Minimum Wages in the UK

Searching for Non-linearities

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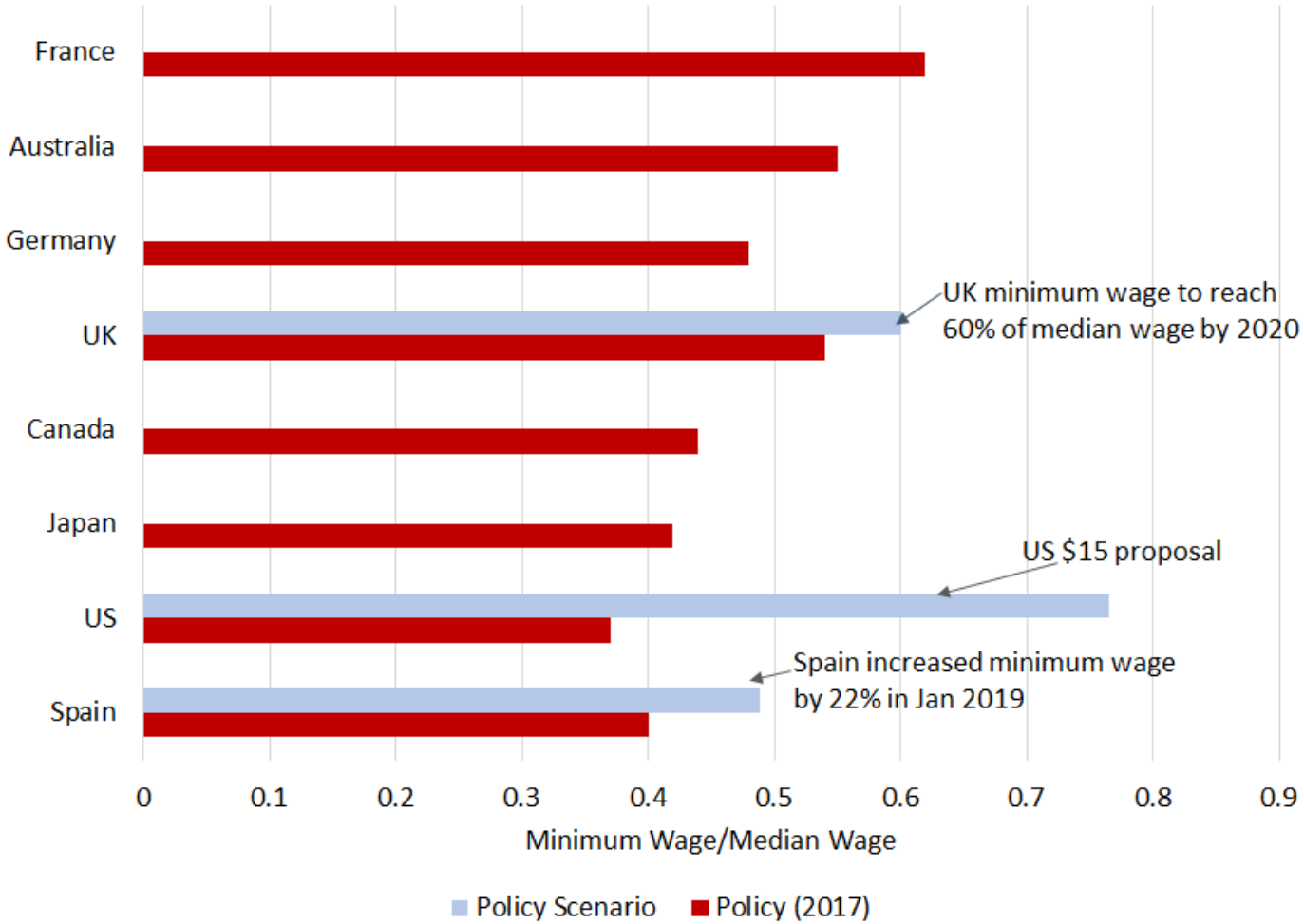
April 2018

Motivation

- Minimum wages are an increasingly popular policy response to low wage growth for low paid workers.

Figure: Minimum wages on the rise



Motivation

- Political logic behind minimum wage (MW) increases in UK seems to be: "introduction of MW doesn't seem to have hit unemployment, so let's put it up some more"
- This is risky: is it a linear relationship or are there **nonlinearities** to be wary of?
- To answer this, **we need a model** to forecast impacts.
- The model presented here can, eventually, help to address this and a wide range of questions:
  1. **Are there significant nonlinearities in minimum wage impacts?**
  2. How does minimum wage compare to other redistributive policies?
  3. What are likely long term impacts on e.g. productivity, capital use, income and wealth inequality?

Preview of Results

- We develop a model that combines search frictions with a production function featuring several margins of substitution between factor inputs.
- Nonlinear unemployment reaction in model from:
  1. **Exogenous nonlinearities**:
     - Non-uniform distribution of skills.
  2. **Endogenous nonlinearities**:
     - Vacancy creation with Cobb-Douglas matching function
     - Imperfect substitution between capital and labour and between labour types
- When calibrated to the UK economy, we find:
  1. quantitatively, imperfect substitution between inputs is most important endogenous source of nonlinearities
  2. nonlinearity in unemployment lies within range of minimum wages planned in UK over next two years.

Outline

Methodology: Key Ingredients

1. **Frictional Labour Markets**. Search frictions can help explain findings of small impacts of UK minimum wage on employment and firm exit.
2. **Capital.** How does the minimum wage affect firms' choice of capital vs. labour?
3. **Heterogeneous Agents.** Will minimum wage hikes cause companies to substitute towards higher skill workers?

Methodology: Ingredients Missing...

1. **Hours Worked**. Labour is entirely discrete, but model could be extended to include hours worked, as chosen by firms and workers.
2. **Participation Margin**. We do not consider positive impact of minimum wage on labour market participation. Again useful extension.
3. **Firm Heterogeneity**. All firms use same technology in this model. But could there be a useful role for minimum wage in eliminating low productivity firms?
4. **Business Cycles**. Should minimum wage increase in recession to provide stimulus to high MPC workers, or decrease to support labour demand?

Related Literature

1. **Structural literature on optimum minimum wage**
   - Search with wage posting - van den Berg and Ridder (1998): no unemployment effects until minimum wage equals productivity level then match is destroyed
   - Search with wage bargaining - Flinn (2006): if vacancy creation is present then smooth unemployment response until minimum wage equals productivity level then match is destroyed
   - **Contribution**: Introduction of decreasing returns to labour in search frameworks, removes cliff-edge effects.
2. **Empirical literature on UK minimum wage** Small employment effects, decrease in firm profits and limited price effects e.g. Leonard et al (2014), Draca and Machin (2011).
   - **Contribution**: Developing a model consistent with these findings, but also capable of examining future risks.

The Model: Environment

Workers.

- Workers differ in observable skill level, which is given (not chosen).
- Two broad skill types - unskilled and skilled (u and s).
- Within broad skill types workers, workers differ by unobservable productivitiy level.
- Unobservable productivity, indexed by i, of a skilled (unskilled) worker is denoted $x_{s,i}$ ($x_{u,i}$), for $i = 1..M$
- Productivity is distributed exogenously according to the pdf $l_s(x_{s,i})$ ($l_u(x_{u,i})$)
- Both workers and firms have a common discount factor, β and are risk neutral

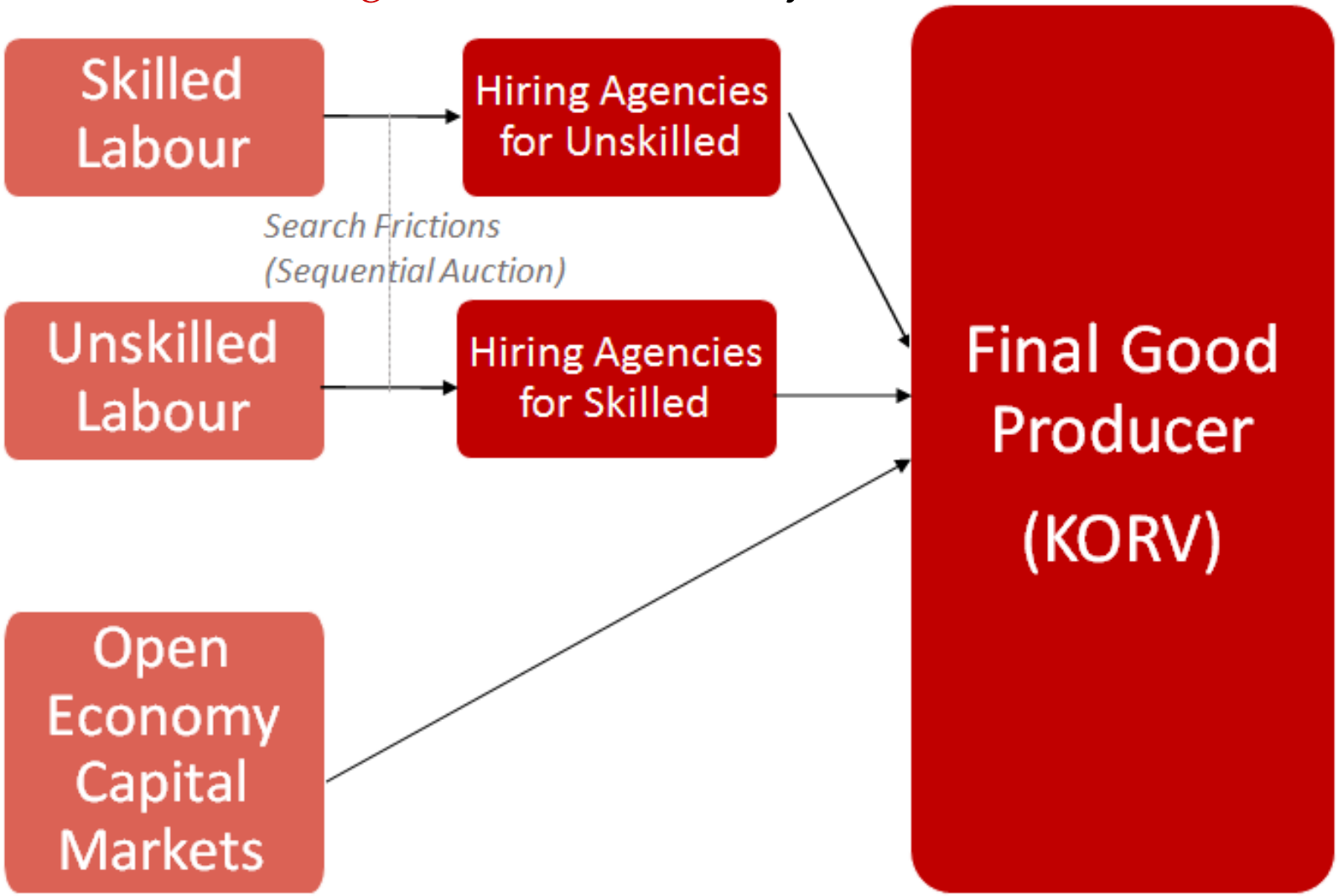
The Model: Environment

Firms

- We wish to allow for both capital to labour substitution in production and substitution between skill types.
- Not easy in pure search/match framework.
- Proposed solution is to have two sectors of production:
  1. **Intermediate sector with search frictions**. Intermediate firms hire labour and sell it onto a final good producer - think of hiring agencies.
     - One segmented intermediate sector for each skill and productivity level of workers.
  2. **Final good sector** that combines labour hired in intermediate sector and capital, with no frictions. Capital-skill complementarity as per Krusell et al (2000) - "KORV" production function.

The Model: Environment

Figure: Model Economy Overview



The Model: Environment

Final Good Firms

- Competitive firms which produce using technology shown below.Inputs used:
  - K_{eq} is amount of capital equipment, K_{st} is amount of capital structures
  - U is effective amount of goods purchased from the low skill intermediate sectors, S is total effective labour from high skill intermediate sectors

$$Y = AK_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} \tag{1}$$

$$U = \left(\sum_{i=1}^{M}(x_{i,u}h_{i,u})^{\frac{\Psi_u - 1}{\Psi_u}}\right)^{\frac{\Psi_u}{\Psi_u - 1}}, S = \left(\sum_{i=1}^{M}(x_{i,s}h_{i,s})^{\frac{\Psi_s - 1}{\Psi_s}}\right)^{\frac{\Psi_s}{\Psi_s - 1}} \tag{2}$$

The Model: Environment

Intermediate Firms

Notation: j will be a vector valued index containing both the broad skill index (u, s) and productivity index $(1..M)$ of a worker.

- One intermediate sector for each worker type j.
- One intermediate firm for every worker (so density of intermediate firms = density of workers)
- Number of matches given by matching function $M(S_j, V_j)$. S_j = number of effective type m job searchers. V_j = vacancies.
- $\theta_j \equiv V_j / S_j$ denotes labour market tightness
- Contact rate for type j firms is $q(\theta_j) \equiv M(S_j, V_j)/V_j$, and $(\theta_j q(\theta_j), \chi\theta_j q(\theta_j))$ are the contact rates for unemployed and employed workers respectively.
- Vacancies determined by free entry : i.e. firms issue a vacancy until expected profit equals vacancy cost.

The Model: Environment

Intermediate Firms: Wage Setting

- Assume that firms and unemployed workers engage in Nash bargaining - the minimum wage acts as a constraint to the Nash maximisation.
- When workers gets poached, incumbent and rival bid-up the wage until the value of employing a poached worker equals the value of carrying a vacancy i.e. zero (Postel-Vinay and Robin (2002))
- Therefore poached workers will get paid their marginal product in final good production.
- **Minimum Wage reduces expected profit from employing not-poached worker, and decreases vacancy creation**

The Model: Behaviour

Workers

A worker of a given type j exist in one of three states:

- unemployed, receiving flow income b,with lifetime value function denoted V_j^{ue}
- employed but not poached, receiving the higher of Nash bargained wage w_j^b and the minimum wage m_w, with value function V_j^{np};
- employed and poached, receiving wage w_j^p, with value function V_j^p

$$V_j^{ue} = b + \beta[\theta_j q(\theta_j) V_j^{np} + (1 - \theta_j q(\theta_j)) V_j^u] \tag{3}$$

$$V_j^{np} = \max(w_j^b, m_w) + \beta\left[\delta_j V_j^{ue} + (1-\delta_j)[\chi\theta_j q(\theta_j) V_j^p + (1 - \chi\theta_j q(\theta_j)) V_j^{np}]\right] \tag{4}$$

$$V_j^p = w_j^p + \beta[\delta_j V_j^{ue} + (1-\delta_j) V_j^p] \tag{5}$$

The Model: Behaviour

Final Good Producers

- The firm's profit maximisation problem is:

$$\max_{K_{st},K_{eq},h_{i,u},h_{i,s}\forall i\in 1..M} \Pi = AK_{st}^{\alpha}\left[\mu U^{\sigma} + (1-\mu)\left(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}}$$

$$-\sum_{i=1}^{M} p_{i,u}h_{i,u} - \sum_{i=1}^{M} p_{i,s}h_{i,s} - r_{st}K_{st} - r_{eq}K_{eq} \tag{6}$$

$$U = \left(\sum_{i=1}^{M}(x_{i,u}h_{i,u})^{\frac{\Psi_u - 1}{\Psi_u}}\right)^{\frac{\Psi_u}{\Psi_u - 1}}, S = \left(\sum_{i=1}^{M}(x_{i,s}h_{i,s})^{\frac{\Psi_s - 1}{\Psi_s}}\right)^{\frac{\Psi_s}{\Psi_s - 1}}$$

- Since final good producer is assumed to be competitive, all inputs are chosen to equalise marginal product is with the price of input.

The Model: Behaviour

Intermediate Firms

- Exist in one of three states:
  - carrying a vacancy, with firm value denoted by J_j^v ,
  - employing a not-poached worker, J_j^{np}, and
  - employing a poached worker, with value J_j^p.

$$J_j^v = -\kappa_j + \beta[q(\theta_j)\{s^n p_j J_j^p + (1 - s_j^u)J_j^p\} + (1 - q(\theta_j))J_j^v] \tag{7}$$

$$J_j^{np} = p_j - \max(w_j^b, m_w) + \beta\Big[(1 - \delta_j)\{\chi\theta_j q(\theta_j)J_j^v + (1 - \chi\theta_j q(\theta_j))J_j^{np}\} + \delta_j J_j^v\Big] \tag{8}$$

$$J_j^p = p_j - w_j^p + \beta[(1 - \delta_j)J_j^p + \delta_j J_j^v] \tag{9}$$

The Model: Behaviour

Intermediate Firms

- Free entry, so $J_j^v = 0$, and Betrand competition between employers implies $J_j^p = 0$ so $w_j^p = p_j$.
- From these we get no entry condition:

$$\kappa_j = \beta q(\theta_j) s_j^u \frac{p_j - \max(w_j^b, m_w))}{1 - \beta(1-\delta_j)(1 - \chi\theta_j q(\theta_j))} \tag{10}$$

- The bargained wage is given below (Φ is the nash bargaining parameter):

$$\begin{aligned} w_j^b &= \underset{w_j^b}{\operatorname{argmax}} (V_j^{np} - V_j^u)^{\Phi_j} (J_j^{np})^{1-\Phi_j} \\ &= \Phi_j p_j + (1-\Phi_j)\left(V_j^u(1-\beta) - \beta(1-\delta_j)\chi\theta_j q(\theta_j)(V_j^p - V_j^u)\right) \end{aligned} \tag{11}$$

The Model: Equilibrium

Equilibrium: a sketch

- Steady State in Labour Markets

$$\delta_j(1 - e_j^{ue}) = \theta_j q(\theta_j) e_j^{ue} \tag{12}$$

$$(\delta_j + \chi\theta_j q(\theta_j)) e_j^{np} = \theta_j q(\theta_j) e_j^{ue} \tag{13}$$

- Solving gives us steady state unemployment and labour market tightness: $e_j^{ue^{ss}}$,θ_j^{ss}
- Intermediate goods market clearing:

$$p_j^s = \max(w_j^b, m_w) + \frac{\kappa_j \left(1 - (\beta(1-\delta_j)(1 - \chi\theta_j^{ss} q(\theta_j^{ss})))\right)}{\beta q(\theta_j^{ss}) s_j^u} \tag{14}$$

$$p_j^d = \frac{\partial Y}{\partial h_j(e_j^{ue^{ss}})} \tag{15}$$

The Model: Minimum Wage Impacts

- From equilibrium conditions:

$$\max(w_j^b, m_w) = \frac{\partial Y}{\partial h_j(e_j^{ue^{ss}})} - \frac{\kappa_j \left(1 - (\beta(1-\delta_j)(1-\chi\theta_j^{ss} q(\theta_j^{ss})))\right)}{\beta q(\theta_j^{ss}) s_j^u} \tag{16}$$

- So wages = **marginal product of labour** minus **recruitment costs**
- Minimum wage increase implies:
  - intermediate firms to decrease vacancies. CD matching function: probability of filling remaining vacancies increaes **reducing recruitment cost**.
  - reducing vacancies decreases employment, increasing **marginal product of labour**.

Calibration Approach

- Standard(ish) macro story: borrow some parameters from literature, estimate others (by SMM).
- We focus on estimating parameters for:
  1. exogenous distributions of worker productivity (log normal), with seperate distributions for unskilled and skilled.
     - *Empirical Targets: Variance of Log Wages and p90-10 ratios*
  2. the elasticities of substitution between workers within these two skill classes, ψ_u, ψ_s,
     - *Empirical Targets: Unemployment Rates*
  3. recruitment costs κ_u, κ_s
     - *Empirical Targets: Unemployment Rates*
  4. the share parameter, μ, in the KORV production function.
     - *Empirical Targets: Graduate Wage Premium*

Calibration Approach: Detail

- Denote the parameters to be estimated as $\Phi = (\psi_u, \psi_s, \kappa_u, \kappa_s, A, \sigma_{u,x}, \sigma_{s,x}, \mu)$.
- Remaining parameters are taken from the literate, data or legislation and are denoted by Ω.
- Estimate the parameters in Φ by SMM, targeting the following empirical moments for unskilled and skilled:
  - median wages,
  - variance of log wages,
  - p90/10 and p50/10 ratios.
  - unemployment rates.
- Let $\hat{M}$ denotes vector of the empirical moments above, and $M(\Phi, \Omega)$ denote the model predictions of these moments. Then:

$$\Phi^{SMM} = \underset{\Phi}{\operatorname{argmin}} (M(\Phi, \Omega) - \hat{M})' (M(\Phi, \Omega) - \hat{M}) \qquad (17)$$

Calibrated Parameters

Table: Estimation Results

Moment	Model Moment	Empirical Moment	% Deviation (Model - Data)
Median Hourly Wage: Unskilled	9.93	9.5	4.44
Median Hourly Wage: Skilled	16.01	15.71	1.94
Var Log Wages: Unskilled	0.45	0.49	-8.29
Var Log Wages: Skilled	0.54	0.57	-5.35
p90/50 Wages: Unskilled	2.01	1.92	4.57
p90/50 Wages: Skilled	2.02	1.96	3.12
p50/10 Wages: Unskilled	1.57	1.57	0.24
p50/10 Wages: Skilled	2.07	2.07	0.19
Min Wage Coverage: Unskilled	0.16	0.16	0.25
Min Wage Coverage: Skilled	0.06	0.06	0.03
Unemployment: Unskilled	0.07	0.07	0.51
Unemployment: Skilled	0.03	0.03	0.76

Calibrated Parameters

Table: Estimated Parameters

Parameter	Description	Source	Value
Ψ_u	Elasticity of substitution between unskilled workers	SMM Estimation	8.251
Ψ_s	Elasticity of substitution between skilled workers	SMM Estimation	14.833
μ	Share parameter determining skill premium in KORV production function	SMM Estimation	0.361
A	Total Factor Productivity	SMM Estimation	6.765
η_u	Variance parameter of worker ability distribution: unskilled workers	SMM Estimation	0.454
η_s	Variance parameter of worker ability distribution: skilled workers	SMM Estimation	0.452
ϕ_u	Nash Bargaining Parameter for unskilled workers	SMM Estimation	0.189
ϕ_s	Nash Bargaining Parameter for skilled workers	SMM Estimation	0.153
κ_u	Hiring cost: unskilled workers	SMM Estimation	162.182
κ_s	Hiring cost: skilled workers	SMM Estimation	3369.239

Calibrated Parameters

Table: Calibrated Parameters

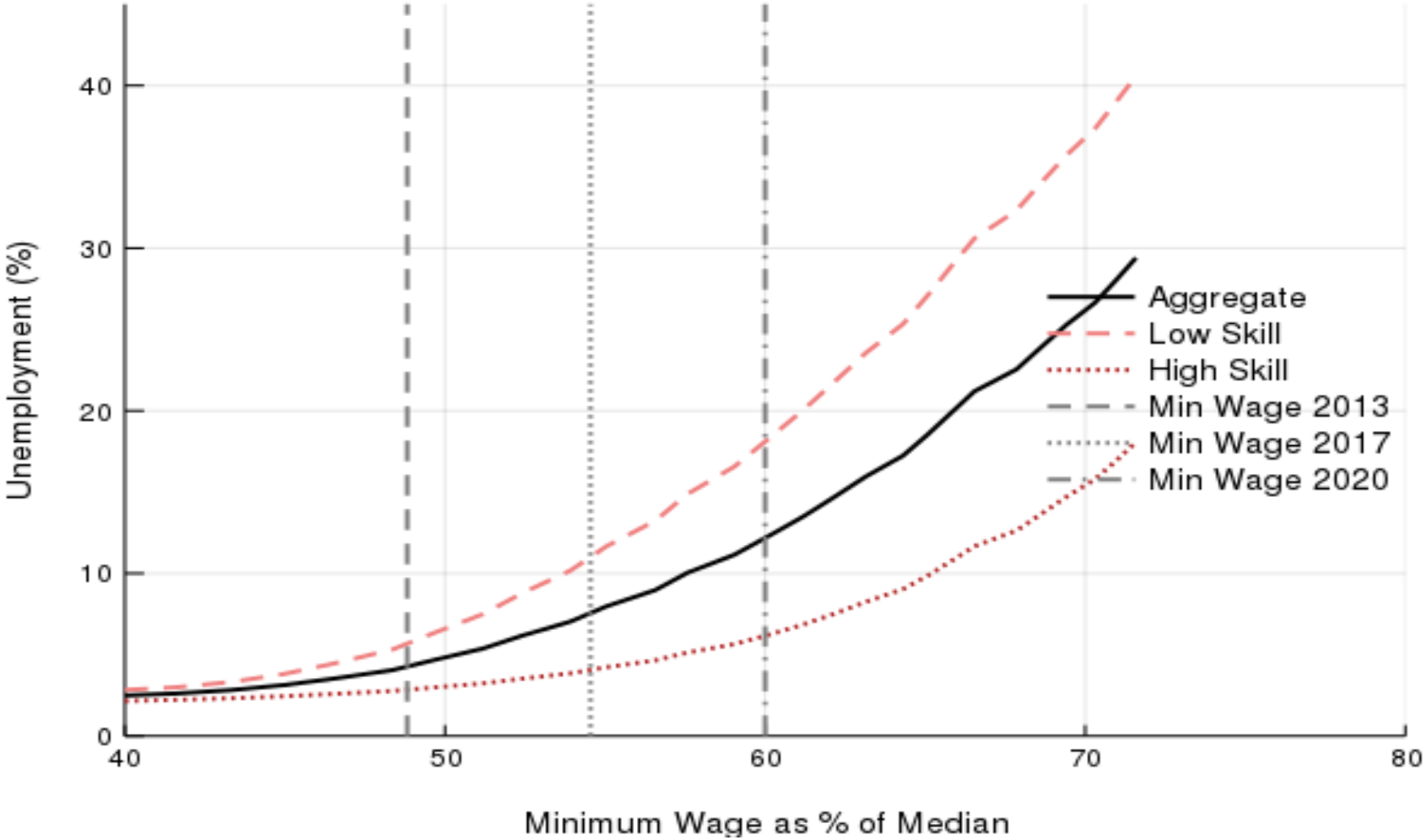
Parameter	Description	Source	Value
δ_u	Job destruction rate: unskilled	LFS 2013q4-2014q3	0.011
δ_s	Job destruction rate: skilled	LFS 2013q4-2014q3	0.007
χ_u	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.112
χ_s	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.075
b	Monthly Unemployment benefits (job seekers allowance)	Legislative level 2013-14	313.492
m_w	Hourly minimum wage	Legislative level 2013-14	6.31
σ	Elasticity of substitution between unskilled and skilled workers	Krusell et al. (2000)	0.401
ρ	Elasticity of substitution between skilled workers and capital equipment	Krusell et al. (2000)	-0.495
α	Capital Structures Parameter	Krusell et al. (2000)	0.117
λ	Input share parameter for capital equipment and skilled labour	Krusell et al. (2000)	0.3
γ	Matching Parameter	Hagedorn and Manovskii	0.407

Results: Matching Reduced Form Evidence

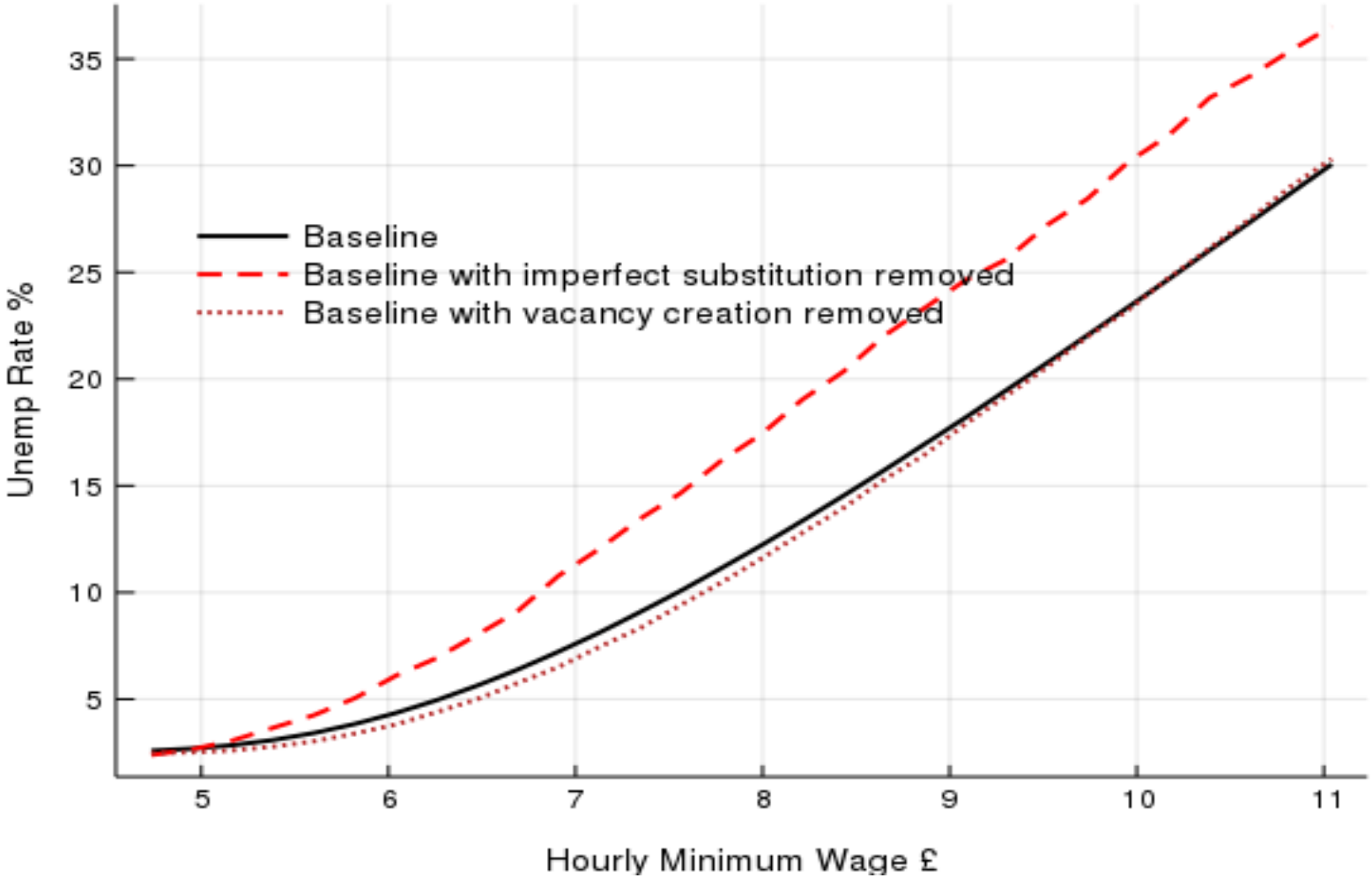
Table: Replicating Reduced Form Evidence

	Dependent Variable		
	Change in ln(average wage)	Abs Change in Profit Margin	% Change in Profit Margin
Results from Model:			
Dummy: Low Wage Firm	0.081	-0.003	-18.3
	(0.0147)	(0.0005)	
-ln(initial average wage)	0.1899	-0.0069	
	(0.0156)	(0.0005)	
Results from Draca et al. (2011):			
Dummy: Low Wage Firm	0.09	-0.029	-22.66
	(0.026)	(0.012)	
-ln(initial average wage)	0.188	-0.032	
	(0.033)	(-0.015)	

Results: Searching for Nonlinearities



Results: Drivers of Nonlinearities



Conclusions and Next Steps

- We develop a model that combines search frictions with a production function featuring several margins of substitution between factor inputs.
- Nonlinear unemployment reaction in model from:
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  2. **Endogenous nonlinearities**:
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