

Wall Street or Main Street: Who to Bail Out?

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Job Market Paper

University of Pennsylvania

This version: November 20, 2017

Abstract

Housing crises are characterized by a sharp increase in foreclosure rates that generates losses to mortgage investors. To preserve the solvency of financial institutions, governments have historically implemented two policies: a) offer bailouts to mortgage investors (Wall Street), and b) subsidize mortgage refinancing to prevent additional foreclosures (Main Street). The implementation of these instruments involves a trade-off, shaped by two frictions. On one hand, houses lose 20% of their value during the foreclosure process because of larger depreciation due to vacancy and vandalism. If the government offers complete bailouts to investors rather than subsidies to households, the economy has to bear the dead-weight loss of the value lost by foreclosed houses. On the other hand, house prices have an idiosyncratic component, so the government does not have perfect information on individual households' decision to default. Households have incentives to engage in strategic default to qualify for benefits. A subsidy policy transfers resources to households that were not planning to default in the absence of the policy but avoids the dead-weight loss of foreclosures. I quantitatively assess the welfare-maximizing policy in a heterogeneous agents' economy and find that a subsidy-only policy would have generated welfare gains of up to 0.4%, measured as the consumption equivalent variation, as compared to the baseline calibration that matches the TARP and HAMP programs implemented during the Great Recession. Households on the left tail of the equity distribution, which benefit the most from a subsidy program, obtain the largest welfare gains. In contrast, a bailout-only policy would have generated a welfare loss of 0.8%.

JEL classification: E21, E32, E44, G21, H24

*Email: davidza@sas.upenn.edu. I would like to thank Hal Cole, Jesús Fernández-Villaverde, Dirk Krueger and Guillermo Ordóñez for their helpful comments and guidance. For great comments, I thank Rhys Bidder, Serdar Birinci, Minsu Chang, Vasco Cúrdia, Mary Daly, Galina Hale, Juan Manuel Hernández, Juan David Herreño, Nicolás Idrobo, Òscar Jordà, John Krainer, Marianna Kudlyak, Kevin Lansing, Thomas Mertens, María José Orraca, Pascal Paul, Nicolas Petrosky-Nadeau, Eugenio Rojas, Daniel Wills, Junyuan Zou, and participants at the Federal Reserve Bank of San Francisco seminars, University of Pennsylvania Macro Club and the Student Research Conference at GWU. All errors are mine. The code for the computations can be found on my [github site](#).

1 Introduction

In the U.S., mortgages are the main source of funding for house purchases, used by two out of three homeowners to finance their home acquisition. Mortgages are collateralized debt contracts in which borrowers pledge their house as collateral. In case of default, the property is foreclosed upon and is legally transferred to the lender, to cover for the losses of the loan. With some exceptions, such as the Great Depression and the Great Recession, the average yearly foreclosure rate has oscillated around 1.5% of total outstanding mortgages. During the recent financial crisis, after house prices plummeted more than 20%, the foreclosure rate almost tripled from an average of 1.52% during 1991-2006, to 4.36% in 2007-2010. This sharp increase in foreclosures generated significant losses to financial institutions, which are the largest investors in the mortgage market (Antoniades, 2015). Figure 1 illustrates the behavior of foreclosure starts since 1992 and the Core Logic housing price index (HPI).

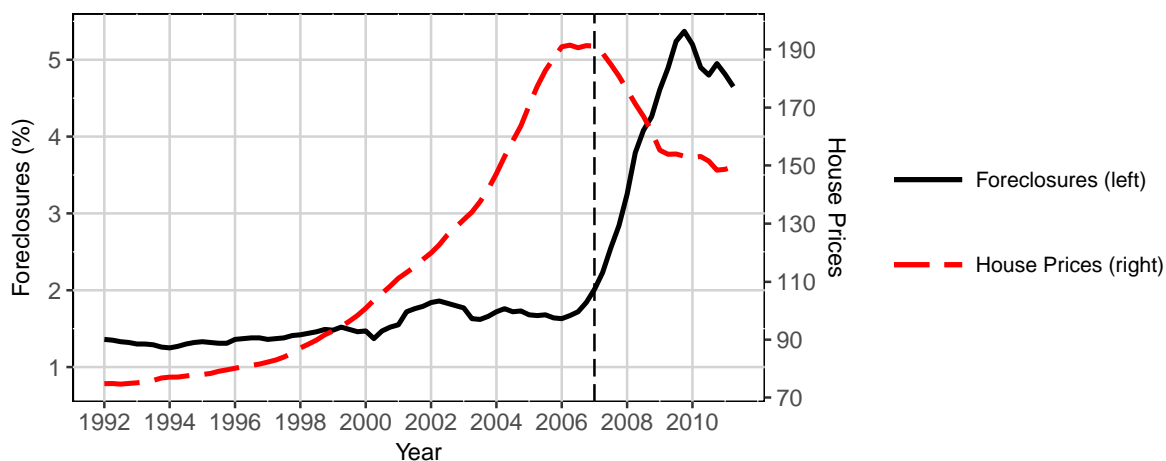


Figure 1: Foreclosure Starts (Mortgage Bankers Association) and House Price Index (Core Logic).
Source: Chatterjee and Eyigungor (2015)

During mortgage crises, governments have historically implemented two sets of policies to preserve the solvency of financial institutions: 1. offer bailouts to cover for the losses of these institutions, and 2. subsidize the mortgage refinancing of households to prevent additional foreclosures. During the Great Recession, through the Emergency Economic Stabilization Act (EESA) of 2008, the Congress authorized the Secretary of the Treasury to implement both types of policies. First, it approved the Troubled Assets Relief Program (TARP), which allocated up to

\$700 billion for investments in institutions that were in financial distress, through the purchase of troubled assets. These investments implicitly bailed out the recipient institutions; of the \$243 billion invested by 2008,¹ the Congressional Budget Office (CBO) estimated a risk-adjusted net present value return of $-\$61$ billion, or 0.5% of the 2007 GDP.² Second, the Treasury committed \$75 billion to subsidize mortgage refinancing through the Home Affordable Modification Program (HAMP), in order to reduce payments-to-income (PTI) ratios and prevent default.

In a frictionless, full-information and representative agent economy, offering bailouts to mortgage investors or subsidies for the mortgage refinancing of households would have exactly the same welfare consequences. If the government could observe which households are planning to default after a house price shock, it could offer them the exact amount that would make them indifferent between defaulting or not. This would prevent the foreclosure rate from rising and financial intermediaries from experiencing losses. Otherwise, the government could allow foreclosures to rise and transfer to banks an amount exactly equal to their losses. In the absence of any friction, both strategies would yield the same welfare outcome.

However, at least two frictions make this policy design non-trivial. On one hand, wide empirical evidence documents the existence of a 20% – 22% price discount on foreclosed houses. Using hedonic regression methods, [Campbell et al. \(2011\)](#) find that in the State of Massachusetts houses that have been foreclosed upon are sold at a price 27% lower, as compared to houses with similar characteristics that are sold by homeowners rather than banks. Out of this 27%, they estimate that 7 percentage points correspond to the discount of banks engaging in fire sales,³ while the remaining 20 percentage points correspond to a loss of value throughout the foreclosure process, mainly because of a larger depreciation due to vacancy and vandalism.⁴ Similarly, using a repeat-sales methodology [Pennington-Cross \(2006\)](#) follows a panel of houses over time and finds that when a

¹ These investments do not include loans to automobile companies.

² Under Section 202 of the EESA, the Office of Management and Budget (OMB) and the Congressional Budget Office (CBO) were required to estimate a semiannual *risk-adjusted* net present value of TARP. The first of such reports published by the CBO in January 2009 estimated a risk-adjusted net present value of the Capital Purchase Program (CPP) equal to $-\$61$ billion. Although TARP was not specifically directed to cover the losses generated by mortgage investments, the primary driver of the crisis was the exposure to the housing sector ([Antoniades, 2015](#)). See [Calomiris and Khan \(2015\)](#) for further details.

³ [Mayer \(1995\)](#) argues that in markets with search frictions, urgent sales are made at lower prices because the quality of the match is lower. This supports the fact that houses in foreclosure are sold at discount, as the real estate market is highly illiquid and banks have incentives to hold the properties for a short time.

⁴ Part of this price discount can also be explained by lower investments of households prior to default. [Melzer \(2017\)](#) finds that homeowners with a high risk of mortgage default cut down substantially on home improvements, consistent with the debt overhang theory. When households cannot appropriate the returns on their investment at every state of the world, investment falls. In particular, households with negative home equity spend 30% less per quarter on home investments, compared to households with positive equity. Similarly, [Harding et al. \(2000\)](#) find evidence that households with high loan-to-value (LTV) ratios invest less in their properties.

house is sold after a foreclosure, its price is 22% lower.⁵ If the government implements a policy that offers bailouts to mortgage holders and does not prevent foreclosures, this 20 – 22% value lost on foreclosed houses represents a dead-weight loss to the economy.

On the other hand, the literature has identified an uninsurable idiosyncratic component in house prices, which is not perfectly observable by the government.⁶ Given that the default decision strongly depends on house prices, the government cannot perfectly observe which households plan to default at every point in time. As a consequence, any policy that subsidizes the mortgage refinancing of households at risk of default generates a moral hazard problem, as households that are not in financial distress might engage in “strategic default” to obtain the benefits. [Mayer et al. \(2014\)](#) estimate that strategic default can be as high as 10% of total default after the announcement of a mortgage modification program. Therefore, any policy that subsidizes mortgage refinancing will avoid the dead-weight loss of foreclosures, but has to cover the costs of subsidizing strategic defaulters, as pointed out by [Foote et al. \(2008\)](#) and [Mayer et al. \(2014\)](#). If taxation is distortionary, levying taxes to subsidize strategic defaulters represents a welfare loss to the economy, through a distortion of the labor decisions of households.

The purpose of this paper is to quantitatively study the welfare-maximizing policy to preserve the solvency of financial institutions during mortgage crises, when governments have the two policy instruments described above. In particular, I study the case of the Great Recession: I assess the welfare costs of the dead-weight loss and strategic default, and evaluate counter-factual policies that yield higher welfare outcomes, when compared to the TARP-HAMP baseline. I use a heterogeneous agents’ life-cycle model with housing and long-term mortgage markets, calibrated in the initial steady state to match the pre-crisis U.S. economy. The model is disciplined by using micro estimates on the dead-weight loss of foreclosures, the magnitude of strategic default and the Frisch elasticity of demand estimated in the macro literature for the calibration. Then, I replicate the financial crisis after unexpected shocks to labor productivity and a loan-to-value restriction at origination hit the economy in 2007, assuming the government implements a policy analogous to TARP and HAMP. This laboratory allows me to perform counter-factual experiments to assess the welfare gains of different policies.

⁵ See [Pennington-Cross \(2006\)](#) for a review of the literature.

⁶ See [Case and Shiller \(1989\)](#); [Flavin and Yamashita \(2002\)](#) and [Piazzesi and Landvoigt \(2015\)](#). There are at least two different sources of idiosyncratic house price risk: a) there are idiosyncratic quality shocks that scale over time, and b) the housing market is illiquid, so there are one-time shocks that are realized at the time of sale reflecting the quality of the match. [Giacoletti \(2016\)](#) finds that the idiosyncratic component of house prices represents 80% of house price variance.

A heterogeneous agents' economy is the appropriate laboratory in which to study the design of a welfare-maximizing policy for two reasons. First, in addition to the frictions already described, offering subsidies to households or bailouts to banks has redistributive consequences that affect welfare and have to be taken into consideration. The welfare consequences of either policy are mainly driven by marginal propensities to consume and insurance motives, which are fully considered in a heterogeneous agents' environment (Mian et al., 2013; Kaplan and Violante, 2014). Second, as pointed out by Foote et al. (2008), the cost of strategic default in a subsidy policy depends on the fraction of eligible households that plan to default after an aggregate house price shock. If a large fraction of the eligible households finds it optimal to default after the aggregate house price shock, the cost of the information friction will not be large, as the fraction of households that can potentially engage in strategic default is small. In contrast, if only a small fraction of eligible households find it optimal to default after the shock, the total amount of subsidies distributed to strategic defaulters is potentially large. In a heterogeneous agents' economy, the welfare-maximizing policy will offer subsidies to certain groups of the population, based on observable characteristics.

The main result of my paper is that a subsidy-only policy dominates the TARP-HAMP combination implemented through the EESA of 2008. The reason is that the distortion generated by the taxes levied to finance strategic default, given the macro estimates of the Frisch elasticity of labor supply, is smaller than the dead-weight loss of foreclosure. For this reason, completely replacing a bailout policy such as TARP with a subsidy policy analogous to HAMP would yield welfare gains equal to 0.25% in consumption equivalent terms. The welfare gains are concentrated on the lower tail of the equity distribution, as households with the highest debt levels receive the largest subsidies. However, households that do not receive benefits also experience welfare gains, as labor taxes are lower under a subsidy-only policy. If, in addition, the government implements a better eligibility rule that targets the households that would not default in the absence of the crisis, and adds an age component to the subsidy, the welfare gains rise to 0.4%. Finally, eliminating HAMP and extending TARP would yield a welfare loss of 0.8%.

This paper abstracts from two additional frictions that arise in this environment. First, by assuming that the crisis happens after a completely unexpected shock, I abstract from the moral hazard problem that exists when financial institutions and households anticipate the government's policy and engage in overly-risky behaviors prior to the crisis. There is anecdotal evidence that policy-makers at the Federal Reserve Board and the Treasury ignored any moral hazard concerns

at the time of the policy design in 2008. As Timothy Geithner stated, *“Trying to mete out punishment to perpetrators during a genuinely systemic crisis—by letting major firms fail or forcing senior creditors to accept haircuts—can pour gasoline on the fire. It can signal that more failures and haircuts are coming, encouraging creditors to take their money and run. Old Testament vengeance appeals to the populist fury of the moment, but the truly moral thing to do during a raging financial inferno is to put it out.”*⁷ Second, I ignore the externalities that foreclosures generate on the price of neighboring houses. The literature has documented this externality, but its effects are very local and do not seem to be very large (see [Campbell et al. \(2011\)](#)). In any case, including a price externality would make foreclosures even costlier, which would reinforce the results obtained in this paper. Including these two frictions is left for future research.

This paper is organized as follows. Section 2 reviews the literature to which this paper is related. Section 3 illustrates the main mechanism driving the welfare-maximizing policy, through a simple example. Section 4 presents the heterogeneous agents’ model used throughout the rest of the paper. Section 5 states the definition of a recursive general equilibrium in this environment. Section 6 describes the calibration of the model to the pre-crisis U.S. economy and the main results in the initial steady state. Section 7 presents the main results of the paper and Section 8 concludes.

2 Related Literature

This paper models the mortgage market, so it is inherently related to the extensive literature on housing and mortgage decisions. My model builds upon [Gervais \(2002\)](#), who models an economy with heterogeneous agents that optimally choose between homeownership and renting. [Jeske et al. \(2013\)](#) add to the model the presence of short-term mortgages, where households have the possibility of defaulting on their debt obligations and the pricing of mortgages is consistent with the probability of default. This paper also models the mortgage market and allows for the possibility of default. However, I model long-term mortgage markets, rather than short-term contracts. In this respect, this paper is close to [Chatterjee and Eyigungor \(2015\)](#), [Corbae and Quintin \(2015\)](#) and [Hedlund \(2016\)](#), who study foreclosures, homeownership, consumption and other macroeconomic aggregates during the recent financial crisis. I depart from all of these works in that I study the welfare consequences of a subsidy-bailout policy aimed at preserving the solvency of mortgage investors.

⁷ See *The Greenspan Legacy* [article](#) in The Economist, published January 5, 2017.

This model is also related to models that study the welfare effects of fiscal policies within financial crises, where the economy is modeled in the steady state and counter-factual fiscal policy experiments are performed after an unexpected aggregate shock. A good example of such a paper is [Kaplan and Violante \(2014\)](#).

As in [Jeske et al. \(2013\)](#), this paper assumes that the idiosyncratic component of house prices is driven by an idiosyncratic depreciation that is partially observable by the government. This assumption is supported by the empirical evidence found by [Case and Shiller \(1989\)](#), [Flavin and Yamashita \(2002\)](#) and [Piazzesi and Landvoigt \(2015\)](#), who estimate large idiosyncratic components in the variance of house prices. The partial observability of these idiosyncratic shocks generates strategic default after the announcement of any mortgage refinancing subsidy policy ([Foote et al., 2008](#); [Mayer et al., 2014](#)).

This paper is also related to the literature that studies bailout policies. In particular, this paper studies the ex-post costs of bailouts ([Acharya et al., 2014](#)), and abstracts from ex-ante moral hazard concerns arising from different policy combinations. For this, I assume that the economy is in the steady state and a one-time, completely unexpected shock hits the economy, so financial institutions and homeowners do not foresee any government policy in the steady state. [Fahri and Tirole \(2012\)](#), [Jeanne and Korinek \(2013\)](#), [Chari and Kehoe \(2016\)](#) and [Faria-e-Castro \(2016\)](#) do take into consideration moral hazard.

Given that I am assuming a completely unexpected aggregate shock to the economy, my model is not appropriate to study any amplification effects of the financial sector during crisis periods. Two such papers that study the effect of household default decisions during the Great Recession on the cost of the funding of banks through a financial accelerator channel are [Faria-e-Castro \(2016\)](#) and [Greenwald \(2016\)](#). Other papers that study fiscal multipliers in New Keynesian settings are [Drautzburg and Uhlig \(2015\)](#) and [Auerbach and Gorodnichenko \(2012\)](#).

3 The Mechanism

This section illustrates the underlying trade-off between bailouts and mortgage refinancing subsidies through a simplified example, emphasizing the role of the dead-weight loss of foreclosures and the information friction on house prices. To isolate the mechanism behind the trade-off, this

example abstracts from many of the determinants of mortgage default, which are later considered in the quantitative model in Section 4.

Assume there are two households, denoted H_1 and H_2 , and one bank in the economy. Each household has a house worth \$100 and an outstanding mortgage of \$100 on the house. Mortgage contracts are short-term and last only for one period, default implies losing the collateral and there is no recourse in the economy. That is, households will default on current mortgages if and only if they are underwater -they owe more than the value of their house-, in which case they lose their house to the bank. Assume the bank, which is the mortgage holder, has liabilities of \$200 and its only assets are the outstanding mortgages of the households. If there are no shocks to house prices, households will repay their debt and the bank will break even. Finally, assume that the government knows the distribution of house prices, but does not observe the price of each individual house.

Suppose there is a completely unexpected one-time shock to house prices that lowers the price of the house owned by H_2 to \$80, while the price of H_1 remains at \$100. After the shock, the house owned by H_2 is worth less than the mortgage debt, so H_2 optimally defaults and its house is foreclosed upon. Given the 20% price discount on foreclosed housing, the bank only recovers \$64 out of the \$80 worth the house of H_2 . Household H_1 will repay its debt, as its house price is still worth \$100. The bank is now insolvent, as its \$164 revenues are not enough to cover the \$200 worth of liabilities.

To preserve the solvency of the bank, the government can choose either of two policies. On one hand, the government can offer subsidies to households to prevent default. Given that the government knows the distribution of house prices, it knows that the household that is underwater would need \$20 to find it optimal to repay its mortgage. However, the government does not observe individual house prices so, if it offers subsidies to mortgage repayments, both households will claim to be underwater to receive the \$20. Therefore, the subsidy policy to prevent the default of H_2 would cost \$40 and would preserve the solvency of the bank, but the government will have to levy \$20 with distortionary taxation to pay to strategic defaulters -household H_1 -. On the other hand, the government could let H_2 default and offer a bailout to the bank. In this case, the economy would assume the cost of the dead-weight loss generated by the foreclosure, equal to \$16. The cost of the dead-weight loss is smaller than the subsidies given to strategic defaulters, so a bailout policy would be welfare maximizing. Figure 2 illustrates the effects of either policy.

Now, assume there are, instead, two households of the type H2 and one household of the type H1 described above. A subsidy policy that prevents default would have to offer \$20 to each household in the economy. The cost of such a policy would be \$60, out of which \$20 correspond to subsidies to strategic defaulters. Otherwise, the government could offer a bailout for each of the two foreclosed houses, and the economy would pay the dead-weight loss, equal to \$32. In this case, the dead-weight loss is larger than the amount necessary to subsidize strategic defaulters, so a subsidy policy is preferable.

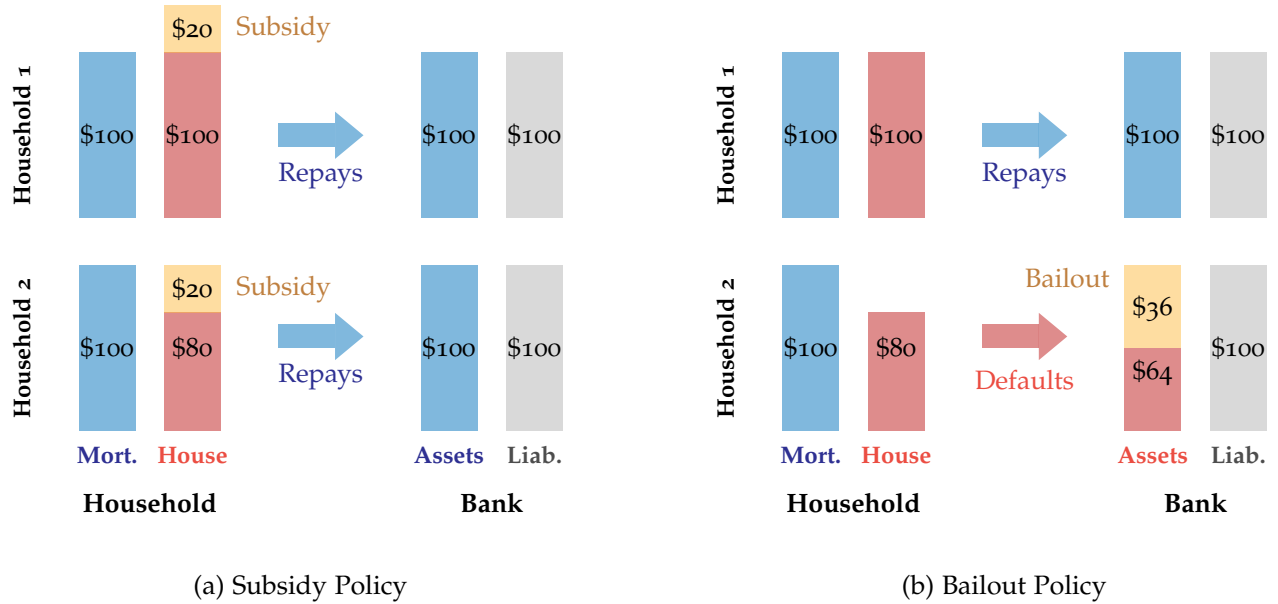


Figure 2: Subsidy vs. Bailout policy

The welfare maximizing policy depends on the size of strategic default, which corresponds to the proportion of households of type H1 versus H2 in the example. If the economy is populated with a large number of households of type H2, who would default absent any subsidy program, the cost of the information friction is low and a subsidy policy is optimal. If, in contrast, the proportion of H1 households is large, a subsidy policy will spend a large amount on strategic defaulters. In this case, it is optimal to bail out the banks.

4 Quantitative Model

In this section, I introduce a heterogeneous agents', life-cycle model with housing and mortgage markets. Section 4.1 describes the economy and defines the problem of the households, Section

4.2 describes the problem of the mortgage holders, Section 4.3 states the problem of production firms, and Section 4.4 defines the problem of the government. Later sections describe a baseline calibration of this economy to the pre-crisis U.S. economy and perform counterfactual experiments to assess the optimal policy.

4.1 Household's Problem

4.1.1 Demographics

The economy is composed of a continuum of households of constant size. Households live for at most T periods, after which they die with probability equal to 1. At age $t \in \{1, \dots, T\}$, the individual faces an exogenous probability π_t of surviving to the next period. By assumption, $\pi_T = 0$. By the law of large numbers, π_t not only is the probability of surviving to the next period, but also the effective amount of people that survive to age $t + 1$. I assume there is no population growth and every period a cohort of newborns of size 1 enters the economy, so at every point in time the total population size remains constant at $\left(1 + \sum_{t=1}^T \prod_{j=1}^t \pi_j\right)$.

4.1.2 Preferences

Households derive utility according to a period utility function $u(c, s, l)$ from non-durable consumption c , housing services s , and labor l . Non-durable consumption is the numeraire good in the economy, and housing can be owned or rented at market-determined prices P_h and q , respectively. Labor is supplied on a labor market with a competitively determined wage w per effective unit of labor supplied. Individuals discount the future according to a discount factor β and maximize their expected lifetime utility $\mathbb{E} \sum_{t=1}^T \beta^t u(c, h, l)$.

4.1.3 Income Dynamics

Every period, the labor productivity of an individual of age t is the product of two components: an idiosyncratic shock $e_t \in \mathcal{E}$, and a deterministic age-dependent productivity \bar{e}_t , so that, given an amount of labor supplied l , the effective units of labor supplied in the market are: $e_t \bar{e}_t l$. As in [Storesletten et al. \(2004\)](#), the idiosyncratic productivity shock e_t follows an autoregressive process, given by:

$$\log(e_{t+1}) = \rho_\epsilon \log(e_t) + \sigma_\epsilon \epsilon_{t+1}, \quad \epsilon_t \sim N(0, 1) \quad (1)$$

where $\rho_e < 1$ is the persistence of the productivity process, and σ_e is the standard deviation of the innovations. The AR(1) process is approximated by a 3-state Markov process with transition probability matrix denoted as $F_e(e_{t+1}|e_t)$, using the procedure described in [Tauchen \(1986\)](#). The age-dependent productivity \bar{e}_t follows an increasing path over the first periods of the cycle, followed by a decreasing path up to age 65 after which individuals retire, as documented by [Hansen \(1993\)](#).

Labor income is subject to a marginal income tax τ_l levied by the government to pay for government expenditures. In addition, the government levies social security taxes τ_{ss} , so total after-tax income is equal to $(1 - \tau_l - \tau_{ss})e_t\bar{e}_t wl$. Taxation is distortionary, so higher government expenditures will generate larger distortions in the economy. After age 65, individuals retire and receive social security benefits b set in equilibrium through a pay-as-you-go social security system.

4.1.4 Housing

There is a perfectly inelastic supply of housing \bar{H} in the economy available for renting and owning at market-determined prices q and P_h , respectively. As in [Jeske et al. \(2013\)](#) and [Elenev et al. \(2016\)](#), I separate the investment and consumption motives of housing and assume that only rental housing can be used for consumption as housing services s , so there is no owner-occupied housing. Households decide to own housing h only for investment purposes and receive benefits through the returns of renting out their property.

The set of available houses for owning and renting is given by $\mathcal{H} := [0, \bar{h}]$. I assume that home-owners can choose to rent any amount $s \in \mathcal{H}$, potentially different from the amount owned $h \in \mathcal{H}$. If households own more than what they rent, $h > s$, I will denote them *net owners*, whereas those that rent more than they own, $s > h$, are *net renters*.

Every period, owned housing is subject to an idiosyncratic depreciation shock $\delta \in \Delta := [\underline{\delta}, \bar{\delta}]$, which is distributed according to a cumulative distribution function $F_\delta(\delta)$ so home-owners face an idiosyncratic shock to their house value. I assume $\underline{\delta} < 0 < \bar{\delta}$ so there is idiosyncratic risk of having an increase or fall in the house value. This assumption embodies the uninsurable idiosyncratic house price risk that home-owners face ([Case and Shiller, 1989](#); [Jeske et al., 2013](#)).

At the end of every period, in order to have a constant housing supply equal to H , I assume that the homeowner must pay for the house depreciation $\delta P_h h$ whenever $\delta > 0$, and receives an additional income equal to $|\delta P_h h|$ whenever $\delta < 0$.

The idiosyncratic depreciation δ is private information for the household and is only partially observed by the government, motivated by [Mayer et al. \(2014\)](#) and [Foote et al. \(2008\)](#). Assuming that the household draws $\delta_0 \in \Delta$, the government observes δ_0 with probability $1 - p$, and $\delta \in \Delta \setminus \{\delta_0\}$ with probability p . Given that the decision to default depends on δ , the government does not perfectly observe which households find it optimal to default. In particular, given that all the other state variables of the household are observable by the government, with probability $1 - p$ the government correctly observes the decision to default of an individual with state variables (t, e, a, h, m) , while with probability p the government potentially observes a different default decision.

4.1.5 Financial Assets and Mortgage Markets

Households can buy one-period risk-free bonds a_t on financial markets at a price $P_a = 1/(1 + r_f)$, where r_f is the risk-free rate of return set in equilibrium. In addition to risk-free bonds, individuals have access to collateralized long-term mortgages to buy h units of housing. A mortgage of size m that pledges h units of housing as collateral, given to a household with productivity e , age t and risk-free bonds a is a contract that delivers $P_m(t, e, a, h, m)m$ in the current period and requires a payment equal to m every period in the future. I assume that every period the mortgage debt disappears with probability $1 - \rho$, so expected mortgage payments follow a decreasing path over time and last for a finite number of periods on average ([Chatterjee and Eyigungor, 2015](#)). The loan-by-loan pricing function $P_m(\cdot)$ is described in section 4.2, where the problem of the mortgage originators is fully characterized. Issuing a mortgage of size m has a one-time cost $F^{issue} \cdot \left(\sum_{j=t}^T \left[\Pi_{i=t}^j \pi_i \right] \left[\frac{\rho}{1+r} \right]^{j-t} \right) m$, equal to a fraction F^{issue} of the total outstanding debt given the existence of search costs in mortgage origination ([Hurst and Stafford, 2004](#)). Finally, individuals face a Loan-To-Value constraint at origination, such that $\left(\sum_{j=t}^T \left[\Pi_{i=t}^j \pi_i \right] \left[\frac{\rho}{1+r} \right]^{j-t} \right) m / P_h h < LTV$. Equivalently, the minimum downpayment required to issue a mortgage is set to $1 - LTV$.

Individuals have the option to default on their mortgage debt after the idiosyncratic depreciation shock δ is realized, subject to losing the property pledged as collateral. In addition to losing the collateral, individuals cannot access the mortgage market in the period of default but can re-enter the market in any future period. Although in the U.S. there is recourse in most States ([Ghent and Kudlyak, 2011](#)), in which case households lose also part of their risk-free assets to cover the difference between the outstanding debt and the house value, I assume there is no

recourse in the economy and leave this for future work. Households also have the option to refinance their mortgage debt. Refinancing a mortgage is equivalent to issuing a new mortgage and using its proceeds to repay the outstanding debt of the original mortgage. This means that households that choose to refinance their mortgage have to pay the proportional fixed cost $F^{ref} \cdot \left(\sum_{j=t}^T \left[\Pi_{i=t}^j \pi_i \right] \left[\frac{\rho}{1+r} \right]^{j-t} \right) m'$ over the newly issued mortgage m' . Note that the cost of issuing a new mortgage F^{issue} is potentially different from the cost of refinancing an existing mortgage F^{ref} .

4.1.6 Government Policies

In steady state, the government plays a passive role in the economy and only manages the social security system: it levies social security taxes τ_{ss} and distributes a pension b to retired households. Whenever there is an unexpected aggregate shock, or a “crisis period”, I assume that two policy instruments become available to the government: 1) distribute subsidies to mortgage refinancing and 2) offer bailouts to mortgage holders. The motivation for this assumption is that historically these instruments have become available to the governments after Congress approval during severe crisis periods like the Great Depression and the Great Recession. During the Great Recession, these instruments became available through TARP and HAMP. In contrast, during “normal” times - that is, in the steady state - governments do not offer bailouts to banks nor implement long-term policies to favor mortgage refinancing.

This section describes the policy instruments available to the government during crisis periods, while the optimization problem the government solves is postponed to section 4.4.

First, to prevent foreclosures the government subsidizes mortgage refinancing at a rate τ , which means that households that choose to refinance have to pay only $(1 - \tau)F \cdot \left(\sum_{j=t}^T \left[\Pi_{i=t}^j \pi_i \right] \left[\frac{\rho}{1+r} \right]^{j-t} \right) m'$ on the new mortgage issued m' . As in the Emergency Economic Stabilization Act, the subsidy is contingent on the mortgage refinancing being executed, to avoid households claiming the subsidy and defaulting on the same period. The purpose of this subsidy is to reduce the number of mortgage defaults by reducing the relative cost of refinancing.

Subsidies are given according to an eligibility rule chosen by the government every period. The eligibility rule potentially depends on all the state variables *observed* by the government, including the idiosyncratic house price shock δ . That is, an eligibility rule is a function $\Gamma : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \rightarrow \{0, 1\}$ that determines which households receive subsidies,

where Δ is the idiosyncratic depreciation observed by the government, which need not be equal to the realized depreciation of the household (recall the information friction described in section 4.1.4). This means that if the government implements an eligibility rule that explicitly targets particular values of observed δ to receive the subsidy, it will potentially make mistakes with probability p : households whose realized δ makes them eligible might not receive the subsidy, while households whose realized δ makes them non-eligible might receive it. As with the eligibility rule, the mortgage refinancing subsidy can also differ across the distribution of households so $\tau : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \rightarrow [0, 1]$.

Second, the government offers bailouts B to the mortgage holders to cover for their losses whenever there is an unexpected rise in foreclosures. Bailouts are pure lump-sum transfers. Henceforth, I will refer to the set of government policies as $\Theta : \{\tau(\cdot), \Gamma(\cdot), B, \tau_l\}$.

4.1.7 Household's Problem

Every period, given the individual state variables (t, e, a, h, m, δ) , aggregate state variables denoted by Ω , government policies Θ , a pricing function for mortgages $P_m(t, e, a, h, m; \Omega, \Theta)$, and prices P_h, q, r_f and w , households choose the amount of savings to take on to the next period a' , housing to own h' , housing services to rent s , consumption c , mortgage units to acquire m' and whether to default, keep or refinance their current mortgage. The recursive formulation of the problem of the household is defined by the following equation:

$$V(t, e, a, h, m, \delta; \Omega, \Theta) = \max\{V^{keep}(t, e, a, h, m, \delta; \Omega, \Theta), V^{def}(t, e, a, h, m, \delta; \Omega, \Theta), \mathbb{E}_p V^{ref}(t, e, a, h, m, \delta; \Omega, \Theta)\} \quad (2)$$

where V^{keep} , $V^{default}$ and $V^{refinance}$ are the value functions associated with keeping, defaulting and refinancing the current mortgage, respectively. The expectation \mathbb{E}_p on the decision to refinance depends on the partial observability of δ by the government and, therefore, the probability of mistakes in subsidy assignation p . A household that is eligible for the refinance subsidy and chooses to refinance might not end up receiving it, whereas a household that is not eligible might

end up receiving it. This will become clear below. The value function associated with keeping the current mortgage is described by the following problem:

$$V^{keep}(t, e, a, h, m, \delta; \mathbf{\Omega}, \Theta) = \max_{\substack{c, s, h', a' \geq 0 \\ l \in [0, 1]}} u(c, s) + \pi_t \beta \mathbb{E}_{e' | e, \delta', m'} V(t+1, e', a', h, m', \delta'; \mathbf{\Omega}', \Theta'), \quad \text{s.t.} \quad (3)$$

$$c + m + qs + P_a a' + \frac{\delta h w}{A} = (1 - \tau_l - \tau_{ss}) e \bar{e}_t w l + a + qh$$

$$m' = \begin{cases} m & \text{w.p. } \rho \\ 0 & \text{w.p. } 1 - \rho \end{cases}$$

$$\delta' \sim F_\delta(\delta'), \quad e' \sim F_e(e' | e), \quad \mathbf{\Omega}' = G(\mathbf{\Omega}), \quad V^{keep}(T+1, \cdot) = 0$$

where G is the law of motion of the aggregate state variables. If the household decides to keep the current mortgage, the payments are equal to m and the mortgage amount will remain outstanding in $t+1$ with probability ρ . With probability $1 - \rho$ the mortgage debt is reset to zero. The last term on the left-hand side of the budget constraint is the cost of the house depreciation $\delta P_h h$, which must be covered by the home-owner at the end of every period. Note that when $\delta < 0$, I am assuming that the household receives an additional income. Given the assumption that households live for at most T periods, I normalize the value at age $T+1$ to be equal to zero so the final period problem is static. The value function for households that default is described by:

$$V^{def}(t, e, a, h, m, \delta; \mathbf{\Omega}, \Theta) = \max_{\substack{c, s, h', a', m' \geq 0 \\ l \in [0, 1]}} u(c, s) + \pi_t \beta \mathbb{E}_{e' | e, \delta', m', \tilde{a}'} V(t+1, e', \tilde{a}', 0, 0, \delta'; \mathbf{\Omega}', \Theta') \quad \text{s.t.} \quad (4)$$

$$c + qs + P_a a' = (1 - \tau_l - \tau_{ss}) e \bar{e}_t w l + a$$

$$\delta' \sim F_\delta(\delta'), \quad e' \sim F_e(e' | e), \quad \mathbf{\Omega}' = G(\mathbf{\Omega}), \quad V^{def}(T+1, \cdot) = 0$$

Households that choose to default do not have to make mortgage payments and lose the house pledged as collateral before paying for the idiosyncratic depreciation, so larger values of δ will make more attractive the option to default. Finally, a household that chooses to refinance faces

uncertainty on whether the government will make mistakes in mortgage assignation, so long as $p > 0$. The expected value for a household that chooses to refinance is given by:

$$\mathbb{E}_p V^{ref}(t, e, a, h, m, \delta; \mathbf{\Omega}, \Theta) = (1 - p) V^{ref, 1-p}(t, e, a, h, m, \delta; \mathbf{\Omega}, \Theta) + p V^{ref, p}(t, e, a, h, m, \delta; \mathbf{\Omega}, \Theta)$$

The value function $V^{ref, 1-p}$ for households that decide to refinance their mortgage and receive the correct mortgage subsidy is:

$$V^{ref, 1-p}(t, e, a, h, m, \delta; \mathbf{\Omega}, \Theta) = \max_{\substack{c, s, h', a', m' \geq 0 \\ l \in [0, 1]}} u(c, s) + \pi_t \beta \mathbb{E}_{e' | e, \delta', \tilde{m}'} V(t + 1, e', a', h', \tilde{m}', \delta'; \mathbf{\Omega}', \Theta'), \quad (5)$$

$$c + \left(\sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left(\frac{\rho}{1+r} \right)^{j-t} \right) (m + (1 - \Gamma(\cdot) \tau(\cdot)) F m') + q s + P_h h' + P_a a' = \\ (1 - \tau_l - \tau_{ss}) e \bar{e}_t w l + a + P_h h - \frac{\delta P_h h w}{A} + q h' + P_m(t, e, a', h', m'; \mathbf{\Omega}, \Theta) m'$$

$$\tilde{m}' = \begin{cases} m' & \text{w.p. } \rho \\ 0 & \text{w.p. } 1 - \rho \end{cases} \quad F = \begin{cases} F^{issue} & \text{if } m = 0, m' > 0 \\ F^{ref} & \text{if } m > 0, m' > 0 \\ 0 & \text{if } m' = 0 \end{cases}$$

$$\delta' \sim F_\delta(\delta'), \quad e' \sim F_e(e' | e), \quad \mathbf{\Omega}' = G(\mathbf{\Omega}), \quad V^{ref}(T + 1, \cdot) = 0$$

$$\left(\sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left[\frac{\rho}{1+r} \right]^{j-t} \right) m' / P_h h' < LTV$$

Households that decide to refinance have to repay the net present value of the outstanding mortgage debt, given by the second term on the budget constraint. Moreover, the household has to pay an after-subsidy fixed cost $\left(\sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left(\frac{\rho}{1+r} \right)^{j-t} \right) (1 - \Gamma(\cdot) \tau(\cdot)) F m'$ on the new mortgage m' : if the household is eligible, $\Gamma(t, e, a, h, m, \delta) = 1$, there is a subsidy equal to $\tau(t, e, a, h, m, \delta)$ on the cost of mortgage refinancing. Otherwise, the household has to pay the full amount. Moreover, the fraction F depends on whether the household is issuing a new mortgage (F^{issue}) or refinancing a current mortgage (F^{ref}). Households that choose to prepay the outstanding debt ($m' = 0$) have to pay no fixed cost (so $F = 0$). The last term on the budget constraint, $P_m(t, e, a', h', m'; \mathbf{\Omega}, \Theta) m'$, is the amount received for the new mortgage m' . The last constraint is a Loan-To-Value constraint.

The value function $V^{ref,p}$ for households that decide to refinance their mortgage and do not receive the correct mortgage subsidy is:

$$V^{ref,1-p}(t, e, a, h, m, \delta; \mathbf{\Omega}, \Theta) = \max_{\substack{c, s, h', a', m' \geq 0 \\ l \in [0,1]}} u(c, s) + \pi_t \beta \mathbb{E}_{e'|e, \delta', \tilde{m}'} V(t+1, e', a', h', \tilde{m}', \delta'; \mathbf{\Omega}', \Theta'), \quad (6)$$

$$c + \left(\sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left(\frac{\rho}{1+r} \right)^{j-t} \right) (m + (1 - (1 - \Gamma(\cdot))\tau(\cdot))Fm') + qs + P_h h' + P_a a' = \\ (1 - \tau_l - \tau_{ss})e\bar{e}_t w l + a + P_h h - \frac{\delta P_h h w}{A} + qh' + P_m(t, e, a', h', m'; \mathbf{\Omega}, \Theta)m'$$

$$\tilde{m}' = \begin{cases} m' & \text{w.p. } \rho \\ 0 & \text{w.p. } 1 - \rho \end{cases} \quad F = \begin{cases} F^{issue} & \text{if } m = 0, m' > 0 \\ F^{ref} & \text{if } m > 0, m' > 0 \\ 0 & \text{if } m' = 0 \end{cases}$$

$$\delta' \sim F_\delta(\delta'), \quad e' \sim F_e(e'|e), \quad \mathbf{\Omega}' = G(\mathbf{\Omega}), \quad V^{ref}(T+1, \cdot) = 0$$

$$\left(\sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left[\frac{\rho}{1+r} \right]^{j-t} \right) m' / P_h h' < LTV$$

Note that the only difference with equation (5) is that, in the case of a mistake, households that receive the subsidy are those for which $\Gamma(t, e, a, h, m, \delta) = 0$. In case of a mistake, eligible households do not receive any subsidy, and ineligible households do receive it. In an economy with perfect information $p = 0$, the household faces no uncertainty on mortgage refinancing subsidy assignation.

The following proposition proves some results on the value function for households in this economy:

Proposition 1. Assume $u(c, s)$ is continuous, strictly increasing and strictly concave. Then V is:

1. Continuous
2. Increasing on a and h
3. decreasing on m and δ

If the productivity persistence parameter $\rho_e > 0$ then V is also increasing on e .

Proof. The proof can be found in the appendix. □

Because of the discrete choice between keeping, refinancing and defaulting on the current mortgage, the value function V need not be concave nor differentiable.

4.2 Mortgage Originators

I assume that in this economy there is no secondary mortgage market, so mortgages are held and serviced by originators. In this way, I assume that mortgage originators are also the owners of each mortgage, which will let me abstract from any possible mismatch of incentives for mortgage refinancing between mortgage investors and servicers. I will refer to them as mortgage investors, mortgage holders or mortgage originators interchangeably throughout the rest of the paper.

There is a continuum of size 1 of mortgage originators, so mortgages are supplied in a competitive market. Mortgage origination firms are owned by all households, so every period each household receives a lump-sum transfer equal to the profits of the firms. Competition is present loan-by-loan, so the origination of each loan is subject to a zero expected profit condition. The pricing function satisfies the following equality:

$$\begin{aligned}
 P_m(t, e, a', h', m'; \Omega, \Theta) m' = & \left(\frac{\pi_t \rho}{1 + r_f} \right) \mathbb{E}_{e', \delta'} \left[\underbrace{d(\mathbf{s}'; \Omega, \Theta) [(1 - \Psi) P_h(1 - \delta') h' - \delta' P_h h']}_{\text{Default}} + \right. \\
 & \underbrace{s(\mathbf{s}'; \Omega, \Theta) \left(\sum_{j=t+1}^T [\Pi_{i=t+1}^j \pi_i] \left(\frac{\rho}{1 + r} \right)^{j-(t+1)} \right) m'}_{\text{Refinance}} + \\
 & \left. \underbrace{(1 - d(\mathbf{s}'; \Omega, \Theta))(1 - s(\mathbf{s}'; \Omega, \Theta))(m' + P_m(t + 1, e', a'', h', m'; \Omega', \Theta') m')}_{\text{Keep mortgage}} \right] \quad (7)
 \end{aligned}$$

where $\mathbf{s}' = (t + 1, e', \delta', a', h', m')$ denotes individual state variables, $d(\cdot)$ is the policy function for default, that takes the value of one if the household chooses to default and zero otherwise, and $s(\cdot)$ is the policy function for mortgage refinancing. The pricing function $P_m(t, e, a', h', m'; \Omega, \Theta)$ is such that the outlays of the bank at t are equal to the expected value of the assets of the bank in $t + 1$ in case the household chooses to: a) default, b) refinance, or c) keep the current mortgage. If the household defaults, the bank receives the after-depreciation value of the house at a price

discount Ψ . I assume that, in order to keep the economy's housing supply constant, in case of default the bank has to pay for the depreciation of the house, represented by the second term on the value of default to the bank. If the household chooses to refinance, the bank receives the total outstanding debt, which is the net present value of future mortgage payments. If the household chooses to keep the current mortgage, the bank receives the payment m' and holds an asset worth $P_m(t+1, e', a'', h', m'; \Omega', \Theta')m'$ at the end of the period. Note that period $t+1$ is discounted taking into account the probability ρ that the mortgage is outstanding at $t+1$ as well as the survival probability of the household π_t .

In steady state, equation (7) holds with equality ex-post by the law of large numbers. The outlays of the mortgage holder on every loan are exactly equal to the realized net present value of payments/assets whenever there are no aggregate unexpected shocks. Note, however, that equation (7) need not hold with equality ex-post whenever there is an aggregate unexpected shock to Ω . An unexpected shock to Ω will potentially change the default and refinance decisions of households, as well as house prices, so the realized net present value of assets to mortgage investors need not equal the outlays of the previous period. Whenever this happens, the government will ensure that the equation holds ex-post.

4.3 Production Firms' Problem

Final good production firms produce according to a simple linear technology $f(L) = AL$, where only labor is used. Firms produce on a perfectly competitive market, so market wages per efficiency unit of labor are directly pinned down by the technology factor A , such that in equilibrium $w = A$.

4.4 Government's Problem

In steady state or, equivalently, whenever there are no aggregate unexpected shocks to the economy (Ω), the government only manages the social security system: levies social security taxes τ_{ss} to working-age households and pays social security proceedings b to retired households.

Whenever an aggregate unexpected shock hits the economy, two instruments become available to the government: subsidies to mortgage refinancing τ and bailouts to mortgage holders B . These instruments are available only during the period of the unexpected shock and disappear thereafter.

The government chooses optimally these instruments to maximize a utilitarian welfare function, subject to keeping the ex-post profits of mortgage originators equal to zero.

In order to finance the expenditures of these policy instruments, the government issues long-term debt worth A after the unexpected shock, in the form of a consol bond that pays a coupon $A \cdot r$ in every period in the future. To pay the coupon every period, the government levies labor income taxes τ_l . At period j , the problem of the government is:

$$\max_{\tau(\cdot), \Gamma(\cdot), B, \tau_l} \int_{\mathbf{s}'} V(\mathbf{s}'; \boldsymbol{\Omega}_j, \tau(\cdot), \Gamma(\cdot), B, \tau_l) d\Phi_j(\mathbf{s}'), \quad \text{s.t.} \quad (8)$$

$$\begin{aligned} \int_{\mathbf{s}} P_m(\mathbf{s}_{-\delta}; \boldsymbol{\Omega}_{j-1}, \Theta_{j-1}) m' d\Phi_{j-1}(\mathbf{s}) = \\ \int_{\mathbf{s}'} \left(\frac{\pi_t \rho}{1+r_f} \right) [d(\mathbf{s}'; \boldsymbol{\Omega}_j, \tau(\cdot), \Gamma(\cdot), B, \tau_l) [(1-\Psi)P_h(1-\delta')h' - \delta'P_h h'] + \\ s(\mathbf{s}'; \boldsymbol{\Omega}_j, \tau(\cdot), \Gamma(\cdot), B, \tau_l) \left(\sum_{i=t+1}^T [\Pi_{i=t+1}^j \pi_i] \left(\frac{\rho}{1+r} \right)^{j-(t+1)} \right) m' + \\ (1-d(\cdot))(1-s(\cdot))(m' + P_m(\mathbf{s}''_{-\delta}; \boldsymbol{\Omega}_j, \tau(\cdot), \Gamma(\cdot), B, \tau_l) m')] d\Phi_j(\mathbf{s}') + B \end{aligned} \quad (\text{Ex-Post Solvency})$$

$$\begin{aligned} (1-p) \int_{\mathbf{s}'} \Gamma(\mathbf{s}') \tau(\mathbf{s}') \left[F \cdot \left(\sum_{i=t}^T [\Pi_{i=t}^j \pi_i] \left[\frac{\rho}{1+r} \right]^{j-t} \right) m' \right] s(\cdot) d\Phi_j(\mathbf{s}') + \\ p \int_{\mathbf{s}'} (1-\Gamma(\mathbf{s}')) \tau(\mathbf{s}') \left[F \cdot \left(\sum_{i=t}^T [\Pi_{i=t}^j \pi_i] \left[\frac{\rho}{1+r} \right]^{j-t} \right) m' \right] s(\cdot) d\Phi_j(\mathbf{s}') + B = A \end{aligned} \quad (\text{Bud. Bal. } j)$$

$$A \cdot r_i = \int_{\mathbf{s}'} \tau_l e \bar{e}_i w l d\Phi_i(\mathbf{s}'), \quad \forall i > j \quad (\text{Bud. Bal. } i > j)$$

$$b \cdot \int_{\mathbf{s}'} \mathbb{1}\{t \geq t^{ret}\} d\Phi_j(\mathbf{s}') = \int_{\mathbf{s}'} \mathbb{1}\{t < t^{ret}\} \tau_{ss} e \bar{e}_i w l d\Phi_j(\mathbf{s}') \quad (\text{Social Security})$$

where j stands for the period of the aggregate unexpected shock, $j-1$ for the previous period, and individual state variables are summarized by $\mathbf{s} = (t, e, a', h', m', \delta)$, $\mathbf{s}' = (t+1, e', a', h', m', \delta')$, $\mathbf{s}''_{-\delta} = (t+1, e', a'', h', m')$ and $\mathbf{s}'_{-\delta} = (t+1, e', a', h', m')$. Note that I explicitly denoted the dependence of the value and policy functions on the instruments of the government $\tau(\cdot), \Gamma(\cdot), B, \tau_l$, rather than with Θ . The ex-post solvency condition is the aggregation of the zero-profit condition

for the mortgage pricing function (equation 7) overall mortgage contracts in the economy at period j . The left-hand side of this equation is the value of the liabilities of mortgage originators, which equal the outlays in period $j - 1$, whereas the right-hand side is the value of the bank's assets in period j plus the bailout transferred to the bank. The budget balance condition at period j states that the total amount of government mortgage refinancing subsidies plus bailouts to mortgage holders must equal the amount of consol bond issued. Note that if $p > 0$, the government might distribute subsidies to households that are not eligible ($1 - \Gamma(\cdot) = 1$). The budget balance condition at $i > j$ states that the government must finance the coupon payments of the consol bonds with labor income taxes. Lastly, the social security restriction describes a balanced budget constraint in the social security system.

Note that the government can achieve the ex-post efficiency constraint through two channels: 1) by choosing B , such that the first restriction holds (Ex-Post Solvency), or 2) by choosing $\tau(\cdot), \Gamma(\cdot)$ and τ_l such that households change their optimal choices and the equality holds.

5 Recursive Competitive Equilibrium

In this section, I state the definition of an equilibrium in this environment. I allow for the possibility of aggregate unexpected shocks from happening, so all functions and equilibrium prices depend on the aggregate states of the economy Ω , as well as on government policies Θ which need not be trivial when an aggregate shocks occur. Index j denotes the period.

Definition 1 (Recursive Competitive Equilibrium). A recursive competitive equilibrium are a value function $V : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \times \{\Omega_j, \Theta\} \rightarrow \mathbb{R}$, policy functions for default and refinancing $d, s : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \times \{\Omega_j, \Theta\} \rightarrow \{0, 1\}$, and all other policy functions $g_c, g_m, g_h, g_r, g_a : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \times \{\Omega_j, \Theta\} \rightarrow \mathbb{R}_+$ for the household, a pricing function $P_m : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \{\Omega_j, \Theta\} \rightarrow \mathbb{R}_+$ for mortgages, government policies $\Theta(\Omega_{j-1}, \Omega_j)$, a price for rental housing $q \in \mathbb{R}_+$, a price for new housing $P_h \in \mathbb{R}_+$, a wage $w \in \mathbb{R}_+$, a risk-free interest rate $r \in \mathbb{R}_+$, an aggregate law of motion $G(\Omega)$ and distributions $\Phi : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \times \{\Omega_j, \Theta\} \rightarrow [0, 1]$ such that:

1. **[Households Optimize]** Given prices $P_m(\cdot), P_h, q, w$ and r , aggregate states Ω , government policy functions Θ and the aggregate law of motion G the value function V solves the household's problem (2), with $d, s, g_{keep}, g_c, g_m, g_h, g_r$ and g_a the respective policy functions.

2. **[Mortgage Originators Optimize]** Given P_h , q , w and r , aggregate states Ω , government policy functions Θ and the aggregate law of motion G , the pricing function $P_m(\cdot)$ for mortgages satisfies an expected zero-profit condition loan by loan, given by equation (7). Any profits are transferred in a lump-sum fashion to households, who own these firms.

3. **[Rental Market Clears]** The rental housing price q is such that demand for rental housing is equal to the total supply of housing in the economy:

$$\int_{\mathbf{s}} g_r(\mathbf{s}) d\Phi(\mathbf{s}) = \bar{H}$$

4. **[Housing Market Clears]** The housing price P_h is such that home-ownership is equal to total housing supply:

$$\int_{\mathbf{s}} g_h(\mathbf{s}) d\Phi(\mathbf{s}) = \bar{H}$$

5. **[Financial Assets Market Clears]** The risk-free rate r is such that savings in the economy equal total mortgage outlays by mortgage investors:

$$\int_{\mathbf{s}} g_a(\mathbf{s}) d\Phi(\mathbf{s}) = \int_{\mathbf{s}} P_m(\mathbf{s}) g_m(\mathbf{s}) d\Phi(\mathbf{s})$$

6. **[Production Firms Optimize]** The wage per efficiency unit of labor w is pinned down by the technology parameter:

$$w = A$$

7. **[Government Optimizes]** If $\Omega' = G(\Omega)$, the government only manages social security and the pension b given to retired households is such that there is budget balance on the social security system:

$$b \cdot \int_{\mathbf{s}} \mathbb{1}\{t \geq t^{ret}\} d\Phi(\mathbf{s}) = \int_{\mathbf{s}} \mathbb{1}\{t < t^{ret}\} \tau_{ss} e \bar{e}_t w g_l(\mathbf{s}) d\Phi(\mathbf{s})$$

If $\Omega' \neq G(\Omega)$ - that is, after an aggregate shock - the government solves problem 8.

8. **[Resource Constraint]** The resource constraint holds:

$$A \cdot \int_{\mathbf{s}} e \bar{e}_t g_l(\mathbf{s}) d\Phi(\mathbf{s}) = \int_{\mathbf{s}} \left[g_c(\mathbf{s}) + \delta P_h h + F \left(\sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left[\frac{\rho}{1+r} \right]^{j-t} \right) g_m(\mathbf{s}) \right] d\Phi(\mathbf{s})$$

9. **[Distribution Consistency]** The distribution $\Phi(t, \cdot)$, for $t \in \{2, \dots, T\}$ is consistent with the policy functions for the households, the Markov transition probabilities for the idiosyncratic shocks, the aggregate law of motion G and the initial distribution $\Phi(1, \cdot)$.
10. **[Aggregate Law of Motion Consistency]** The aggregate law of motion is consistent with the distribution Φ , policy functions for the households and the Markov transition probabilities for the idiosyncratic shocks.

Agents in this economy assign zero probability to any aggregate shock and, therefore, do not have expectations with respect to the policies the government might implement during mortgage crises. For this reason, agents do not take overly risky behaviors in steady state, and the model completely abstracts from moral hazard.

Note that this definition of equilibrium allows for the possibility of aggregate shocks from happening. Definition 2 in the appendix defines the steady state equilibrium in this economy, where no aggregate shocks occur, so Definition 2 is a particular case of Definition 1.

6 Calibration

This section describes the functional forms and parameter values used to calibrate the model to the pre-crisis steady state U.S. economy. Details on the computation of the model are available in Section B in the appendix.

Demographics

I set $T = 30$, such that one period in my model corresponds to 2 years of life. Individuals enter the economy at the age of 20 ($t = 1$) and live up to age 80 ($t = 30$). This timing captures the life-cycle patterns of consumption and housing investments while keeping the computation of the model tractable. Every period, an individual of age t survives to period $t + 1$ according to an exogenous probability π_t , which corresponds to the empirical survival probabilities from the Actuarial Life Tables of the Social Security Administration⁸. I assume that individuals of age $T = 30$ (80 years old) die with probability exactly equal to one. I assume that households retire at age 65, which means that $t^{ret} = 22$ in my model.

⁸ The life table and survival probabilities can be found at [this link](#).

Preferences

I use the following parameterization for the utility function:

$$\sum_{t=1}^T \beta^t \pi_t \left[\frac{(\psi c^\kappa + (1 - \psi) h^\kappa)^{\frac{1-\sigma}{\kappa}}}{1 - \sigma} - \theta_l \frac{l^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}} \right]$$

Where the elasticity of substitution between consumption and housing is equal to $1/(1 - \kappa)$. [Fernández-Villaverde and Krueger \(2011\)](#) make a review of the findings in the literature and argue that this elasticity varies according to the model specification and samples used for estimation. I set the value of κ to -0.1 , which is close to the estimates presented in their review. The Frisch elasticity is given by η . Following the macro estimates of the parameter η on life cycle models, I set $1/\eta = 0.5$ (for a review on the macro and micro estimates, see [Keane and Rogerson \(2015\)](#)). Finally, I set $\sigma = 2$ and $\beta = 0.96$ which are standard in the literature, and the weight parameter of the disutility of labor $\theta_l = 5$ to match an average labor supply equal to 0.34.

Labor Productivity

The productivity of households has two components: 1) an idiosyncratic shock, and 2) a life-cycle component. The idiosyncratic component follows the AR(1) process described by equation (1), where I set $\rho_y = 0.95$ and $\sigma_\epsilon = 0.22$, which correspond to 2-year values close to the estimates of [Storesletten et al. \(2004\)](#)⁹. In order to compute the model, I discretize the idiosyncratic productivity process with a 3-state Markov Chain approximation using the method by [Tauchen \(1986\)](#). I set $m_y = 1$, so the points in the Markov Chain are at ± 1 standard deviations from the mean, resulting in a grid for the stochastic productivity given by $\{0.602, 1.0, 1.661\}$, and a transition probability matrix:

$$\begin{bmatrix} 0.9507 & 0.0492 & 0.0 \\ 0.036 & 0.9272 & 0.036 \\ 0.0 & 0.0492 & 0.9507 \end{bmatrix}$$

The life-cycle component of productivity is set to match the estimates of [Hansen \(1993\)](#), so average productivity is hump-shaped over the life cycle.

⁹ $\rho_y = 0.98^2$ and $\sigma_\epsilon = 0.1 \times \sqrt{2}$.

Financial Markets

The parameter ρ , which corresponds to the rate at which mortgage debt survives over time, is chosen so the average duration of mortgage payments lasts 25 years, which in the current calibration corresponds to 12 periods. That is, I choose $\rho = 0.92$. I set the costs of issuing new debt and refinancing existing debt to $F^{issue} = 0.015$ and $F^{ref} = 0.025$, respectively, which fall inside the range of estimates by [Hurst and Stafford \(2004\)](#). This means that to issue new debt, the household must pay 1.5% of the value of the total outstanding debt, while to refinance it has to pay 2.5%. Lastly, as in [Garriga and Hedlund \(2016\)](#), a Loan-To-Value limit of $LTV = 1.25$ is assumed. In equilibrium, this limit will not be binding.

Table 1 summarizes the values of the parameters chosen for the initial steady state calibration.

	Value	Target/Source
<i>Preferences</i>		
β	0.96	Literature
σ	2	Literature
κ	-0.1	Fernández-Villaverde and Krueger (2011)
η	0.5	Keane and Rogerson (2015)
θ_l	5	Average $L = 0.34$
π_t	-	U.S. Actuarial Life Tables
<i>Demographics</i>		
T	30	One period = 2 years
t^{ret}	22	Retirement at 65 years
<i>Productivity</i>		
λ_ϵ	0.95	Storesletten et al. (2004)
σ_ϵ	0.22	Storesletten et al. (2004)
m	1	± 1 standard deviation
\bar{e}_t	-	Hansen (1993)
<i>Financial Market</i>		
ρ	0.92	25 years average mortgage duration
F^{issue}	0.015	Hurst and Stafford (2004)
F^{ref}	0.025	Hurst and Stafford (2004)
LTV	1.25	Garriga and Hedlund (2016)

Table 1: Parameter values

Endogenously Calibrated Parameters

To make the computation feasible I set the idiosyncratic depreciation shock to take values on a three-point grid distributed uniformly over the interval $\delta \in [\underline{\delta}, \bar{\delta}]$. The values of $\underline{\delta}$ and $\bar{\delta}$ are endogenously calibrated so a) the foreclosure rate matches the two-year average of 3.0% during the pre-crisis period, 1991-2007, and the average homeownership rate matches 66.5% for the same period, corresponding to the estimates by the Federal Reserve Bank of St. Louis¹⁰.

The parameter ψ is the weight of non-durable consumption with respect to overall consumption. According to the NIPA tables and the Bureau of Economic Analysis' Personal Consumption Expenditure data, the expenditures on housing services account for around 14.1% – 15% of overall expenditures (Jeske et al., 2013; Corbae and Quintin, 2015). I set ψ such that the ratio of nominal expenditures on housing services with respect to nondurable consumption is equal to 0.141. Table 2 shows the values of $\underline{\delta}$, $\bar{\delta}$ and ψ .

Given that the information problem arises when the government offers subsidies to mortgage refinancing, the calibration of the parameter p is postponed to Section 7, where I calibrate the model to match the Great Recession and HAMP and TARP are implemented.

6.1 Steady State

Table 2 summarizes the moments of the baseline calibration in the initial steady state and the data.

Parameter	Value	Variable	Data	Model
		<i>Targeted Moments</i>		
$\bar{\delta}$	0.119	Default rate	3.0%	2.4 %
$\underline{\delta}$	-0.345	Ownership rate	66.5%	65.5 %
ψ	0.84	Rent/Cons expenditures	14.1%	14.2 %
		<i>Untargeted Moments</i>		
		Std. Dev. of Idiosyncratic Component	10 – 14.5%	13.4 %
		Average home equity	62%	64.5 %
		Median size (sq ft.): owned/rented	1.51	1.52

Table 2: Model and Data Moments

¹⁰Estimates can be found on [this link](#).

In addition to matching the targeted moments described above, the model matches untargeted moments related to the housing and mortgage markets. First, the model's standard deviation of the idiosyncratic house price shock lies within the range estimated by the literature. As estimated by [Giacoletti \(2016\)](#), using the CoreLogic repeat-sales data from San Francisco, Los Angeles and San Diego, the standard deviation of the idiosyncratic component of house price risk over a period of up to two years is between 10% and 14.5%. The standard deviation that results from the endogenously chosen grid for idiosyncratic house price risk, which corresponds to a uniform distribution in the model, is equal to 13.4 %. Second, the model's average home equity, equal to 64.5 %, is close to the 62% observed in the data. Furthermore, the home equity cumulative distribution function generated by the model, illustrated in Figure 3, closely matches the distribution obtained from the Survey of Consumer Finances 2007, as in [Chatterjee and Eyigungor \(2015\)](#). In the model, the lower tail of the distribution is heavier, but the rest of the distribution is close to the data. A potential explanation why the model does not closely match the left tail of the distribution is that, in the model, I assume that households start their life with no assets or housing. In the data, there are households at age 20 that hold financial assets and housing mainly because they receive inter-vivos transfers and bequests, or because they start working before age 20. Figure 13 in the appendix illustrates the life-cycle averages of home equity in the model and the data and shows this mismatch at the beginning of the life-cycle. Finally, the model closely matches the ratio of the median size of owner-occupied versus rental housing, obtained by [Chatterjee and Eyigungor \(2015\)](#) from the American Community Survey of 2007.

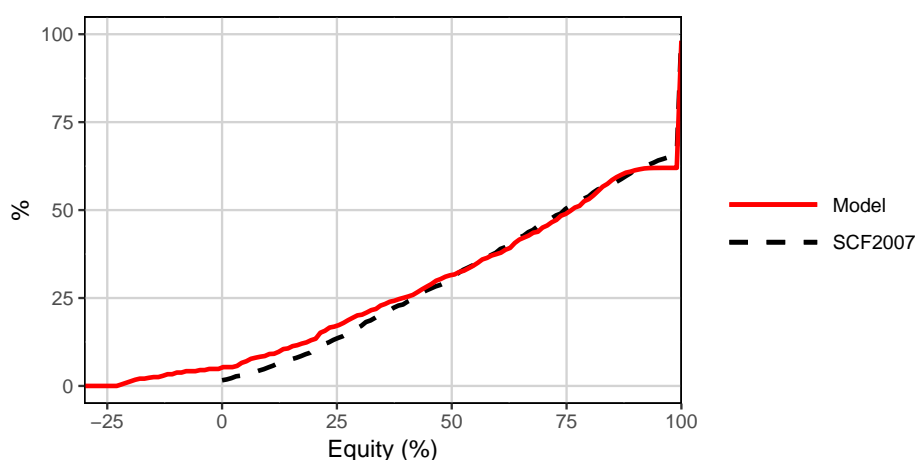
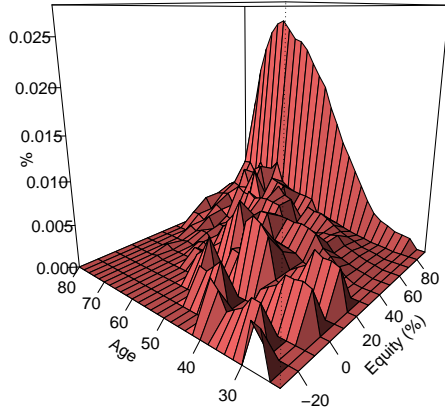
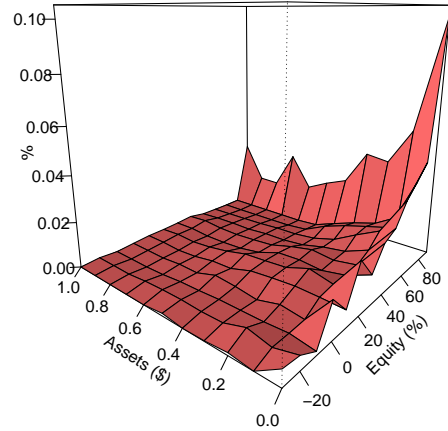


Figure 3: Home Equity Cumulative Distribution - Model vs. Data (SCF 2007)

Figure 4a shows the steady state distribution of home-owners, according to their age and home equity. There is a positive correlation between age and equity, with a correlation coefficient equal to 0.56 in the model. As households grow over their life cycle and make their mortgage payments, they increase the percentage owned of their house and, therefore, their home equity. There is a mass of 36.8% of home-owners that have no mortgage debt, so their home equity is equal to 100%. These households have made all of their mortgage payments. Figure 4b illustrates the distribution of home-owners according to their savings and home equity. The correlation coefficient between both variables is only slightly positive (equal to 0.06), as both financial assets and housing are substitutes of each other. Some households accumulate savings until they have enough assets to repay their mortgage debt, which increases their equity but reduces their financial assets.



(a) Distribution of home-owners on the age-equity state space



(b) Distribution of home-owners on the savings-equity state space

Figure 5 illustrates the behavior of net renters in my model, which are households that rent more than what they own. The percentage of net renters follows a decreasing path at the beginning of the life cycle, close to what happens with renters on the Survey of Consumer Finances of 2007. However, given that households know they are going to die with probability one on their last period of life, they sell all of their owned housing at the end of the cycle. This is opposite to what is observed in the data, as in my model there are no bequests and the probability of death is equal to one at age 80. This is not true in the data, because households have bequest motives, and individuals do not die with certainty at age 80. Figure 13 in the appendix illustrates other life-cycle averages of the model.

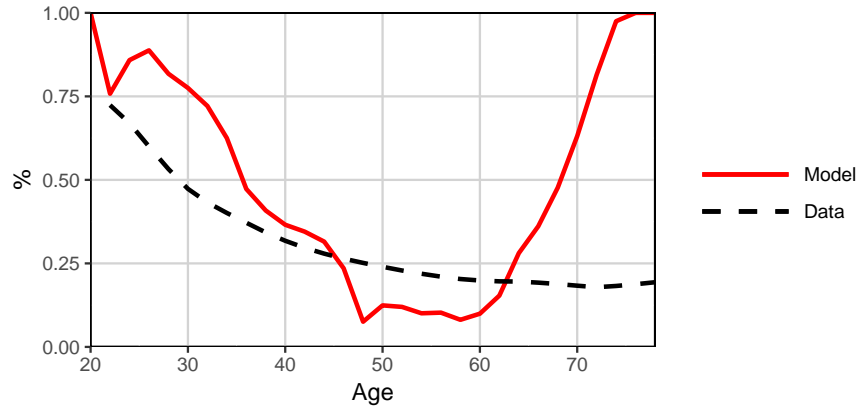


Figure 5: Home Equity Cumulative Distribution - Model vs. Data (SCF 2007)

Default in Steady State

In the model, there is default in steady state. Households that choose to default lose their collateral, are left out of the mortgage market for the period of default, equal to two years, and have to pay the fixed cost if they want to issue a new mortgage in the future. Table 3 characterizes households that choose to default, according to their level of savings a and the income and house value shocks during the period of default.¹¹ Of the individuals that default, 100% do so after drawing a negative shock on their house value, corresponding to the largest depreciation $\delta = \bar{\delta}$. Within defaulters, there are individuals with and without savings. Of those households without savings, 79.61 % choose to default after a negative income shock. These households find it impossible to continue repaying their mortgage, as this would imply negative levels of non-durable consumption during the period of the shock. In contrast, of the defaulters with positive savings, only 21.17 % choose to default after a negative income shock. These households decide to default on their mortgage, even though their income and assets would allow them to continue making mortgage payments.

	Without savings: $a = 0$	With savings: $a > 0$
House value shock	100 %	100 %
Income shock	79.61 %	21.17 %
Total default	0.78 %	1.7 %

Table 3: % of defaulters with income and price shocks on the period of default.

¹¹For this exercise, an income shock occurs when the household draws the lowest possible productivity shock. A price shock occurs when the household draws the largest possible depreciation shock.

Panel (a) in Figure 6 illustrates the average default rate by age, as a percentage of home-owners that hold a mortgage in each age group. Default is largest among young households and decreases over the life-cycle. This happens because households have low levels of savings at the beginning of their life cycle. After a negative income shock these households cannot continue making mortgage payments, either because in the present their income is too low, or because, given the persistence of the shock, their future income will not be sufficient to repay the loan. In contrast, older households have savings that serve as a buffer in case of a negative income shock.

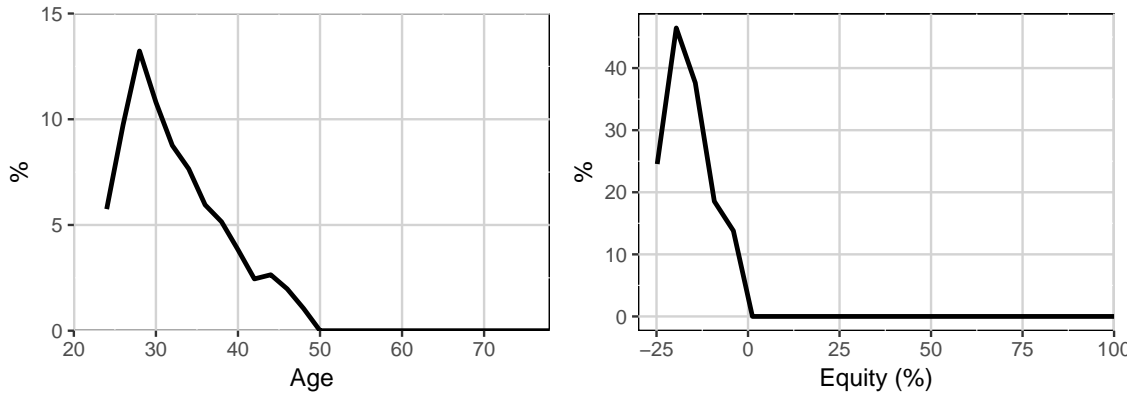


Figure 6: Model average default rate (%) (a) by age, and (b) home equity (%)

Panel (b) in Figure 6 illustrates the default rate by home equity. Default occurs only for households with negative equity, or underwater, and reaches 46% for households with sufficiently negative equity. Only 31.5% of households underwater choose to default. As the literature has pointed out, being underwater is a necessary, but not sufficient, condition for default. It is necessary because if the household is not underwater it can always choose to sell the house, repay the mortgage and obtain positive profit. This is not true when the household is underwater and the outstanding debt is larger than the value of the house. Being underwater is not a sufficient condition to default for the following reasons. First, there are fixed costs of mortgage origination, so a household that defaults today will have to pay a fixed cost to originate a new mortgage in the future. This might prevent it from defaulting on the mortgage. Second, in my model mortgages are long-term contracts, whose terms are fixed at the moment of origination. A household that is underwater might prefer to keep the current contract whenever the terms of the mortgage, that were set at the origination period, are better than the expected terms of a new mortgage in the future. For example, before the crisis, a wealthy and productive household would originate

a mortgage with prime interest rates. If in the future this household receives a low-probability, negative productivity shock that reduces its future credit-worthiness, such as a long unemployment spell, it might prefer to stick to its original contract, as any future mortgage will be underwritten at higher interest rates.

Mortgage Refinancing in Steady State

Figure 7 illustrates the percentage of homeowners that decide to refinance their mortgage on the age-equity state space. In contrast to default, mortgage refinancing occurs at a later stage in the life cycle and is made by households that have positive home equity. For these households with positive equity, it is never optimal to default, as they can always sell their house, repay their total outstanding debt and keep the profits. However, they might prefer to refinance to get a mortgage with better terms.

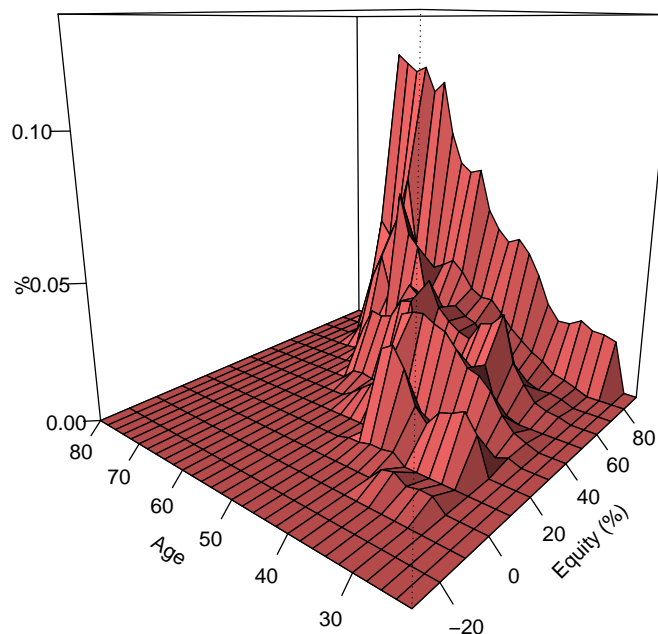


Figure 7: Mortgage refinancing as a % of home-owners by age and equity

Mortgages are refinanced for two reasons. They can be refinanced to decrease the size of the mortgage, such that future mortgage payments are lower, or can be refinanced to borrow funds, which increases the size of the mortgage and the future mortgage payments. Figure 8 illustrates

the average mortgage refinancing rate for households that choose to refinance upwards, or increase their debt, and for those that choose to refinance downwards, or decrease their debt.

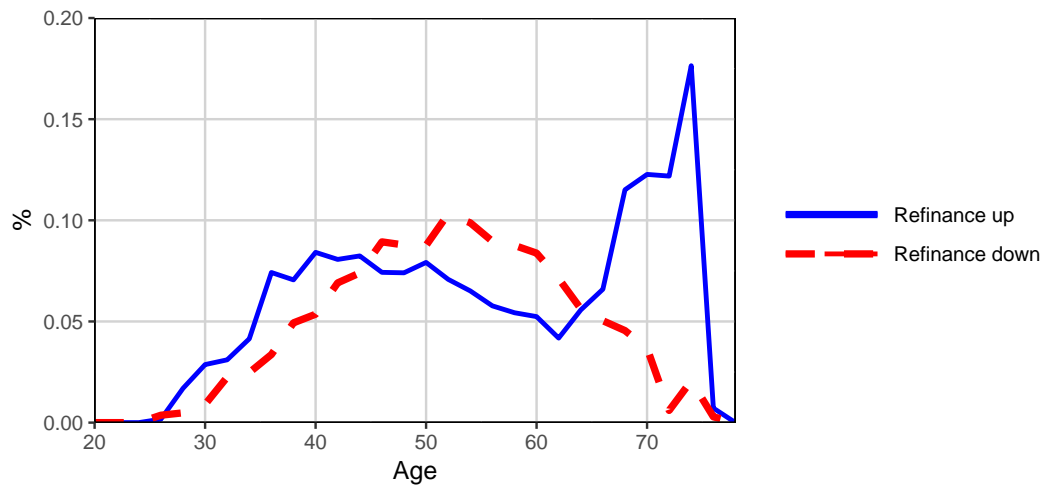


Figure 8: Mortgage refinancing as a (%) of home-owners by age. Solid line: refinancing to increase mortgage size. Dashed line: refinancing to decrease mortgage size.

Refinancing a mortgage upwards implies increasing present consumption at the expense of increasing mortgage payments and, thus, lower future consumption. In contrast, refinancing a mortgage downwards implies increasing future consumption at the expense of present consumption. Given that productivity follows a hump-shaped path over the life cycle and households smooth consumption, the mortgage refinancing rate to increase debt is larger early in the life cycle, whereas the refinancing rate to decrease debt is larger later in the cycle.

In particular, households that have the highest income at the beginning of their life cycles are the ones that choose to refinance their mortgages upwards. Given that the idiosyncratic component of productivity is persistent and the age component is hump-shaped, these households expect a high income path in the future, so choose to refinance upwards early in their lives to smooth consumption. Similarly, households that have high productivity shocks in the peak of the productivity cycle, between ages 45-55, choose to refinance downwards, to substitute present consumption with future consumption.

7 The Great Recession

In order to generate a shock that induces a large drop in house prices and an increase in the foreclosure rate, I assume that while the economy is in steady state agents face two completely unexpected shocks that last for 3 periods: 1) a drop in the age component of productivity, so the drop in earnings by age group is close to the findings of [Glover et al. \(2014\)](#) during the Great Recession, summarized in Table 4, and 2) an increase in the minimum downpayment requirement to 25% of the house price, such that the maximum Loan-To-Value at origination falls from its initial steady state value $LTV = 1.25$ to 0.75. The second shock is justified by the Federal Reserve's *Willingness to Lend Survey*, according to which the number of banks that declared tightening of credit standards rose from almost zero to above 50% in 2007, and the median downpayment requirement more than doubled, from a pre-crisis level of 5% to 13% ([Boz and Mendoza, 2014](#)).

Age group	Per capita earnings
20 – 29	–12.8%
30 – 39	–11.1%
40 – 49	–8.8%
50 – 59	–9.6%
60 – 69	–4.4%
70+	+0.3%

Table 4: Productivity Shock

For the baseline calibration, I assume that the government implements a subsidy-bailout policy analogous to TARP and HAMP, described below.

7.1 HAMP

Through the Home Affordable Modification Program (HAMP), the government offered subsidies to mortgage refinancing for households in financial distress. Households with mortgages that satisfied the following conditions were eligible: 1. payments-to-income had to be greater than or equal to 31% of monthly gross income, 2. the household must have been delinquent or in danger of falling behind on mortgage payments, 3. the property pledged as collateral had to be owner-occupied and the primary residence of the household, 4. the property had to be a

single-family property with 1-4 units and an unmodified first-lien mortgage balance of up to \$729,750, 5. the mortgage had to be originated before January 1st, 2009, and 6. the mortgage modification had to pass a net present value test that would make the mortgage holder better off after the modification (Agarwal et al., 2017). The program aimed at reducing mortgage payments to exactly 31% of monthly income. When designed, the government allocated \$75 billion to HAMP.

In the baseline calibration of the model, households that satisfy conditions analogous to 1. and 2. are eligible for a mortgage refinancing subsidy. That is, access to HAMP is granted to households that satisfy two conditions. First, they must have payments-to-income above some threshold described below. Second, given that I don't model strategic complementarities that lead some households to "strategically default", while others don't, I assume that once the subsidy program is announced, all households claim to be "in danger of falling behind on mortgage payments". The government then observes the households' state variables \tilde{s} and grants subsidies to those for whom default is optimal according to \tilde{s} . Given that the government does not perfectly observe the individual state variables, it will subsidize households for which default is optimal according to the observed \tilde{s} , but is not according to the realized s , which are the *strategic defaulters* in my model. In equilibrium, the probability of observing an incorrect state, p , will be calibrated such that strategic default accounts for 10% of total default, as described in Section 7.2.

7.2 The Information Parameter p

Mayer et al. (2014) estimate the magnitude of strategic default when mortgage refinancing programs are implemented. The authors use an exogenous settlement between the U.S. Federal Government and Countrywide Financial Corporation in 2008 - before HAMP -, in which Countrywide agreed to "offer modifications to seriously delinquent borrowers" with subprime, first-lien mortgages. Immediately after the announcement, delinquency rates of eligible households increased 10% yearly, compared to non-eligible mortgages and mortgages of the same class from other financial institutions. Furthermore, the increase was larger among households with large available credit on credit cards and lower current LTV, who were least likely to default otherwise.

Even though 10% is a point estimate of strategic default for the specific case of the settlement between Countrywide and the Federal Government, given the lack of other estimates for strategic default during the Great Recession, I assume that in the case of HAMP strategic default accounted for exactly 10% of total default. This assumption might overstate total strategic default, as HAMP

only allowed refinancing that reduced payments-to-income exactly to 31%, while the Countrywide settlement allowed for unconstrained refinancing. In this sense, households might have had more incentives to strategically default during the Countrywide settlement.

To obtain a 10% strategic default, I set the parameter $p = 0.016$ such that, in equilibrium, in 1.6% of the cases the government observes states \tilde{s} that are associated with default for households that would not choose to default according to their realized s . These households receive the benefits of HAMP, even though they would not have defaulted in the absence of the program.

7.3 TARP

The Troubled Assets Relief Program was implemented to preserve the solvency of systemically important financial institutions. Although, initially \$700 billion were allocated to the purchase of troubled assets, by late 2008 TARP had only invested \$243 billion through the Capital Purchase Program. Under Section 202 of the EESA, the Office of Management Budget (OMB) and the Congressional Budget Office (CBO) were required to perform a semiannual *risk-adjusted* net present value of TARP. The first of such reports, published by the CBO in January, 2009, estimated a risk-adjusted net present value of the Capital Purchase Program (CPP) equal to $-\$61$ billion (Calomiris and Khan, 2015). In the baseline calibration, I assume that TARP is a one-time transfer from the government to the mortgage investors to cover for the losses after the aggregate shock.

7.4 Results and Counter-factual Experiments

I model HAMP and TARP as one-time programs that are implemented in the same period of the aggregate shock and last only for one period, which corresponds to two years. The reason for a one-period duration is that, once the aggregate shock takes place, there is no uncertainty in the economy, as agents perfectly foresee the transition path of the economy towards the steady state. Therefore, any policy to preserve the solvency of mortgage originators can be implemented in the first period of the shock. The duration I am assuming is not far from what happened with TARP, where most of the investments undertaken by late 2008 were reversed in less than a year, as the financial institutions recovered. In contrast, HAMP lasted through December 2016, and was modified over the years.

In the baseline calibration, the government implements TARP and HAMP so as to preserve ex-post solvency of mortgage originators, where 45% of the resources correspond to direct transfers to the mortgage investors (TARP), and 55% as subsidies to mortgage refinancing (HAMP). These numbers match the proportion of expenditures during the Great Recession, where the government expected to spend \$60 billion on implicit bailouts through TARP, and allocated \$75 billion to HAMP. For these proportions to hold in the model, the payments-to-income eligibility threshold for HAMP is set at 27.5%, which is close to the 31% threshold observed in the data. The subsidy is contingent on mortgage refinancing, which means that households cannot obtain the subsidy and default in the same period. The size of the subsidy is such that, in the period of the transfer, payments to income would be exactly 27.5% if the household continued under the same debt contract. In my model, however, households receive this transfer and have to refinance their mortgage without constraint.

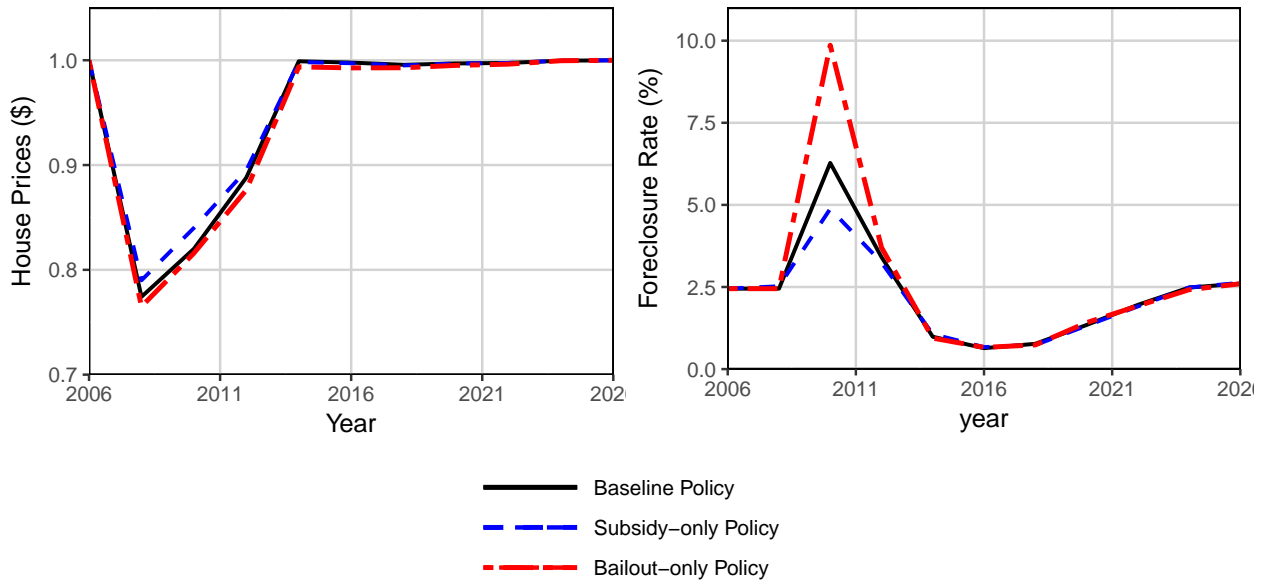


Figure 9: Transitional dynamics of (a) house prices and (b) foreclosure rate.

The solid lines in Figure 9 illustrate the behavior of housing prices and foreclosures in the baseline calibration, when the government implements HAMP and TARP. The aggregate shock generates a decrease in house prices of 21%, while foreclosure rates in the model almost tripled from 2.4 % to 6.8%. This behavior is close to what happened during the Great Recession, illustrated in Figure 1. As an external check, the model does well at replicating the percentage of households

underwater during the recession, which is a non-targeted moment, equal to 16.2% in the model and 15% in the data (Melzer, 2017).

It is worth explaining the behavior of the foreclosure rate during the transition path. Initially, there is a sharp increase in the foreclosure rate, which rapidly reverts and falls below the pre-crisis level, finally returning back to the initial steady state after 20 years. This happens because during the peak of the crisis a large fraction of households is left with negative home equity and lower incomes due to the productivity shock. Some of them choose to default and others choose to refinance. Those who receive HAMP benefits are able to modify their mortgages to better terms and move out of the lower part of the equity distribution, having a lower risk of default in the future. For this reason, default is below the pre-crisis level for some periods after the crisis. As the economy recovers and the distribution of households returns back to steady state, the default rate rises to the initial levels. Figure 10 illustrates the behavior of the equity distribution over the transition. In the first period of the shock, the distribution shifts to the left. Given that default and refinancing rates increase during the crisis, after some periods the equity distribution shifts to the right and slowly converges to the pre-crisis distribution.

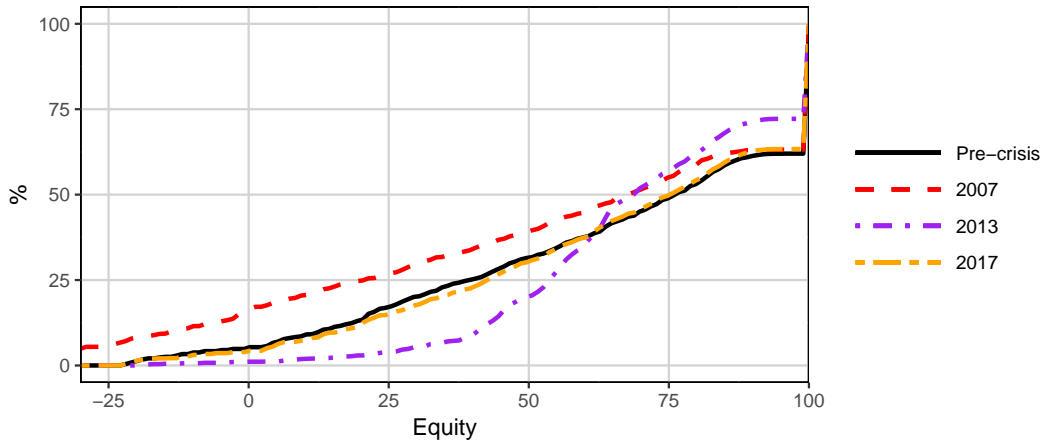


Figure 10: Equity distribution during transitional dynamics.

Figure 9 illustrates two counter-factual experiments. In the first one, denoted “Subsidy-only Policy” in the figure, the government completely eliminates TARP and lowers the payments-to-income eligibility threshold of HAMP to 22.5%, so as to reduce foreclosures up to the level where mortgage originators have no losses. Note that, since the aggregate shock lasts for three periods and households default and refinance while the shock lasts, foreclosures are below the pre-crisis

level for some years after the shock disappears. Given this, in order to preserve the solvency in the first period, default need not be reduced to the pre-crisis level. Using a utilitarian welfare function, welfare increases 0.2% in consumption equivalent terms, as compared to the baseline calibration.

There are two reasons for this. First, the welfare cost of the dead-weight loss associated with foreclosures is larger than the welfare cost caused by the distortion generated by the taxation levied to subsidize strategic defaulters. This happens because the Frisch elasticity of labor supply is not sufficiently large, so increasing taxes to subsidize an additional 10% of mortgage holders - strategic defaulters - does not generate a large welfare cost in the economy. Moreover, redistributing resources to strategic defaulters is not bad *per se*, so the welfare gain of strategic defaulters outweighs part of the distortion generated by taxation. In contrast, a 20% dead-weight loss of foreclosed houses generates unambiguously negative welfare consequences that outweigh the cost of strategic default. In this sense, it is welfare preferable to implement a subsidy policy that prevents foreclosures from happening, than to allow households to default and offer bailouts to mortgage holders to cover the ex-post losses.

The second reason is more subtle. As discussed at the end of Section 6.1, being underwater is a necessary, but not a sufficient condition for mortgage default, and only households that are *sufficiently* underwater choose the default. For this reason, after a negative house price shock, the government has to spend fewer resources to prevent foreclosures through mortgage refinancing subsidies, than to cover for the losses of that default, given that the default could be prevented with a transfer that is smaller than the negative balance. For this reason, the government can preserve the solvency of the banks by spending fewer resources on mortgage refinancing subsidies, than by covering its ex-post losses.

In the second experiment, denoted “Bailout-only Policy” on the figure, the government completely eliminates HAMP and only makes a bailout to mortgage originators to preserve ex-post solvency. In this case, default increases, as illustrated on the right panel of Figure 9, and welfare decreases 0.8% in consumption equivalent terms.

Given that subsidies redistribute resources within the economy, there are some groups that benefit more from a subsidy-only policy. Figure 11 illustrates the average welfare gains of implementing the subsidy-only policy instead of the baseline HAMP-TARP, according to home equity. Home-owners with -40% to -20% of equity have the largest gains, as they are the most indebted ones and receive the largest subsidies. This group would be willing to reduce its

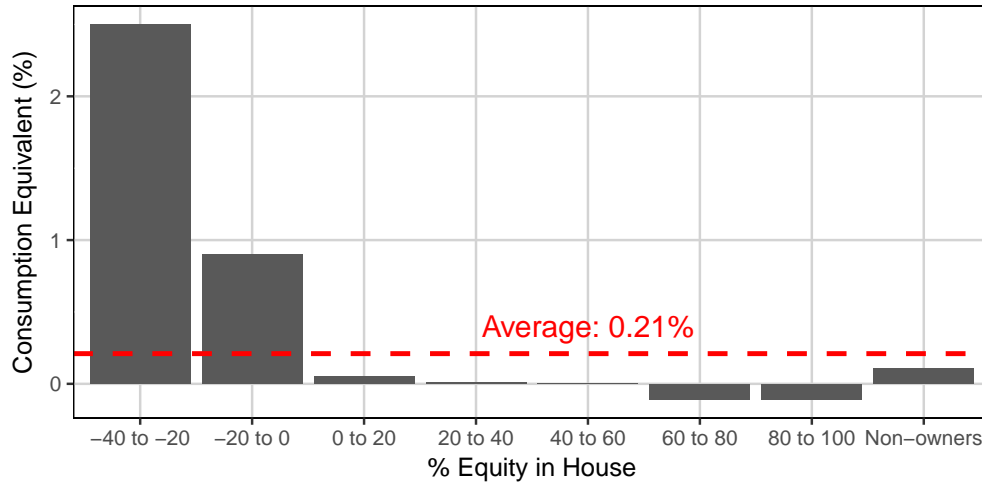


Figure 11: Welfare gains in consumption equivalent terms (%) under a subsidy-only policy, by home equity (%).

lifetime consumption up to 2.5% in the HAMP-TARP baseline, to be in the subsidy-only scenario. Households with equity between -20% and 0% also receive a large portion of the subsidies and would be willing to reduce their consumption up to 0.9% in order to be in the subsidy-only policy. The welfare gains of households with equity between 0% and 60% are small, as only a small fraction of them receive subsidies. Households that have equity above 60% on their home experience negative welfare gains, as under the subsidy-only case house prices do not fall as much, when compared to the baseline scenario (see Figure 9). Given that these households are the largest house investors, higher prices during the crisis reduce the future returns on their investments. Moreover, they do not receive subsidies so, on average, the subsidy-only program reduces their welfare. Finally, the group of households that do not own a house experience a welfare gain of 0.11% in consumption terms, because on the subsidy-only scenario the government levies fewer taxes.

Figure 12 illustrates default by age, before and during the Great Recession. Given that young households suffered the largest drop in earnings and were the most indebted ones, the default rate increased more among them, when compared to the pre-crisis level. Although the HAMP policy targeted households with high Payments-to-Income, which is a variable that is correlated with age, a better eligibility rule should have an age component that targets the youngest households. In that way, the government could achieve a larger reduction in foreclosures, at a lower cost.

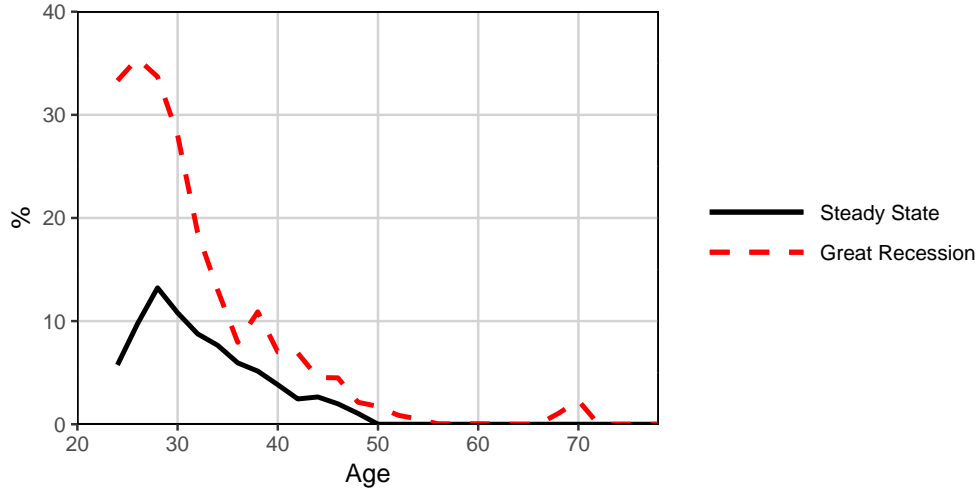


Figure 12: Default rate by age, before and after the Great Recession.

A final counter-factual experiment does precisely that, by eliminating TARP and implementing a subsidy-only policy, where the eligibility rule is modified. Under the new policy, only households that were not defaulting in the initial steady state and choose to default after the shock are eligible. That is, the subsidies are targeted mostly at the youngest homeowners. Moreover, the size of the subsidy is the same as is HAMP but has an additional component which decreases with age. Under this scenario, using a utilitarian welfare function yields welfare gains of 0.4% in consumption equivalent terms. This means that a better-designed subsidy policy based on observable characteristics can yield welfare improvements during mortgage crises.

8 Concluding Remarks

Housing crises are events in which a decrease in aggregate house prices leads to higher-than-expected foreclosure rates, generating potentially large losses to mortgage investors. To preserve the solvency of financial institutions, which are the largest investors in the mortgage market, governments have historically implemented two policies: a) offer bailouts to institutions to cover for their losses, and b) subsidize the mortgage refinancing of households to prevent additional foreclosures. During the Great Recession, foreclosures tripled after a house price drop of over 20%. Through the Emergency Economic Stabilization Act of 2008, the government implemented the Troubled Assets Relief Program (TARP), through which it implicitly bailed out troubled financial

institutions through asset purchases, and the Home Affordable Modification Program (HAMP), which offered subsidies for the mortgage refinancing of households in risk of default.

This paper studies the welfare-maximizing policy to preserve the solvency of financial institutions, assuming that the planner can offer bailouts and mortgage refinancing subsidies. The use of these instruments involves a trade-off, determined by two frictions. On one hand, foreclosed houses lose 20% of their value during the foreclosure process, as houses get damaged during the process and depreciation is larger because of vacancy. Therefore, if the government offers bailouts to investors rather than subsidies to households to prevent foreclosures, the economy has to bear the welfare cost of the dead-weight loss.

On the other hand, the literature has identified an idiosyncratic component in house prices, which is not perfectly observable by the government. Given that mortgage default depends on house prices, the government does not have perfect information on individual households' decision to default. When offered subsidies, households have incentives to engage in strategic default to qualify for the benefits of the subsidy program. A subsidy policy avoids the dead-weight loss of foreclosed houses, since it prevents mortgage default, but subsidizes households that would not default in the absence of the program. If taxation is distortionary, the taxes levied to subsidize strategic default generate a welfare loss in the economy, through a distortion in the labor decision of households.

This paper quantitatively assesses the welfare maximizing-policy during mortgage crises. Specifically, a heterogeneous agents' model is calibrated to the pre-crisis U.S. economy, where the trade-off between bailouts and subsidies is disciplined using empirical micro estimates on the dead-weight loss of foreclosed houses and the size of strategic default. In equilibrium, the welfare cost generated by the dead-weight loss is larger than the distortion generated by the taxation levied to subsidize strategic defaulters, given the Frisch elasticity of labor estimated in the macro literature. For this reason, a subsidy-only policy would have generated welfare gains of up to 0.4%, measured as the consumption equivalent variation, when compared to the baseline calibration that matches the TARP and HAMP programs. Given that a subsidy policy implies a redistribution of resources within the economy, the welfare gains are heterogeneous. Households on the left tail of the equity distribution, which benefit the most from a subsidy program, obtain the largest welfare gains. In contrast, a bailout-only policy would have generated a welfare loss of 0.8%.

The set-up I use has three main shortcomings. First, I assume that the Great Recession was generated by exogenous shocks to fundamentals. In my model, any bailout offered to financial institutions has the sole effect of transferring funds to cover the losses, without any effect on house prices. Different results would be obtained under the assumption that the Great Recession was generated by changes in expectations. In that alternative setting, a bailout policy could be a mechanism to align expectations toward an equilibrium in which housing prices do not fall as much. This means that, to the extent that the Great Recession was expectations-driven, my paper understates the potential welfare gains of using bailouts over subsidies.

Second, I explicitly ignore the moral hazard concerns of bailing out banks or subsidizing households. Clearly, both policies can generate incentives for banks and households to engage in overly-risky behaviors, affecting the likelihood of future crises and, therefore, having welfare consequences. To avoid moral hazard issues, I assume that the crisis was a zero-probability event, so neither households nor banks had expectations about the policies the government would implement during a crisis. The moral hazard consequences of government policies during mortgage crises are a topic left for future research.

Finally, a recent strand of the literature evaluates the impact of government bailouts during crises, through the financial accelerator channel. Bailouts may reduce the risk of banks running out of equity, which reduces the financing costs of the economy and increases economic activity. For computational reasons, this paper does not model the amplification effects of government policies through their effect on the balance sheets of banks. I abstract from this effect by assuming that aggregate shocks are zero-probability events in the economy, so in the steady state banks, finance their activities at the risk-free rate. In this sense, my paper under-estimates the benefits of a bailout policy.

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Appendices

A Steady State Equilibrium

This definition is a particular case of Definition 1. In the initial and final steady states, there is perfect foresight of house prices and foreclosures, so banks need no bailouts and the government does not have access to the emergency policies described above. This means that Ω and Θ are constant, so I will omit the dependency of the problem on the aggregate variables. The following is the definition of a steady state equilibrium of this economy.

Definition 2 (Stationary Recursive Competitive Equilibrium). A stationary recursive competitive equilibrium are a value function $V : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \rightarrow \mathbb{R}$, policy functions for default and refinancing $d, s : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \rightarrow \{0, 1\}$, and all other policy functions $g_c, g_m, g_h, g_r, g_a : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \rightarrow \mathbb{R}_+$ for the household, a pricing function $P_m : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \rightarrow \mathbb{R}_+$ for mortgages, a price for rental housing $q \in \mathbb{R}_+$, a price for new housing $P_h \in \mathbb{R}_+$, a wage $w \in \mathbb{R}_+$, a risk-free interest rate $r \in \mathbb{R}_+$ and distributions $\Phi : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \Delta \rightarrow [0, 1]$ such that:

1. **[Households Optimize]** Given prices $P_m(\cdot)$, P_h , q , w and r the value function V solves the household's problem (2), with d , s , g_{keep} , g_c , g_m , g_h , g_r and g_a the respective policy functions.
2. **[Mortgage Originators Optimize]** Given P_h , q , w and r , the pricing function $P_m(\cdot)$ for mortgages satisfies an expected zero-profit condition loan by loan, given by equation (7).
3. **[Rental Market Clears]** The rental housing price q is such that demand for rental housing is equal to the total supply of housing in the economy:

$$\int_{\mathbf{s}} g_r(\mathbf{s}) d\Phi(\mathbf{s}) = \bar{H}$$

4. **[Housing Market Clears]** The housing price P_h is such that home-ownership is equal to total housing supply:

$$\int_{\mathbf{s}} g_h(\mathbf{s}) d\Phi(\mathbf{s}) = \bar{H}$$

5. **[Financial Assets Market Clears]** The risk-free rate r is such that savings in the economy equal total mortgage outlays by mortgage investors:

$$\int_{\mathbf{s}} g_a(\mathbf{s}) d\Phi(\mathbf{s}) = \int_{\mathbf{s}} P_m(\mathbf{s}) g_m(\mathbf{s}) d\Phi(\mathbf{s})$$

6. **[Production Firms Optimize]** The wage per efficiency unit of labor w is pinned down by the technology parameter:

$$w = A$$

7. **[Social Security Budget Balance]** The pension b given to retired households is such that there is budget balance on the social security system:

$$b \cdot \int_{\mathbf{s}} \mathbb{1}\{t \geq t^{ret}\} d\Phi(\mathbf{s}) = \int_{\mathbf{s}} \mathbb{1}\{t < t^{ret}\} \tau_{ss} e \bar{e}_t w g_l(\mathbf{s}) d\Phi(\mathbf{s})$$

8. **[Resource Constraint]** The resource constraint holds:

$$A \cdot \int_{\mathbf{s}} e \bar{e}_t g_l(\mathbf{s}) d\Phi(\mathbf{s}) = \int_{\mathbf{s}} \left[g_c(\mathbf{s}) + \delta P_h h + F \left(\sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left[\frac{\rho}{1+r} \right]^{j-t} \right) g_m(\mathbf{s}) \right] d\Phi(\mathbf{s})$$

9. **[Distribution Consistency]** The distribution $\Phi(t, \cdot)$, for $t \in \{2, \dots, T\}$ is consistent with the policy functions for the households, the Markov transition probabilities for the idiosyncratic shocks and the initial distribution $\Phi(1, \cdot)$.

This is the definition in steady state, where no aggregate shocks occur, so banks perfectly forecast the foreclosure rate in the economy and price the mortgages accordingly. In equilibrium, by the law of large numbers, profits are exactly equal to zero, loan by loan, and there is no government intervention.

B Computation of the Model

As described in the Definition 2, in equilibrium the household's problem must be solved and the zero-profit condition for mortgages must hold. Note that the policy and value functions for the household's problem depend on the mortgage pricing function, and vice versa. This means that both the household's functions and mortgage pricing function must be solved as a fixed point: *a*) given $P_m(t, e, a', h', m'; \Omega)$, the household's problem must be solved and, *b*) given the household's policy functions, the zero-profit condition for mortgages must hold.

To solve the fixed point, the computation of this model is composed by two sub-routines: *a*) given a pricing function $P_m(t, e, a', h', m'; \Omega)$, solve the household's problem described in the recursive formulation in equation (2), by starting at the last period of life and iterating backwards; and *b*) given the policy functions that solve the household's problem, solve the fixed point of the pricing function $P_m(t, e, a', h', m'; \Omega)$ given by equation (7), by doing pricing function iteration.

Given the large number of state variables, I perform the computation using massive parallel GPU computing, as described in Aldrich et al. (2010). Given a set of parameters, the following algorithm is used to compute the equilibrium of the economy:

B.1 Stationary Recursive Competitive Equilibrium

- o. Given parameters $\beta, \sigma, \kappa, \psi, \eta, \theta_l, \pi_t, T, t^{ret}, \lambda_\epsilon, \sigma_\epsilon, m, \bar{e}_t, \rho, \dots$, start with an initial guess for prices P_h^0, q^0, r^0
1. Set $n = 0$ and an initial guess for:

Mortgage pricing function:	$P_m^0(t, e, a', h', m'; \Omega)$
Policy functions:	$D^0(t, e, a, h, m, \delta; \Omega), C^0(t, e, a, h, m, \delta; \Omega), \dots$
Value function:	$V^0(t, e, a, h, m, \delta; \Omega)$

2. Iteration n :

Subroutine 1 - Pricing Function Computation:

Given $D^n(t, e, a, h, m, \delta; \Omega), C^n(t, e, a, h, m, \delta; \Omega), \dots$

- (a) Set $j = 0$ and $P_m^{n,j}(t, e, a', h', m'; \Omega) = P_m^n(t, e, a', h', m'; \Omega)$.
- (b) Given j and the policy functions $D^n(t, e, a, h, m, \delta; \Omega), C^n(t, e, a, h, m, \delta; \Omega), \dots$, compute $P_m^{n,j+1}(t, e, a', h', m'; \Omega)$, according to equation (7).
- (c) If $\|P_m^{n,j+1}(t, e, a', h', m'; \Omega) - P_m^{n,j}(t, e, a', h', m'; \Omega)\| < \epsilon$, stop and set $P_m^{n+1}(t, e, a', h', m'; \Omega) = P_m^{n,j+1}(t, e, a', h', m'; \Omega)$. Otherwise, set $j = j + 1$ and repeat (b).

Subroutine 2 - Value and Policy Functions Computation:

Given $P_m^{n+1}(t, e, a', h', m'; \Omega)$:

- (a) For $j = T$ compute $V^{n+1}(j, e, a, h, m, \delta; \Omega), D^{n+1}(j, e, a, h, m, \delta; \Omega), C^{n+1}(j, e, a, h, m, \delta; \Omega), \dots$, by solving the static problem in equation (2)
 - (b) For $j = T - 1, \dots, 1$, given $V^{n+1}(j + 1, e, a, h, m, \delta; \Omega)$ compute $V^{n+1}(j, e, a, h, m, \delta; \Omega), \dots$, by solving the equation (2)
3. If $\|P_m^{n+1}(\cdot) - P_m^{n,j}(\cdot)\| + \|V^{n+1}(\cdot) - V^n(\cdot)\| < \epsilon$, continue. Otherwise, set $n = n + 1$ and repeat 1.
 4. If $P_m^{n+1}(t, e, a', h', m'; \Omega)$ and $V^{n+1}(j, e, a, h, m, \delta; \Omega), D^{n+1}(j, e, a, h, m, \delta; \Omega), C^{n+1}(j, e, a, h, m, \delta; \Omega), \dots$ are such that markets clear (Definition 2), stop. Otherwise, set a new guess for P_h^0, q^0 and r^0 and go to 0.

B.2 Recursive Competitive Equilibrium

To compute the transitional dynamics of the model, assume that after the unexpected shock the economy takes S periods before getting to the new stationary steady state, after which it stays with certain probability.

- o. Given parameters $\beta, \sigma, \kappa, \psi, \eta, \theta_l, \pi_t, T, t^{ret}, \lambda_\epsilon, \sigma_\epsilon, m, \bar{e}_t, \rho, \dots$, start with an initial guess for prices along the transition $\{P_s, q_s, r_s\}_{s=1}^S$
1. Compute the pre-shock steady state and the corresponding distribution of households $\Phi_0(t, e, a', h', m'; \Omega)$
2. Iterate Backwards:

- (a) $s = S$: Given P_s, q_s, r_s , compute $P_m^s(t, e, a', h', m'; \Omega)$ and $V^s(t, e, a', h', m'; \Omega), \dots$, using the algorithm for the Stationary Recursive Competitive Equilibrium (Section B.1)
 - (b) Compute $P_m^{s-1}(t, e, a', h', m'; \Omega)$ given $P_m^s(t, e, a', h', m'; \Omega)$ and $V^s(t, e, a', h', m'; \Omega), \dots$
 - (c) Compute $V^{s-1}(t, e, a', h', m'; \Omega), \dots$, given $P_m^{s-1}(t, e, a', h', m'; \Omega)$ and $V^s(t, e, a', h', m'; \Omega), \dots$
 - (d) If $s > 1$, set $s = s - 1$ and go to (b). Otherwise, continue.
3. Iterate Forward:
- (a) $s = 1$: Set $\Phi_1(t, e, a', h', m'; \Omega) = \Phi_0(t, e, a', h', m'; \Omega)$
 - (b) Compute $\Phi_{s+1}(t, e, a', h', m'; \Omega)$, given $\Phi_s(t, e, a', h', m'; \Omega)$ and the policy functions $A^s(t, e, a', h', m'; \Omega), \dots$
 - (c) Compute aggregate quantities at $s + 1$, given prices $P_{s+1}, q_{s+1}, r_{s+1}$, distribution of individuals $\Phi_{s+1}(t, e, a', h', m'; \Omega)$ and policy functions $A^{s+1}(t, e, a', h', m'; \Omega), \dots$, as described in Definition 1.
 - (d) If $s < S - 1$, set $s = s + 1$ and go to (b). Otherwise, continue.
4. If markets clear at $s = 1, \dots, S$ (Definition 1), stop. Otherwise, set a new guess for $\{P_s, q_s, r_s\}_{s=1}^S$ and go to 1.

The tolerance level is given by $\epsilon > 0$, and the number of periods the economy takes to arrive to the new steady state is assumed to be $S = 10$, that correspond to 20 years in the data.

C Proves of Propositions

Proof. The proof is done by induction on t . Define the following budget correspondence, based on the household's problem:

$$\Gamma^{keep}(t, e, a, h, m, \delta; \Omega) = \{ (l, c, a', s, h') \in [0, 1] \times \mathbb{R}_+^2 \times \mathcal{H}^2 \mid \\ c + (1 - \tau_m)m + qs + P_a a' + P_h \delta h \leq (1 - \tau_l)e\bar{e}_l w l + a + qh \}$$

In period $t = T + 1$, $V^{keep}(T + 1, \cdot) = 0$, so the value function $V^{keep}(T, \cdot)$ only depends on a through the budget correspondence $\Gamma^{keep}(t, e, a, h, m, \delta; \Omega)$. Let $a_1 \leq a_2$. Given that $\Gamma^{keep}(T, e, a_1, h, m, \delta; \Omega) \subseteq \Gamma^{keep}(T, e, a_2, h, m, \delta; \Omega)$, it is straightforward that $V^{keep}(T, e, a_1, h, m, \delta; \Omega) \leq V^{keep}(T, e, a_2, h, m, \delta; \Omega)$, as the maximization is done over a larger set. Analogously, $V^{def}(T, \cdot)$ and $V^{ref}(T, \cdot)$ are increasing on a . Given that the maximum of three functions that are increasing on a is itself increasing on a , then $V(T, \cdot)$ is increasing on a . By an analogous argument, $V(T, \cdot)$ is increasing on h and decreasing on m and δ . Continuity of $V(T, \cdot)$ follows from Berge's Maximum Theorem, given that $u(c, s)$ is continuous, the correspondence $\Gamma^{keep}(t, e, a, h, m, \delta; \Omega)$ is non-empty, compact, upper- and lower-semicontinuous.

Now, assume $V(t + 1, \cdot)$ is increasing on a and h and decreasing on m and δ . Note that $V(t, \cdot)$ only depends on a, h, m and δ through the budget set. By the same argument as above, $V(t, \cdot)$ is increasing on a and h and decreasing on m and δ .

To prove that $V(t, \cdot)$ is increasing on e , assume $V(t + 1, \cdot)$ is increasing on e' . Note that for positive values of ρ_e , and $e_1 \leq e_2$ the conditional distribution $F_e(e'|e_2)$ first-order stochastically dominates $F_e(e'|e_1)$, given that $F_e(e'|e_2) \leq F_e(e'|e_1), \forall e'$. Given that $V(t + 1, \cdot)$ is increasing on e , this implies that $\mathbb{E}_{e'|e_2, \delta', m'} V(t + 1, e', a', h', m', \delta'; \Omega') \geq \mathbb{E}_{e'|e_1, \delta', m'} V(t + 1, e', a', h', m', \delta'; \Omega')$ for all e', a', h', m', δ' and Ω' .

□

D Other Figures

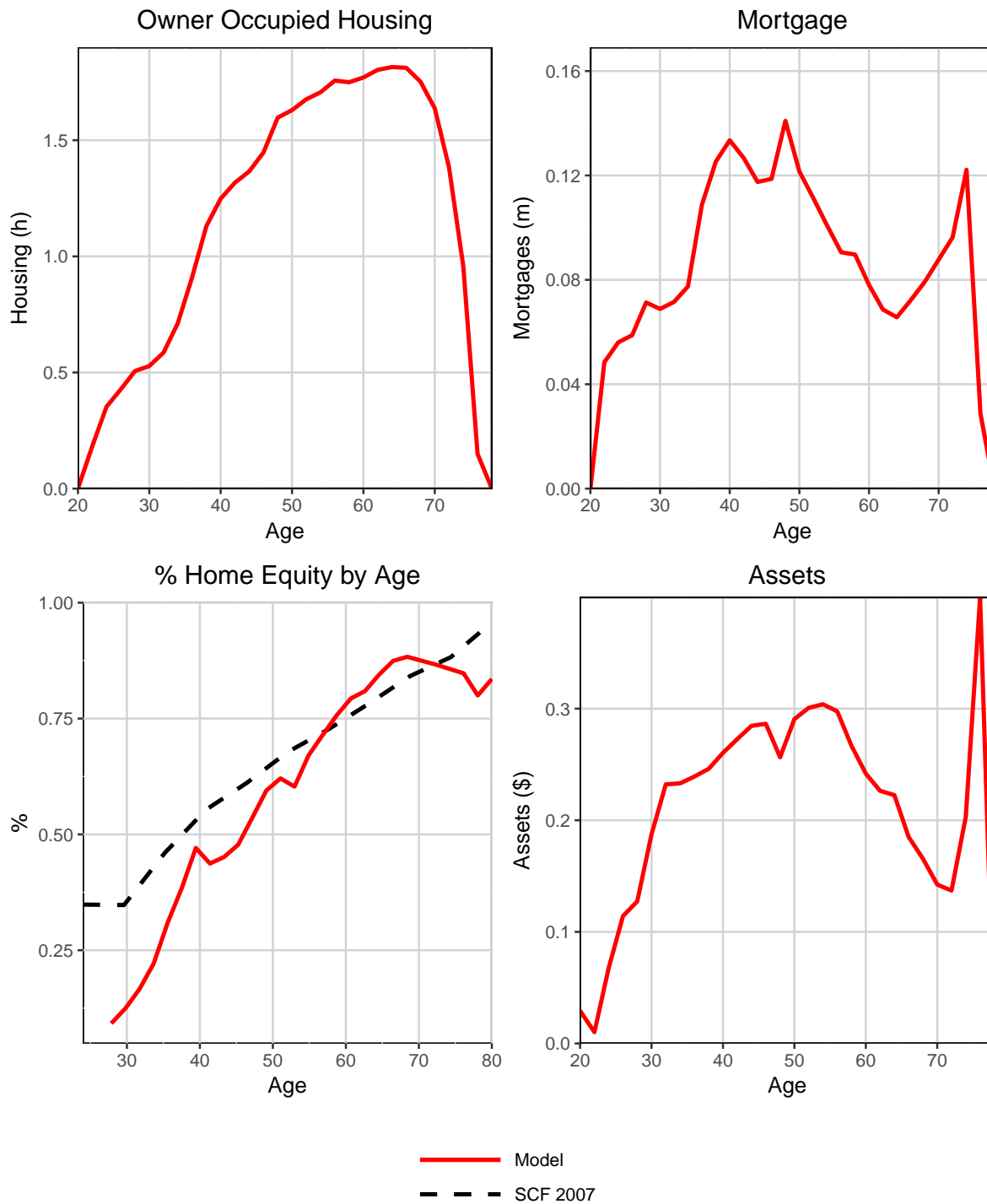


Figure 13: Life cycle averages of (a) Owner-occupied housing, (b) Mortgage size, (c) Home equity, and (d) Savings