Problem Set II

Dynamic Macroeconomics I

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1 Heterogeneous Agents (30 points)

The economy is populated by a continuum of households of measure 1. Assume there is no population growth. Households have preferences given by:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{c^{1-\sigma}}{1-\sigma}$$

Every period, households have one unit of labor, which is supplied inelastically, and receive an idiosyncratic productivity shock z_t, such that the labor income is $e^{z_t}w_t$, and the idiosyncratic shock follows an AR(1) process, described by:

$$z_{t+1} = \rho z_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, \sigma_\epsilon^2) \tag{1}$$

In addition to labor income, households own risk-free bonds a_t that pay a net interest r_t. Households can hold a negative amount of assets $a_t < 0$, which means they can hold debt. However, there is an exogenous borrowing constraint $\bar{A}$, such that $a_t \geq -\bar{A}$. Households are born with initial assets $a_0 = 0$, and their initial productivity is $z_0 = 0$. Every $t \geq 0$, households choose consumption c_t and asset accumulation a_{t+1} to maximize their utility, subject to a period budget constraint:

$$c_t + a_{t+1} = e^{z_t}w_t + (1+r_t)a_t$$

This means that the economy is populated by *heterogeneous households*, that differ across labor productivity z_t and asset holdings a_t. In addition to households, there is a representative firm that produces according to a Cobb-Douglas production function: $y_t = k_t^{\alpha} l_t^{1-\alpha}$.

1.1 Recursive Representation

Formulate the recursive representation of the household's problem. Be sure to point out what the individual and aggregate state variables are.

1.2 Discretization of AR(1) Process

Assume the stochastic productivity process has persistence $\rho = 0.9$ and standard deviation $\sigma_\epsilon = 0.0872$. Using Tauchen (1986)'s method, compute a 5-point grid $\mathcal{Z}$ and a 5×5 Markov transition probability matrix Π, such that the resulting discrete process resembles the AR(1) process in equation (1). Use $m = 1.5$ in Tauchen's method.

1.3 Simulation

Assume for now that $K = 31$, such that prices are $w = 2.2$ and $r = 0.04$, and agents are borrowing constrained, such that $\bar{A} = 0$. Use the following parameter values:

$$\beta = 0.96, \quad \sigma = 3, \quad \alpha = 0.36, \quad \delta = 0.08$$

Solve the households problem with a 100-point grid for assets that goes from $\bar{A} = 0$ to 70. Simulate the consumption and asset accumulation paths of an agent in the economy during $110,000$ periods.

1. Plot the consumption path during the first 100 periods.
2. Discard the first $10,000$ simulations and plot the stationary distribution.
3. Compute and describe the main moments in this economy.
4. Compute the Lorenz curve in asset holdings and the gini coefficient.
5. For different values of the productivity risk $\sigma \in \{0.04, 0.06, 0.08, 0.1, 0.12, 0.14, 0.16\}$, plot the gini coefficient. What is the relation between productivity risk and inequality?

1.4 Impulse Response Functions

Take a representative agent with non-stochastic steady state asset holdings and productivity.

1. Compute the impulse response function after the agent receives the largest positive productivity shock during one period.
2. Compute the impulse response function after the agent receives the largest negative productivity shock during one period.
3. Are the responses to both shocks symmetric? If not, argue why.

1.5 Equilibrium

Are $w = 1$ and $r = 0.03$ equilibrium prices? If not, compute the equilibrium prices and describe the algorithm used to compute them.

1.6 Relaxing Borrowing Limits

Describe the consequences of relaxing borrowing limits in this economy to $\bar{A} = 1$. What is the impact on prices, gini coefficient, and total savings in the economy?