly traversed up to a different bound at each step, resulting in correspondingly different outputs—this cannot be detected by MemoizeIt. On the other hand, our technique does not detect memoization opportunities outside data-structure traversals.

2) Performance test generation: SpeedGun [25] generates performance regression tests for multi-threaded programs to discover how changes to source code influence the amount of synchronization required. PerfPlotter [13] uses symbolic execution to generate distributions of a program's performance under different inputs. WISE [12] uses symbolic execution to automatically generate tests that exercise worst-case behaviour of a program. These techniques aim to reason about program performance across all inputs and to automatically generate test programs or test inputs. Our goal is not to automate test generation, but to identify program functions that contain data-structure traversals and to aid developers in writiing performance tests that exercise these traversals.

3) Data-structure analysis: A number of techniques have been developed to analyze data structures using dynamic analysis. HeapViz [11] summarizes complex relationships between Java collections to provide a concise visualization of the heap. MG++ [31] generates representations of dynamically evolving data-structures. Pheng and Verbrugge [24] measure the number of data-structures created and modified over time in Java programs. Laika [14] detects data-structures in executing binaries using Bayesian unsupervised learning. DSI [35] identifies pointer-based data-structures in C programs by combining a multitude of observations over time. Raman and August [26] detect recursive data-structures and profile structural modifications in order to measure their stability.

Similarly, several static analysis techniques aim to discover abstract representations of data-strucures used in a program, and this body of work usually falls into the category of *shape analysis* [17]. Sophisticated frameworks can be used to prove complex data-structure invariants [27].

In all these techniques, the central theme has been identifying the type of data-structures or their representation in pro-

gram memory, and not on identifying functions that *traverse* these data-structures to perform work.

4) Execution Contexts and AECs: In dynamic analysis, execution indexing [36] allows uniquely identifying a point in a program execution. Such execution indices are too fine-grained for TRAVIOLI. The problem of reasoning about a potentially unbounded number of calling contexts in recursive programs has been studied for a long time in the field of static analysis [29, 30]. Our approach of removing cycles in execution contexts to construct AECs is similar to the approach employed by Whaley and Lam [34] for context-sensitive pointer analysis, where connected-components in the call-graph are collapsed to a single node. A subtle difference is that we retain the sequence of functions on the sub-path from the entry of a connected component to its exit; therefore, the resulting AEC is a valid sequence of call-sites that can be used for debugging, similar to stack-traces.

5) Access Graphs: Access graphs have been previously used for static liveness analysis [19] to concisely capture unbounded sets of access paths that may be live at a program point. Our access graphs are similar in that a node can represent a regular pattern of access paths. However, the access graphs we construct capture dynamic information regarding data-structure traversals. Moreover, we distinguish nodes based on AECs rather than program-locations; therefore, our access graphs are context-sensitive.

APPENDIX

Theorem 1. The problem of determining if an arbitrary function contains a traversal is undecidable.

Proof: Assume we have a function called `traverses(f)` that determines if an input function `f` is a traversing function. Now, we can construct another function `halts` that takes as input another function `p` and some input `x` that and returns true if an only if `p` halts when provided with the input `x`:

```
1  function halts(p, x) {
2    var f = function(arr) {
3      p(x); // Must halt for 'arr' to be traversed
4      for (var i = 0; i < arr.length; i++) {
5        print(arr[i]);
6      }
7    }
8    return traverses(f); // True iff p(x) halts
9  }
```

But we know that the halting problem is undecidable. Hence, the function `traverses` cannot exist. ∎

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