RISING WAGE INEQUALITY: TECHNOLOGICAL CHANGE AND SEARCH FRICTIONS

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Abstract

I investigate whether labor market frictions can explain rising wage inequality in the US. I combine the production framework in Krusell et al. (2000) with the sequential auction model of Postel-Vinay and Robin (2002). The presence of search frictions provides a range of explanations for rising wage inequality not present in competitive models i.e. changes to job flows, firm heterogeneity or bargaining power. I find that differences in search frictions between skilled and unskilled workers can explain the presence of a positive skill premium but not its growth. Estimates of capital-skill complementarity in Krusell et al. (2000) are therefore robust to including search frictions.

Keywords: Search Frictions, Monopsony, Labor Markets, Wage Inequality, Technological Change

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1. Introduction

This paper re-asses the role of technology in explaining rising wage inequality in a model where, a priori, changes to wage inequality can be driven both by changes to the technology of production and changes to search frictions. Including search frictions as a candidate explanation for rising wage inequality in our model is motived by a number of empirical explanations that so far have not been grounded in theory. For example, the importance of firm heterogeneity in explaining growing wage inequality found in Song et al. (2015) would be difficult to incorporate in a perfectly competitive model where workers (and consumers) would instantly relocate to firms that are more productive. Similarly the quantitative importance of institutions such as the minimum wage and trade unions for explaining the rise in wage inequality, as documented in Card and DiNardo (2002) and Lee (1999), is difficult to reconcile with perfectly competitve models (see Flinn (2006), Manning (2003) and Teulings (2000)).

I therefore relax the assumption of perfect competition between employers, which runs through much of the literature that seeks to explain the rise in US wage inequality over the last four decades. This assumption is present both in relatively early studies such as Katz and Murphy (1992) and later contributions such as Krusell et al. (2000) and Acemoglu and Autor (2011). A competitive labor market framework, while restrictive in some dimensions, does not preclude a rich variety of explanations for the rise in wage inequality; ranging from the capital-skill complementarity channel in Krusell et al. (2000) to cohort specific supply changes emphasized in Card and Lemieux (2001). However, in a competitive framework these explanations are naturally restricted to two broad categories: those based on changes to the technology governing production of output; and those based on changes to the skill distribution of workers.

A key contribution of this paper is to develop a structural model that can incorporate both technological based explanations for rising wage inequality and explanations reflecting labor market frictions and institutional change as outlined above. Specifically, I do this by combining the production framework specified in Krusell et al. (2000) with the sequential auction wage bargaining model developed in Postel-Vinay and Robin (2002) and Cahuc et al. (2006). Both frameworks have been very influential in explaining different dimensions of inequality: dynamics of inequality in the case of Krusell et al. (2000) and cross-sectional levels in the case of Cahuc et al. (2006). Marrying the two frameworks has value for a number of reasons, not least that they touch on issues where there is a great deal of overlap e.g. capital skill complementarity may have both increased wages of high skilled workers directly, by raising the marginal product of labor, and indirectly by inducing greater vacancy creation for skilled workers, lowering the search frictions they faced.

Krusell et al. (2000) relate the rise in the graduate wage premium to the fall in the price of capital equipment. They show that a production specification where skilled and unskilled labor combine with capital to produce output can provide an explanation for the rise in the graduate wage premium when capital is more complementary with skilled than unskilled labor. Quantitatively, the authors show that this capital skill complementarity channel, when combined with the large falls in the price of capital equipment observed in the data, is able to explain almost all of the rise in the graduate wage premium seen over the 1980s and early 1990s. One contribution of this paper is to update the sample used in Krusell et al. (2000) and show that their parsimonious model continues to provide a good fit to the data.

In the sequential auction model of the labor market in Cahuc et al. (2006), average wages of a given skill type of worker depend on the worker's marginal productivity at a given firm - as in the competitive framework - but also on job to job transition rates, bargaining strength, the distribution of firm heterogeneity and outside options in unemployment. The eventual goal of this research agenda is to evaluate the contribution of changes to each of these factors to the rise in wage inequality, and see whether the estimates of capital skill complementarity from Krusell et al. (2000) are materially different once these factors are accounted for. This could be the case if, for example, job market frictions have significantly worsened for unskilled workers relative to skilled. This would mean my model, because it allows for these change of frictions, would be less reliant on the technology channel emphasised in Krusell et al. (2000) to explain the growth in the graduate wage premium, and would deliver parameter estimates suggesting a smaller degree of capital skill complementarity.

This has important consequences for policy; the technological explanation for rising wage inequality implies governments face a relatively acute trade-off if they wish to boost living standards of low skill workers through policies such as the minimum wage or increasing unionization rates. On the one hand such policies can improve the incomes of those in work but, if low skilled labor is indeed significantly substitutable with capital, these policies risk pushing more workers into unemployment. Any findings suggesting a lower level of substitutability between unskilled labor and capital will therefore have an important bearing on how acute this trade-off is. Indeed in a working paper, I again marry the frameworks in Krusell et al. (2000) and Cahuc et al. (2006) to investigate the impact of the minimum wage on unemployment, and whether there are significant nonlinearities in a model where both search frictions and capitl skill complementarirty are present.

My strategy for taking the model to the data has been to maintain consistency with Krusell et al. (2000) by focusing on the graduate wage premium as my measure of labor market inequality, and by using national accounts and the Current Population Survey (CPS) data. I find that differences in search frictions between graduates and non-graduates can explain the presence of a graduate wage premium due to higher estimated job contact rates and lower job destruction rate for graduates, but cannot explain the growth of the wage premium. I therefore find that estimates of capital-skill complementarity in the production framework of Krusell et al. (2000) are not significantly changed by allowing for labor market frictions.

This finding is driven by the fact that the empirical measures of labor market frictions I use, such as job-to-job mobility and job destruction rates, do not show any trends favouring skilled workers (graduates) relative to unskilled, indeed if anything the reverse is true, and so do not provide an alternative explanation for the rise in the graduate wage premium. This also suggests that the potential amplification mechanism discussed above - that increased demand for skilled workers raised wages directly via an increase in their productivity and indirectly via a reduction in their search frictions - does not appear to have strong support in the data. I therefore do not include endogenous vacancy creation in the model, which would significantly complicate analysis. It also important to stress that this paper focuses on changes to job mobility as a candidate explanation for rising skill premiums rather than changes to institutions, such as declining union and minimum wage coverage, which represent a promising area for future research.

The rest of this paper is organised as follows. Section 2 will present the model, starting first with an overview of both the production technology in Krusell et al. (2000) (henceforth KORV) and the sequential model of Cahuc et al. (2006) before examining how I combine them in my model. Section 3 discusses the data I use to estimate the combined model, before Section 4 presents my econometric approach. Section 5 presents findings and Section 6 concludes.

2. The Model

Introducing search frictions, with wage bargaining, into the production technology in KORV comes up against a key theoretical challenge, which is that doing so directly would mean firms bargaining with many workers i.e. a multi-player game as per Stole and Zwiebel (1996). These multi-player games seem unlikely to be relevant for considering aggregate dynamics in the labor market. I therefore abstract from such effects by specifying a competitive final good firm, where production is as in KORV, and an intermediate good sector with a labor market structure identical to the sequential auction model of Cahuc et al. (2006) i.e. with random search by unemployed and employed workers, firm heterogeneity, and where incumbent employers can respond to job offers made to their employees by rivals. There are segmented intermediate goods sectors for unskilled and skilled labor, and firms within each intermediate goods sector have heterogeneous quality.

I will first present an overview of the KORV production environment in its original form, before explaining how I incorporate intermediate goods sectors with search frictions into the KORV production environment. Finally, I explain how search frictions and wage bargaining operate within the intermediate goods sector.

2.1. **KORV Production Function: No Frictions or Intermediate Goods.**

In the original formulation of KORV, final good in period t, Y_t is produced using capital structures, $K_{st,t}$, capital equipment, $K_{eq,t}$, and skilled and unskilled labor, S_t & U_t, as inputs, as shown in equation (1).

$$\begin{aligned} Y_t &= A_t G(K_{st,t}, K_{eq,t}, U_t, S_t) \\ &= A_t K_{st,t}^{\alpha} [\mu U_t^{\sigma} + (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda) S_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} \end{aligned} \tag{1}$$

with $\sigma, \rho < 1$ and $\alpha, \lambda, \mu \in (0,1)$. The elasticity of substitution between unskilled labor input and capital equipment, denoted by $\varepsilon_{u,k_{eq}}$, equals $1/(1-\sigma)$. The elasticity of substitution between unskilled and skilled labor inputs, denoted $\varepsilon_{u,s}$, is also given by $1/(1-\sigma)$. Finally, the elasticity of substitution between the skilled labor input and capital equipment, denoted by $\varepsilon_{s,k_{eq}}$, is given by $1/(1-\rho)$. The parameter, α, together with λ, determine the capital share of output, and μ determines the output share of unskilled workers.

Unskilled and skilled labor input are hours worked by non-graduates and graduates in efficiency units e.g $U_t \equiv \Psi_{u,t} h_{u,t}$, $S_t \equiv \Psi_{s,t} h_{s,t}$, where $\Psi_{i,t}$ is the efficiency of labor input of a given skill level, where skill is indexed by $i \in \{u, s\}$, and $h_{i,t}$ is the total amount of hours worked. Krusell et al. (2000), in their baseline model, impose that $\Psi_{u,t}$ and $\Psi_{s,t}$ both follow stationary stochastic processes (iid) as allowing for any time trend would introduce an unexplained source of skills-biased technical change, contrary to the aim of their paper which is to examine the extent to which increased capital use can explain the rise in the graduate wage premium.

The final good is used for consumption c_t, investment in capital equipment $x_{eq,t}$ and investment in capital structures $x_{st,t}$, as shown in equation (2), where q_t is the relative efficiency of producing capital equipment from the final good (or equivalently $1/q_t$ is the relative price of capital equipment).

$$Y_t = c_t + x_{st,t} + \frac{x_{eq,t}}{q_t} \tag{2}$$

The final good producer has the following profit maximisation problem, where $(w_{u,t}, w_{s,t})$ denote the wages for unskilled and skilled workers respectively, and $(r_{st,t}, r_{eq,t})$ denote the rental rates for capital structures and equipment respectively:

$$\begin{aligned} \max_{K_{st,t}, K_{eq,t}, h_{u,t}, h_{s,t}} \Pi = & A_t K_{st,t}^{\alpha} [\mu U_t^{\sigma} + (1-\mu)(\lambda K_{eq,t}^{\gamma} + (1-\lambda) S_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} \\ & - w_{u,t} h_{u,t} - w_{s,t} h_{s,t} - r_{st,t} K_{st,t} - r_{eq,t} K_{eq,t} \end{aligned} \tag{3}$$

In both KORV's original model and in my adaptation the final good producer is assumed to be competitive, so the first order conditions (FOCs) for its profit maximisation problem are as shown in Equations (4) through (7).

$$\begin{aligned}
w_{u,t} &= A_t(1-\alpha)K^{\alpha}_{st,t}[\mu U^{\sigma}_t + (1-\mu)(\lambda K^{\gamma}_{eq,t} + (1-\lambda)S^{\gamma}_t)^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (4)\\
&\quad \times \mu U^{\sigma-1}\Psi_{u,t}\\
w_{s,t} &= A_t(1-\alpha)K^{\alpha}_{st,t}[\mu U^{\sigma}_t + (1-\mu)(\lambda K^{\gamma}_{eq,t} + (1-\lambda)S^{\gamma}_t)^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (5)\\
&\quad \times(1-\mu)(\lambda K^{\gamma}_{eq,t} + (1-\lambda)S^{\gamma}_t)^{\frac{\sigma-\gamma}{\gamma}}(1-\lambda)S^{\gamma-1}_t\Psi_{s,t}\\
r_{eq,t} &= A_t(1-\alpha)K^{\alpha}_{st,t}[\mu U^{\sigma}_t + (1-\mu)(\lambda K^{\gamma}_{eq,t} + (1-\lambda)S^{\gamma}_t)^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (6)\\
&\quad \times(1-\mu)(\lambda K^{\gamma}_{eq,t} + (1-\lambda)S^{\gamma}_t)^{\frac{\sigma-\gamma}{\gamma}}K^{\gamma-1}_{eq,t}\\
r_{st,t} &= \alpha A_t K^{\alpha-1}_{st,t}[\mu U^{\sigma}_t + (1-\mu)(\lambda K^{\gamma}_{eq,t} + (1-\lambda)S^{\gamma}_t)^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} && (7)
\end{aligned}$$

In the absence of frictions, growth in the graduate wage premium (denoted by $\pi_t = w_{s,t}/w_{u,t}$) is given in equation (8), where g_z denotes the growth rate in variable z.[1]

$$\begin{aligned}
g_{\pi_t} \simeq (1-\sigma)(g_{h_{u,t}} - g_{h_{s,t}}) + \sigma(g_{\Psi_{s,t}} - g_{\Psi_{u,t}}) + \\
(\sigma-\gamma)\lambda\Big(\frac{K_{eq,t}}{S_t}\Big)(g_{K_{eq,t}} - g_{\Psi_{s,t}} - g_{h_{s,t}})
\end{aligned} \tag{8}$$

2.2. **KORV production function: Incorporating Intermediate Goods.** I now interpret U_t and S_t as the effective amount of intermediate goods produced in the unskilled and skilled intermediate goods sectors respectively. Specifically I define $U_t \equiv \Psi_{u,t}y_{u_t}$ and $S_t \equiv \Psi_{s,t}y_{s_t}$ where $y_{i,t}$ is the volume of intermediate goods produced in skill sector $i \in \{u, s\}$ and $\Psi_{i,t}$ is, analogously to the KORV environment, the efficiency level of that intermediate good.

In each segmented intermediate goods market, unemployed workers are randomly matched to intermediate firms of quality ν (I refer to this as match quality), and with a sampling distribution $F_{i,t}(\nu)$ and pdf, $f_{i,t}(\nu)$. I denote the cdf and pdf of the cross-section distribution of match quality across all workers as $L_{i,t}(\nu)$ and $l_{i,t}(\nu)$, which differs from the offer distribution as workers can search for higher quality matches on the job.

A worker in a match of quality ν produces exactly ν units of intermediate good for every hour they work, though hours worked are assumed to be fixed for each skill type of worker.[2] The effective input of intermediate goods from the unskilled

[1]This by derived by taking logs of the graduate wage premium - given by the final goods firm's FOCs - and then differentiating with respect to time to give equation (8).

[2]I make this assumption to maintain consistency with the original formulation of the KORV production function where labor inputs are measured in efficiency units of total hours worked.

and skilled intermediate sectors are therefore as shown in equation (9), where $h_{i,t}$ is again the raw total amount of hours worked by workers of skill type i.

(9)
$$U_t \equiv \Psi_{u,t} y_{u,t} = \Psi_{u,t} h_{u,t} \int_{\nu_{inf}}^{\nu_{max}} \nu \ell_{t,u}(\nu) d\nu \, , \, S_t \equiv \Psi_{s,t} y_{s,t} = \Psi_{s,t} h_{s,t} \int_{\nu_{inf}}^{\nu_{max}} \nu \ell_{t,s}(\nu) d\nu$$

Final good producers are again assumed to be competitive and so pay a price, p_i, for a unit of type i intermediate good given by $p_i = \frac{\partial Y}{\partial y_i} \Psi_i$. An intermediate good firm of match quality ν in intermediate sector i receives revenue equal to $p_i \nu$.

2.3. **Intermediate Goods Sector.** All intermediate firms and workers have common discount rate, ρ, and are risk neutral. As is standard in the search literature, I assume firms can employ a maximum of one worker so intermediate firms become synonymous to matches or jobs. Job destruction rates are exogenously given, but allowed to vary by skill sectors and are denoted by $\delta_{i,t}$. Workers receive flow income in unemployment equal to $b_{i,t} * p_{i,t}$, where $b_{i,t}$ is their replacement rate and $p_{i,t}$ is the price of the intermediate good they produce as defined above.[3]

The job offer arrival rates in unemployment and employment are denoted $\lambda_{0,i,t}$,$\lambda_{1,i,t}$ and will be assumed to be exogenously given.

Intermediate Goods Sector: Wage Bargaining with Unemployed Workers

I start by stating the Bellman equation for an unemployed worker of skill type i in equation (10), where $V_{0,i}(p_i)$ is the expected lifetime utility of an unemployed worker, $\phi_0(p_i, \nu)$ is the wage paid to a previously unemployed worker now in a match of quality ν and $V(p_i, \phi_0(p_i, \nu), \nu)$ is the expected lifetime utility of that worker.

$$(\rho + \lambda_{0,i}) V_{0,i}(p_i) = p_i b_i + \lambda_{0,i} \int_{\nu_{inf_i}}^{\nu_{\max}} V(p_i, \phi_0(p_i, x), x) \, dF_i(x) \tag{10}$$

Equation (10) indicates that the unemployed worker receives flow income $b_i p_i$ in the current period and in the next period, which is discounted at rate ρ, they encounter a match with probability $\lambda_{0,i}$, where the match quality is drawn from the distribution $F_i(\nu)$ and lies in the interval $[\nu_{inf_i}, \nu_{max}]$.

As in Cahuc et al. (2006), I assume that there is a latent vacancy posting cost, which ensures that intermediate firms won't post a match unless it will be accepted by a worker, so the lower bound of the match quality distribution is the workers reservation match quality ν_{inf_i}.

[3]This implies that unemployment income is independent of the match quality that the worker had in their previous employment. Reemployment wages are therefore not path dependent, which aids tractability in the model - see Cahuc et al. (2006) for further discussion.

Cahuc et al. (2006) propose a generalized form of Nash bargaining both for unemployed and employed workers. For unemployed workers, this takes the standard form whereby previously unemployed workers (I henceforth refer to these as 'entrant' workers) are paid a wage, $\phi_0(p_i, \nu)$, that equalizes the expected lifetime utility of working at a match of quality ν with the expected lifetime utility of being unemployed plus a share, β (the bargaining parameter), of match surplus $V(p_i, p_i\nu, \nu) - V_{0,i}(p_i)$, as expressed in equation (11).[4]

$$V\left(p_i, \phi_0(p_i, \nu), \nu\right) = V_{0,i}(p_i) + \beta\left[V(p_i, p_i\nu, \nu) - V_{0,i}(p_i)\right] \tag{11}$$

From equations (11) and (10), Cahuc et al. (2006) derive the closed form solution for entrant wages shown in equation (12), where $\bar{F} \equiv 1 - F$.

$$\phi_0(p_i, \nu) = p_i \cdot \left(\nu_{inf_i} - (1-\beta)\int_{\nu_{inf_i}}^{\nu} \frac{\rho + \delta + \lambda_1 \bar{F}_i(x)}{\rho + \delta + \lambda_1 \beta \bar{F}_i(x)} dx\right) \tag{12}$$

Note that Cahuc et al. (2006) also use equation (11) and 10 to derive the expression for the reservation match quality, ν_{inf_i}, shown in equation (13).

$$\nu_{inf_i} = b_i + \int_{\nu_{inf_i}}^{\nu_{max}} \frac{\beta(\lambda_{0,i} - \lambda_{1,i})\bar{F}_i(x)}{\rho + \delta_i + \beta_i \lambda_{1,i} \bar{F}_i(x)} dx \tag{13}$$

Intermediate Goods Sector: Wage Bargaining with Employed Workers

A key novelty in the sequential auction model of Cahuc et al. (2006) is that incumbent employers can respond to rival job offers made to their employees, in contrast to wage posting models such as Burdett and Mortensen (1998). In this environment, the wage paid to an employee will depend on (i) the match quality of the highest ranked match they have encountered in their employment spell, ν^+ , which will be at their current employer, (ii) the match quality at their outside option, ν^-, which is the second highest match they have encountered in their employment spell, and (iii) the price of the intermediate good they produce, p_i, which will be the same for all workers in a given skill group i . I denote this wage $\phi(p_i, \nu^-, \nu^+)$.

In order to state the Bellman equation for the employed worker, I must first specify what happens to a worker employed at a match of quality ν encounters a match of quality ν', and is currently paid a wage w. First, if $\nu' > \nu$, then the employee moves to higher quality match and gets wage $\phi(p_i, \nu, \nu')$. Encountering a match of quality $\nu' < \nu$ will trigger a renegotiation of the employees wage contract at their

[4] I have assumed there is zero value to a firm from having a vacancy i.e. a free entry condition holds, which when combined with the assumption of a common discount rate for firms and workers and risk neutrality of all agents means the match suprlus can be expressed as $V(p_i, p_i\nu, \nu) - V_{0,i}(p_i)$ i.e. the match suprlus equals the worker surplus when they are paid a wage equal to their marginal product.

current employer if ν' exceeds a threshold, denoted $\chi(p_i, w, \nu)$, where $\chi(p_i, w, \nu)$ is defined by the equality $\phi(p_i, \chi(p_i, w, \nu), \nu) = w$.

The Bellman equation for a worker employed at a match of quality ν and paid a wage, w, is therefore as shown in equation (14).

$$\begin{aligned}
&\left[\rho + \delta_i + \lambda_{1,i}\bar{F}_i(\chi(p_i, w, \nu))\right] V_i(p_i, w, \nu) \\
&= w + \delta_i V_{0,i}(p_i) + \lambda_{1,i} \int_{\chi(p_i,w,\nu)}^{\nu} V_i(p_i, \phi(p_i, x, \nu), x) dF_i(x) \\
&+\lambda_{1,i} \int_{\nu}^{\nu_{\max}} V_i(p_i, \phi(p_i, \nu, x), x) dF_i(x)
\end{aligned} \tag{14}$$

Equation (14) indicates that the worker, after receiving wage w in the current period, will either lose their job with probability δ_i or, failing that, make contact with a match that triggers a renegotiation of their wage in the next period with probability $\lambda_{1,i}\bar{F}_i(\chi(p_i, w, \nu))$. If the match quality at the alternative match, x, lies in the region $(\chi(p_i, w, \nu), \nu]$ the worker stays at their current employer and receives a pay rise $\phi(p_i, x, \nu) - w$. If $x > \nu$ the worker moves to the alternative match and gets a wage $\phi(p_i, \nu, x)$. Note that this wage need not be greater than their previous wage as workers may be willing to take a pay cut if the possibility of future wage increases at the higher quality match employer is sufficiently greater than at their incumbent employer.

All this is left to do is specify the result of wage bargaining that occurs between an employee who encounters a match of sufficient quality to trigger a wage renegotiation. I denote the higher of the incumbent and rival employer's match quality as ν^+ , and the lower match quality as ν^-. The worker will supply their labor to the higher quality match, and the lower quality match becomes their outside option. Cahuc et al. (2006) adapt the Nash bargaining game of Osborne and Rubinstein (1990) to an environment with rival bidders, and show the bargained wage must satisfy equation (15).

$$\begin{aligned}
V(p_i, \phi(p_i, \nu^-, \nu^+), \nu^+) =& V(p_i, p_i\nu^-, \nu^-) + \\
& \beta[V(p_i, p_i\nu^+, \nu^+) - V(p_i, p_i\nu^-, \nu^-)]
\end{aligned} \tag{15}$$

Equation (15) indicates that a worker receives their outside option - the value of working at the firm with productivity ν^- at a wage equal to their marginal product, $p_i\nu^-$, plus a share, β, of the match surplus from working at the higher productivity firm.

Cahuc et al. (2006) prove that the wage, $\phi(p_i, \nu^-, \nu^+)$, satisfying equation (15) has the form shown in equation (16) when value functions are as defined as in equation

(14).

$$\phi(p_i, \nu^-, \nu^+) = p_i\left(\nu^+ - (1-\beta)\int_{\nu^-}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx\right) \tag{16}$$

Intermediate Goods Sector: Wage and Employment Distributions

The key objects of interest in the model are the wage distributions for each skill type of worker. The analysis of the preceding sections indicates that a worker's wage depends on two stochastic variables: their current match quality, ν, and that of their outside option, χ. As in Cahuc et al. (2006), I impose that the labor market is in steady state in order to derive expressions for the cross section distributions of ν, $L_i(\nu)$, and of χ conditional on ν, $L_i(\chi|\nu)$.

Steady state in the labor market requires equations (17) through to (19) to hold (where e_i^{ue} denotes the unemployment rate of skill type i, and I have suppressed that there is a total population, N_i, of type i workers that would multiply both sides of each equation).

$$\delta_i(1 - e_i^{ue}) = \lambda_{0,i} e_i^{ue} \tag{17}$$

$$\lambda_{0,i} F_i(\nu) e_i^{ue} = [(\lambda_{1,i}\bar{F}_i(\nu) + \delta_i)](1 - e_i^{ue}) L_i(\nu) \tag{18}$$

$$\begin{aligned}\lambda_{0,i} e_i^{ue} f(\nu) + \lambda_{1,i} L_i(\chi) f(\nu)(1 - e_i^{ue}) = {} & [(\lambda_{1,i}\bar{F}_i(\chi) + \delta_i)]\times \\ & (1 - e_i^{ue}) L_i(\chi|\nu)\ell_i(\nu)\end{aligned} \tag{19}$$

Equation (17) requires that the inflows of workers into unemployment - the left hand side (LHS) of the equation - equals the outflow from unemployment. Equation (18) requires the inflow into the measure of workers employed at a match of quality less than ν equals the outflow: the inflow consists of unemployed workers who make contact with a match of quality less than ν (LHS of equation (18)) and the outflow is employed workers with match quality below μ who either lose their job, with probability δ_i or make contact with a higher quality match, with probability $\lambda_{1,i}\bar{F}_i(\nu)$. Finally equation (19) requires that the inflow into the measure of workers employed a match quality equal to ν and with an outside option of quality less than χ equals the outflow: the inflow again consists of unemployed workers meeting a match of quality ν (by definition their outside option, i.e. unemployment, has a match quality less than all feasible values of χ), plus workers employed at a match of quality less than χ who make contact with a match of quality ν; the outflow is employed workers with a match quality equal to ν and with an outside option of quality less than χ who either lose their job or receive an offer of a match of quality exceeding χ.

The expressions for the steady state cross sectional distribution of workers across matches and outside options derived from the steady state requirements above are shown in equations (20) and (21), where $\kappa_{1,i} \equiv \frac{\lambda_{1,i}}{\delta_i}$.

$$L_i(\nu) = \frac{F_i(\nu)}{1 + \kappa_{1,i}\bar{F}_i(\nu)} \tag{20}$$

$$L_i(\chi|\nu) = \left[\frac{1 + \kappa_{1,i}L_i(\chi)}{1 + \kappa_{1,i}L_i(\nu)}\right]^2 \tag{21}$$

The expected wage for a worker of type i is given by:

$$E(w_i) = p_i \int_{\underline{\nu}}^{\nu_{max}} \left[\nu - [1 + \kappa_{1,i}\bar{F}_i(\nu)]^2 \times \int_{\nu_{inf}}^{\nu} \frac{(1-\beta)[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\bar{F}_i(x)]}{[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\beta\bar{F}_i(x)][1 + \kappa_{1,i}\bar{F}_i(x)]^2}dx\right]\ell_i(\nu)d\nu \tag{22}$$

The graduate wage premium will therefore depend on the same variables as in Krusell et al. (2000), which influence the price of the intermediate good produced by skill type i, p_i , but also on relative job mobility rates, outside options in unemployment, distributions of match quality, and bargaining strength. In this paper I use the same data as in Krusell et al. (2000) which limits my ability to identify the impact of all these potential channels on the graduate wage premium: I can however consider the impact of changes to relative job mobility rates, outside options in unemployment and to the distribution of match quality.

3. Data

3.1. **Data: Krusell et al. (2000).**

In keeping with KORV's original study, I use labor market data from the Current Population Survey (CPS) and data on capital inputs and the labor share of income from U.S national accounts. Skilled labor is defined as total hours worked by graduates and unskilled labor input is the total hours worked by non-graduates. The authors split each skill type down further into education, gender and race cells to impute hours for those with missing data.I follow their exact approach for comparability.[5]

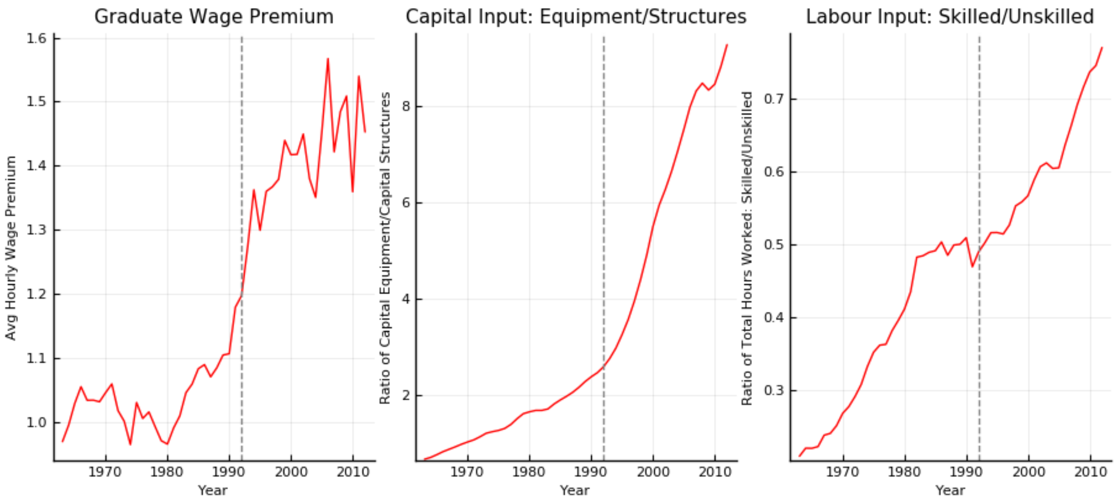
The authors differentiate between capital equipment, such as machinery, hardware and software, and capital structures e.g. buildings. The theoretical basis for doing so is presumably that capital skill complementarity is much more likely to occur with equipment than with structures. An important element of KORV's approach is their use of a relative price deflator for capital equipment that is based on the approach of Gordon (1990), which they use to calculate the real value of the stock of capital equipment (all other variables are deflated using a GDP deflator). This relative price of equipment falls significantly over KORV's

[5]See Appendix 1 of Krusell et al. (2000) for details of this approach

sample period, which in turn implies that the real value of capital equipment used by firms increases appreciably faster than capital structures. Polgreen and Silos (2008) show that use of alternative price series suggest significantly less capital skill complementarity.

The key trends driving results in KORV are summarised in Figure 1. The rise in the graduate wage premium happens despite an increase in relative supply of skilled labor: given the authors assume constant relative labor efficiency in their baseline specification, the only possible driver of the rise in the graduate wage premium can be the growing use of capital equipment combined with some degree of capital skill complementarity, which is indeed what their results suggest. The authors estimate an elasticity of substitution between capital equipment and unskilled labor of 1.67 vs an equivalent elasticity of 0.67 for skilled labor.

FIGURE 1. Key Data Trends in KORV

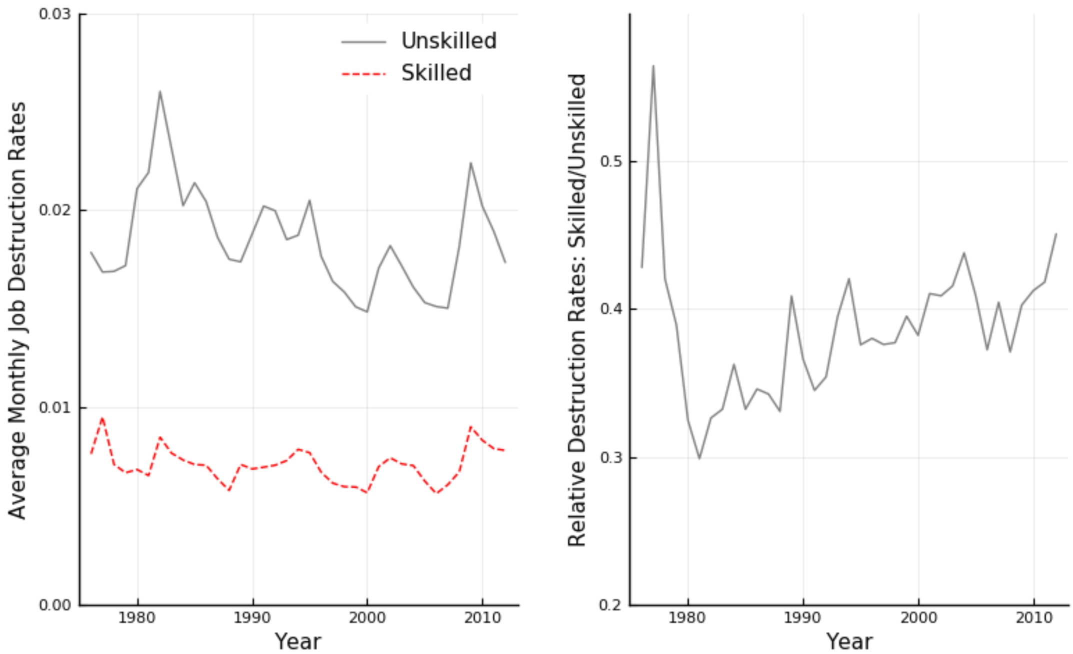


Notes: The grey dashed line represents end of sample period used in Krusell et al. (2000). Wage and hours worked data series are taken from the Annual Social and Economic Supplement of the CPS. Series for capital structures and equipment are constructed recursively as per Krusell et al. (2000): I use a starting stock of capital equipment and structures in 1963 using data from National Accounts data and adding annual investment data in equipment and structures also from the National Accounts. Both investment series are deflated by the non-durable consumption deflator used in DiCecio (2009), updates of which are available on FRED, Federal Reserve Bank of St. Louis. Capital equipment investment is additionally deflated using the relative capital equipment deflator from DiCecio (2009), which is based on the approach of Gordon (1990). I use the the same depreciation rates of capital stock as Krusell et al. (2000): 12.5% for equipment and 5% for structures.

3.2. **Data: labor Market Frictions.** I supplement the core KORV data with data on labor market frictions. With each measure of labor market friction, the key dimension of interest will be the trend in frictions for skilled workers relative to unskilled. In the absence of distinct trends in relative frictions it is unlikely that incorporating labor market frictions into KORV will offer a different explanation for the rise in the graduate wage premium than the original KORV specification.

A key friction will be the degree of competitive intensity, $\kappa_{1,i}$ the rate of job to job contact rates relative to job destruction rates, which determines how quickly workers proceed up the job ladder. Job destruction rates can easily be taken from the panel element of the CPS, with the results given in Figure 2, which shows that, if anything, this measure of job market frictions shifted in favour of unskilled workers relative to skilled.

FIGURE 2. Job destruction rates

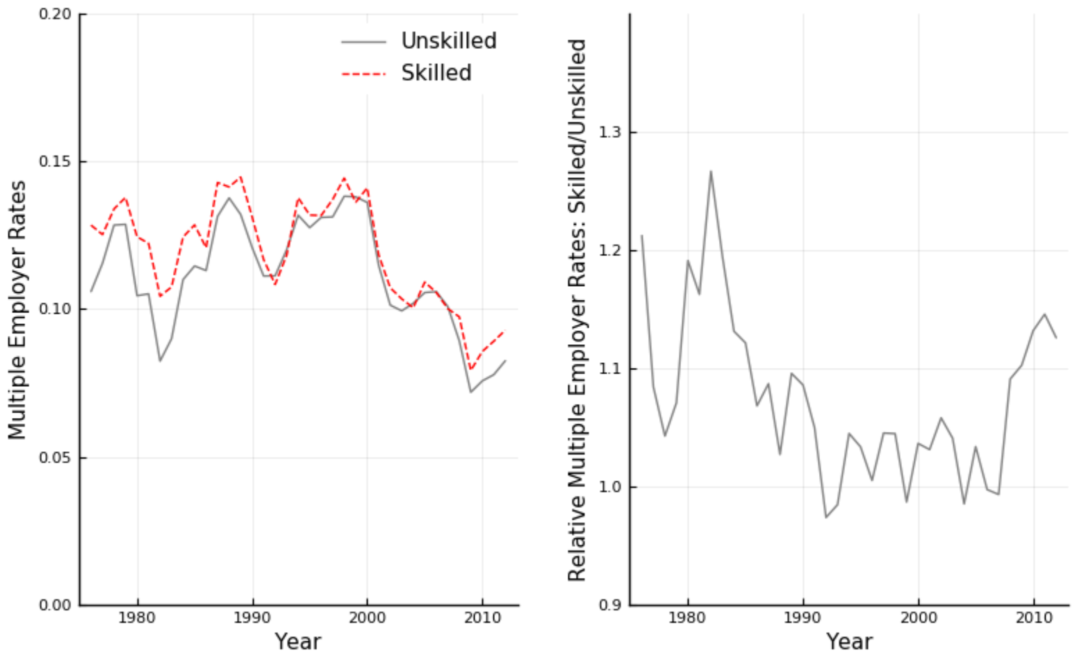


Notes: Figure shows the average monthly transition rates of workers from employment to unemployment for a given year. Source: Basic Monthly files of the CPS.

Job contact rates are not readily observable in the CPS and there has only been a question on change of employers since 1994, which hampers comparison with KORV since their original sample period finished in 1992. Since in any case job-to-job transitions would be used to infer job contact rates (not all contacts result in a transition), I use an alternative measure of job mobility which is the proportion of continuously employed individuals in a year that report having at least two employers (not concurrently) in that year, which is shown in Figure 3 and is referred to henceforth as the multiple employer rate. Figure 4 shows that movements in the multiple employer rate track movements to job-to-job mobility very closely. [6]

In relative terms, the multiple job rate rises falls for skilled workers: in Appendix B I show that the multiple employer rate is an increasing monotonic function of the job contact rate for employees, $\lambda_{1,i}$, so the downward trend in multiple employer rates for skilled workers relative to unskilled workers, taken together with the increase in relative job destruction rates, suggests a decrease in relative competitive intensity for skilled workers.

FIGURE 3. Multiple Employer Rates



Notes: Figure shows the proportion of workers with no spells of unemployment who have had more than one non-concurrent employer in the previous year. Source: Annual Social and Economic Supplement of the CPS.

Of course the importance of movements up and down the job ladder for changes to the graduate wage premium depends on the dispersion of match quality: for example if there is little dispersion then even big changes in frequency of movements are unlikely to make much of a difference to changes in relative wage levels.

[6]The CPS question on number of employers in the last year started in 1976, so I am forced to start my sample 13 years after Krusell et al. (2000) start theirs.

FIGURE 4. Multiple Employer and Job-to-Job Mobility Rates

Notes: Figure compares the "Multiple Employer Rate" defined in Figure 3 to the average monthly transition rate of employees to new jobs ("Job-to-Job Mobility Rate" from the Basic Monthly files of the CPS . Both series are normalised with reference to their value in 1995.

I use the standard deviation of log residual wages as my measure of dispersion - that is wages controlling for education, race, sex and year. I purposefully do not control for age since there are some endogenous returns to job tenure and total employment duration in my model, both of which are correlated with age (and are not directly measured in the CPS over the duration of my sample period).[7] This measure of dispersion increases in relative terms for skilled workers as shown in Figure 5.[8]

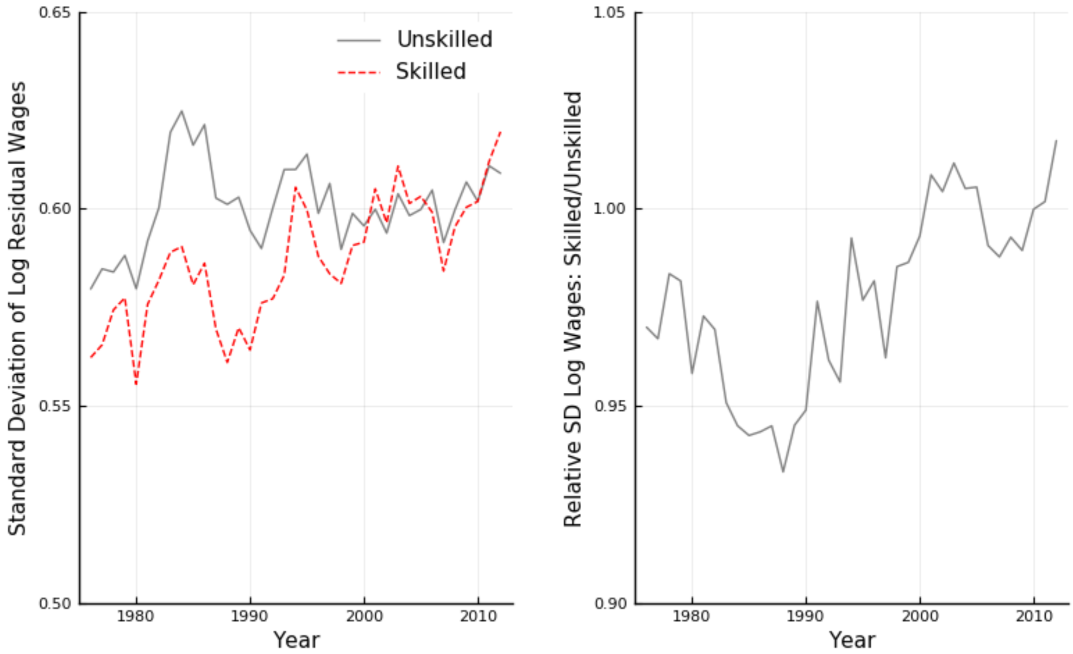
Controlling for ex-ante differences in agents' human capital levels has the advantage of maintaining consistency with the datasets used in Krusell et al. (2000), and so facilitating comparison with their results. However, it risks attributing some unobservable differences in human capital to job ladder impacts, so a longer

[7]I acknowledge this risks attributing some variance in wages driven by human capital accumulation over a worker's career to job ladder effects. However, I find that my estimates of the parameters of the KORV production function do not change if I control for age when calculating residual wage variance - see Appendix C for this and other robustness checks.

[8]Note that using other measures of wage dispersion, such as the interquartile range, does not change my estimates of the KORV production function parameters - see Appendix C.

term research goal is to estimate this model in the context of matched employer-employee data so I can better separate out individual and firm fixed effects.

FIGURE 5. Wage Dispersion

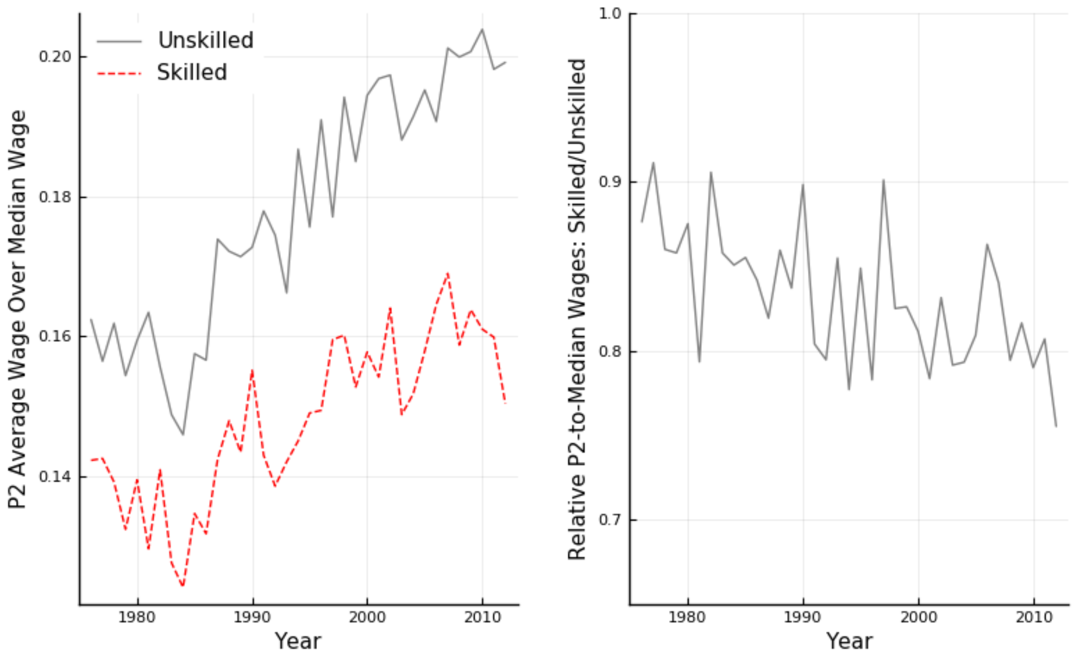


Finally the environment workers face in unemployment, both in terms of unemployment flow income and job contact rates, has an impact on the average quality of matches and wages for employed workers. This impact is less in models with on-the-job search than in models without, but it nonetheless must be accounted for.

Specifically I will need to estimate or infer the lower bound of the match quality distribution for each skill type, $\nu_{inf,i}$. In principle, this could be done by exploiting the tractable relationship between the lower bound of the sampling distribution of match quality and unemployment replacement rates and job contact rates shown previously in equation (13).

However, replacement rates are determined not only by legislative framework but also by the degree of insurance provided by asset accumulation, family/social relationships and many other factors beyond this making it difficult to observe in practice. I therefore directly estimate $\nu_{inf,i}$ by targeting the ratio of average wages of workers in the first two percentiles of the wage distribution to the median wage: the next section on my estimation approach and Appendix B discuss identification in more detail. The empirical moment is shown in Figure 6, where the trends shown suggest changes to outside options have compressed the tail of the unskilled wage distribution more than for skilled workers.[9]

FIGURE 6. Lower Bound of Wages Relative To Median



Note that when I simply replicate KORV's estimation approach I use exactly the same treatment of the data as they do, however when it comes to incorporating frictions I will trim the bottom and top percentile of the wage distribution out of the sample to minimise measurement error.

Perhaps surprisingly, the balance of the data on labor market frictions considered here has, if anything, moved in favour of unskilled workers relative to skilled workers, when skill is equated with having a degree. Skilled workers have seen their relative job destruction rates increase, and their relative job to job mobility rates (using the proxy measure discussed above) decease.

The one potential mitigating factor that may has moved in the favour of skilled workers, at least in the context of a job ladder model, is an increase in their residual wage variance, both in absolute terms and relative to unskilled workers. I will see

[9]The estimated parameters of the KORV production function are not sensitive to using different measures of the lower bound of the wage distribution i.e. the actual minimum, or different wage percentiles - see Appendix C.

that this means my estimates of the variance parameter of the match quality sampling distribution increases in relative terms for skilled workers, which leads to a relative increase in the cross sectional average of their match quality.[10]

4. Estimation Approach

As with my exposition of the model, I will first present the original estimation approach used by Krusell et al. (2000), i.e. under perfect competition and with no intermediate goods sectors. I then set-out a two stage strategy for estimating the KORV parameters in the context of my model. The first stage is to estimate the parameters of the sequential auction model in the intermediate goods markets. The second stage is to incorporate results from the previous step to estimate the parameters of the KORV production function in the final good sector.

4.1. **KORV Estimation: Without Frictions.** Krusell et al. (2000) estimate their model by simulated pseudo maximum likelihood (SPML), targeting the model's predictions for the labor share of output and the wage bill ratio of skilled workers relative to unskilled workers, denoted lsh_t and wbr_t respectively, to their empirical counterparts.[11] Both of these model moments come from the first order conditions of the final good firm's profit maximisation condition. In addition, Krusell et al. (2000) impose a no arbitrage condition between capital structures and equipment, i.e. their empirical strategy aims to minimise the difference between the model's predictions for the rate-of-return (RoR) on capital structures and the predicted RoR for capital equipment, alongside the other empirical targets mentioned above.[12] This estimation strategy is summarised in equations (23), (24) and (25) respectively, where X_t is the set of factor inputs $(K_{st,t}, K_{eq,t}, U_t, S_t)$, $(\kappa_{eq},\kappa_{st})$ are the depreciation rates for capital equipment and structures respectively, and ϕ is the vector of all parameters to be estimated.

[10]This is a function of job to job contact rates that significantly exceed job destruction rates in all years, which means that $L_i(\nu)$ will generally first order stochastically dominate $F_i(\nu)$. This in turn means a mean preserving increase in the spread of the $F_i(\nu)$, which is implied by my estimation strategy of matching the increasing dispersion of skilled workers wages while keeping the mean of the sampling distribution constant, will lead to a increase in the cross section mean of match quality.

[11]SPML is generally attributed to Laroque and Salanie (1993) and is used when a closed form solution for the exact likelihood or quasi likelihood are both unavailable. Just as MLE can be viewed as a specific form of GMM (where the expectation of the score is the relevant moment), so SPML can be viewed as specific form of SMM where I am taking the expectation of a set of moments across both simulations and across time.

[12]This is done as neither RoR is directly observable in the data.

$$\frac{w_{u,t}h_{u,t} + w_{s,t}h_{s,t}}{Y_t} = lsh_t(X_t, \psi_t; \phi) \tag{23}$$

$$\frac{w_{s,t}h_{s,t}}{w_{u,t}h_{u,t}} = wbr_t(X_t, \psi_t; \phi) \tag{24}$$

$$0 = (1 - \kappa_{st}) + A_{t+1}G_{K_{s,t}}((X_t, \psi_t; \phi) - E_t(\frac{q_t}{q_{t+1}})(1 - \kappa_{eq}) -$$

$$q_t A_{t+1} G_{k_{eq,t}}((X_t, \psi_t; \phi) \tag{25}$$

Equations (23), (24) and (25) can be represented in vector form as $Z_t = f(X_t, \psi_t, \epsilon_t; \phi)$, where Z_t is a vector of the empirical moments on the left hand side of equations (23), (24) and (25) and $f(X_t, \psi_t, \epsilon_t; \phi)$ is a vector of the model moments on the right hand side of these equations.

Note that there are two stochastic elements in this system of estimation equations. First ψ_t is a (2×1) vector of the log of the efficiency levels of unskilled and skilled labor respectively, and follows a stationary process in KORV's benchmark estimation as set out in equation (26).

$$\psi_t = ln(\Psi_t), \psi_t = \psi_0 + \omega_t \text{ and } \Psi_t \equiv (\Psi_{u,t}, \Psi_{s,t}) \tag{26}$$

ω_t is a vector shock process to the log of labor efficiency that is assumed to be multivariate normal and *iid* with covariance matrix Ω i.e. $\omega \overset{i.i.d}{\sim} N(0, \Omega)$, and ψ_0 is a vector of the log of intial values of unskilled and skilled labor efficiency $(\psi_{0,u}, \psi_{0,s})$. In the benchmark estimation, the authors impose that there is no covariance between the two labor efficiency shocks and that they have a common variance so Ω can be rewritten as $\Omega = \eta_\omega^2 I$.[13]

The other stochastic process in this estimation procedure is in the no arbitrage condition, equation (25), where the third term on the right hand side of this equation $E_t(\frac{q_t}{q_{t+1}})(1-\kappa_{eq})$ is the undepreciated capital equipment multiplied by the expected rate of change in the relative price of equipment. Krusell et al. (2000) make the simplifying assumption that this term can be replaced with $\frac{q_t}{q_{t+1}}(1 - \kappa_{eq}) + \epsilon_t$, where $\epsilon_t \sim N(0, \eta_{\epsilon_t})$.

[13]In fact, as a robustness check, Krusell et al. (2000) do allow for a non-zero covariance between the two efficiency shocks and differing variances, but the estimated covariance is very small and there is little difference between the estimated variances so they opt for a benchmark estimation with zero covariance and a common variance.

In principle, the vector of parameters to be estimated, ϕ, contains 11 elements :$\{\kappa_{st}, \kappa_{eq}, \alpha, \mu, \lambda, \sigma, \gamma, \eta_\varepsilon, \eta_\omega, \psi_{0,u}, \psi_{0,s}\}$. However the authors calibrate $(\kappa_{st}, \kappa_{eq})$ using estimates from the literature, estimate η_ε separately , and normalize $\psi_{0,s} = 0$.[14] [15] This leaves $\phi = \{\alpha, \mu, \lambda, \sigma, \gamma, \eta_\omega, \psi_{0,u}\}$ to be estimated i.e seven parameters: given the are targeting three moments for each year of their 30 year dataset, the model is over-identified.

Finally, the authors construct an instrument for hours worked, $\hat{h}_{u,t}, \hat{h}_{s,t}$ to allow for potential endogeneity between relative hours worked and relative wages. While such endogoneity would be irrelevant if the sole goal was to match the model to the data, I presume the authors employ this strategy so that they can more credibly give the parameters economic interpretations, i.e. as elasticities of substitution, and hence use the model for counter-factual analysis. The exogenous factor inputs used in model estimation are therefore $\hat{X}_t = (K_{st,t}, K_{eq,t}, \hat{h}_{u,t}, \hat{h}_{s,t})$. Estimation then proceeds in three steps:

(1) Draw S values of the vector of shocks to labor efficiency, ω_t^j, and of the forecast error in expected price gains of capital equipment, ϵ_t^j,(where j indexes the realization of the shock) to get S realizations of $f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi)$ from the model for each time period t,

(2) Use these S realizations to obtain the following moments:

$$m_s(\hat{X}_t, \phi) = \frac{1}{S}\sum_{i=1}^{S} f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi)$$

$$V_s(\hat{X}_t, \phi) = \frac{1}{S-1}\sum_{i=1}^{S}(f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi) - m_s(\hat{X}_t, \phi))(f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi) - m_s(\hat{X}_t, \phi))'$$

(3) Minimise the following objective function:

$$\begin{aligned} l_s(\hat{X}_t, \phi) = & \frac{1}{2T}\sum_{t=1}^{T}\Big\{(Z_t - m_s(\hat{X}_t, \phi))' V_s(\hat{X}_t, \phi) \\ & \times (Z_t - m_s(\hat{X}_t, \phi)) + ln(det(V_s(\hat{X}_t, \phi)))\Big\} \end{aligned} \tag{27}$$

In a companion paper to Krusell et al. (2000), Ohanian et al. (1997) look at how successfully the estimation approach above identifies the true parameters of the model in Monte Carlo simulations, and find very small median and mean biases in estimators even when using relatively few simulations in estimation i.e. for $S = 10$. They find that for $S = 50$ the mean bias is “essentially zero”.

[14]The authors set $\kappa_{eq} = 0.125$ and $\kappa_{st} = 0.05$ following Greenwood et al. (1997).
[15]The authors estimate η_ε via an ARMA regression of q_t.

4.2. **Incorporating Frictions into KORV Estimation.** I proceed in two steps to incorporate the sequential auction model of Cahuc et al. (2006) into the KORV production set-up. First I separately estimate the parameters of the sequential auction model, which include job contact rates for employed workers of each skill type $\lambda_{1,i,t}$ and the parameters of their match distribution. Appendix B examines identification of these parameters in greater detail, proving exact identification of the job contact rates using the empirical strategy outlined here and showing evidence from Monte Carlo simulations that my strategy for estimating the parameters of the match quality distribution also successfully identifies the true parameters of the model.

In the second part of my estimation approach, I estimate the parameters of the KORV production function in a way the incorporates the changes to labor market frictions implied by my estimation of the sequential auction model of the intermediate goods sectors. This step is, in econometric terms, a minor modification of the original approach of Krusell et al. (2000), as presented above, that uses two key outputs from the sequential auction model: the average match quality and wage of each skill type which are both identified up to a common scaling factor, which is the price of the intermediate good produced in a given skill sector. The rest of this section describes each of these steps in greater detail.

Sequential Auction Estimation: Job Contact Rates

The monthly job contact rate for employees, $\lambda_{1,i,t}$, is chosen so that the model matches the empirical proportion of individuals continuously employed in a year who have more than one employer (denoted $\tau_{i,t}$).This moment is given in the model by equation (28)

$$\tau_{i,t} = 1 - \int_{\nu_{inf_{i,t}}}^{\nu_{max}} (1 - \lambda_{1,i,t}\bar{F}_{i,t}(\nu))^{12} dL_{i,t}(\nu) \tag{28}$$

In Appendix B, I show that this expression is independent of the match quality distribution - this can be seen by change of variable in the integration - meaning I can estimate job contact rates separately of distributional parameters. The expression is also an increasing monotonic function of $\lambda_{1.i}$ which implies this parameter is indeed identified when I estimate it by simulated method of moments, as set out in equation (29) (where $\hat{x}$ denotes the empirical counterpart of model moment x).

$$\lambda^*_{1,i,t} = \underset{\lambda_{1,i,t}}{\operatorname{argmin}} \big(\tau_t(\lambda_{1,i,t}) - \hat{\tau}_t\big)^2 \tag{29}$$

Sequential Auction Estimation: Distribution of Match Heterogeneity

I assume that sampling distribution of match heterogeneity can be characterised by a lower truncated log normal distribution, and therefore can be fully described by three parameters: the mean and variance parameters, $\zeta^{\nu}_{i,t}$, $\eta^{\nu}_{i,t}$, and lower truncation point, $\nu_{inf_{i,t}}$. Note that by estimating the lower bounds directly I bypass the need to estimate job contact rates for the unemployed or replacement rates. This follows because my principal interest interest is to estimate the distribution of wages and match quality for workers in the intermediate goods market; unemployment conditions influence these variables through the lower bound of the match quality distribution only.

Given I only have data on employees, and not employers, a natural option to estimate $\zeta^{\nu}_{i,t}$ and $\eta^{\nu}_{i,t}$ is to use moments of the wage distribution for workers of each skill type $i \in u, s$. Note, however, that all wages of a given skill type are scaled by the price of the intermediate good, p_i (see equation (16)), which depends on the parameters of the KORV production function that I have yet to estimate. I therefore require the moment of the wage distribution that I will target to be scale invariant, and so choose the variance of log wages.

As both $\zeta^{\nu}_{i,t}$ and $\eta^{\nu}_{i,t}$ have a positive monotonic impact on match quality dispersion in the model, they will not be separately identified using the variance of log wages. I therefore set the value of $\zeta^{\nu}_{i,t}$ to target the mean of the sampling distribution, $\mathbb{E}^{F_{i,t}}(\nu)$, to an arbitrary fixed value ($= 1$). Note that this also avoids introducing a "black-box" source of skills biased technological change via an increase in the relative means of the sampling distribution of match quality $\mathbb{E}^{F_{s,t}}(\nu)/\mathbb{E}^{F_{u,t}}(\nu)$ (Krusell et al. (2000) impose that the relative labor efficiency of skilled to unskilled workers is constant for the same reason) but doesn't rule out an endogenous increase in the mean of the cross section distribution of match quality $\mathbb{E}^{L_{i,t}}(\nu)$. The variance parameter of the sampling distribution of match quality, $\eta^{\nu}_{i,t}$, is therefore left free to match the dispersion of log wages within a skill type i in the model to it's empirical counterpart.

Finally I must estimate the lower bound of the distribution of match quality, $\nu_{inf_{i,t}}$. Provided the bargaining parameter is sufficiently high, a worker at a match of quality $\nu = \nu_{inf_{i,t}}$ will earn the lowest wage in the model's wage distribution, denoted $\underline{w}_{i,t}$, where $\underline{w}_{i,t} = \nu_{inf_{i,t}} \times p_{i,t}$.[16] Since all wages are scaled by the price

[16] In the model, the minimum wage in the population of workers will be paid to workers with match quality equal to the lower bound of the match distribution in the model when $\beta > \frac{\lambda_1}{\rho+\delta+two\lambda_1}$. This condition is derived from observing first that the wage expression in equation (16) is always decreasing in ν^-, so the lowest wage observable wage will certainly belong to those who have come from unemployment i.e. who have $\nu^- = \nu_{inf}$. Such workers will have a wage precisely equal to ν_{inf} when they are matched with the lowest match quality firms i.e. $\nu^+ = \nu^- = \nu_{inf}$. Finally a sufficient condition for this to be the lowest wage in the

of the intermediate good, $p_{i,t}$, which will not be estimated at this stage, rather than target the lower bound of the wage distribution I target the ratio of the lower bound to the median wage: $\underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}})$. When it comes to the empirical counterpart of this moment, I choose to use the average wages of workers in the bottom two percentiles of the wage distribution (again relative to the median) rather than the lower bound of the empirical wage distribution as this is likely to subject to significant measurement error.

In summary, I estimate the parameters of the sampling distribution, $\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$, by solving the minimisation problem shown in equation (30), where $\hat{x}$ denotes the empirical counterpart of model moment x, and W is the weighting matrix.[17]

$$(\zeta^{\nu*}_{i,t}, \eta^{\nu*}_{i,t}, \nu^{*}_{inf_{i,t}}) = \underset{\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}}{\operatorname{argmin}} \; (m_t - \hat{m}_t)^T W (m_t - \hat{m}_t) \tag{30}$$

$$m_t \equiv \Big(var_{\log(w_{i,t})}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}), \underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}), \mathbb{E}^{F_{i,t}}(\nu)(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}) \Big)$$

I calculate the moments of the wage distribution in the model from a given guess for parameters by generating a sample of workers using the cross section distributions in equations (18) and (19) to give the match quality of the workers' employers and outside options, and then using equation (16) to then generate the wages of these workers.

Sequential Auction Estimation: Other Parameters to be Calibrated

In the absence of matched employee and employer data, I set the bargaining parameter to $\beta = 0.95$. I find lower levels of the bargaining parameters mean the model struggles to hit the level of the labor share and rise in the graduate wage premium seen in the data. This occurs because the sequential auction part of the model sets an upper bound on the labor share in the overall model, since incorporating a final goods sector with the KORV production function will always decrease the labor share, relative to its level in the intermediate goods sector, due to the presence of capital inputs. The upper bound on the labor share implied by the sequential auction results may be close to or even below the empirical labor share that I am targeting if the bargaining parameter is set too low. This issue is explored quantitatively in Appendix A. Although this calibrated bargaining parameter value appears high compared to some results in the micro literature, for

population provided the derivative of the wage expression is positive for this worker and the second derivative is always positive. The latter condition always holds, and former holds when β is greater than the threshold shown above.

[17]The weighting matrix W, is chosen so I effectively minimise the percentage deviation of model moments from their empirical moments, which avoids the scale of absolute moment deviations biasing estimates i.e. $W = I.\frac{1}{\hat{m}}$.

example Cahuc et al. (2006), many of these estimates come from structural models that do not feature capital and so are not directly comparable to ours. Finally, I arbitrarily set the monthly discount rate to 0.004.

Adding Sequential Auction Results to KORV Estimation

I adopt essentially the same empirical approach as Krusell et al. (2000) i.e matching the models predictions for the evolution of the graduate wage premium and labor share to their empirical counterparts and imposing a zero rate-of-return (RoR) difference between capital structure and capital equipment in the model. However, I make two key modifications to incorporate results from the sequential auction estimation.

The first modification comes about because the unskilled and skilled labor inputs are not simply hours worked by the two types but rather the amount of intermediate goods from each skill type sector. The inputs U_t and S_t therefore become as defined in equation (9) where I multiply the labor inputs that KORV use (total hours in efficiency units) by the average match quality in each skill sector, which are derived from estimation of the sequential auction part of my model. I denote estimated average match equality as $\mathbb{E}^{\hat{L}_{i,t}}(\nu) \equiv \int_{\hat{\nu_{inf}}_{i,t}}^{\nu_{max}} \nu \hat{\ell_{i,t}}(\nu)$ (where hats denote estimated variables/parameters).

Second average wages for a given skill type i are no longer simply the marginal product of that skill type in production of the final good, but determined as specified in equation (22). I decompose this expression into two parts, as shown below.

$$\begin{aligned}
\mathbb{E}^{L_{i,t}}(w_{i,t}) \quad &= p_{i,t} \times \mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1) \qquad (31)\\
\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1) \equiv &\int_{\underline{\nu}}^{\nu_{max}} \Bigg[\nu - [1 + \kappa_{1,i}\bar{F}(\nu)]^2 \times \\
&\int_{\nu_{inf}}^{\nu} \frac{(1-\beta)[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\bar{F}(x)]}{[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\beta\bar{F}(x)][1 + \kappa_{1,i}\bar{F}(x)]^2} dx \Bigg] \ell_i(\nu) d\nu
\end{aligned}$$

Thus average wages are calculated by multiplying the price of the intermediate good $p_{i,t}$ (which equals its marginal product in the production of the final good) by the average wages in the intermediate good sector when the price of the intermediate good is normalised to one $E(w_{i,t}, p_{i,t} = 1)$. I estimate $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$ and $\mathbb{E}^{L_{i,t}}(\nu)$ by using estimation results from the sequential auction part of my model. Specifically, I simulate a sample of workers, using equations (18) and (19) to generate the match quality of these workers' employers and outside options and

Table 1. Parameter Estimates: KORV vs Replication

Parameter	KORV findings	Replication Estimates
α	0.117	0.121
γ	-0.495	-0.459
σ	0.401	0.39

equation (16) to generate their wages, and then taking the average wage and match quality of the simulated sample of workers.

A short hand way of stating the points made above is that, to adapt KORV's original methodology to including an intermediate goods sector with a sequential auction labor market, I scale the labor input of skill type i used in KORV by a 'productivity scale' which is my estimate of $\mathbb{E}^{L_{i,t}}(\nu)$ and I calculate average wages by multiplying the marginal product of a given intermediate good in the KORV production function by a 'wage scale' that is my estimate of $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$. Otherwise, estimation of the parameters of the KORV production function proceeds exactly as described in section 4.1.

5. Results

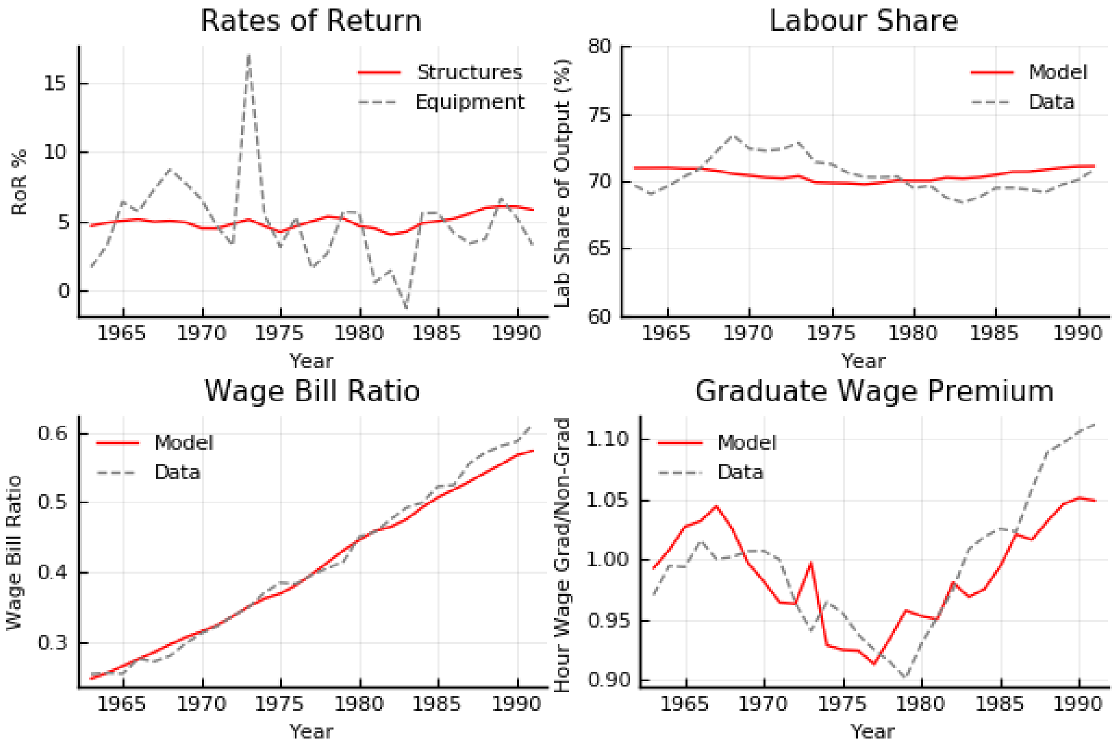
This section starts by verifying that I can replicate the results provided in Krusell et al. (2000) when I use their estimation strategy and data. I then show results from estimation of the sequential auction model of the intermediate goods sector, and finally I show the impact of incorporating search frictions into the KORV production process. When considering this impact my focus will be on how, if at all, estimates of capital skill complementarity change and how that changes explanations for the rises in the graduate wage premium.

5.1. **Replication of KORV methodology.** I am able to replicate results from KORV both in terms of fit to the author provided data - see Figure 7 for my fit to the data and Figure 8 for the equivalent Figure in Krusell et al. (2000) - and parameter estimates - see Table 1.[18] When I include more recent data in my replication of the KORV methodology, rather than using only their original sample period of 1963-1992, I find the model again fits the data well - see Figure 9.[19] Table 2 shows inclusion of more recent data decreases estimates of capital skill

[18]Note that KORV provide estimates of α, σ, γ only so I focus on these parameters in Table 1.

[19]I have to switch from author provided data to publicly available data to extend the time period, which is why the parameter estimates for the original sample period shown in Table 2 differ from those in Table 1.

FIGURE 7. Replication of KORV



complementarity (as captured by the difference in the elasticity of substitution between unskilled labor and capital equipment, denoted $\varepsilon_{U,K_{eq}}$, and the elasticity of substitution between skilled labor and capital equipment, denoted $\varepsilon_{S,K_{eq}}$).[20] An explanation for the reduction in estimated capital skill complementarity could be that the growth in the graduate wage premium remains steady after 1992, despite an sharp acceleration in capital equipment growth (see Figure 1); to reconcile these two patterns the model requires a lower estimate of capital skill complementarity than in the original sample period.

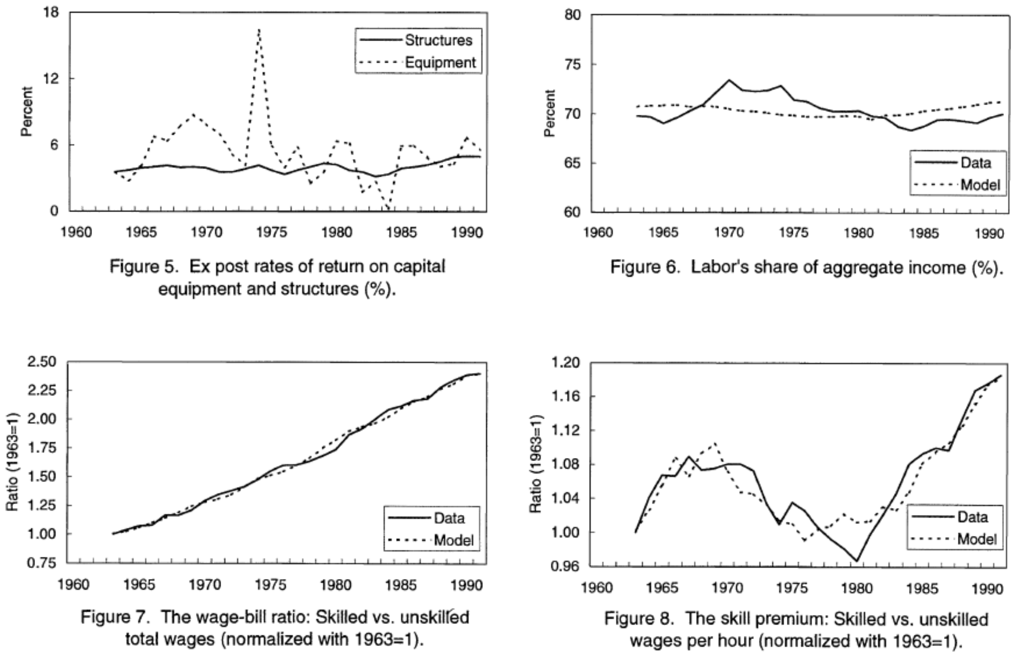
5.2. **Sequential Auction Results.**

Sequential Auction Results: Job Contact Rates

The first row of Figure 10 shows my estimates of job contact rates for unskilled and skilled workers, in absolute and relative terms (I plot a six year rolling average of estimated relative contact rates to emphasise the trend). The second row of the same figure shows the empirical targets these estimates are based on - the

[20] $\varepsilon_{U,K_{eq}} \equiv \frac{1}{1-\sigma}$, $\varepsilon_{S,K_{eq}} \equiv \frac{1}{1-\gamma}$.

FIGURE 8. KORV's original fit



Notes: I am unable to directly provide model predictions using KORV's parameter estimates, as the author's only provide a subset of the relevant estimates, so I directly reproduce the figure from Krusell et al. (2000) showing the fit of their model to the data.

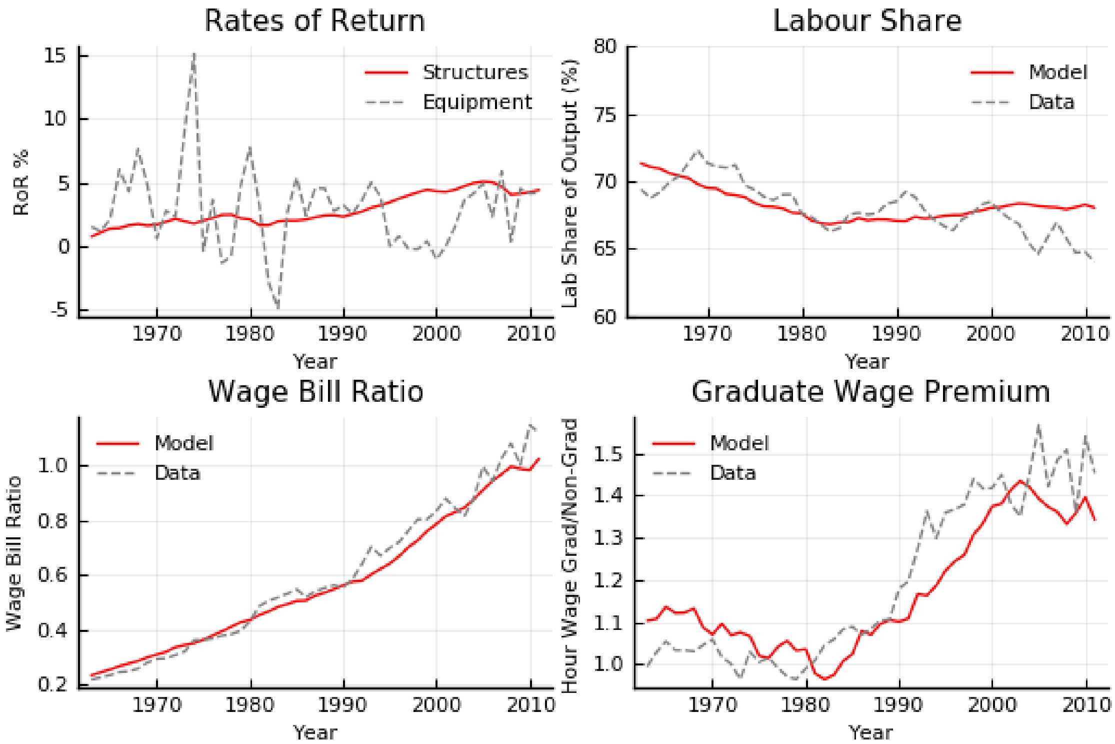
TABLE 2. Parameter Values with Extended Sample Period

Parameter	**Original Sample Period**	**Extended Sample Period**
λ	0.925	0.92
μ	0.767	0.866
α	0.099	0.094
γ	-0.477	-0.313
σ	0.454	0.439
$\varepsilon_{S,K_{eq}}$	0.677	0.762
$\varepsilon_{U,K_{eq}}$	1.833	1.781
CSC Strength: $\boldsymbol{\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}}$	**1.156**	**1.02**

proportion of individuals with more than one (non concurrent) employer in a year ("multiple employer rate") - and the corresponding model moments.

I am able to exactly match the model moments to their empirical counterparts. Estimated job contact rates do not exactly track the data on multiple employer

FIGURE 9. Replication of KORV: Extended Sample Period



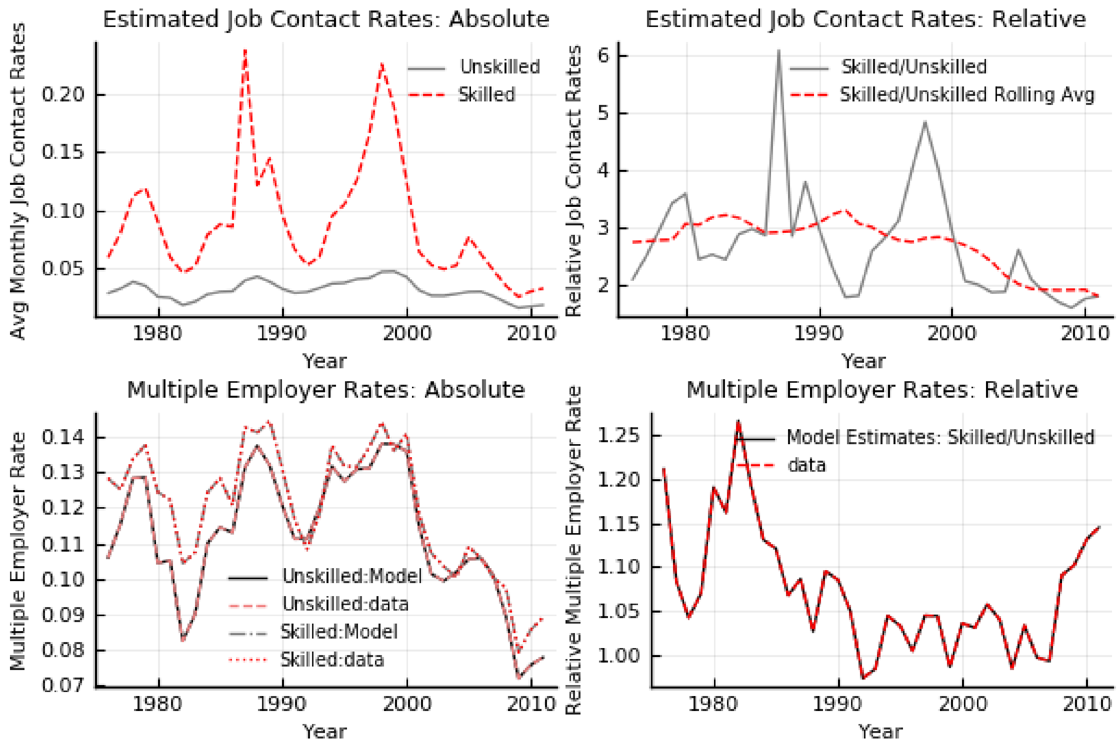
rates because job contact rates are not the sole determinant of the multiple employer rate: job destruction also plays a role, as shown in equation (28). The intuition here is that workers who exit and enter the labor market more frequently will spend more time at the bottom of the job ladder and hence move employers more often.

However, overall the trend is for estimated job contact rates for skilled workers to decrease over time relative to those of unskilled workers, which mirrors the trend in the multiple employer rate.

Sequential Auction Results: Distribution of Match Quality

For each of my two skill types, I estimate three parameters of the match quality distribution, which is assumed to take a truncated log normal form: the mean, variance and lower bound parameters, $\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ respectively. I am able to match the model to the targeted empirical moments precisely in the case of both $\eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ where the relevant targets are log wage variance and the ratio of average wages of workers in the bottom two percentiles of the wage distribution to median wages respectively - the top row of figures 11 and 12 show parameter

FIGURE 10. Job Mobility

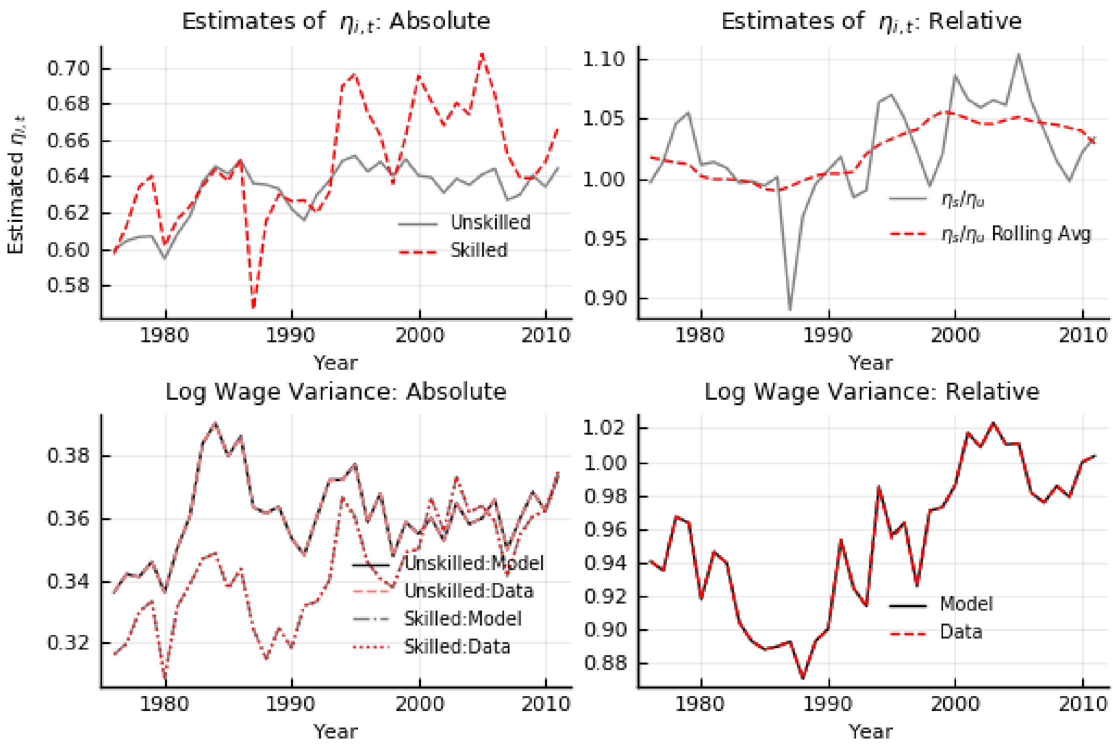


estimates for $\eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ respectively, and the bottom rows show the close fit of model moments to the data. Estimates of $\zeta^{\nu}_{i,t}$ are in a sense less relevant since they are simply set at the level necessary to keep the mean of the sampling distribution of match quality constant at an arbitrary target ($\mathbb{E}^{F_{i,t}}(\nu) = 1$).

The estimated variance parameter of the sampling distribution of match quality, $\hat{\eta}^{\nu}_{i,t}$, for skilled workers increases over time relative to the equivalent parameter for unskilled workers, mirroring changes in the empirical target (residual log wage variance). Estimates of the lower bound of the match quality distribution, $\hat{\nu}_{inf_{i,t}}$ decrease in relative terms for skilled workers, again mirroring the trend in the empirical target.

5.3. **Impact of Sequential Auction Estimates on KORV results.** As argued in section 4.2, the results of my estimation of the sequential auction structure of the intermeidate goods market can be fully characterised by two series for the purposes of estimating the parameters of the KORV production function. The first series is the 'productivity scale', which is my estimate of the average match quality by skill $\mathbb{E}^{L_{i,t}}(\nu)$ that I use to scale labor inputs. The second series is

FIGURE 11. Match Quality Dispersion



the'wage scale', $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$, which relates to average wages of skill type i via the identity $\mathbb{E}^{L_{i,t}}(w_{i,t}) = p_{i,t}\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$. These series are plotted in absolute and relative terms in Figure 13, with a rolling 6 year average of the relative series added to emphasise the relevant trends.

Figure 13 shows that the presence of search frictions can explain a positive skill premium since the relative wage scale is estimated to be consistently above one. Reflecting trends in job contact rates, the relative productivity and wage scaling factors of the high skill workers are increasing, albeit very mildly, until around the early 1990s, but then decrease after this. However, this trend is not strong enough to significantly change estimates of the parameters in the KORV production function as shown in Table 3. In particular, the estimate of capital skill complementarity (the difference between the elasticity of substitution between unskilled labor and capital equipment and skilled labor and capital equipment, $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$ is very similar. The model with frictions seems to fit the data slightly less accurately than the original KORV formulation as shown in Figure 14.

FIGURE 12. Match Quality Lower Bound

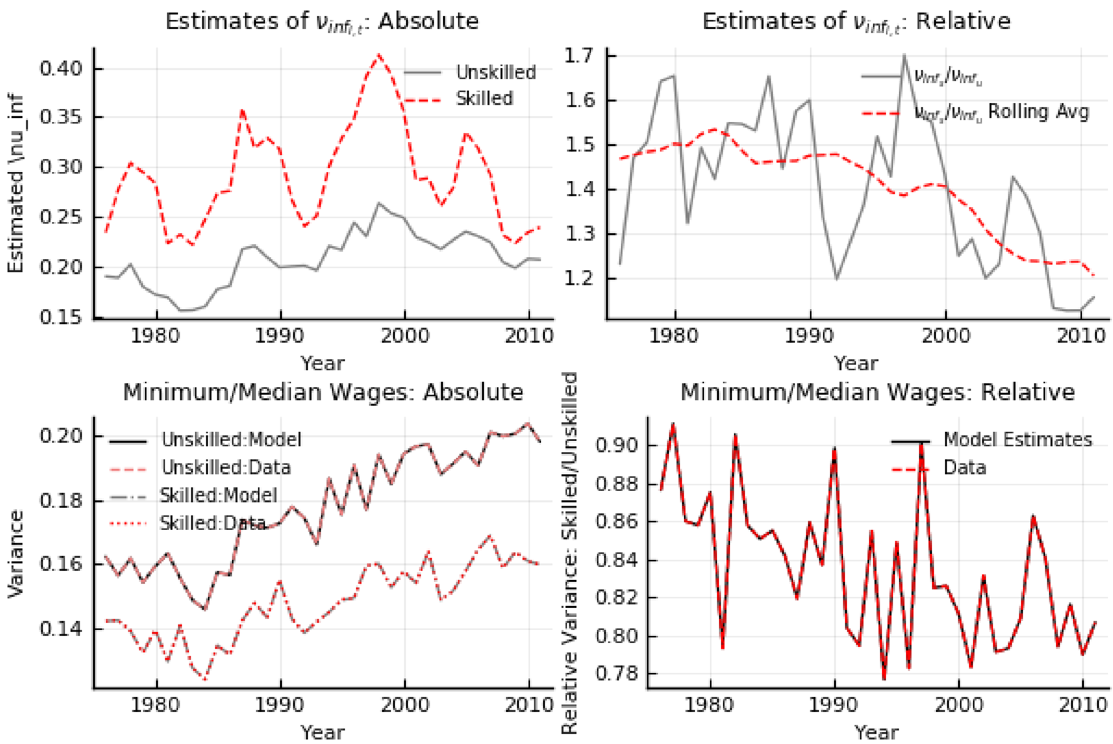
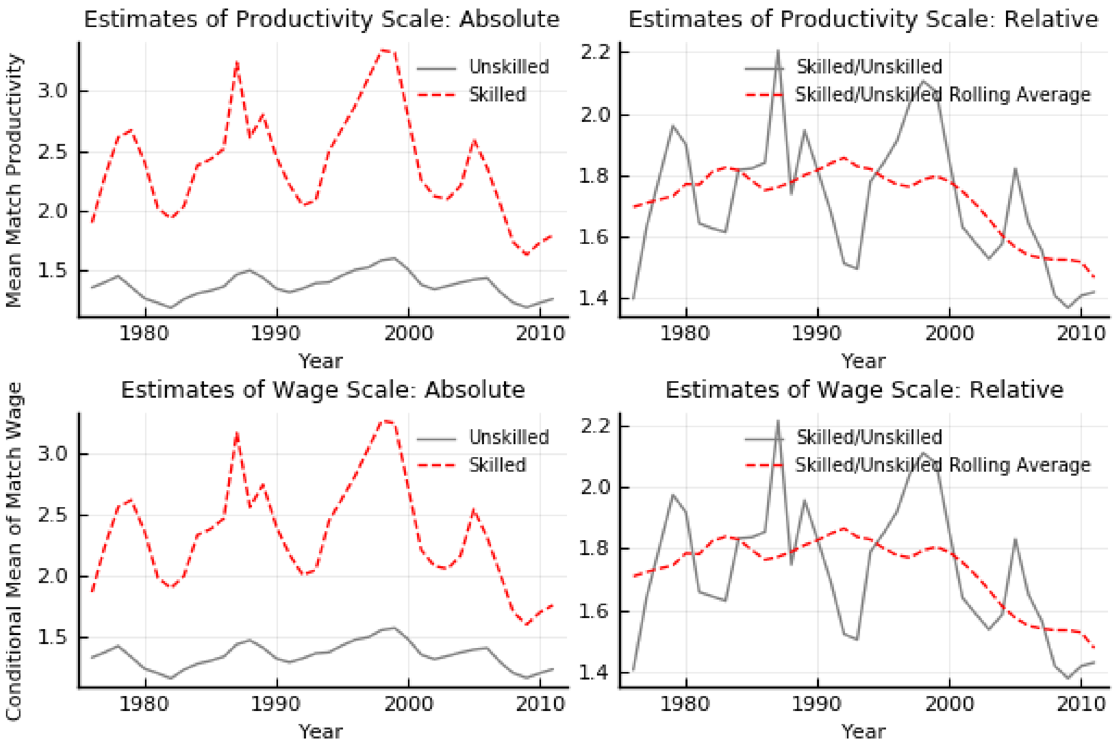


TABLE 3. KORV parameter values: the importance of frictions

Parameter	With Frictions	Without Frictions
λ	0.505	0.568
μ	0.833	0.806
α	0.083	0.091
γ	-0.186	-0.209
σ	0.329	0.352
Elas. of Subs. btw S and K_{eq}, $\varepsilon_{S,K_{eq}}$ $(= 1/1-\gamma))$	0.843	0.827
Elas. of Subs. btw U and K_{eq}, $\varepsilon_{U,K_{eq}}(= 1/1-\sigma))$	1.489	1.544
CSC Strength: $\boldsymbol{\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}}$	**0.646**	**0.716**

The model with frictions is still entirely reliant on the capital skill complementarity channel to generate an increase in the graduate wage premium, as can be seen by examining model predictions when I shut down this channel by imposing $\sigma = \gamma$: Figure 15 shows that both the model with frictions and without in fact predict large falls in the graduate wage premium, due to the increase in the relative supply of graduates, when there is no capital skill complementarity.

FIGURE 13. Scale Factors



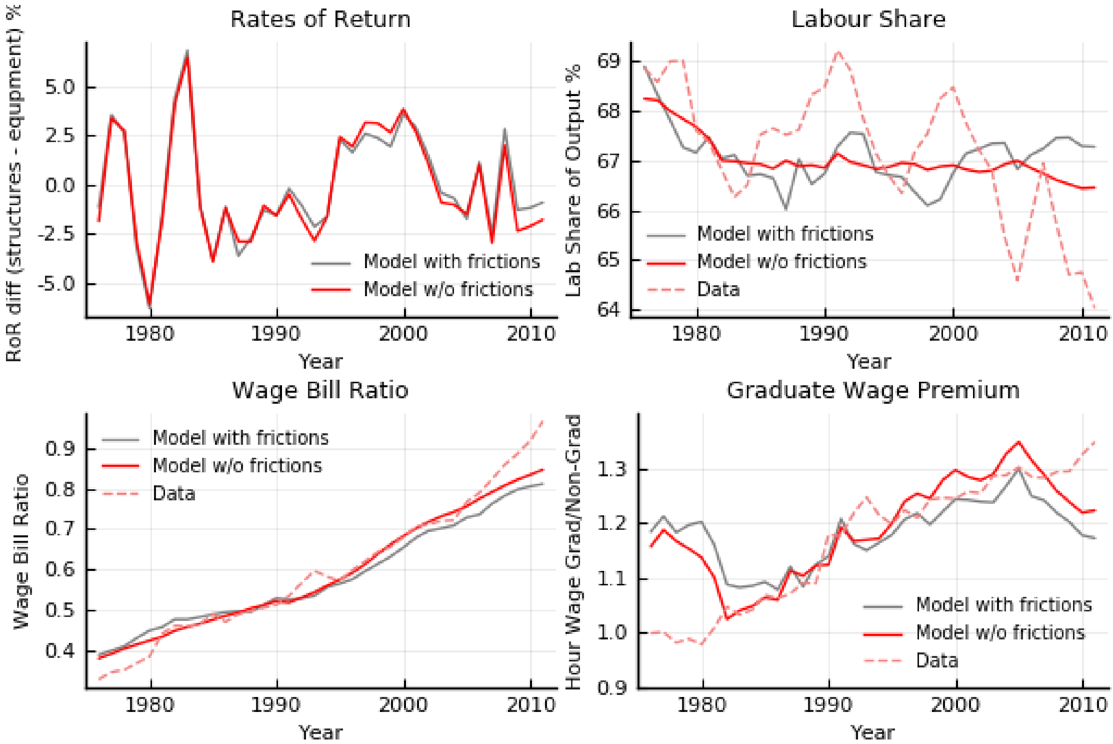
6. Conclusion

I developed an empirically testable model that combines the production framework specified in Krusell et al. (2000) with the sequential auction model of Cahuc et al. (2006). This model has the potential to identify the contribution of institutions, frictions and technology to growing wage inequality, though I focus on the latter two dimensions in this paper.

The empirical contribution of this paper is to examine whether adding a sequential auction labor market structure to Krusell et al. (2000) materially changes estimates of capital-skill complementarity. If I maintain consistency with Krusell et al. (2000) by using CPS labor market data only i.e. where no employer side information is used, I find that estimates of capital-skill complementarity are not significantly changed by allowing for labor market frictions.

This reflects the fact that my empirical measures of job market frictions, taken as a whole, do not move decisively in favour of either skilled or unskilled workers. Contrary to my expectations, both measures of job mobility rates and job destruction rates move in favour of unskilled workers relative to skilled workers, however, this is partly offset by an increase in estimated match quality dispersion for skilled

FIGURE 14. Model Fit: With and Without Frictions

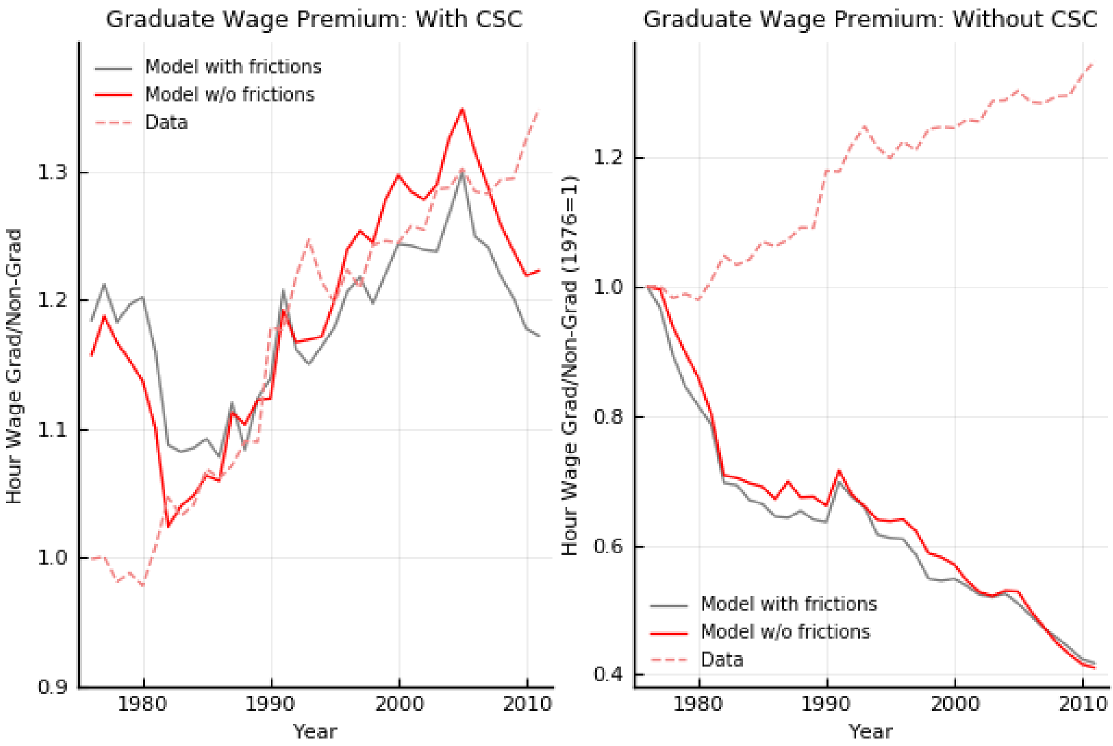


workers which means climbing the job ladder brings greater rewards and boosts their average pay.

To maintain consistency and comparability with Krusell et al. (2000) I do not use any direct data on firm heterogeneity, which limits my ability to identify changes to the distribution of match heterogeneity and also constrains identification of bargaining parameters, which could be one way to capture changes to the institutional environment. Adapting this framework to matched employer employee data is therefore a promising line of future research.

I also make the simplifying assumption that job contact rates are exogenous in my model, in keeping with Cahuc et al. (2006). However, this means changes in the use of capital, and hence the relative demand for skilled labor, have no impact on the job market frictions these workers face. Adding endogenous vacancy creation to the model, while representing a significant theoretical and empirical challenge, would therefore shed light on the links and feedback mechanisms between wage inequality, technology and labor market frictions.

FIGURE 15. Model Fit: No Capital Skill Complementarity



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Appendix A. Model Results and Bargaining Power

This appendix discusses the sensitivity of my results to the choice of the bargaining parameter. In my baseline estimation I impose a high level of bargaining power for both worker types ($\beta_u = \beta_s = 0.95$). I find that when I set the bargaining parameter at significantly lower levels, i.e. 0.75 or 0.5, and estimate the parameters of the KORV production function there is an acute tension between the model's ability to match both the rise in the graduate wage premium and the level of the labor share of output. The rest of this appendix explains this tension and its quantitative impact. Overall I find that only when I assume a relatively high bargaining parameter are I able to satisfactorily match the relevant trends in the data.

I first consider the intuition for why there might be a tension between matching the rise in the graduate wage premium and level of the labor share at lower levels of the bargaining parameter. First recall that the original, competitive, version of the KORV model is relatively successful at matching both the rise in the graduate wage premium and the labor share: see Figure 16. When I introduce the sequential auction model into this set-up, average wages will now be lower than the marginal product of labor if the bargaining parameter is significantly less than unity and for realistic job contact rates. In other words, the labor share will be lower in the model with frictions than in the original KORV environment for a given set of production function parameters. When I estimate the KORV parameters in my frictional labor market model, and have a low level of bargaining power, the estimation approach compensates for the downwards pressure this puts on the labor share by making labor more important (and capital less important) in the production of output. However, this jeapordises the ability of the model to match the graduate wage premium since the increased use of capital equipment is the main channel that pushes the wage premium up.

To illustrate this quantitative impact of this tension, let us consider estimates of the KORV production function parameters when I set the bargaining parameter to 0.5 for both unskilled and skilled workers - see Table 4 and Figure 17. The estimate of α, the exponent of capital structures (K_{st}), hits the zero lower bound, and it also delivers lower levels of λ, the coefficient of capital equipment since this too increases the labor share. However a lower level of λ limits the channel of capital skill complementarity - see

FIGURE 16. KORV with perfect competition

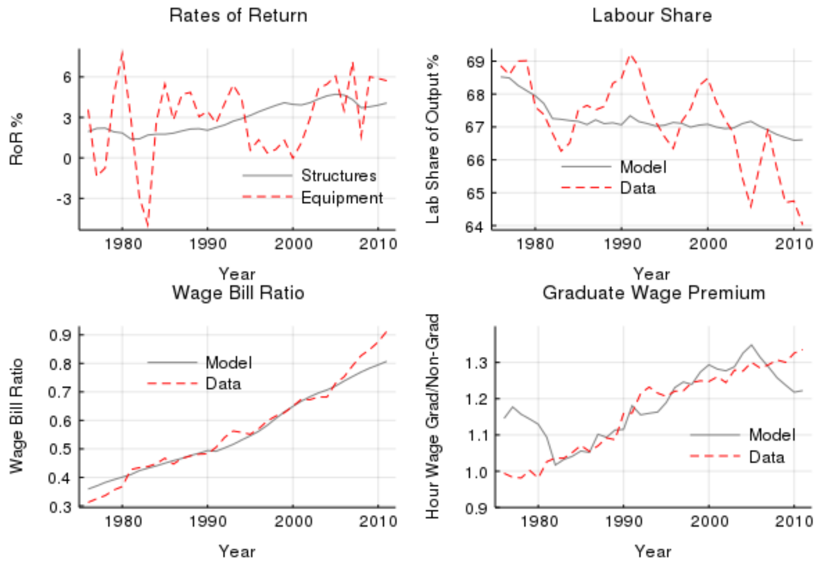


TABLE 4. KORV parameter values: bargaining parameter impact

Parameter	No Frictions (KORV)	Baseline ($\beta = 0.95$)	$\beta = 0.5$
λ	0.568	0.507	0.17
μ	0.806	0.644	0.401
α	0.091	0.083	0.0
γ	-0.209	-0.188	0.043
σ	0.352	0.33	-0.166

equation (8) - and means that though the model can fit the labor share to a reasonable approximation, it completely misses the rise in GWP: see Figure 17. Indeed the fit is much worse than that of the purely competitive set-up in KORV: see Figure 16.

Increasing the bargaining parameter from 0.5 (Figure 17 is based on this) to 0.95 improves the results significantly - see Figure 18. While much of the micro evidence points to much lower levels of the bargaining parameter, generally such estimates are highly model dependent.

APPENDIX B. IDENTIFICATION

There are two sets of parameters to identify in my model: the parameters of the KORV production function, and those in the sequential auction model of the labor market. While Krusell et al. (2000) do not explicitly discuss identification in their paper, they do refer to the results of a companion empirical paper Ohanian et al. (1997) which shows

FIGURE 17. KORV with frictions: $\beta = 0.5$

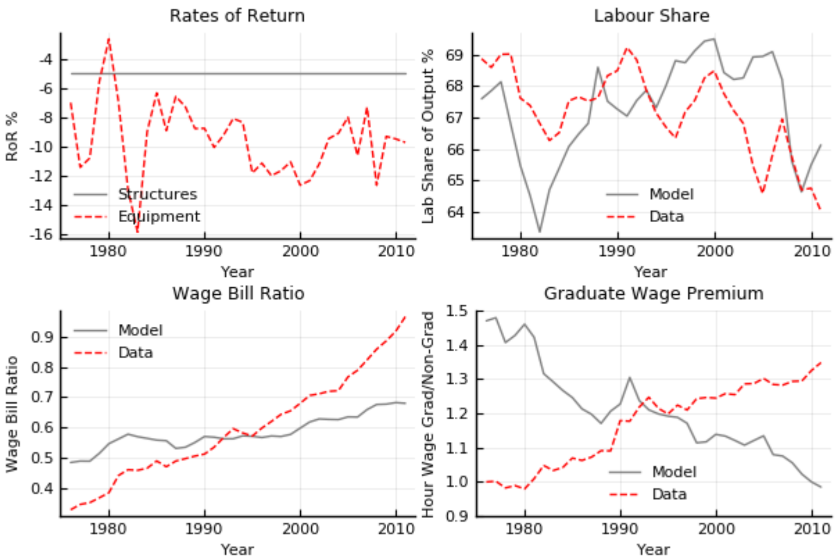
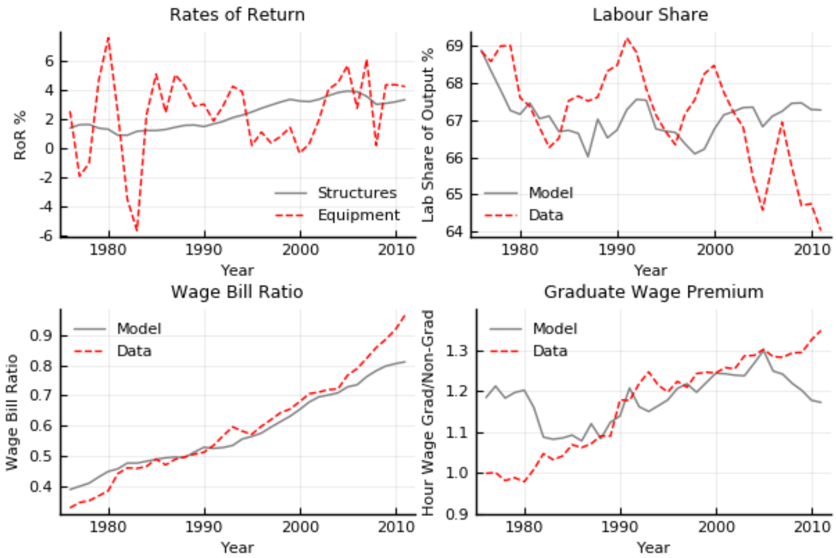


FIGURE 18. KORV with frictions - baseline bargaining power: $\beta = 0.95$



their estimation strategy is succesful at identifying the true parameters in Monte Carlo simulations. As my estimation of the parameters of the KORV production function very closely follows their method, and is done separately and subsequently to estimation of the sequential auction parameters, I do not repeat that exercise here and instead rely on their identification results.

The sequential auction structure of the labor market in my model is no different from Cahuc et al. (2006), however I use employee reported data (from the CPS) to estimate the relevant parameters, whereas Cahuc et al. (2006) used matched-employee-employer (MEE) data. I chose to use CPS data because a key motivation for this paper is to test the robustness of findings in Krusell et al. (2000) to incorporating frictions; I therefore sought to maintain as much consistency as possible to their estimation approach which used CPS data for wages and labor input. However, the MEE data that Cahuc et al. (2006) use plays a key role in their identification strategy so it is worth considering whether the parameters I wish to identify in the sequential auction model are identified when using employee data only.

First bargaining parameters by worker skill level are much more difficult, if not impossible, to identify without some form of employer information. In the absence of such data, match output or suplus becomes much more difficult to estimate and hence reliable estimates of bargaining parameters are not readily available. This is why I choose to set bargaining parameters by assumption.[21] The remaining objects of interest in the sequential auction model are job contact rates, (note job destruction rates come straight from the data) and the distribution of match quality, where I will consider the possibility of both non-parametric and parameteric identification.

B.1. **Job Contact Rates.** There are two job contact rates in the sequential auction model for each skill type of worker: those for the unemployed and employed: $\lambda_{0,i}$ and $\lambda_{1,i}$ respectively. $\lambda_{0,i}$ determines the unemployment rate and, because it influences the outside option of workers, the minimum match quality of firm that a worker will accept an offer at. However, the unemployment rate does not play a role in the estimation of the KORV parameters (since labor input is total hours worked by workers and is taken straight from the data) or in the estimation of any other parameters in the sequential auction model, and I will estimate the lower bound of acceptable match quality directy, as described in the next section. I therefore have no need to estimate $\lambda_{0,i}$.

I instead focus on estimation of $\lambda_{1,i}$, which is key for determining both average match quality, and average wages of worker of a given skill type. Both variables play a role in estimating the parameters of the KORV production function, as described in section 5.3.

I estimate $\lambda_{1,i}$ using SMM and targeting the proportion of continuous employed workers in a given year who have moved employers at least once. I denote this proportion τ_i. In the model, the expression for this moment is given in equation 32, which is obtained by substituting the expression for the cross section distribution of match quality in equation (20) into equation (28).

[21]The analysis of Appendix A suggests the labor share in the economy is informative about the average bargaining power of all workers, though would not help to estimate bargaining parameters by skill level.

$$\tau_i = 1 - \int_{\nu_{inf_i}}^{\nu_{max}} (1 - \lambda_{1,i,}\bar{F}_i(\nu))^{12} \frac{1 + \kappa_{1,i}}{[1 + \kappa_{1,i}\bar{F}_i(\nu)]^2} f_i(\nu) d\nu \tag{32}$$

As I am estimating $\lambda_{1,i}$ separately, and prior to, the estimation of the match quality distribution F, I require that equation (32) is independent of F. This can be proven by integrating by change of variable i.e. if I let $r = \bar{F}_i(\nu)$ so that $\frac{dr}{d\nu} = -f(\nu)$ the expression for τ becomes as shown in equation (33), which is independent of F.

$$\tau_i = 1 - \int_0^1 (1 - \lambda_{1,i,t} r)^{12} \frac{1 + \kappa_{1,i}}{[1 + \kappa_{1,i} r]^2} dr \tag{33}$$

The presence of the twelfth order polynomial in equation (33) hinders an analytical proof of identification, however it is easy to verify with a symbolic equation solver that this expression is a positive monotonic function of $\lambda_{1,i}$ which given the quadratic objective function in SMM proves identification of $\lambda_{1,i}$. This result is not surprising given it is possible to prove (analytically) that the monthly steady state job mobility rate is increasing in $\lambda_{1,i}$.

B.2. **Distribution of Match Quality.** There are two considerations when discussing identification the distribution of match quality. First, I need to consider whether the distribution can be non-parametrically identified or not. I will argue that it can be, but only by relying heavily on the structure of the model. Therefore when it comes to estimation I prefer to assume a log normal distribution of match quality. I will estimate the parameters of the match quality distribution by targeting moments of the empirical wage distribution. However, higher order moments of the wage distribution in the model are not tractable, hindering an analytical proof of identification, so I instead present evidence from Monte Carlo simulations that my estimation strategy can identify the "true" parameters of the match quality distribution.

I start by showing that, in theory, the match distribution could be identified non-parametrically. Consider the expression for the wage earned by worker of skill type i, whose current employer has match quality ν^+ and whose outside option match quality (the second highest quality match they've had contact with) is ν^- as shown in equation (34). It is immediately clear that if I were to use the empirical wage distribution to try and identify ν^+ I encounter the problem that wages depend not only on ν^+ but ν^- so I can't simply invert equation (34) to back out the quality of the current match, ν^+. Note I assume that I do know the other parameter values in the equation due to the identification arguments presented above for job contact rates, and because other parameters either come straight from the data, like job destruction rates, or are set by assumption, like bargaining parameters.

$$\phi(p_i, \nu^-, \nu^+) = p_i \left(\nu^+ - (1 - \beta) \int_{\nu^-}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx \right) \tag{34}$$

However, I do know that all employees who were unemployed in the previous period and then get a job (I refer to these workers as entrants) have a common level of ν^-, which equals ν_{inf_i}, the lower bound of the match quality distribution. Entrants will therefore be paid the wage shown in equation (35).

$$\phi(p_i, \nu_{inf_i}, \nu^+) = p_i\left(\nu^+ - (1-\beta)\int_{\nu_{inf_i}}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx\right) \tag{35}$$

I argued previously that if the bargaining parameter is high enough to guarantee that wages are an increasing function of the employer's match quality (which is the case in my baseline), then ν_{inf} is identified as the lower bound of wages in the empirical wage distribution. Therefore, in principle, I could identify the distribution of ν by inverting equation (12) for each wage in the empirical distribution of entrants' wages. This inversion can be done as follows: I start by letting $w = \phi(p_i, \nu_{inf_i}, \nu^+)$ and differentiating w with respect to ν^+ to get:

$$\begin{aligned}
\frac{dw}{d\nu+} &= p_i\left[1 - (1-\beta)\frac{\rho + \delta_i + \lambda_{1,i}\bar{F}_i(\nu^+)}{\rho + \delta_i + \lambda_{1,i}\beta\bar{F}_i(\nu^+)}\right] \\
&= p_i\left[\frac{\beta(\rho + \delta_i) + (2\beta - 1)\lambda_{1,i}\bar{F}_i(\nu^+)}{\rho + \delta_i + \lambda_{1,i}\beta\bar{F}_i(\nu^+)}\right] \\
\implies \quad \frac{d\nu+}{dw} &= \frac{1}{p_i}\frac{\rho + \delta_i + \lambda_{1,i}\beta\bar{F}_i(\nu^+)}{\beta(\rho + \delta_i) + (2\beta - 1)\lambda_{1,i}\bar{F}_i(\nu^+)}
\end{aligned} \tag{36}$$

Further note that under the assumption I have made about the bargaining parameter, a worker's wage is an increasing function of the match quality of their employer (ν_+), which implies that $\bar{F}_i(\nu^+) = \bar{F}_i^w(w(\nu^+))$. This is helpful since, while $\bar{F}_i(\nu^+)$ is not observable in the data, $\bar{F}_i^w(w(\nu^+))$ is. Substituting $\bar{F}_i(\nu^+) = \bar{F}_i^w(w(\nu^+))$ into equation (36) I can then derive an expression for $\nu+$ in terms of w by solving this differential equation.

However, this relies heavily on the structure of the model and, moreover, on part of the structure - the entrant wage distribution - that was not a particular focus of Cahuc et al. (2006). I therefore choose to make a parametric assumption for the distribution of match quality, and assume it is log normal.

I must now show that I can identify the parameters of this log normal distribution i.e. the mean parameter, ζ_i, the variance parameter, η_i and the lower bound ν_{inf_i}. Recall that my estimation of these parameters is based on a SMM approach as summarised in equation (37), where $\underline{w_i}$ is the lowest wage in the wage distribution, $Q^{50}_{w_i}$ is the median wage and $\mathbb{E}^{F_{i,t}}(\nu)$ is the mean of the match quality sampling distribution, which will be targeted at a fixed value (I impose $\mathbb{E}^{F_{i,t}}(\nu) = 1$).

$$(\zeta^{\nu*}_{i,t}, \eta^{\nu*}_{i,t}, \nu^*_{inf_{i,t}}) = \underset{\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}}{\text{argmin}} \; (m_t - \hat{m}_t)^T W (m_t - \hat{m}_t) \tag{37}$$

$$m_t \equiv \big(var_{\log(w_{i,t})}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}), \underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}),$$
$$\mathbb{E}^{F_{i,t}}(\nu)(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}) \big)$$

Proof of identification is hindered by the lack of tractability of the higher order moments of the wage distribution i.e. consider the expression for the second moment of the wage distribution $\mathbb{E}(w_i^2)$, which is given in equation (38) and already incorporates two simplifying assumptions: $\beta = 0$ and $\rho = 0$.

$$\mathbb{E}(w_i^2) = p_i^2 \int_{\underline{\nu}}^{\nu_{max}} \Bigg[\nu^2 - 2[1 + \kappa_{1,i}\bar{F}_i(\nu)]^2 \times \tag{38}$$
$$\int_{\nu_{inf}}^{\nu} \{ q - \int_q^{\nu} (\kappa_{1,i}\bar{F}_i(x)dx) \} \frac{dq}{1 + \kappa_{1,i}\bar{F}_i(q)} \Bigg] \frac{1 + \kappa_{1,i}}{[1 + \kappa_{1,i}\bar{F}_i(\nu)]^2} f_i(\nu) d\nu$$

Given the intractable nature of this expression, I test whether my estimation procedure correctly identifies the true parameters of the model by Monte Carlo methods. That is I simulate a cross-section sample of wages for 50,000 workers (i.e. slightly less than the 60,000 that feature in the CPS) from the model with an arbitrary choice of parameters (henceforth the "true" parameters). I then estimate the model using this simulated data to see if I recover the true parameters.

Before considering results, recall that I estimate the lower bound of the match quality distribution by targeting the ratio of the lower bound of wages in my sample relative to the median. As argued above this gives exact identification of the $\nu_{inf,i}$. I therefore feed the true parameter for the lower bound of the match quality into my estimation procedure directly, since it is exactly identified, rather than the mininum of simulated wages i.e. I set $\underline{\hat{w}_{i,t}}/Q^{50}_{\hat{w}_{i,t}}$ - the empirical moment I am targeting - to $\nu_{inf_{i,t}}/Q^{50}_{\hat{w^{sim}}_{i,t}}$, where the superscript sim denotes simulated wage data.[22]

The results of my Monte Carlo simulation exercise - where I estimate 50 sets of parameters (corresponding to 50 simulations of data from the true model) - are shown in Figure 19. I see that my estimation strategy is reasonably successful in recovering the true parameters, though not perfect: while there are some biases in the estimates, in each case they are very small in size.

[22] As in my actual empirical estimation of the sequential auction parameters, I normalise the price of the intermediate good, p_i to one when performing the Monte Carlo test of identification.

Figure 19. Monte Carlo Analysis of Identification

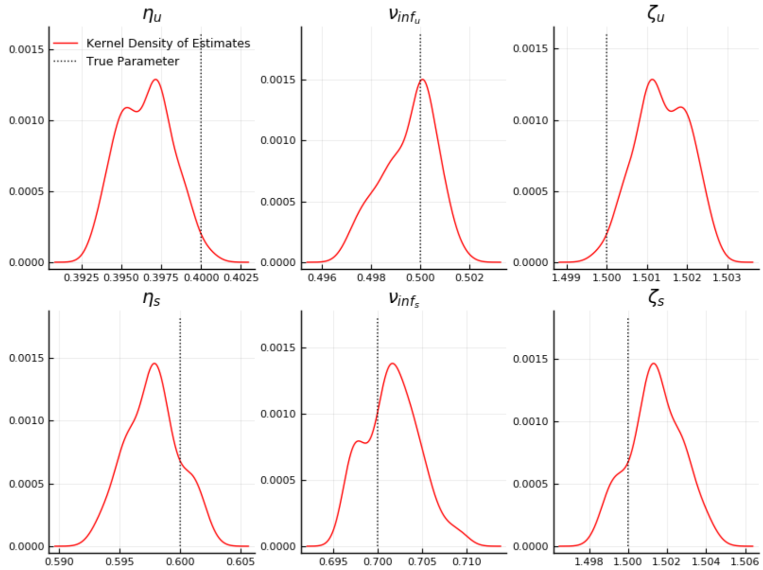


Table 5. KORV parameter values with frictions: robustness

(1) Parameter	(2) Without Frictions	(3) With Frictions: Baseline	(4)	(5)	(6)	(7)
λ	0.568	0.507	0.504	0.506	0.507	0.505
μ	0.806	0.644	0.457	0.405	0.277	0.488
α	0.091	0.083	0.083	0.083	0.083	0.083
γ	-0.209	-0.188	-0.186	-0.187	-0.188	-0.184
σ	0.352	0.33	0.329	0.331	0.33	0.322
$\varepsilon_{S,K_{eq}}$	0.827	0.841	0.843	0.842	0.841	0.844
$\varepsilon_{U,K_{eq}}$	1.544	1.493	1.49	1.495	1.493	1.474
CSC Strength: $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$	**0.716**	**0.651**	**0.646**	**0.653**	**0.651**	**0.63**

Appendix C. Robustness

This section tests the robustness of parameter estimates of the KORV production function in my model to changes to my empirical strategy for estimating the parameters of the sequential auction model of the intermediate goods markets. In particular, I consider the impact of: (i) estimating the lower bound of the match quality distribution by targeting the average wage of workers in the first percentile of the wage distribution (rather than average wage of the bottom two percentiles) - see column 4 of Table 5, (ii) estimating the lower bound of the match quality distribution by targeting the average wage of workers in bottom 5 percentiles - see column 5, (iii) estimating the variance parameter of the (log normal) sampling distribution of match quality by targeting residual wage variance, where I will now control for age as well as race, sex and years of education in calculating this residual variance - see column 6, (iv) estimating the variance parameter of the sampling distribution by targeting the interquartile range of residual log wages, rather than the variance - see column 7 None of these changes to my empirical strategy make a significant difference to my results, as illustrated in the table below.