Rising Wage Inequality: Technological Change and Search Frictions

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Abstract

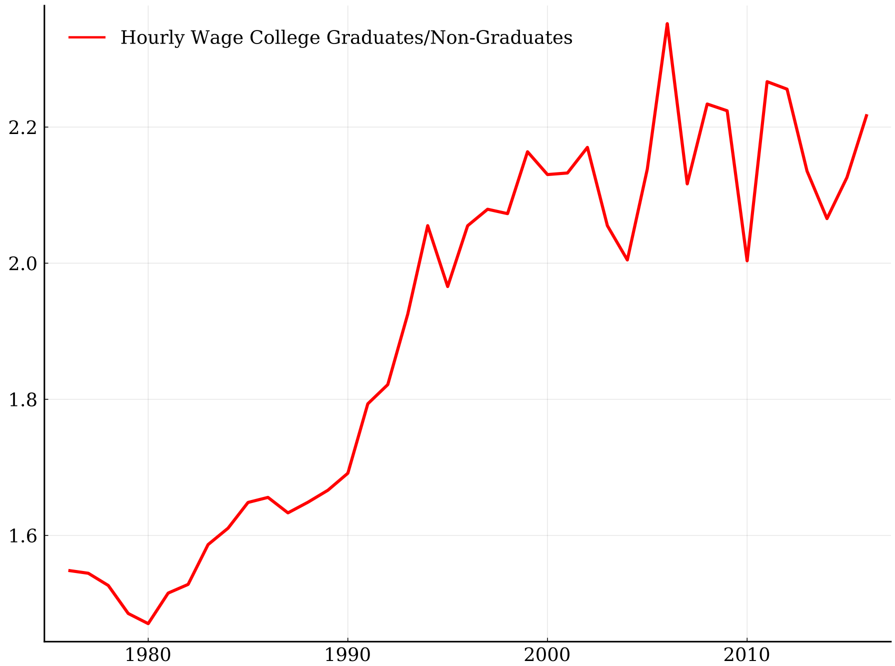
This paper examines whether labor market frictions can explain the level and growth of the college wage premium in the US. I develop a novel model where both capital skill complementarity and differences in the search frictions faced by college and non-college workers drive the college wage premium. The presence of search frictions, and hence monopsonistic power, provides a range of explanations for rising college premiums not present in competitive models i.e. changes to relative job offer rates, to firm heterogeneity or to bargaining power between education groups. College workers enjoy substantially lower job destruction rates and higher job offer rates than non-college workers, which generates the presence of a significant, and relatively stable, college wage premium in my model. I also find that bargaining strength, as captured by unionization rates, starts off at similar levels for college and non-college workers but declines more severely for non-college workers. This trend explains a substantial portion of the growth in the college wage premium in my baseline model.

Keywords: Search Frictions, Monopsony, Labor Markets, Wage Inequality, Technological Change.

1. Introduction

The large literature that seeks to explain rising college wage premiums (see Figure 1) in the United States since the 1980s has generally emphasized a process of technological change favoring college workers.[1] This paper investigates whether changes to search frictions and the wage bargaining environment can provide an alternative explanation. To this end, I develop and estimate a novel structural model featuring capital skill complementarity and on-the-job search with wage bargaining.

Figure 1. The College Wage Premium



Notes: This plots the raw hourly wage premium with wage and hours worked data from the Annual Social and Economic (ASEC) Supplement of the CPS. The empirical analysis presented later in this paper adjusts for composition changes within college and non-college workers as per Krusell, Ohanian, Rios-Rull, and Violante (2000).

The empirical application of my model to U.S data over the last four decades suggests differences in job destruction rates and job offer rates faced by college

[1]For example, Katz and Murphy (1992) attribute the growth in the college wage premium to skill-biased technological change in labor efficiency. Krusell, Ohanian, Rios-Rull, and Violante (2000) explain the rising premium by the increased use of capital equipment and a production function where capital equipment is more complementary to college labor than to non-college labor.

college and non-college workers can explain the presence of a large college wage premium, but not its growth. This is because the differences in these search frictions, while sizable, are relatively stable over my sample period. In contrast, declines in unionization generate substantial growth in the college wage premium in my model. This is both because of the underlying trends in the data - unionization declines more significantly for non-college workers - and the endogenous workings of the model, which predicts that non-college workers are more sensitive to changes in their bargaining strength than college workers as they face lower competition for their labor by employers (i.e. lower job offer rates).[2]

Including search frictions as a potential explanation for rising college wage premiums is motivated by a number of empirical findings that, while suggestive of monopsony power, have so far not been grounded in theory. For example, the importance of firm heterogeneity in explaining growing wage inequality (Song, Price, Guvenen, Bloom, and von Wachter (2018), Card, Heining, and Kline (2013)) is inconsistent with a perfectly competitive model where workers would instantly relocate to more productive firms. Similarly the quantitative importance of institutions such as the minimum wage and trade unions for rising wage inequality (Card and DiNardo (2002), Lee (1999)) is difficult to reconcile with perfectly competitive models. Differences in the level and growth of job offer rates between education groups have received less attention in the literature. However, innovations such as web-based job platforms (e.g. LinkedIn) plausibly represent a form of skill biased technological change in the matching function that could contribute to the growth in the college wage premium in wages. Equally the network effects of participating in college education may aid job search, and lower job destruction risks, contributing to the level of the college wage premium (and its growth if these effects have increased over time).

I therefore depart from much of the structural literature on college wage premiums and relax the assumption of perfect competition between employers. Specifically, I develop a structural model that can investigate the role of labor market frictions and institutional changes for rising college wage premiums, while still accounting for technological and supply-based explanations. The benefit of nesting these explanations in a structural setting is that I can use counter-factual model predictions to quantify the relative importance of each channel. The model set-out in this paper features capital skill complementarity, i.e. capital is more complementary to college labor than non-college labor. This means increased capital use by firms raises the college wage premium, in the spirit of Krusell, Ohanian, Rios-Rull, and Violante (2000). Labor markets in my model feature on-the-job search and sequential auction wage bargaining as in Cahuc, Postel-Vinay, and Robin (2006). In this environment, average wages of a given worker type depend on the worker's

[2]This prediction is consistent with evidence from the empirical literature e.g. Farber, Herbst, Kuziemko, and Naidu (2018) find that the union wage premium is greater for non-college workers.

marginal product, as in a competitive framework, but also on their job offer rates, bargaining strength, the distribution of match quality and outside options in unemployment. Both frameworks have been very influential in explaining different dimensions of inequality: the dynamics of inequality *between* different education groups in the case of Krusell, Ohanian, Rios-Rull, and Violante (2000) and cross-sectional levels of inequality *within* given education groups in the case of Cahuc, Postel-Vinay, and Robin (2006).

Developing a model featuring both capital skill complementarity and a sequential auction model of labor markets has value for two main reasons. First, introducing a production framework with imperfect substitution between inputs into the sequential auction model makes the latter relevant for questions concerning the level and growth of between-group inequality. Previously the sequential auction model, like many search models of wage inequality, had principally been used to examine the level of within-group inequality.[3] Relaxing the assumption of perfect substitution between labor inputs is key as it means that relative scarcity of labor inputs influences relative wages. This is likely to be an important channel when considering between group inequality.[4,5] Second, the richness of my model allows me to investigate whether the strength of the capital skill complementarity channel found in competitive models - such as Krusell, Ohanian, Rios-Rull, and Violante (2000) - remains valid when allowing for labor market imperfections. This is a crucial question for policy since the degree of substitutability between workers of different education levels and capital determines the labor market effects of many policies e.g. the minimum wage or capital taxes.

The empirical part of this paper considers two key questions. First, I examine the contribution of search frictions to the level and growth of the college wage premium. In particular, I focus on the contribution of differences in job offer rates, unionization rates and the distribution of match quality between college and non-college workers. Second, I examine whether estimates of capital skill complementarity are materially different once search frictions are accounted for, as compared to estimates from an equivalent model assuming perfect competition.

[3] For example, Burdett, Carrillo-Tudela, and Coles (2016) decompose within skill-group wage inequality into firm, worker and sorting components in the context of a wage posting model estimated on low, medium and high education workers separately. Cahuc, Postel-Vinay, and Robin (2006) do likewise but in the context of a wage-bargaining model, and looking at within-occupation wage inequality.

[4] For example, see Goldin and Katz (2008) and Card and Lemieux (2001) for evidence on the empirical importance of the relative scarcity of labor supply for college wage premiums.

[5] Bilal, Engbom, Mongey, and Violante (2019) also develop a sequential auction model of the labor market in an environment with decreasing returns to labour input, where the latter allows them to have a well defined notion of firm size.

My first key finding is that differences in search frictions between college and non-college workers can explain just under one third of the average *level* of the college wage premium over my sample period (1976-2016). This is in large part due to college workers facing estimated job contact rates in employment more than double those for non-college workers, and job destruction rates that are less than half those of non-college workers.[6] However, the advantages college workers enjoy in terms of higher job contact rates and lower job destruction rates stay relatively constant over the sample period meaning they cannot explain the *growth* of the college wage premium.

In contrast, relative bargaining strength, which I set equal to the unionization rates of college workers relative to non-college workers, has changed substantially over time.[7] The decline in unionization has a double-whammy impact on the wages of non-college workers. First, in the data, the decline in union membership is more severe for non-college workers over my sample period. [8] Second, in the model, non-college workers are more sensitive to changes in their bargaining power than college workers, due to their lower estimated job offer rates. Job offers are particularly important in the sequential auction model both because they move employees to higher quality matches, and because they trigger bidding wars between rival and incumbent employers over the worker (even when the rival employer has a lower match quality). Non-college workers' lower job offer rates mean their exogenous bargaining strength plays a bigger role in determining their wages than is the case for college workers. My second key finding is therefore that the relative decline in unionization of non-college workers can explain just under a quarter of the rise in the college wage premium, and means estimates of capital-skill complementarity in the production framework of Krusell, Ohanian, Rios-Rull, and Violante (2000) are somewhat lower than in an equivalent competitive model.

The lower estimates of capital-skill complementarity are driven by the fact that rising relative bargaining strength for college workers means my baseline model can match the rise in the college wage premium with less capital skill complementarity

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[6]Job offer rates are not directly observable in the CPS: these estimates therefore refer to model estimates. The estimated average monthly job offer rate for graduate employees is 0.07 over my sample period versus 0.03 for non-graduate employees. In contrast, job destruction rates are estimated directly from employment to unemployment (EU) flow rates in the data: the average monthly EU rate is 0.007 for college workers and 0.018 for non-college workers.

[7]This is done as an illustrative scenario to investigate the impact of one form of institutional change. The absence of matched employer employee data with an education variable in the U.S means conventional ways of estimating bargaining strength, e.g. as in Cahuc, Postel-Vinay, and Robin (2006), are not available.

[8]This empirical trend is emphasized in Stansbury and Summers (2020). Farber, Herbst, Kuziemko, and Naidu (2018) also note that the relationship between years of schooling and probability of union membership was strongly negative in the 1960s but over time has weakened and become positive in recent years.

than the equivalent competitive model. However, it is notable that the parameter estimates in my baseline model are of the same order of magnitude to those in the equivalent competitive model. Both models require the capital skill complementarity channel to be present and relatively strong as, without it, both predict a counter-factual fall in the college wage premium in response to the growing supply of college college workers.

The rest of this paper is organized as follows. Section 2 will present the model, starting first with an overview of Krusell, Ohanian, Rios-Rull, and Violante (2000) (henceforth KORV) before explaining how I incorporate search frictions and wage bargaining as per Cahuc, Postel-Vinay, and Robin (2006). Section 3 discusses the data I use to estimate my model, before Section 4 presents my econometric approach. Section 5 presents findings and Section 6 concludes.

2. The Model

Introducing search frictions and wage bargaining into the production technology in KORV presents a key theoretical challenge: doing so directly would mean firms bargaining with many workers i.e. a multi-player game as per Stole and Zwiebel (1996). These multi-player games seem unlikely to be relevant for considering aggregate inequality dynamics. I therefore abstract from these considerations by specifying a competitive final good sector, where production is as in KORV, and an intermediate good sector with random search by unemployed and employed workers, firm heterogeneity, and where incumbent employers can respond to job offers made to their employees by rivals (as in Cahuc, Postel-Vinay, and Robin (2006)). There are segmented intermediate goods sectors for non-college and college workers.

To build intuition, I first present an overview of the KORV model in its original form before describing how I incorporate intermediate goods sectors with search frictions. Finally, I show how search frictions and wage bargaining operate within the intermediate goods sector.

2.1. **KORV Production Function: No Frictions or Intermediate Goods.** In the original formulation of KORV, final good in period t, Y_t is produced using capital structures, $K_{s,t}$, capital equipment, $K_{e,t}$, and college and non-college labor, C_t & N_t, as inputs, as shown in equation (1).

$$
\begin{aligned}
Y_t &= A_t G(K_{s,t}, K_{e,t}, N_t, C_t) \\
&= A_t K_{s,t}^{\alpha}[\mu N_t^{\sigma} + (1-\mu)(\lambda K_{e,t}^{\gamma} + (1-\lambda)C_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} \qquad (1)
\end{aligned}
$$

with $\sigma, \gamma < 1$ and $\alpha, \lambda, \mu \in (0,1)$. The elasticity of substitution between non-college labor input and capital equipment, denoted by ε_{n,k_e}, is equal to $1/(1-\sigma)$. The elasticity of substitution between non-college and college labor, denoted $\varepsilon_{n,c}$, is also equal to $1/(1-\sigma)$. Finally, the elasticity of substitution between the college labor and capital equipment, denoted by ε_{c,k_e}, is equal to $1/(1-\gamma)$. Capital-skill complementarity is present when $\sigma > \gamma$. The parameter, α, together with λ, determine the capital share of output, and μ determines the output share of non-college workers.

Labor input is hours worked in efficiency units e.g $N_t \equiv \Psi_{n,t}h_{n,t}$, $C_t \equiv \Psi_{c,t}h_{c,t}$, where $\Psi_{i,t}$ is the efficiency of labor input of worker type $i \in \{n, c\}$, and $h_{i,t}$ is the total amount of hours worked by that worker type. Krusell, Ohanian, Rios-Rull, and Violante (2000), in their baseline model, impose that $\Psi_{n,t}$ and $\Psi_{c,t}$ both follow stationary stochastic processes. They do not allow for any time trend in relative labour efficiency as this would introduce an unexplained source of skills-biased technical change, contrary to the aim of their paper which is to examine the contribution of increased capital use to the rise in the college wage premium.

The final good is used for consumption c_t, investment in capital equipment $x_{eq,t}$ and investment in capital structures $x_{st,t}$, as shown in equation (2), where q_t is the relative efficiency of producing capital equipment from the final good (or equivalently $1/q_t$ is the relative price of capital equipment).

$$Y_t = c_t + x_{st,t} + \frac{x_{eq,t}}{q_t} \tag{2}$$

The final good producer has the following profit maximisation problem, where $(w_{n,t}, w_{c,t})$ denote the wages for non-college and college workers respectively, and $(r_{st,t}, r_{eq,t})$ denote the rental rates for capital structures and equipment respectively:

$$\begin{aligned} \max_{K_{s,t},K_{e,t},h_{n,t},h_{c,t}} \Pi = {} & A_t K_{s,t}^{\alpha}[\mu N_t^{\sigma} + (1-\mu)(\lambda K_{e,t}^{\gamma} + (1-\lambda)C_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} \\ & - w_{n,t}h_{n,t} - w_{c,t}h_{c,t} - r_{st,t}K_{s,t} - r_{eq,t}K_{e,t} \end{aligned} \tag{3}$$

In both KORV's original model and in my adaptation the final good producer is assumed to be competitive, so the first order conditions (FOCs) for its profit maximisation problem are as follows:

$$\begin{aligned} w_{n,t} &= A_t(1-\alpha)K_{s,t}^{\alpha}[\mu N_t^{\sigma} + (1-\mu)(\lambda K_{e,t}^{\gamma} + (1-\lambda)C_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (4) \\ & \quad \times \mu U^{\sigma-1}\Psi_{n,t} \\ w_{c,t} &= A_t(1-\alpha)K_{s,t}^{\alpha}[\mu N_t^{\sigma} + (1-\mu)(\lambda K_{e,t}^{\gamma} + (1-\lambda)C_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (5) \\ & \quad \times (1-\mu)(\lambda K_{e,t}^{\gamma} + (1-\lambda)C_t^{\gamma})^{\frac{\sigma-\gamma}{\gamma}}(1-\lambda)C_t^{\gamma-1}\Psi_{c,t} \\ r_{eq,t} &= A_t(1-\alpha)K_{s,t}^{\alpha}[\mu N_t^{\sigma} + (1-\mu)(\lambda K_{e,t}^{\gamma} + (1-\lambda)C_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (6) \\ & \quad \times (1-\mu)(\lambda K_{e,t}^{\gamma} + (1-\lambda)C_t^{\gamma})^{\frac{\sigma-\gamma}{\gamma}} K_{e,t}^{\gamma-1} \\ r_{st,t} &= \alpha A_t K_{s,t}^{\alpha-1}[\mu N_t^{\sigma} + (1-\mu)(\lambda K_{e,t}^{\gamma} + (1-\lambda)C_t^{\gamma})^{\frac{\sigma}{\gamma}}]^{\frac{1-\alpha}{\sigma}} && (7) \end{aligned}$$

In the absence of frictions, growth in the college wage premium (denoted by $\pi_t = w_{c,t}/w_{n,t}$) is given in equation (8), where g_z denotes the growth rate in variable z.[9]

$$g_{\pi_t} \simeq (1-\sigma)(g_{h_{n,t}} - g_{h_{c,t}}) + \sigma(g_{\Psi_{c,t}} - g_{\Psi_{n,t}}) + (\sigma-\gamma)\lambda\Big(\frac{K_{e,t}}{C_t}\Big)(g_{K_{e,t}} - g_{\Psi_{c,t}} - g_{h_{c,t}}) \tag{8}$$

The first term on the right-hand-side of equation (8), $(1-\sigma)(g_{h_{n,t}} - g_{h_{c,t}})$, captures the impact of changes to the relative scarcity of college labor and depends on the substitutability between college and non-college labor. Greater substitutability (higher σ) implies changes to relative scarcity are less important for relative wages. The second term, $\sigma(g_{\Psi_{c,t}} - g_{\Psi_{n,t}})$, captures the impact of relative efficiency of college labor. When $\sigma > 0$ the elasticity of substitution between college and non-college labor is greater than 1 and improvements in the labor efficiency of college workers will increase their relative wages: the reverse holds when $\sigma < 0$. Finally, the third term, $(\sigma-\gamma)\lambda\big(\frac{K_{e,t}}{C_t}\big)(g_{K_{e,t}} - g_{\Psi_{c,t}} - g_{h_{c,t}})$, captures the capital-skill-complementary effect. When $\sigma > \gamma$ capital equipment complements college labor more than non-college labor and increases in the growth rate of capital equipment per college worker will increase the college wage premium.

2.2. **KORV production function: Incorporating Intermediate Goods.** I incorporate search frictions into the model by introducing two segmented intermediate goods sectors that employ non-college and college workers. The sector and corresponding education group are indexed by $i \in \{n, c\}$. I now interpret N_t and C_t as the effective amount of intermediate goods produced in the non-college and college intermediate goods sectors respectively. Specifically I define $N_t \equiv \Psi_{n,t} y_{n,t}$ and $C_t \equiv \Psi_{c,t} y_{c,t}$ where $y_{i,t}$ is the volume of intermediate goods produced in skill sector i and $\Psi_{i,t}$ is the efficiency level of that intermediate good.

In each segmented intermediate goods market, unemployed workers are randomly matched to intermediate firms of quality ν (I refer to this as match quality), and with a sampling distribution $F_{i,t}(\nu)$ and pdf, $f_{i,t}(\nu)$. I denote the cdf and pdf of the cross-section distribution of match quality across all employed workers as $L_{i,t}(\nu)$ and $l_{i,t}(\nu)$, which differs from the sampling distribution as workers can search on the job.

A worker in a match of quality ν produces exactly ν units of intermediate good for every hour they work. Hours worked are assumed to be fixed for each worker type

[9] This by derived by taking logs of the college wage premium - given by the final goods firm's FOCs - and then differentiating with respect to time to give equation (8).

and do not vary with match quality.[10] The effective input of intermediate goods from the non-college and college intermediate sectors are therefore as shown below, where $h_{i,t}$ is again the raw total amount of hours worked by type i workers.

$$N_t \equiv \Psi_{n,t} y_{n,t} = \Psi_{n,t} h_{n,t} \int_{\nu_{inf,u}}^{\nu_{max}} \nu \ell_{t,u}(\nu) d\nu \tag{9}$$

$$C_t \equiv \Psi_{c,t} y_{c,t} = \Psi_{c,t} h_{c,t} \int_{\nu_{inf,s}}^{\nu_{max}} \nu \ell_{t,s}(\nu) d\nu \tag{10}$$

Final good producers are again assumed to be competitive and so pay a price, p_i, for a unit of type i intermediate good given by $p_i = \frac{\partial Y}{\partial y_i}$. An intermediate good firm of match quality ν in intermediate sector i receives revenue equal to $p_i \nu$. The following subsection describes how search frictions and wage bargaining operates within the intermediate goods sectors.

2.3. **Intermediate Goods Sectors.** All intermediate firms and workers have common discount rate, ρ, and are risk neutral. As is standard in the search literature, I assume firms can employ a maximum of one worker so intermediate firms become synonymous to matches or jobs. Job destruction rates are exogenously given, but allowed to vary by worker type and are denoted by $\delta_{i,t}$. Workers receive flow income in unemployment equal to $b_{i,t} * p_{i,t}$, where $b_{i,t}$ is their replacement rate and $p_{i,t}$ is the price of the intermediate good they produce as defined above.[11]

The job offer arrival rates in unemployment and employment are denoted by $\lambda_{0,i,t}$ and $\lambda_{1,i,t}$, and are assumed to be exogenously given.

Intermediate Goods Sector: Wage Bargaining with Unemployed Workers

Equation (11) represents the Bellman equation for an unemployed worker of worker type i, where $U(p_i, b_i)$ is the expected lifetime utility of an unemployed worker, $\phi_0(p_i, \nu)$ is the wage paid to a previously unemployed worker now in a match of quality ν and $V\left(p_i, \phi_0(p_i, \nu), \nu\right)$ is the expected lifetime utility of that worker.

$$(\rho + \lambda_{0,i})\, U(p_i, b_i) = p_i b_i + \lambda_{0,i} \int_{\nu_{inf_i}}^{\nu_{\max}} V\left(p_i, \phi_0(p_i, x), x\right) dF_i(x) \tag{11}$$

Unemployed workers receive flow income $b_i p_i$ in the current period and in the next period, which is discounted at rate ρ, they encounter a match with probability

[10] I make this assumption to maintain consistency with the original formulation of the KORV production function where labor inputs are measured in efficiency units of total hours worked.

[11] This implies that unemployment income is independent of the match quality that the worker had in their previous employment. Re-employment wages are therefore not path dependent, which aids tractability in the model - see Cahuc, Postel-Vinay, and Robin (2006) for further discussion.

$\lambda_{0,i}$, where the match quality is drawn from the distribution $F_i(\nu)$ and lies in the interval $[\nu_{inf_i}, \nu_{max}]$.

As in Cahuc, Postel-Vinay, and Robin (2006), I assume that there is a latent vacancy posting cost, which ensures that intermediate firms won't post a match unless it will be accepted by a worker, so the lower bound of the match quality distribution is the workers reservation match quality ν_{inf_i}.

I use the generalized form of Nash bargaining proposed in Cahuc, Postel-Vinay, and Robin (2006) for unemployed and employed workers. Previously unemployed workers are paid a wage, $\phi_0(p_i, \nu)$, that equalizes the expected lifetime utility of working at a match of quality ν with the expected lifetime utility of being unemployed plus a share, β (the bargaining parameter), of match surplus $V(p_i, p_i\nu, \nu) - U(p_i, b_i)$, as shown below: [12]

$$V\left(p_i, \phi_0(p_i, \nu), \nu\right) = U(p_i, b_i) + \beta\left[V(p_i, p_i\nu, \nu) - U(p_i, b_i)\right] \tag{12}$$

Intermediate Goods Sectors: Wage Bargaining with Employed Workers

A key novelty in the sequential auction model of Cahuc, Postel-Vinay, and Robin (2006) is that incumbent employers can respond to rival job offers made to their employees, in contrast to wage posting models such as Burdett and Mortensen (1998). In this environment, the wage paid to an employee will depend on (i) the match quality of the highest ranked match they have encountered in their employment spell, ν^+, which will be at their current employer, (ii) the match quality at their outside option, ν^-, which is the second highest match they have encountered in their employment spell, and (iii) the price of the intermediate good they produce, p_i, which will be the same for all workers in a given skill group i . I denote this wage $\phi(p_i, \nu^-, \nu^+)$.

Suppose a worker employed at a match of quality ν encounters a match of quality ν', and is currently paid a wage w. If $\nu' > \nu$, the employee moves to higher quality match and gets wage $\phi(p_i, \nu, \nu')$. Encountering a match of quality $\nu' < \nu$ will trigger a renegotiation of the employees wage contract at their current employer if ν' exceeds a threshold, denoted $\chi(p_i, w, \nu)$, where $\chi(p_i, w, \nu)$ is defined by the equality $\phi(p_i, \chi(p_i, w, \nu), \nu) = w$.

[12] I have assumed there is zero value to a firm from having a vacancy i.e. a free entry condition holds, which when combined with the assumption of a common discount rate for firms and workers and risk neutrality of all agents means the match surplus can be expressed as $V(p_i, p_i\nu, \nu) - U(p_i, b_i)$ i.e. the match surplus equals the worker surplus when they are paid a wage equal to their marginal product.

The Bellman equation for a worker of worker type i employed at a match of quality ν and paid a wage w is shown in equation (13).

$$\begin{aligned}
&\left[\rho + \delta_i + \lambda_{1,i}\bar{F}_i(\chi(p_i, w, \nu))\right] V_i(p_i, w, \nu) \\
&= w + \delta_i U(p_i, b_i) + \lambda_{1,i} \int_{\chi(p_i,w,\nu)}^{\nu} V_i(p_i, \phi(p_i, x, \nu), x) dF_i(x) \\
&+\lambda_{1,i} \int_{\nu}^{\nu_{\max}} V_i(p_i, \phi(p_i, \nu, x), x) dF_i(x) \qquad (13)
\end{aligned}$$

Equation (13) tells us that the worker receives wage w in the current period and will either lose their job with probability δ_i or, failing that, make contact with a match that triggers a renegotiation of their wage in the next period with probability $\lambda_{1,i}\bar{F}_i(\chi(p_i, w, \nu))$. If the match quality at the alternative match, x, lies in the region $(\chi(p_i, w, \nu), \nu]$ the worker stays at their current employer and receives a pay rise $\phi(p_i, x, \nu) - w$. If $x > \nu$ the worker moves to the alternative match and gets a wage $\phi(p_i, \nu, x)$. Note that this wage need not be greater than their previous wage as workers may be willing to take a pay cut if the potential for future wage increases at the higher quality match is sufficiently greater than at their incumbent employer.

Equation (14) shows the result of the wage bargaining that occurs when an employee encounters a match of sufficient quality to trigger a wage renegotiation. I denote the higher of the incumbent and rival employer's match quality as ν^+ , and the lower match quality as ν^-. The worker will supply their labor to the higher quality match, and the lower quality match becomes their outside option. Cahuc, Postel-Vinay, and Robin (2006) adapt the Nash bargaining game of Osborne and Rubinstein (1990) to an environment with rival bidders, and show the bargained wage must satisfy equation (14).

$$\begin{aligned}
(14) \qquad V(p_i, \phi(p_i, \nu^-, \nu^+), \nu^+) =& V(p_i, p_i\nu^-, \nu^-) + \\
& \beta[V(p_i, p_i\nu^+, \nu^+) - V(p_i, p_i\nu^-, \nu^-)]
\end{aligned}$$

The worker receives their outside option, which is the value of working at the firm with productivity ν^- at a wage equal to their marginal product, $p_i\nu^-$, plus a share, β, of the match surplus from working at the higher productivity firm.

Cahuc, Postel-Vinay, and Robin (2006) prove that the wage, $\phi(p_i, \nu^-, \nu^+)$, satisfying equation (14) has the form shown in equation (15) when value functions are as defined as in equation (13).

$$(15) \qquad \phi(p_i, \nu^-, \nu^+) = p_i \left(\nu^+ - (1 - \beta) \int_{\nu^-}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx \right)$$

Intermediate Goods Sectors: Wage and Employment Distributions

The key objects of interest in the model are the wage distributions for each worker type. The analysis of the preceding sections indicates that a worker's wage depends on two stochastic variables: their current match quality, ν, and the match quality of their outside option, χ. As in Cahuc, Postel-Vinay, and Robin (2006), I impose that the labor market is in steady state in order to derive expressions for the cross section distributions of ν, $L_i(\nu)$, and of χ conditional on ν, $L_i(\chi|\nu)$.

Steady state in the labor market requires equations (16) through to (18) to hold, where e_i^{ue} denotes the unemployment rate of worker type i:

$$\delta_i(1 - e_i^{ue}) = \lambda_{0,i}e_i^{ue} \tag{16}$$

$$\lambda_{0,i}F_i(\nu)e_i^{ue} = [(\lambda_{1,i}\bar{F}_i(\nu) + \delta_i)](1 - e_i^{ue})L_i(\nu) \tag{17}$$

$$\lambda_{0,i}f(\nu)e_i^{ue} + L_i(\chi)\lambda_{1,i}f(\nu)(1 - e_i^{ue}) = (\lambda_{1,i}\bar{F}_i(\chi) + \delta_i)(1 - e_i^{ue})L_i(\chi|\nu)\ell_i(\nu) \tag{18}$$

Equation (16) requires that the inflows of workers into unemployment, the left hand side (LHS) of the equation, equals the outflow from unemployment shown on the right hand side (RHS). Equation (17) pins down the cross-sectional distribution of workers across match quality $L_i(\nu)$ and requires the inflow of workers into matches of quality less than ν equals the outflow. The inflow (LHS of equation (17)) consists of unemployed workers who make contact with a match of quality less than ν with probability $\lambda_{0,i}F_i(\nu)$. The outflow (RHS of equation (17)) is employed workers with match quality below ν who either lose their job with probability δ_i or make contact with a higher quality match with probability $\lambda_{1,i}\bar{F}_i(\nu)$. Finally equation (18) pins down the cross-sectional distribution of workers' outside options given they are at a match of quality ν: $L_i(\chi|\nu)$. It requires that the inflow of workers into matches of quality ν and with an outside option of match quality less than χ equals the outflow. The inflow (LHS of equation (18)) consists of unemployed workers meeting a match of quality ν with probability $\lambda_{0,i}f(\nu)$ (by definition their outside option, i.e. unemployment, has a match quality less than all feasible values of χ), plus workers previously employed at a match of quality less than χ who make contact with a match of quality ν. The outflow (LHS of equation (17)) is employed workers with a match quality equal to ν and with an outside option of match quality less than χ who either lose their job with probability δ_i or receive an offer of a match of quality exceeding χ with probability $\lambda_{1,i}\bar{F}_i(\chi)$.

The expressions for the steady state cross sectional distribution of workers across matches and outside options derived from the steady state requirements above are shown in equations (19) and (20), where $\kappa_{1,i} \equiv \frac{\lambda_{1,i}}{\delta_i}$.

$$L_i(\nu) = \frac{F_i(\nu)}{1 + \kappa_{1,i}\bar{F}_i(\nu)} \tag{19}$$

$$L_i(\chi|\nu) = \left[\frac{1 + \kappa_{1,i}L_i(\chi)}{1 + \kappa_{1,i}L_i(\nu)}\right]^2 \tag{20}$$

The expected wage for a worker of type i is given by:

$$E(w_i) = p_i \int_{\underline{\nu}}^{\nu_{max}} \Bigg[\nu - [1 + \kappa_{1,i}\bar{F}_i(\nu)]^2 \times \tag{21}$$

$$\int_{\nu_{inf}}^{\nu} \frac{(1-\beta)[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\bar{F}_i(x)]}{[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\beta\bar{F}_i(x)][1 + \kappa_{1,i}\bar{F}_i(x)]^2} dx\Bigg] \ell_i(\nu) d\nu$$

The college wage premium in the model ($E(w_c)/E(w_n)$) will therefore depend on the same variables as in Krusell, Ohanian, Rios-Rull, and Violante (2000), which influence the price of the intermediate good produced by worker type i, p_i, but also on relative job destruction and offer rates, outside options in unemployment, distributions of match quality, and bargaining strength.

3. Data

This section describes the data I use to estimate the parameters in the KORV production function and in the sequential auction labor markets in the intermediate goods sectors. I then present my estimation approach in detail in Section 4 with identification further examined in Appendix B.

3.1. **Data: Krusell, Ohanian, Rios-Rull, and Violante (2000).** In keeping with KORV's original approach, I use labor market data from the Current Population Survey (CPS) and data on capital inputs and the labor share of income from U.S national accounts. College (non-college) labor input is defined as total hours worked by college workers (non-college workers).[13] The authors split each worker type down further into education, gender and race cells to impute hours for those with missing data. I follow their exact approach for comparability.[14]

The authors differentiate between capital equipment, such as machinery, hardware and software, and capital structures e.g. buildings, as capital-skill complementarity

[13] All CPS data is taken from the Integrated Public Use Microdata Series (IPUMS) distribution: https://cps.ipums.org/cps/

[14] See Appendix 1 of Krusell, Ohanian, Rios-Rull, and Violante (2000) for details of this approach

is more likely to occur with the former than the latter.[15] An important element of KORV's approach is using a relative price deflator for capital equipment that is based on the approach of Gordon (1990), which they use to calculate the real value of the stock of capital equipment (all other variables are deflated using a GDP deflator). This relative price of equipment falls significantly over KORV's sample period, which in turn implies that the real value of capital equipment used by firms increases appreciably faster than capital structures. Polgreen and Silos (2008) show that use of alternative price series suggest significantly less capital skill complementarity.

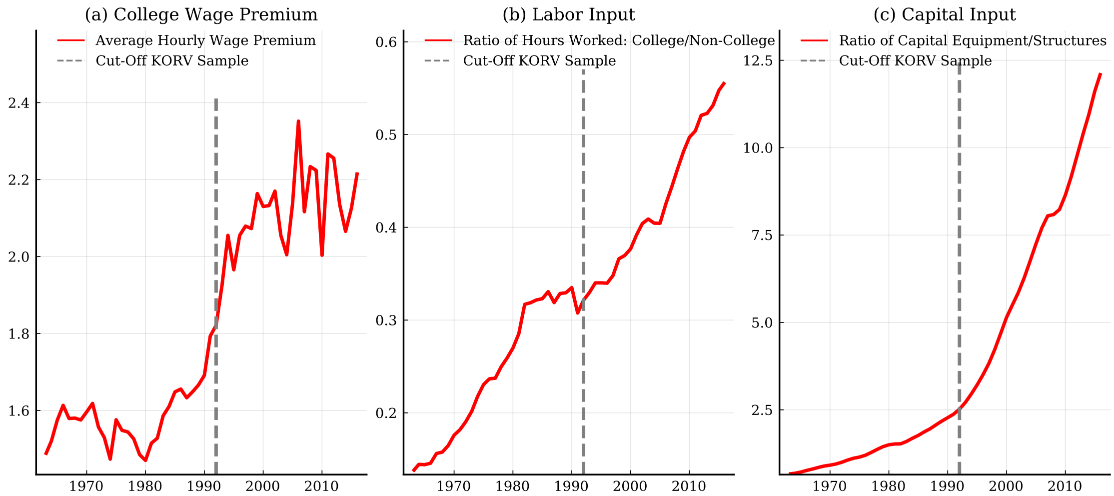
The key trends driving results in KORV are summarised in Figure 2. The rise in the college wage premium shown in Panel (a) of Figure 2 happens despite the increase in relative supply of college labor shown in Panel (b): given the authors assume constant relative labor efficiency in their baseline specification, the only possible driver of the rise in the college wage premium is the growing use of capital equipment shown in Panel (c) combined with some degree of capital skill complementarity, which is indeed what their results suggest. The authors estimate an elasticity of substitution between capital equipment and non-college labor of 1.67 vs an equivalent elasticity of 0.67 for college labor.

3.2. **Data: Labor Market Frictions.** I supplement the data used by Krusell, Ohanian, Rios-Rull, and Violante (2000) with data on labor market frictions, also from the CPS. With each measure of labor market friction, the key dimension of interest will be the time trend in frictions for college workers relative to the trend faced by non-college workers. In the absence of distinct trends in relative frictions it is unlikely that incorporating labor market frictions into KORV will offer a different explanation for the rise in the college wage premium than the original KORV specification.

A crucial search friction is the degree of competitive intensity, $\kappa_{1,i}$ which is the rate of job to job contact rates relative to job destruction rates ($\kappa_{1,i} \equiv \frac{\lambda_{1,i}}{\delta_i}$). This determines how quickly workers proceed up the job ladder. I take job destruction rates from the monthly panel element of the CPS, as shown in Figure 3. While

[15] The capital stock series are constructed recursively as per Krusell, Ohanian, Rios-Rull, and Violante (2000): I use a starting stock of capital equipment and structures in 1963 using data from National Accounts data and add annual investment data in equipment and structures also from the National Accounts. Both investment series are deflated by the non-durable consumption deflator used in DiCecio (2009) (available on the Federal Reserve Bank of St. Louis' Economic Data website (FRED): https://fred.stlouisfed.org/series/CONSDEF). Capital equipment investment is additionally deflated using the relative capital equipment deflator from DiCecio (2009) (also available on FRED: https://fred.stlouisfed.org/series/PERIC), which is based on the approach of Gordon (1990). I apply the same depreciation rates for capital stock as Krusell, Ohanian, Rios-Rull, and Violante (2000): 12.5% for equipment and 5% for structures.

Figure 2. Key Data Trends in KORV



Notes: The grey dashed line in all panels represents the end of the sample period used in Krusell, Ohanian, Rios-Rull, and Violante (2000). Wage and hours worked data (shown in panels (a) and (b) respectively) are taken from the Annual Social and Economic (ASEC) Supplement of the CPS, and modified as per Krusell, Ohanian, Rios-Rull, and Violante (2000). Panel (c) shows the relative stock of capital equipment to capital structures.

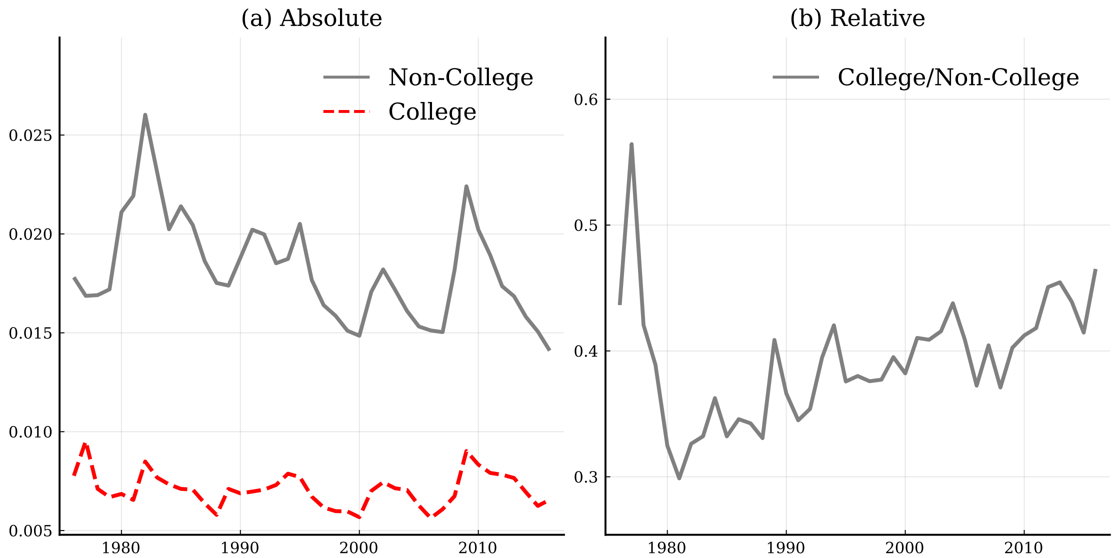
college workers have lower job destruction rates than non-college workers in all years, this difference has narrowed over time as non-college workers' job destruction rates have trended downwards while those of college workers have a relatively stable trend.

Job contact rates are not readily observable in the CPS and there has only been a question on change of employers since 1994, which hampers comparison with KORV since their original sample period finished in 1992. Since in any case job-to-job transitions would be used to infer job contact rates (not all contacts result in a transition), I use an alternative measure of job mobility which is the proportion of continuously employed individuals that report having at least two non-concurrent employers in the previous year (the 'multiple employer' rate) which is shown in Figure 4. Figure 5 shows that movements in the multiple employer rate track movements to job-to-job mobility rates very closely. [16]

In absolute terms, college workers have generally had slightly higher multiple employer rates than non-college workers. This difference has been broadly stable over time, though it appears to be relatively counter-cyclical. Both job-to-job mobility

[16] The CPS question on number of employers in the last year started in 1976.

Figure 3. Job destruction rates



Notes: Job destruction rates are the average monthly transition rates of workers from employment to unemployment for a given year. Panel (a) shows the series separately for non-college and college workers, Panel (b) shows the series for college workers relative to that of non-college workers. Source: Basic Monthly files of the CPS.

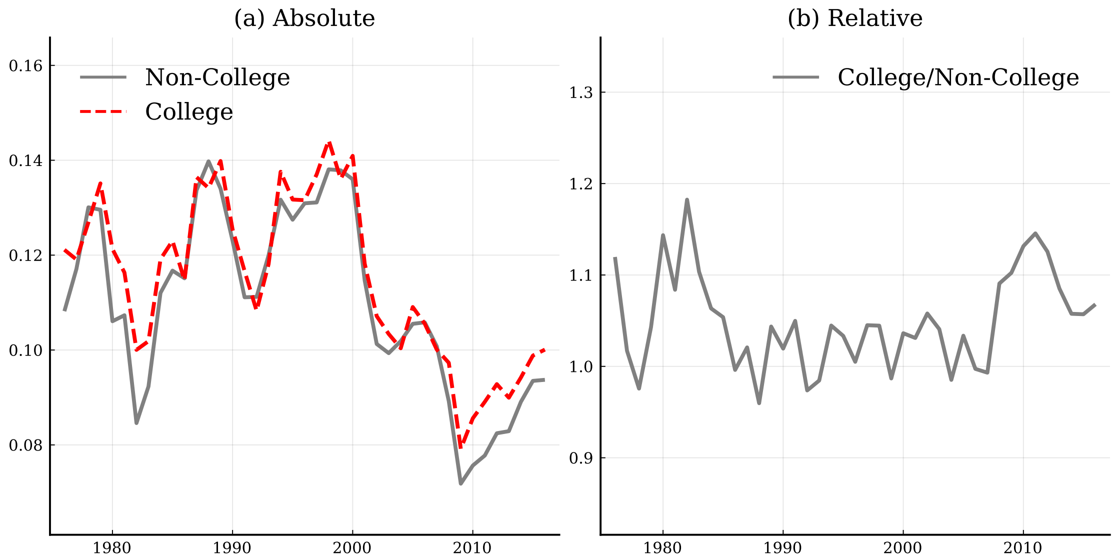
and multiple employer rates have declined significantly since the early 2000s for college and non-college workers alike.[17]

The importance of movements up and down the job ladder for mean wages depends on the dispersion of match quality. I use the standard deviation of log residual wages as a target to identify the distribution of match quality (Section 4 and Appendix B discuss estimation and identification in more detail). I calculate the residual wage after controlling for years of education, race, sex and year in Mincer type wage regression. I do this separately for college and non-college workers. I do not control for age since there are some endogenous returns to job tenure and total employment duration in my model, both of which are correlated with age (and are not directly measured in the CPS over the duration of my sample period).[18] This

[17]The aggregate trend of declining job flows is documented in e.g. Decker, Haltiwanger, Jarmin, and Miranda (2016) and Molloy, Smith, Trezzi, and Wozniak (2016)

[18]This risks attributing some variance in wages driven by human capital returns to experience to job ladder effects. However, I find that my estimates of the parameters of the KORV production function do not change if I control for age when calculating residual wage variance - see Appendix C for this and other robustness checks. Nevertheless there may still be some unobservable human capital differences not picked up by either years of education or age that get attributed to job ladder effects using this approach. This represents a disadvantage of

FIGURE 4. Multiple Employer Rates



Notes: The multiple employer rate is the proportion of workers with no spells of unemployment who have had more than one non-concurrent employer in the previous year. Panel (a) shows the series separately for non-college and college workers, Panel (b) shows the series for college workers relative to that of non-college workers. Source: Annual Social and Economic Supplement of the CPS.

measure of dispersion increases in relative terms for college workers as shown in Figure 6.[19]

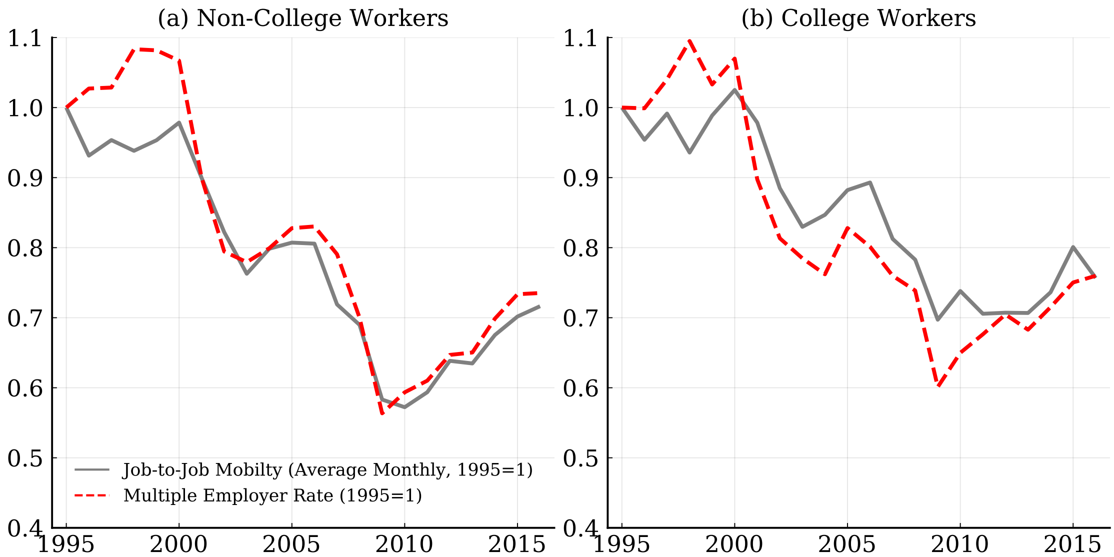
A key advantage of investigating college wage premiums in a labor market model with wage bargaining is that this environment allows us to explore the impact of institutions that influence the relative wage bargaining strength of college and non-college workers. As an illustrative scenario, I focus on the evolution of unionization rates as a measure of institutional change and will assume bargaining strength for college workers relative to non-college workers moves in line with their relative unionization rates, as described in further detail in Section 4.2.[20] Figure 7 shows that unionization rates (% of individuals in a trade union) start of at similar

maintaining consistency with KORV by using employee data only rather than using matched employee-employer data to better distinguish worker and firm fixed effects.

[19]Note that using other measures of wage dispersion, such as the interquartile range, does not change my estimates of the KORV production function parameters - see Appendix C.

[20]I refer to this as an illustrative scenario since bargaining strength in an model of individual wage bargaining, such as mine, is likely determined by many institutional and individual characteristics other than unionization. A promising avenue for future research would be to estimate relative strength using the labor share in sectors with large concentrations of non-college and college workers.

FIGURE 5. Multiple Employer and Job-to-Job Mobility Rates



Notes: The figure compares the multiple employer rate defined in Figure 4 to the average monthly transition rate of employees to new jobs ('Job-to-Job Mobility Rate') from the Basic Monthly files of the CPS. Panel (a) shows this comparison for non-college workers, Panel (b) shows the comparison for college workers. Both series are normalised with reference to their value in 1995.

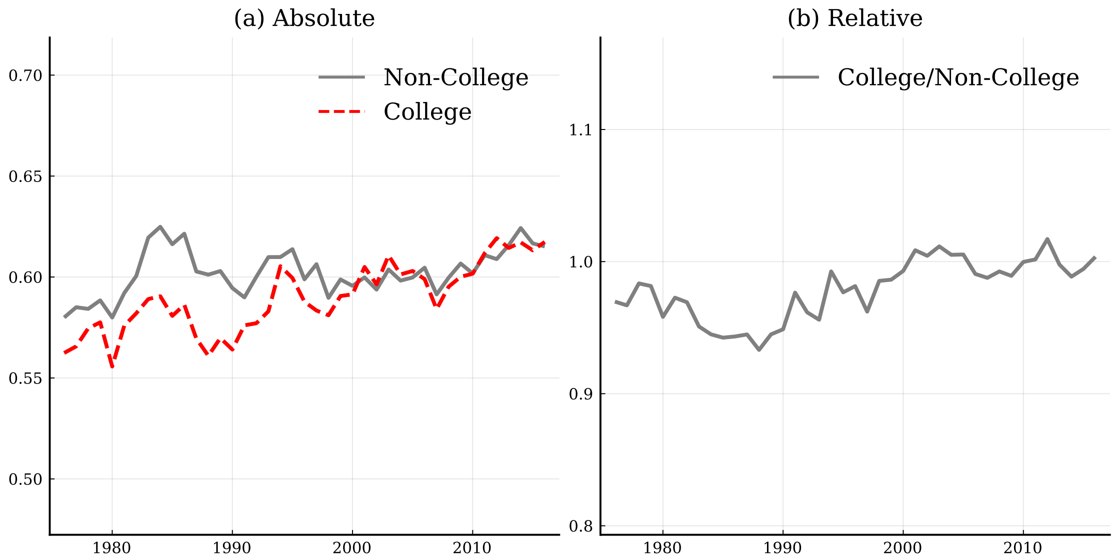
levels for college and non-college workers but decrease more severely for non-college workers, a pattern which is documented using a number of data sources in Farber, Herbst, Kuziemko, and Naidu (2018).

Finally the environment workers face in unemployment, both in terms of unemployment flow income and job contact rates, has an impact on the average quality of matches and wages for employed workers. This impact is less in models with on-the-job search than in models without, but it nonetheless must be accounted for.

Specifically I will need to estimate or calculate the lower bound of the match quality distribution for each worker type, $\nu_{inf,i}$. In principle, this could be done by exploiting the tractable relationship between the lower bound of the sampling distribution of match quality and unemployment replacement rates and job contact rates shown previously in equation (**??**).

However, replacement rates are determined not only by legislative framework but also by the degree of insurance provided by asset accumulation, family/social relationships and many other factors beyond this making it difficult to observe in practice. I therefore directly estimate $\nu_{inf,i}$ by targeting the ratio of average wages of workers in the first five percentiles of the wage distribution to the median wage.

Figure 6. Residual Log Wage Dispersion



Notes: The figure shows the variance of residual log hourly wages from a Mincer wage regression of wages against education, race, sex and year. Hourly wages are again calculated as per Krusell, Ohanian, Rios-Rull, and Violante (2000) using ASEC CPS data, but are now trimmed (by dropping hourly wages in the bottom and top percentiles) to minimise measurement error. Panel (a) shows the series separately for non-college and college workers, Panel (b) shows the series for college workers relative to that of non-college workers.

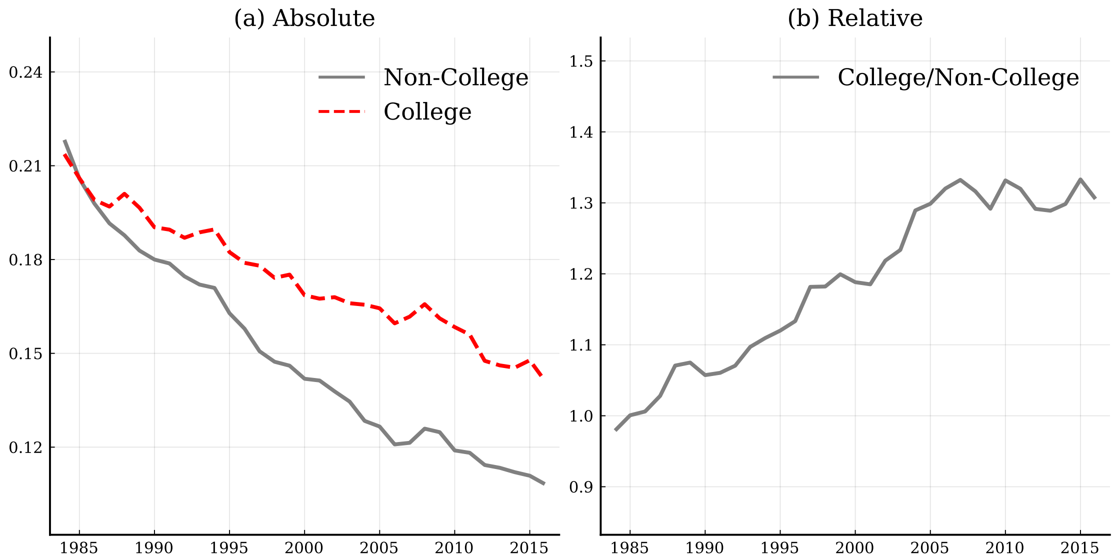
This empirical moment is shown in Figure 8, where the trends shown suggest changes to outside options have compressed the tail of the non-college wage distribution more than for college workers.[21]

Note that when I simply replicate KORV's estimation approach I use exactly the same treatment of the data as they do, however when it comes to incorporating frictions I will trim the bottom and top percentile of the wage distribution out of the sample to minimize measurement error, which is more of a concern for estimating the sequential auction part of my model since I target higher order wage moments than when estimating the KORV parameters.

Overall, the key patterns we see are that job destruction rates are substantially lower for college workers, but that this advantage erodes over the sample period. This stands in contrast to unionization rates - which will drive my baseline scenario for bargaining strength- which start at similar values for non-college and college workers but decline more rapidly for the former. Multiple employer rates - my

[21]The estimated parameters of the KORV production function are not sensitive to using different measures of the lower bound of the wage distribution i.e. the actual minimum, or different wage percentiles - see Appendix C.

FIGURE 7. Unionization Rates



Notes: The figure shows the % of individuals in a trade union, as measures in the CPS. A consistent measure of this is available in the CPS from 1984 onward.

preferred measure of job mobility - exhibit similar levels and trends for non-college and college workers. [22]

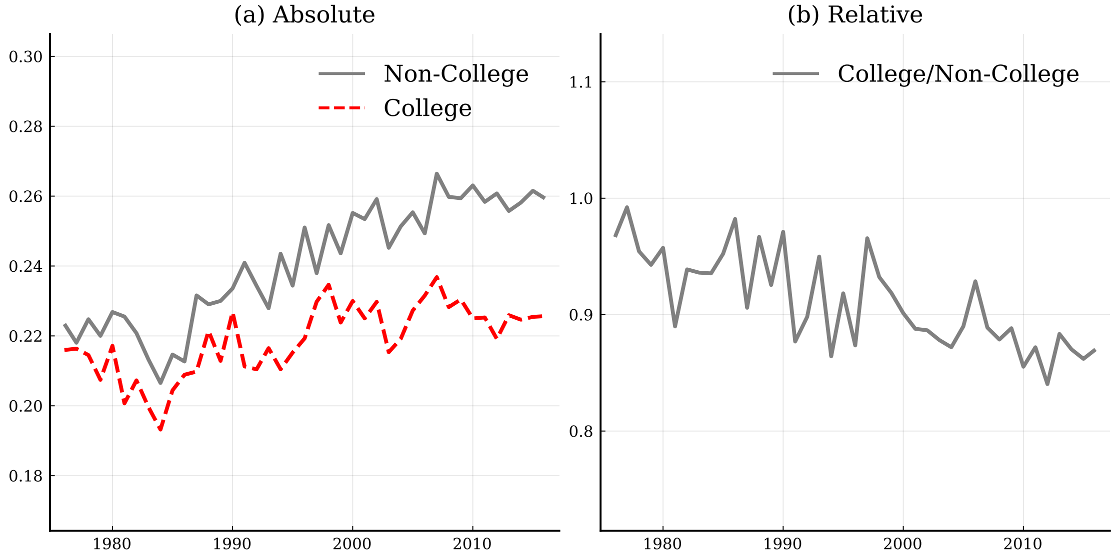
4. Estimation Approach

As with my exposition of the model, I will first present the original estimation approach used by Krusell, Ohanian, Rios-Rull, and Violante (2000), i.e. under perfect competition and with no intermediate goods sectors. I then set-out a two stage strategy for estimating the KORV parameters in the context of my model. The first stage is to estimate the parameters of the sequential auction model in the intermediate goods markets. The second stage is to incorporate results from the first stage to estimate the parameters of the KORV production function in the final good sector.

4.1. **KORV Estimation: Without Frictions.** Krusell, Ohanian, Rios-Rull, and Violante (2000) estimate their model by simulated pseudo maximum likelihood (SPML), matching the model's predictions for the labor share of output and the wage bill ratio of college workers relative to non-college workers, denoted

[22]This does not imply job offer rates are similar for non-college and college workers however since job mobility is a function of job offer rates and job destruction rates: this is discussed further in Section 5.2.

FIGURE 8. Lower Bound of Wages Relative To Median



Notes: The figure shows the lower bound of the wage distribution (defined as the average wage of workers in the bottom five percentiles of the wage distribution) divided by the median wage. Hourly wages are again calculated as per Krusell, Ohanian, Rios-Rull, and Violante (2000) using ASEC CPS data, but are now trimmed (by dropping hourly wages in the bottom and top percentiles) to minimise measurement error. Panel (a) shows the series separately for non-college and college workers, Panel (b) shows the series for college workers relative to that of non-college workers.

lsh_t and wbr_t respectively, to their empirical counterparts.[23] In addition, Krusell, Ohanian, Rios-Rull, and Violante (2000) target a no arbitrage condition between capital structures and equipment, i.e. their empirical strategy aims to minimise the difference between the model's predictions for the rate-of-return (RoR) on capital structures and the predicted RoR for capital equipment, alongside the other empirical targets mentioned above.[24] All model moments come from the first order conditions of the final good firm's profit maximisation condition given in equations (4) through (7).This estimation strategy is summarised in equations (22), (23) and (24) respectively, where X_t is the set of factor inputs $(K_{s,t}, K_{e,t}, N_t, C_t)$, $(\kappa_{eq},\kappa_{st})$ are the depreciation rates for capital equipment and structures respectively, and ϕ is the vector of all parameters to be estimated.

[23]SPML is generally attributed to Laroque and Salanie (1993) and is used when a closed form solution for the exact likelihood or quasi likelihood are both unavailable. Just as MLE can be viewed as a specific form of GMM (where the expectation of the score is the relevant moment), so SPML can be viewed as specific form of SMM where I am taking the expectation of a set of moments across both simulations and across time.

[24]This is done as neither RoR is directly observable in the data.

$$\frac{w_{n,t}h_{n,t} + w_{c,t}h_{c,t}}{Y_t} = lsh_t(X_t, \psi_t; \phi) \tag{22}$$

$$\frac{w_{c,t}h_{c,t}}{w_{n,t}h_{n,t}} = wbr_t(X_t, \psi_t; \phi) \tag{23}$$

$$\begin{aligned} &0 = (1 - \kappa_{st}) + A_{t+1}G_{K_{c,t}}((X_t, \psi_t; \phi) - E_t(\frac{q_t}{q_{t+1}})(1 - \kappa_{eq}) - \\ &q_t A_{t+1} G_{k_{eq,t}}((X_t, \psi_t; \phi) \end{aligned} \tag{24}$$

Equations (22), (23) and (24) can be represented in vector form as $Z_t = f(X_t, \psi_t, \epsilon_t; \phi)$, where Z_t is a vector of the empirical or targeted moments on the left hand side of equations (22), (23) and (24) and $f(X_t, \psi_t, \epsilon_t; \phi)$ is a vector of the model moments on the right hand side of these equations.

Note that there are two stochastic elements in this system of estimation equations. First ψ_t is a (2×1) vector of the log of the efficiency levels of non-college and college labor respectively, and is assumed to follow a stationary process in KORV's benchmark estimation as set out in equation (25).

$$\psi_t = \psi_0 + \omega_t, \ \psi_t \equiv \log .(\Psi_{n,t}, \Psi_{c,t}) \tag{25}$$

ω_t is a vector shock process to the log of labor efficiency that is assumed to be multivariate normal and *iid* with covariance matrix Ω i.e. $\omega \overset{i.i.d}{\sim} N(0, \Omega)$, and ψ_0 is a vector of the log of initial values of non-college and college labor efficiency $(\psi_{0,u}, \psi_{0,s})$. In the benchmark estimation, the authors impose that there is no covariance between the two labor efficiency shocks and that they have a common variance so Ω can be rewritten as $\Omega = \eta_\omega^2 I$.[25]

The other stochastic process in this estimation procedure is in the no arbitrage condition, equation (24), where the third term on the right hand side of this equation $E_t(\frac{q_t}{q_{t+1}})(1 - \kappa_{eq})$ is the undepreciated capital equipment multiplied by the expected rate of change in the relative price of equipment. Krusell, Ohanian, Rios-Rull, and Violante (2000) make the simplifying assumption that this term can be replaced with $\frac{q_t}{q_{t+1}}(1 - \kappa_{eq}) + \epsilon_t$, where $\epsilon_t \sim N(0, \eta_{\epsilon_t})$.

[25]As a robustness check, Krusell, Ohanian, Rios-Rull, and Violante (2000) do allow for a non-zero covariance between the two efficiency shocks and differing variances, but the estimated covariance is very small and there is little difference between the estimated variances so they opt for a benchmark estimation with zero covariance and a common variance.

In principle, the vector of parameters to be estimated, ϕ, contains 11 elements :$\{\kappa_{st}, \kappa_{eq}, \alpha, \mu, \lambda, \sigma, \gamma, \eta_\varepsilon, \eta_\omega, \psi_{0,u}, \psi_{0,s}\}$. However the authors calibrate $(\kappa_{st}, \kappa_{eq})$ using estimates from the literature, estimate η_ε separately , and normalize $\psi_{0,s} = 0$.[26] This leaves $\phi = \{\alpha, \mu, \lambda, \sigma, \gamma, \eta_\omega, \psi_{0,u}\}$ to be estimated i.e seven parameters: given the estimation approach targets three moments for each year of their 30 year dataset, the model is over-identified.

Finally, the authors construct an instrument for hours worked, $\hat{h}_{n,t}, \hat{h}_{c,t}$ to allow for potential endogeneity between relative hours worked and relative wages.[27] While such endogeneity would be irrelevant if the sole goal was to match the model to the data, instrumenting labor inputs means one can more credibly give the parameters economic interpretations, i.e. as elasticities of substitution, and hence use the model for counter-factual analysis. The exogenous factor inputs used in model estimation are therefore $\hat{X}_t = (K_{s,t}, K_{e,t}, \hat{h}_{n,t}, \hat{h}_{c,t})$. Estimation then proceeds in three steps:

(1) Draw S values of the vector of shocks to labor efficiency, ω_t^j, and of the forecast error in expected price gains of capital equipment, ϵ_t^j,(where j indexes the realization of the shock) to get S realizations of $f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi)$ from the model for each time period t,

(2) Use these S realizations to obtain the following moments:

$$m_s(\hat{X}_t, \phi) = \frac{1}{S} \sum_{i=1}^{S} f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi)$$

$$V_s(\hat{X}_t, \phi) = \frac{1}{S-1} \sum_{i=1}^{S} (f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi) - m_s(\hat{X}_t, \phi))(f(\hat{X}_t, \psi_t^j, \epsilon_t^j; \phi) - m_s(\hat{X}_t, \phi))'$$

(3) Minimise the following objective function:

$$\begin{aligned} l_s(\hat{X}_t, \phi) = & \frac{1}{2T} \sum_{t=1}^{T} \Big\{ (Z_t - m_s(\hat{X}_t, \phi))'(V_s(\hat{X}_t, \phi))^{-1} \\ & \times (Z_t - m_s(\hat{X}_t, \phi)) + \log(\det(V_s(\hat{X}_t, \phi))) \Big\} \end{aligned} \tag{26}$$

[26] The authors set $\kappa_{eq} = 0.125$ and $\kappa_{st} = 0.05$ following Greenwood, Hercowitz, and Krusell (1997) and estimate η_ε via an ARMA regression of q_t.

[27] The instruments are constructed by regressing hours worked of each worker type against a constant, current, and lagged stock of capital equipment and structures, the lagged relative price of equipment, a trend, and the lagged value of the U.S. business cycle indicator produced by the Economic Cycle Research Institute: https://www.businesscycle.com/ecri-reports-indexes/all-indexes.

In a companion paper to Krusell, Ohanian, Rios-Rull, and Violante (2000), Ohanian, Violante, Krusell, and Ros-Rull (1997) look at how successfully the estimation approach above identifies the true parameters of the model in Monte Carlo simulations, and find very small median and mean biases in estimators even when using relatively few simulations in estimation i.e. for $S = 10$. They find that for $S = 50$ the mean bias is “essentially zero”.

4.2. **Incorporating Frictions into KORV Estimation.** I proceed in two steps to incorporate the sequential auction model of Cahuc, Postel-Vinay, and Robin (2006) into estimation of the KORV production function parameters. First I separately estimate the parameters of the sequential auction model, which include job contact rates for employed workers of each worker type $\lambda_{1,i,t}$ and the parameters of their match distribution. Appendix B examines identification of these parameters in greater detail, showing exact identification of the job contact rates using the empirical strategy outlined here and providing evidence from Monte Carlo simulations that my strategy for estimating the parameters of the match quality distribution also successfully identifies the true parameters of the model.

In the second part of my estimation approach, I estimate the parameters of the KORV production function incorporating the changes to labor market frictions implied by the first stage of my estimation process. This second step is, in econometric terms, a minor modification of the original approach of Krusell, Ohanian, Rios-Rull, and Violante (2000), as presented above, that uses two key outputs from the sequential auction model: the average match quality and wage of each worker type, which are both identified up to a scaling factor in the first stage of estimation. This scaling factor is the price of the intermediate good produced in a given skill sector, which is determined by the parameters of the KORV production function. The rest of this section describes each of these steps in greater detail, starting with estimation of the sequential auction model.

Sequential Auction Estimation: Job Contact Rates

The monthly job contact rate for employees, $\lambda_{1,i,t}$, is chosen so that the model matches the empirical proportion of individuals continuously employed in a year who have more than one employer (the multiple employer rate, denoted $\tau_{i,t}$). This moment is given in the model by equation (27).

$$\tau_{i,t} = 1 - \int_{\nu_{inf_{i,t}}}^{\nu_{max}} (1 - \lambda_{1,i,t}\bar{F}_{i,t}(\nu))^{12} dL_{i,t}(\nu) \tag{27}$$

In Appendix B, I show that this expression is independent of the match quality distribution meaning I can estimate job contact rates separately of distributional parameters. The expression is also an increasing monotonic function of $\lambda_{1.i}$ which implies this parameter is indeed identified when I estimate it by simulated method of moments, as set out in equation (28) (where $\hat{x}$ denotes the empirical counterpart of model moment x).

$$\lambda^*_{1,i,t} = \underset{\lambda_{1,i,t}}{\operatorname{argmin}}\big(\tau_t(\lambda_{1,i,t}) - \hat{\tau}_t\big)^2 \tag{28}$$

Sequential Auction Estimation: Distribution of Match Heterogeneity

I assume that sampling distribution of match heterogeneity can be characterized by a lower truncated log normal distribution, and therefore can be fully described by three parameters: the mean and variance parameters, $\zeta^{\nu}_{i,t}$, $\eta^{\nu}_{i,t}$, and lower truncation point, $\nu_{inf_{i,t}}$. Note that by estimating the lower bounds directly I bypass the need to estimate job contact rates for the unemployed or replacement rates. This follows because my principal interest is to estimate the distribution of wages and match quality for workers in the intermediate goods market; unemployment conditions influence these variables through the lower bound of the match quality distribution only.

Given I have data on employees only, and not employers, a natural option to estimate $\zeta^{\nu}_{i,t}$ and $\eta^{\nu}_{i,t}$ is to use moments of the wage distribution for workers of each worker type $i \in n, c$. Note, however, that all wages of a given worker type are scaled by the price of the intermediate good, p_i (see equation (15)), which depends on the parameters of the KORV production function that I have yet to estimate. I therefore require the moment of the wage distribution that I will target to be scale invariant, and so choose the variance of log residual wages.

As both $\zeta^{\nu}_{i,t}$ and $\eta^{\nu}_{i,t}$ have a positive monotonic impact on match quality dispersion in the model, they will not be separately identified using the variance of log wages. I therefore set the value of $\zeta^{\nu}_{i,t}$ to target the mean of the sampling distribution, $\mathbb{E}^{F_{i,t}}(\nu)$, to an arbitrary fixed value (= 1). Note that this also avoids introducing a 'black-box' source of skills biased technological change via an increase in the relative means of the sampling distribution of match quality $\mathbb{E}^{F_{c,t}}(\nu)/\mathbb{E}^{F_{n,t}}(\nu)$ (Krusell, Ohanian, Rios-Rull, and Violante (2000) impose that the relative labor efficiency of college to non-college workers is constant for the same reason). This does not rule out an endogenous increase in the mean of the cross section distribution of match quality $\mathbb{E}^{L_{i,t}}(\nu)$. The variance parameter of the sampling distribution of match quality, $\eta^{\nu}_{i,t}$, is left free to match the dispersion of residual log wages within a worker type i in the model to its empirical counterpart.

Finally I must estimate the lower bound of the distribution of match quality, $\nu_{inf_{i,t}}$. Provided the bargaining parameter is sufficiently high, a worker at a match of quality $\nu = \nu_{inf_{i,t}}$ will earn the lowest wage in the model's wage distribution, denoted $\underline{w}_{i,t}$, where $\underline{w}_{i,t} = \nu_{inf_{i,t}} \times p_{i,t}$.[28] Since all wages are scaled by the price of the intermediate good, $p_{i,t}$, which will not be estimated at this stage, rather than target the absolute lower bound of the wage distribution I target the ratio of the lower bound to the median wage: $\underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}})$. When it comes to the empirical counterpart of this moment, I choose to use the average wages of workers in the bottom five percentiles of the wage distribution (again relative to the median) rather than the minimum of the empirical wage distribution as this is likely to be subject to significant measurement error.

In summary, I estimate the parameters of the sampling distribution, $\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$,by solving the minimization problem shown in equation (29), where $\hat{x}$ denotes the empirical counterpart of model moment x, and W is the weighting matrix.[29]

$$\begin{aligned} (\zeta^{\nu*}_{i,t}, \eta^{\nu*}_{i,t}, \nu^{*}_{inf_{i,t}}) &= \underset{\zeta^{\nu}_{i,t},\eta^{\nu}_{i,t},\nu_{inf_{i,t}}}{\operatorname{argmin}} \ (m_t - \hat{m}_t)^T W (m_t - \hat{m}_t) \qquad (29) \\ m_t \equiv \Big(& var_{\log(w_{i,t})}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}), \underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}), \\ & \mathbb{E}^{F_{i,t}}(\nu)(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}) \Big) \end{aligned}$$

I calculate the moments of the wage distribution in the model for a given guess of parameters by generating a sample of workers using the cross section distributions of workers' match quality and outside options given in equations (17) and (18) respectively, and then using equation (15) to derive the wages of these workers. Note that for notational convenience equation (29) suppresses the dependence of the model moments on job destruction rates, δ_i, and job contact rates, $\lambda_{1,i}$, which are taken from the data and estimated in the previous step respectively.

[28]In the model, the minimum wage in the population of workers will be paid to workers with match quality equal to the lower bound of the match distribution in the model when $\beta > \frac{\lambda_1}{\rho+\delta+2\lambda_1}$. This condition is derived from observing first that the wage expression in equation (15) is always decreasing in ν^-, so the lowest wage observable wage will certainly belong to those who have come from unemployment i.e. workers who have $\nu^- = \nu_{inf}$. Such workers will have a wage precisely equal to ν_{inf} when they are matched with the lowest match quality firms i.e. $\nu^+ = \nu^- = \nu_{inf}$. Finally a sufficient condition for this to be the lowest wage in the population is that the derivative of the wage expression with respect to the current match quality, ν^+, is positive for this worker and the second derivative is always positive. The latter condition always holds, and former holds when β is greater than the threshold shown above.

[29]The weighting matrix W, is chosen so I effectively minimize the percentage deviation of model moments from their empirical moments, which avoids the scale of absolute moment deviations biasing estimates i.e. $W = I.\frac{1}{\hat{m}}$.

Sequential Auction Estimation: Bargaining Parameters

I examine the impact of changing institutions on the wage bargaining environment by calibrating the bargaining parameters using relative unionization rates in the data. This is meant as an illustrative scenario since unionization rates are likely just one of the many determinants of individual bargaining strength represented by the parameter $\beta_{i,t}$ in the model. However, the empirical evidence on union wage premiums, e.g. in Farber, Herbst, Kuziemko, and Naidu (2018), does suggest union membership continues to have a significant impact on wages. This evidence also suggests union membership has a more significant effect on the wages of non-college workers than on the wages of college workers ,a result that will be consistent with my counterfactual simulations.

Given the illustrative nature of the exercise, I set $\beta_{c,t} = \beta_{c,0} = 0.95\ \forall t$, and adjust $\beta n, t$ such that $\beta_{c,0}/\beta n, t$ equals the empirical ratio of unionization rates in each period. [30] I find lower levels of the bargaining parameters mean the model struggles to simultaneously hit the level of the labor share and the rise in the college wage premium seen in the data. This occurs because the sequential auction part of the model sets an upper bound on the labor share in the overall model: incorporating a final goods sector with capital inputs will always lower the labor share relative to its level in the intermediate goods sector where labor is the only input. The upper bound on the labor share implied by the sequential auction results may be close to or even below the empirical labor share that I am targeting if the bargaining parameter is set too low. This issue is discussed in more detail in Appendix A. Although this calibrated bargaining parameter value appears high compared to some results in the micro literature, for example Cahuc, Postel-Vinay, and Robin (2006), many of these estimates come from structural models that do not feature capital and so are not directly comparable to ours.

Sequential Auction Estimation: Discount Rate

Finally, I arbitrarily set the monthly discount rate to 0.004.

Adding Sequential Auction Results to KORV Estimation

I adopt essentially the same empirical approach as Krusell, Ohanian, Rios-Rull, and Violante (2000) i.e matching the models predictions for the evolution of the wage bill of college workers relative to that of non-college workers and the labor share of income to their empirical counterparts and targeting a zero rate-of-return (RoR) difference between capital structure and capital equipment in the model.

[30] An alternative approach is to estimate $\beta_{c,0}$, exploiting its impact on the labor share by estimating it jointly with the KORV production parameters. I have found this approach significantly increases computation time and the complexity of estimation, without substantially improving model fit. It also yields an estimate of $\beta_{c,0} = 0.90$ i.e. not far away from my calibrated value.

However, I make two key modifications to incorporate results from the sequential auction stage of my estimation approach.

The first modification is necessary because the non-college and college labor inputs are not simply hours worked by the two types but rather the amount of intermediate goods from each worker type sector. The inputs N_t and C_t therefore become as defined in equation (**??**) where I multiply the labor inputs that KORV use (total hours in efficiency units) by the average match quality in each skill sector. Estimates of average match quality are derived from simulations using estimated parameters from the sequential auction part of my model, and are denoted $\mathbb{E}^{\hat{L}_{i,t}}(\nu) \equiv \int_{\hat{\nu_{inf_{i,t}}}}^{\nu_{max}} \nu \hat{\ell_{i,t}}(\nu)$ (where hats denote estimated variables or parameters).

Second average wages for a given worker type i are no longer simply the marginal product of that worker type in production of the final good, but are determined as specified in equation (21). I decompose this expression into two parts, as shown below.

$$\begin{aligned}
\mathbb{E}^{L_{i,t}}(w_{i,t}) &= p_{i,t} \times \mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1) \\
\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1) &\equiv \int_{\underline{\nu}}^{\nu_{max}} \Bigg[\nu - [1 + \kappa_{1,i}\bar{F}(\nu)]^2 \times \\
&\int_{\nu_{inf}}^{\nu} \frac{(1-\beta)[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\bar{F}(x)]}{[1 + \frac{\delta_i}{\delta_i+\rho}\kappa_{1,i}\beta\bar{F}(x)][1 + \kappa_{1,i}\bar{F}(x)]^2} dx \Bigg] \ell_i(\nu) d\nu
\end{aligned} \tag{30}$$

Thus average wages are calculated by multiplying the price of the intermediate good $p_{i,t}$ (its marginal product in the production of the final good) by the average wages in the intermediate good sector when the price of the intermediate good is normalized to one $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$. As with estimates for average match quality, I estimate $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$ by using estimation results from the sequential auction part of my model.

To summaries, I adapt KORV's original methodology to include intermediate goods sectors with sequential auction labor markets via two modifications. First I scale the labor input of a given worker type i by a 'productivity scale' which is my estimate of the average match quality in their intermediate good sector, $\mathbb{E}^{\hat{L}_{i,t}}(\nu)$. Second, I then calculate average wages by multiplying the marginal product of a given intermediate good in the KORV production function (for a given parameter guess) by a 'wage scale' that is my estimate of average normalized wages in the relevant intermediate goods sector, $\mathbb{E}^{\hat{L}_{i,t}}(w_{i,t}, p_{i,t} = 1)$. Otherwise, estimation of the parameters of the KORV production function proceeds exactly as described in Section 4.1.

Table 1. Parameter Estimates: KORV vs Replication

Parameter	KORV findings	Replication Estimates
α	0.117	0.122
γ	-0.495	-0.45
σ	0.401	0.402
$\varepsilon_{S,K_{eq}}$ $(= 1/1-\gamma))$	0.669	0.689
$\varepsilon_{U,K_{eq}}$ $(= 1/1-\sigma))$	1.669	1.674
CSC Strength: $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$	1.001	0.984

Notes: Rows 2-4 show estimates of the subset of primitive parameters of the KORV production function that were published in Krusell et al. (2000). Row 5 shows the implied elasticity of substitution between capital equipment and skilled labor input, $\varepsilon_{S,K_{eq}}$. Row 6 shows the implied elasticity of substitution between capital equipment and unskilled labor input, $\varepsilon_{U,K_{eq}}$. Row 7 shows the implied strength of the capital skill complementarity (CSC) channel, as measured by $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$.

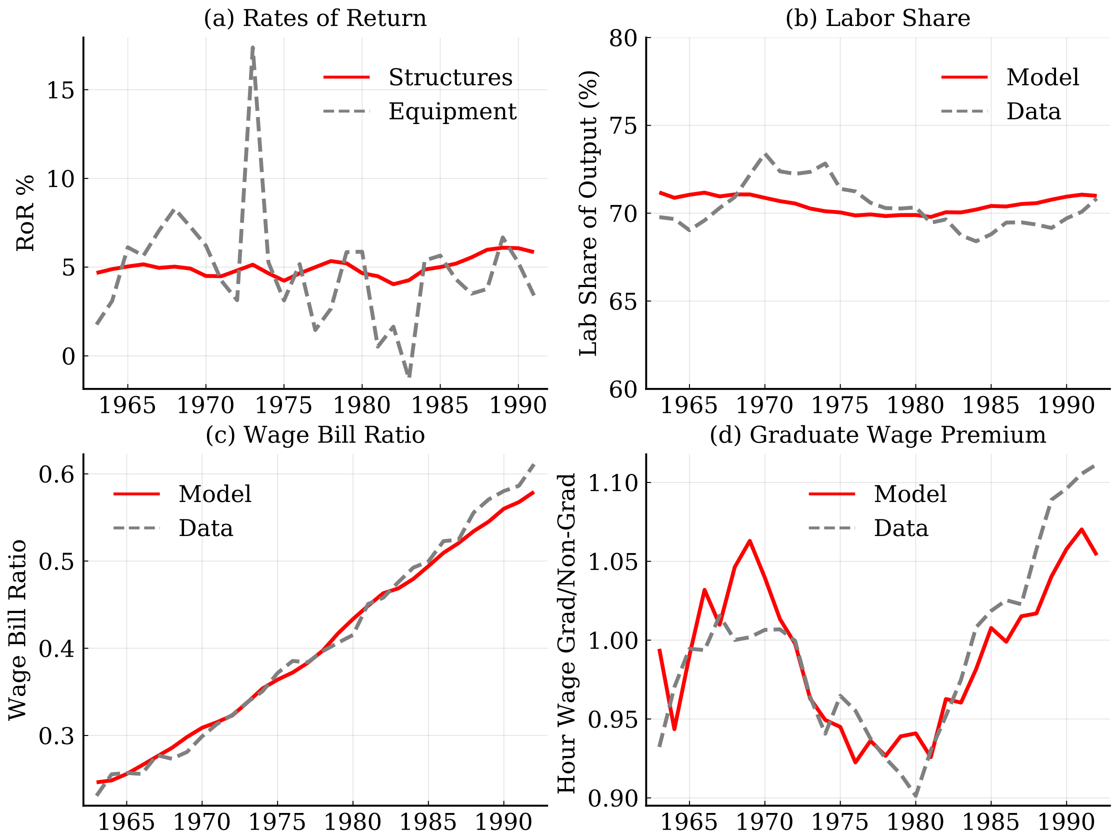
5. Results

This section starts by verifying that I can replicate the results provided in Krusell, Ohanian, Rios-Rull, and Violante (2000) when I use their estimation strategy and data. I also present the results of replicating KORV's methodology, i.e. with no frictions, for an updated sample period. I then show results from estimation of the sequential auction model of the intermediate goods sectors, and finally I show the impact of incorporating intermediate goods sectors with search frictions into estimation of the KORV production function. When considering this impact my focus will be on how, if at all, estimates of capital skill complementarity change and how that changes explanations for the rise in the college wage premium.

5.1. **Replication of KORV methodology.** I am able to replicate results from KORV both in terms of fit to the author provided data (see Figure 9 for my fit to the data and Figure 10 for the equivalent figure in Krusell, Ohanian, Rios-Rull, and Violante (2000)) and in terms of parameter estimates (see Table 1). In particular, I estimate very similar levels of capital skill complementarity (as measured by the difference in the elasticity of substitution between non-college labor and capital equipment, denoted $\varepsilon_{U,K_{eq}}$, and the elasticity of substitution between college labor and capital equipment, denoted $\varepsilon_{S,K_{eq}}$).

When I include more recent data in my replication of the KORV methodology, rather than using only their original sample period of 1963-1992, I find the model

Figure 9. Replication of KORV



Notes: All model moments displayed in the figure are generated using estimated parameter values using my replication of KORV's methodology. Panel (a) of the figure compares the ex-post rates of return (RoR) on capital structures and equipment predicted by the model. Panels (b) through (d) compare model moments to their empirical counterparts for the labor share of income, the wage bill of college workers relative to that of non-college workers, and the college wage premium respectively.

again fits the data well - see Figure 11.[31] Table 2 shows inclusion of more recent data decreases estimates of capital skill complementarity. A potential explanation for this is that the empirical growth in the college wage premium remains steady after 1992 despite an sharp acceleration in capital equipment growth (see Figure 2); to reconcile these two patterns the model requires a lower estimate of capital skill complementarity than in the original sample period.

[31] I have to switch from author provided data to publicly available data to extend the time period, which is why the parameter estimates for the original sample period shown in Table 2 differ from those in Table 1.

FIGURE 10. KORV's original fit

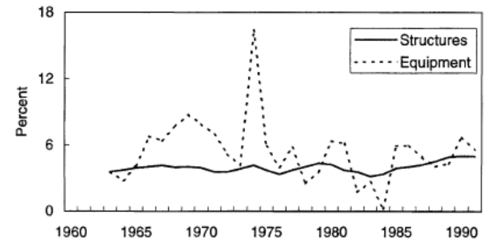


Figure 5. Ex post rates of return on capital equipment and structures (%).

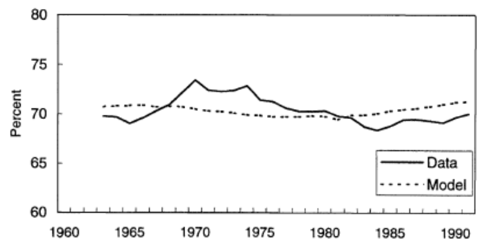


Figure 6. Labor's share of aggregate income (%).

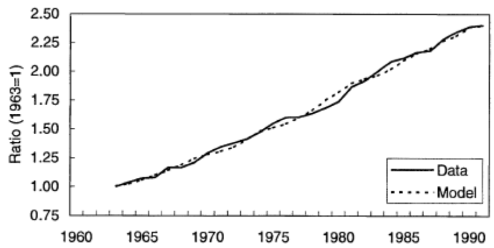


Figure 7. The wage-bill ratio: Skilled vs. unskilled total wages (normalized with 1963=1).

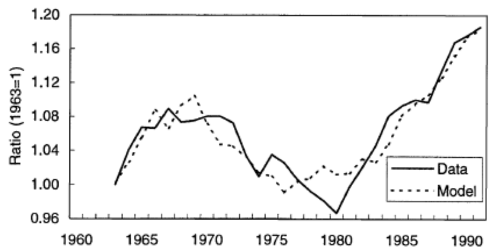


Figure 8. The skill premium: Skilled vs. unskilled wages per hour (normalized with 1963=1).

Notes: I am unable to directly provide model predictions using KORV's parameter estimates, as the author's only provide a subset of the relevant estimates, so I directly reproduce the figure from Krusell, Ohanian, Rios-Rull, and Violante (2000) showing the fit of their model to the data.

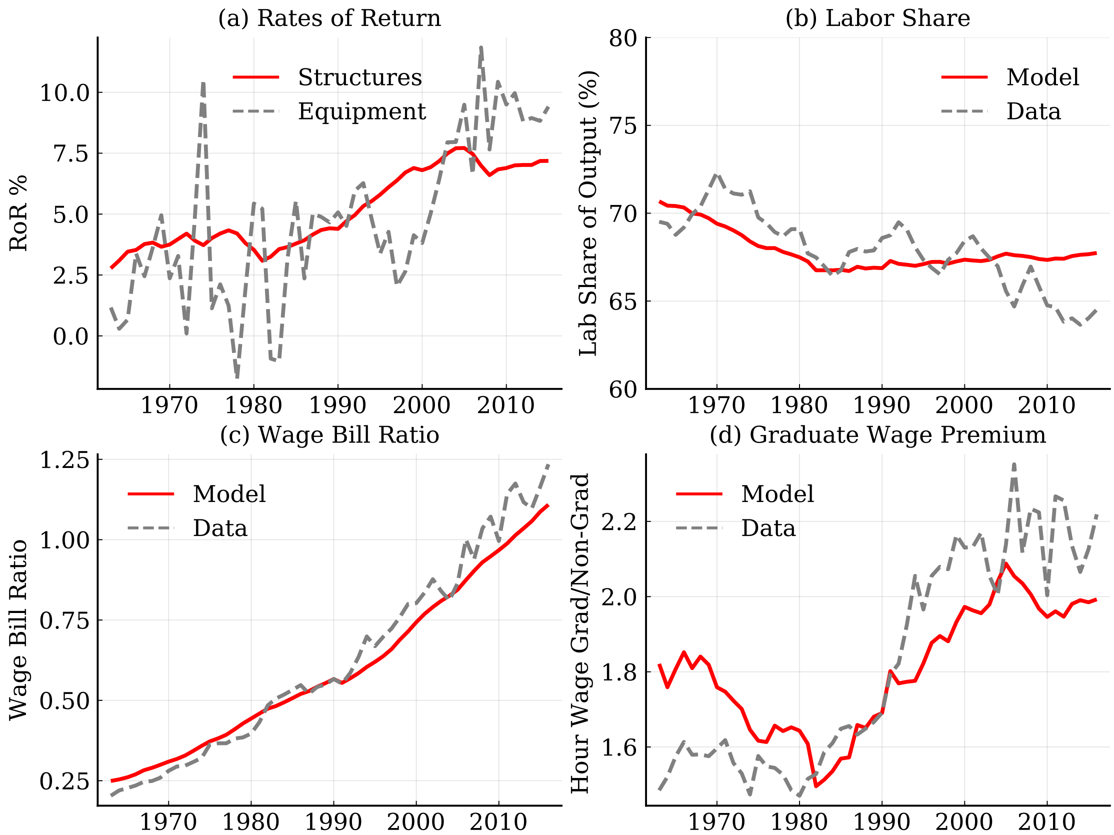
5.2. **Sequential Auction Results.**

Sequential Auction Results: Job Contact Rates

The first row of Figure 12 shows my estimates of job contact rates for non-college and college workers, in absolute and relative terms (I plot a six year rolling average of estimated relative contact rates to emphasize the time trend). The second row of the same figure shows the empirical targets these estimates are based on - the multiple employer rate - and the corresponding model moments.

I am able to exactly match the model moments to their empirical counterparts. Estimated job contact rates do not exactly track the data on multiple employer rates because job contact rates are not the sole determinant of the multiple employer rate: job destruction also plays a role, as shown in equation (27). The intuition here is that workers who exit the labor market more frequently will spend more time at the bottom of the job ladder and hence move employers more often due to favorable job offers. This explains why college workers are estimated to have

Figure 11. Replication of KORV: Extended Sample Period



Notes: All model moments displayed in the figure are generated from estimated parameter values using my replication of KORV's methodology and updating sample data to the latest available. Panel (a) of the figure compares the ex-post rates of return (RoR) on capital structures and equipment predicted by the model. Panels (b) through (d) compare model moments to their empirical counterparts for the labor share of income, the wage bill of college workers relative to that of non-college workers, and the college wage premium respectively.

significantly higher job contact rates than non-college workers despite similar multiple employer rates: the higher job contact rate for college workers offsets their lower job destruction rates to leave the multiple employer rate at a similar level to non-college workers. So while casual inspection of multiple employer rates alone might suggest similar levels of employer competition for college and non-college workers, the findings here illustrate that interpreting the data through a structural model leads to a quite different conclusion i.e. that college workers enjoy much higher job contact rates than non-college workers.

Table 2. Parameter Values with Extended Sample Period

Parameter	Original Sample Period	Extended Sample Period
λ	0.594	0.551
μ	0.757	0.324
α	0.111	0.123
γ	-0.461	-0.267
σ	0.452	0.424
ε_{c,k_e} $(= 1/1-\gamma))$	0.684	0.79
ε_{n,k_e} $(= 1/1-\sigma))$	1.823	1.736
CSC Strength: $\varepsilon_{n,k_e} - \varepsilon_{c,k_e}$	1.139	0.947

Notes: Rows 2-6 show estimates of the primitive parameters of the KORV production function. Row 7 shows the implied elasticity of substitution between capital equipment and skilled labor input, ε_{c,k_e}. Row 8 shows the implied elasticity of substitution between capital equipment and unskilled labor input, ε_{n,k_e}. Row 9 shows the implied strength of the capital skill complementarity (CSC) channel, as measured by $\varepsilon_{n,k_e} - \varepsilon_{c,k_e}$.

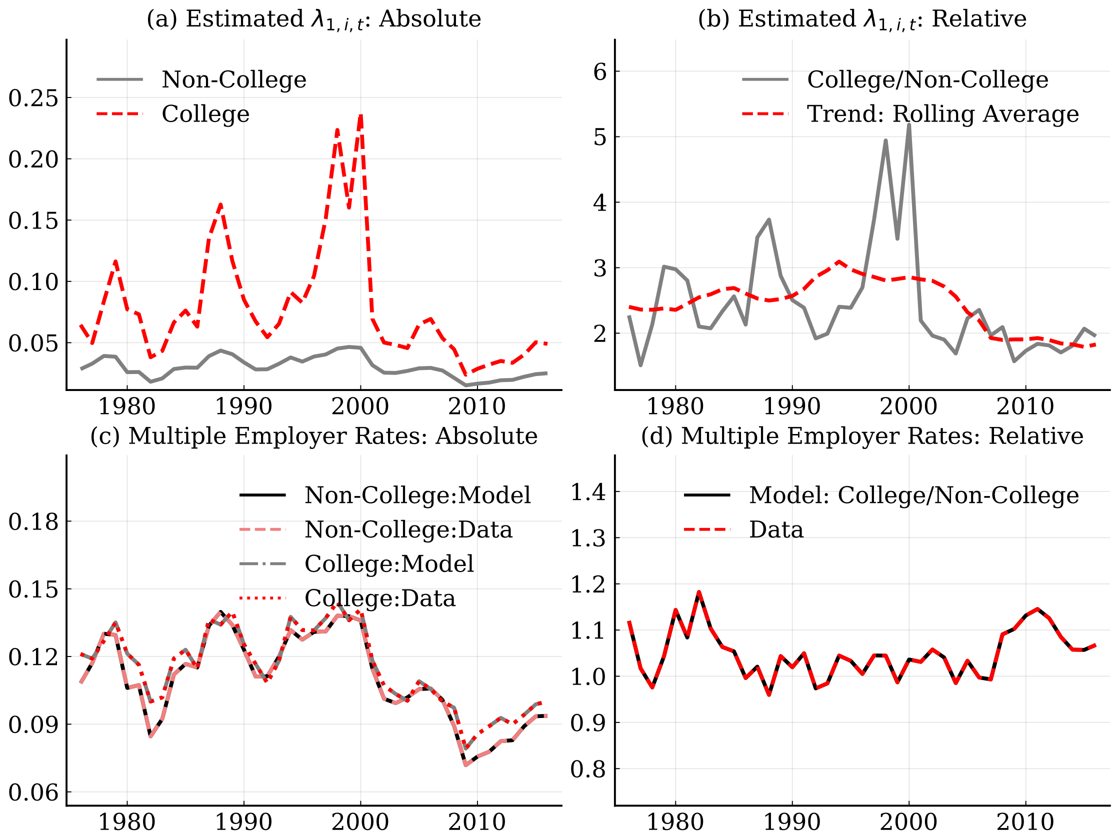
However, the time trend is for estimated job contact rates for college workers to increase relative to those of non-college workers until the late 1990s and then decrease thereafter.

Sequential Auction Results: Distribution of Match Quality

For each of my two worker types, I estimate three parameters of the match quality distribution, which is assumed to take a truncated log normal form: the mean, variance and lower bound parameters, $\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ respectively. I am able to match the model to the targeted empirical moments precisely in the case of both $\eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ where the relevant targets are log residual wage variance and the ratio of average wages of workers in the bottom five percentiles of the wage distribution to median wages respectively. Figures 13 and 14 show parameter estimates for $\eta^{\nu}_{i,t}$ and $\nu_{inf_{i,t}}$ respectively, and illustrate the close fit of the model moments to the data. Estimates of $\zeta^{\nu}_{i,t}$ are in a sense less relevant since they are simply set at the level necessary to keep the mean of the sampling distribution of match quality constant at an arbitrary target ($\mathbb{E}^{F_{i,t}}(\nu) = 1$).

The estimated variance parameter of the sampling distribution of match quality, $\hat{\eta}^{\nu}_{i,t}$, for college workers increases over time relative to the equivalent parameter for non-college workers, mirroring changes in the empirical target (residual log wage variance). Estimates of the lower bound of the match quality distribution, $\hat{\nu}_{inf_{i,t}}$ decrease in relative terms for college workers, again mirroring the trend in the empirical target.

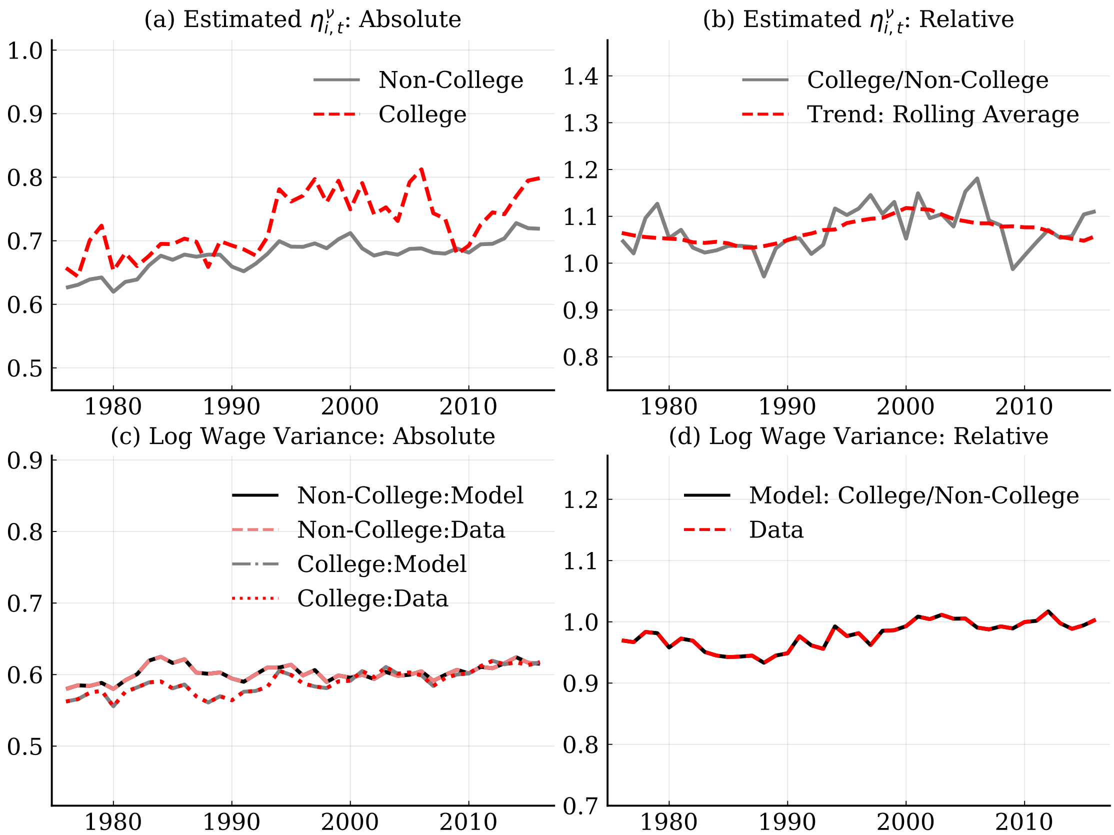
Figure 12. Job Mobility



Notes: The first row of the figure shows estimated job contact rates for employees, $\lambda_{1,i,t}$, in absolute terms for non-college workers and college workers - Panel (a) - and for college workers relative to non-college workers - Panel (b). The second row shows implied model predictions for the multiple employer rate, and their empirical counterparts, shown in absolute and relative terms in Panels (c) and (d) respectively.

5.3. **KORV production function: parameter estimates.** As argued in Section 4.2, the results of my estimation of the sequential auction structure of the intermediate goods market can be fully summarized by two series for the purposes of estimating the parameters of the KORV production function. The first series is the 'productivity scale', which is the estimate of the average match quality by skill $\mathbb{E}^{L_{i,t}}(\nu)$ that I use to scale labor inputs. The second series is the'wage scale', $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$, which relates to average wages of worker type i via the identity $\mathbb{E}^{L_{i,t}}(w_{i,t}) = p_{i,t}\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$. These series are plotted in absolute and relative terms in Figure 15, with a rolling 6 year average of the relative series added to emphasize the relevant trends.

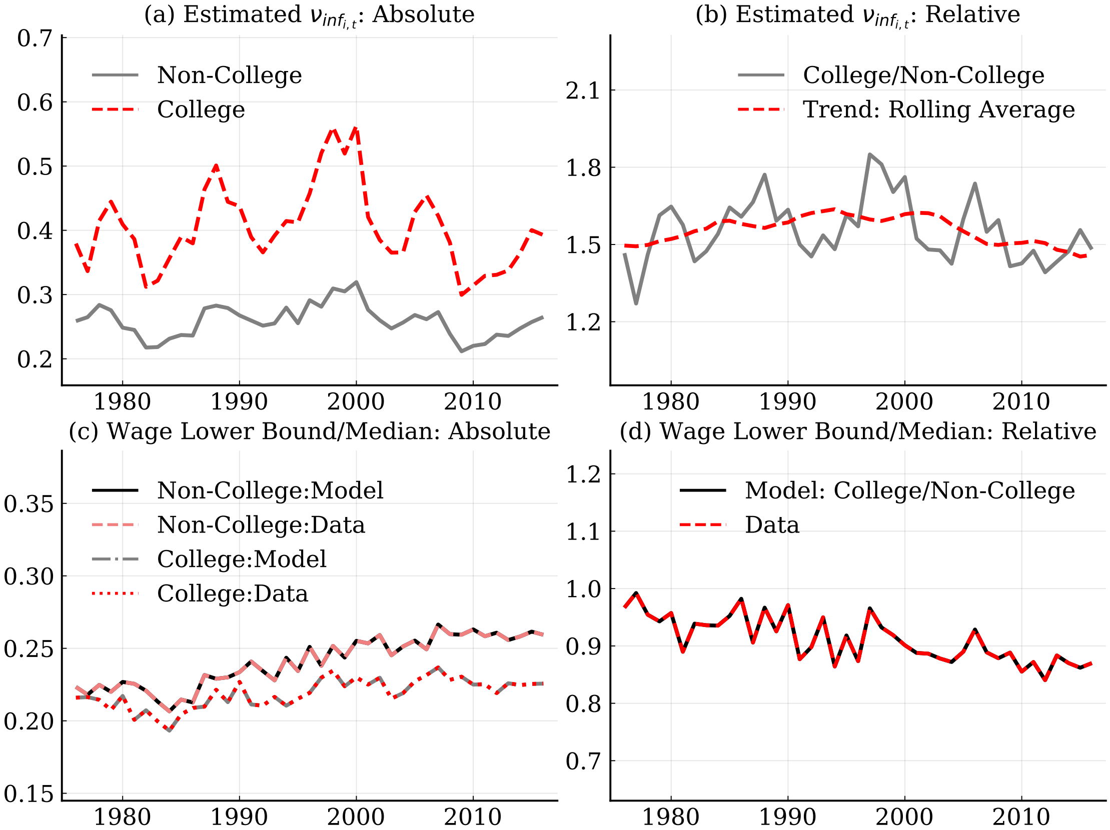
FIGURE 13. Match Quality Dispersion



Notes: The first row of the figure shows estimates of the variance parameter of the match quality distribution, $\eta^{\nu}_{i,t}$, in absolute terms for non-college workers and college workers - Panel (a) - and for college workers relative to non-college workers - Panel (b). The second row shows implied model predictions for the variance of log wages and their empirical counterparts (where the empirical variance is given for residual log wages, after controlling for education, race, sex and year). This moment is shown in absolute and relative terms in Panels (c) and (d) respectively.

Figure 15 shows that the presence of search frictions can explain a positive college wage premium since the relative wage scale is estimated to be consistently above one (see Panel (d)). Reflecting trends in estimated job contact rates and match quality distributions, the average match quality (the 'productivity scale') of college workers relative to non-college workers (see Panel (b)) increases moderately until the late 1990s but then decreases after this. The upwards trend in the wage scale of college workers relative to non-college workers (Panel (d)) is more pronounced due to the increase in their relative bargaining strength, driven by increasing relative unionization rates. This trend means my baseline model is less reliant on the capital skill complementarity channel for generating an increase in the college wage premium, as compared to an equivalent competitive model (i.e with no frictions).

FIGURE 14. Match Quality Lower Bound

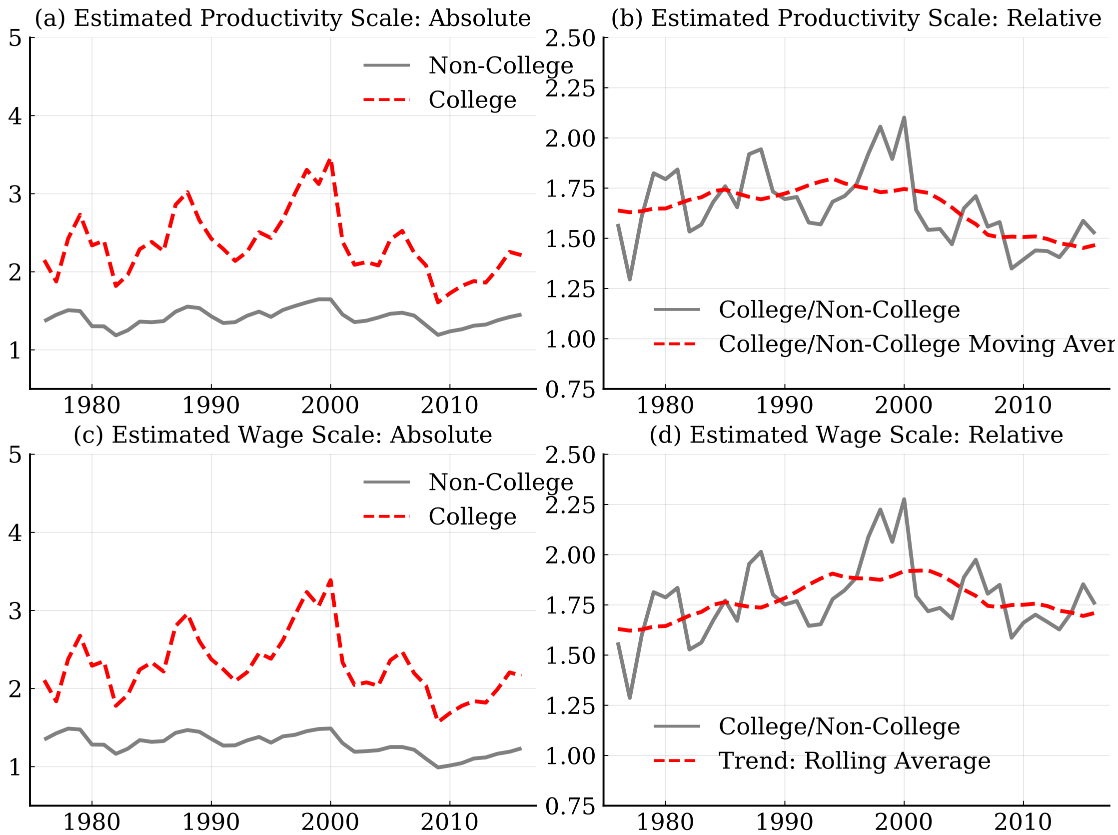


Notes: The first row of the figure shows estimates of the lower bound of the match quality distribution, $\nu_{inf,i,t}$, in absolute terms for non-college workers and college workers - Panel (a) - and for college workers relative to non-college workers - Panel (b). The second row shows implied model predictions for the lower bound of the wage distribution as ratio of the median wage, and their empirical counterparts, shown in absolute and relative terms in Panels (c) and (d) respectively.

In particular, the estimate of capital skill complementarity is smaller in my baseline model (column 2 of Table 3) compared to the equivalent model without frictions (column 3 of Table 3). This is driven largely by a lower estimate of the elasticity of susbtitution between non-college labor and capital equipment in my baseline model. This finding is very policy relevant since this parameter is likely key when considering the impacts of labor market policies to help low college workers e.g. minimum wages.

The extent to which estimates of capital skill complementarity are lower in my baseline model than in an equivalent model with no frictions is demonstrated in Figure 17. This plots the counterfactual scenario where search frictions are

FIGURE 15. Scale Factors



Notes: The first row of the figure shows estimates of the mean of the match quality distribution, $\mathbb{E}^{L_{i,t}}(\nu)$ (the 'productivity scale'), in absolute terms for non-college workers and college workers - Panel (a) - and for college workers relative to non-college workers - Panel (b). The second row of the figure shows estimates of the mean wage of workers when the price of the intermediate good they produce is normalized to one, $\mathbb{E}^{L_{i,t}}(w_{i,t}, p_{i,t} = 1)$ (the 'wage scale'), in absolute terms for non-college workers and college workers - Panel (c) - and for college workers relative to non-college workers - Panel (d).

as estimated in my baseline model but the parameters of the KORV production function are as estimated in a model without frictions. The predicted rise in the college wage premium is stronger in this counterfactual scenario than in my baseline model precisely due to stronger capital skill complementarity channel when the model is estimated without allowing for search frictions.

The model with frictions fits the data more accurately than the original KORV formulation as shown in Figure 18. Both models predict a counter-factual decline in the college wage premium from the onset of the 2007-08 financial crisis due to

Table 3. KORV parameter values: the importance of frictions

Parameter	With Frictions	Without Frictions
λ	0.499	0.526
μ	0.556	0.601
α	0.098	0.126
γ	-0.327	-0.208
σ	0.247	0.337
ε_{c,k_e} $(= 1/1-\gamma)$	0.754	0.828
ε_{n,k_e} $(= 1/1-\sigma)$	1.328	1.507
CSC Strength: $\varepsilon_{n,k_e} - \varepsilon_{c,k_e}$	0.575	0.68

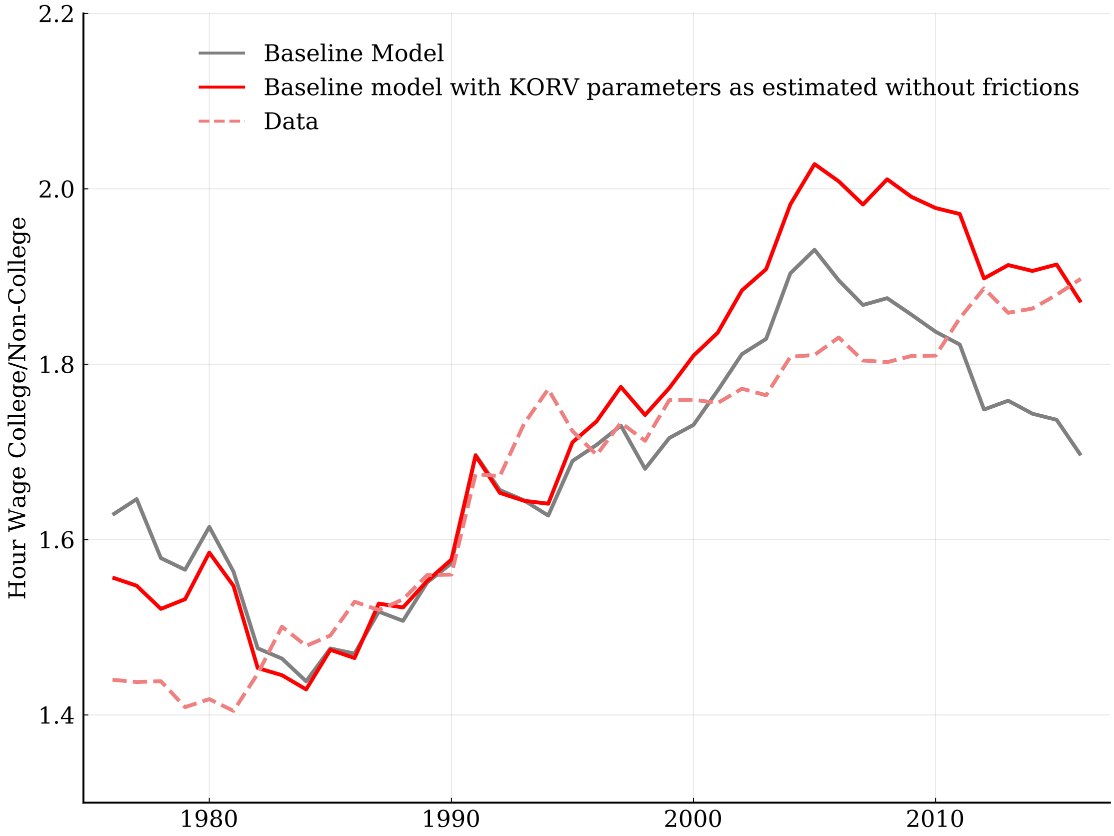
Notes: Rows 2-6 show estimates of the primitive parameters of the KORV production function. Row 7 shows the implied elasticity of substitution between capital equipment and college labor input, ε_{c,k_e}. Row 8 shows the implied elasticity of substitution between capital equipment and non-college labor input, ε_{n,k_e}. Row 9 shows the implied strength of the capital skill complementarity (CSC) channel, as measured by $\varepsilon_{n,k_e} - \varepsilon_{c,k_e}$.

a slowdown in the growth rate in the capital equipment seen in the data, and an increase in the relative supply of college workers.

5.4. **Model Simulations: Counterfactual Analysis.** I now consider which channels in my baseline model make the greatest contribution to the level and growth of the college wage premium. To build intuition, I start by showing the impact of varying the key search parameters in my model on the predicted average wages of non-college and college workers. This is done in Figure 19, which focuses on the impact of changing job contact rates, job destruction rates, the variance of match quality (again in a mean preserving way) and bargaining power. Note that varying any search parameter for worker type i that affects their average match quality has an impact both on the average wages of that worker type (e.g. an own-wage impact) and on the average wages of the other worker type (e.g. a cross-wage impact). This is due to the imperfect substitution of factor inputs in the final good production function. Figure 19 focuses on the own-wage impacts, while Appendix D describes the (much smaller) cross-wage impacts.

Three key patterns emerge from Figure 19. First that changing bargaining power has a larger impact on average wages than changing any other search parameter. This is true for non-college and college workers. This is because the positive impact of higher job contact rates (lower job destruction rates) on the match quality and outside option distribution of workers is partly offset by a reduction in the price of the intermediate good they produce. This reduction occurs because there is

FIGURE 16. Strength of Capital Skill Complementarity Channel

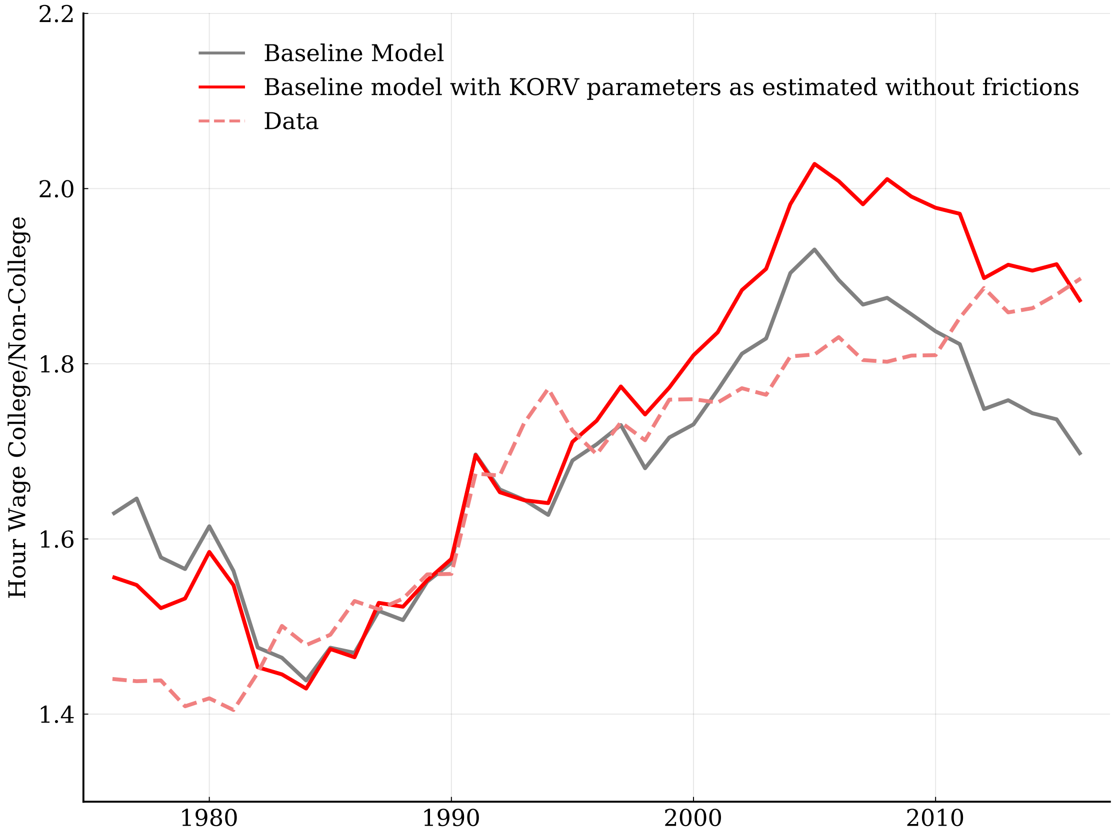


diminishing marginal product in the production function for each factor input. In contrast, increases in bargaining power have no such offsetting changes in the price of the intermediate good as they do not affect the match quality distribution.

The second key finding from this analysis is that changing job contact rates and job destruction rates for non-college workers has a bigger (own-wage) impact than changing these parameters for college workers. This again reflects the price reaction of intermediate goods: the parameter estimates of the final good production function suggest that the marginal product of intermediate goods produced by college workers diminishes at a faster rate than for non-college workers.

Finally, and crucially for results to follow, we see that average wage of non-college workers is more sensitive to changes in their bargaining power than is the case for college workers. This is driven entirely by the endogenous workings of the sequential auction model of labor markets I use, and lower estimated job offer rates for non-college workers. Job offers are particularly important in the sequential auction model both because they move employees to higher quality matches, and because they trigger bidding wars between rival and incumbent employers over

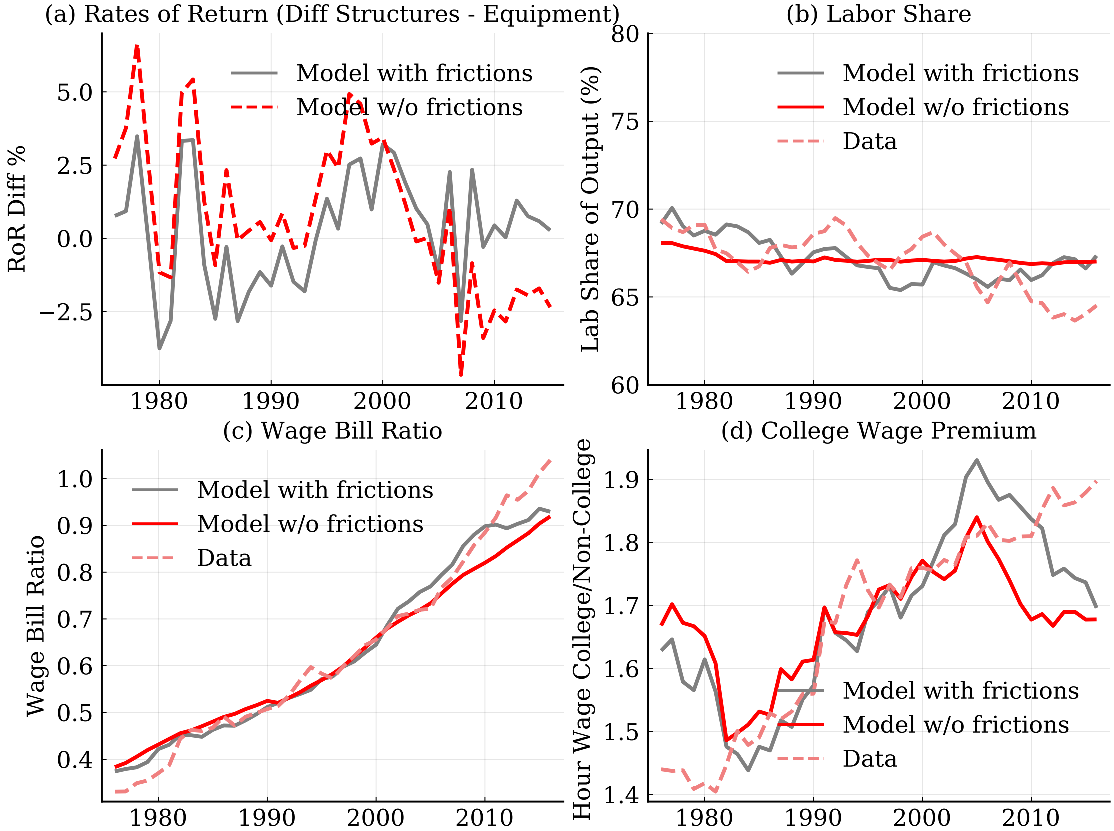
Figure 17. Strength of Capital Skill Complementarity Channel



the worker (even when the rival employer has a lower match quality). Non-college workers' lower job offer rates mean their exogenous bargaining strength plays a bigger role in determining their wages than is the case for college workers.

This is again done by using counterfactual scenarios: Figures 20 and 21 show that eliminating the differences between college and non-college workers' job destruction rates and job offer rates respectively lower the predicted level of the college wage premium by roughly similar amounts, without changing it's growth rate. Figure 22 shows the impact of eliminating differences in bargaining parameters between college and non-college workers, which are entirely driven by differences in their respective unionization rates. We see that the differences in bargaining parameters are responsible for a substantial component of both the average level of and growth of the predicted college wage premium in my model.

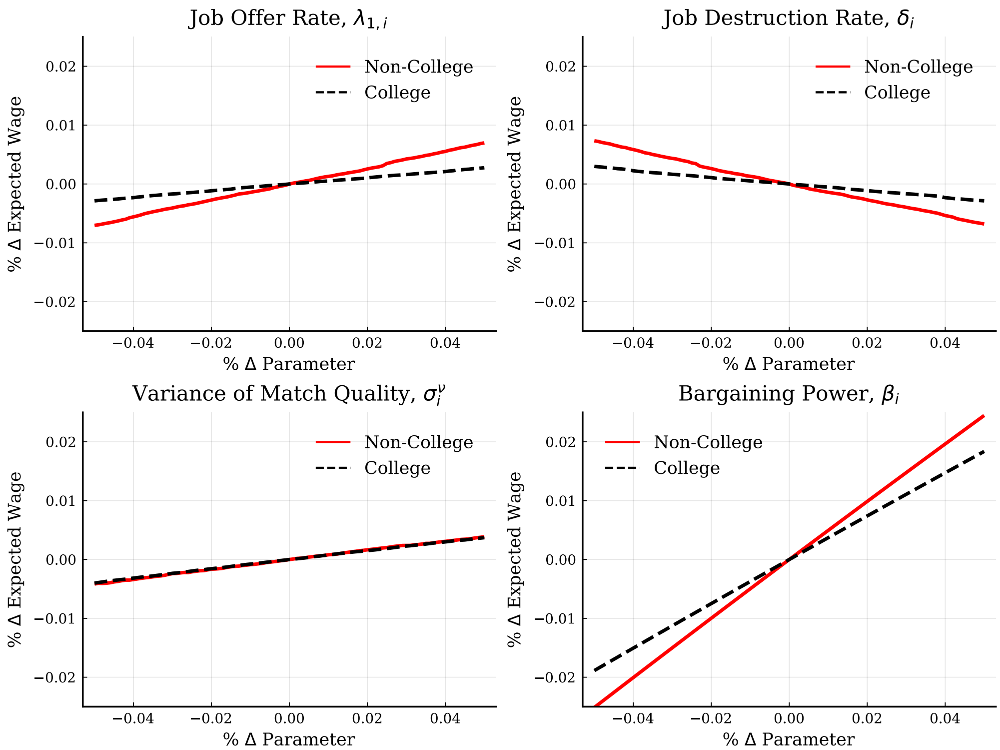
FIGURE 18. Model Fit: With and Without Frictions



Notes: The model moments displayed in the figure are generated both from estimated parameter values using my replication of KORV's methodology i.e without frictions (the 'w/o frictions' series) and using estimates when incorporating frictions (the 'with frictions' series). Panel (a) of the figure compares the difference in ex-post rates of return (RoR) between capital structures and equipment predicted by the two versions of the model. Panels (b) through (d) compare model moments to their empirical counterparts for the labor share of income, the wage bill of college workers relative to that of non-college workers, and the college wage premium respectively.

Despite the importance of search frictions found in the above analysis, my baseline model is still reliant on the capital skill complementarity (CSC) channel to generate an increase in the college wage premium. This can be seen by examining model predictions when I shut down the CSC channel by imposing $\hat{\sigma} = \hat{\gamma}$: Figure 23 shows that both the model with frictions and without in fact predict large falls in the college wage premium. This is due to the increase in the relative supply of college workers, when there is no CSC. This illustrates that the CSC channel is responsible for offsetting the negative impact of the rise in the relative supply of college workers in my baseline model.

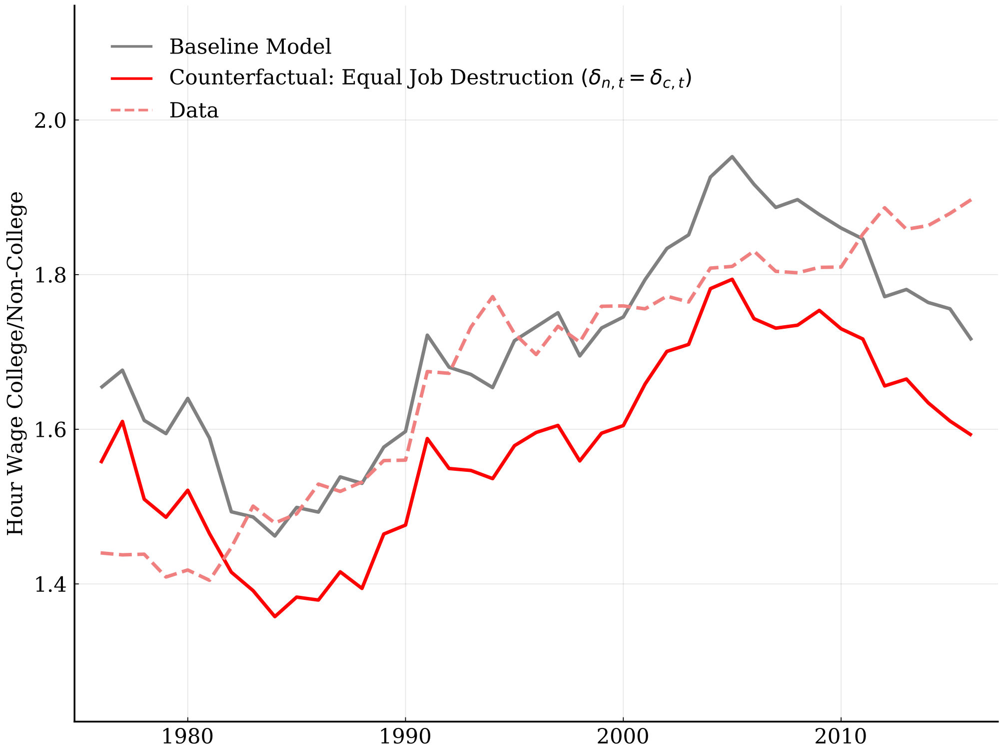
Figure 19. Parameter Impacts on Expected Wages



Notes: The average expected wages shown on the y-axis are wages averaged over both the match quality distribution in my model in each time period and averaged over all time periods. Deviations (of wages and parameters) are shown with respect to a baseline where all parameters are set a constant level in each time period equal to the average level of each parameter over the sample period. I then separately vary each parameter by $\pm 5\%$ in each time period, and simulate the impact on average expected wages.

Table 5 summarizes the impact of all the key channels in my baseline model on the level and growth of the college wage premium ($CWP \equiv \%$ difference in hourly wages: college graduate - non-graduate). First we see my baseline model almost exactly matches the average level of the college wage premium seen in the data over my sample period (1976-2018): the average CWP is 70.6% in my model versus 67.3% in the data. Absent any differences in search frictions, my baseline model suggests college workers would have 40% higher average hourly wages than non-college workers. In other words, the model suggests search frictions make a positive contribution of 30% points to the average level of the CWP, with that contribution being roughly equally due to college workers' higher job offer rates, higher bargaining strength and lower job destruction rates.

Figure 20. The Impact of Differences in Job Destruction Rates



Turning to the growth of the college wage premium, my model predicts just under two thirds of the growth seen in the data, predicting a rise of 25.5% points in the CWP over the sample period versus 39.8% point growth in the data.[32] The decline in the relative bargaining strength of non-college workers to college workers - set equal to the decline in their relative unionization rates - increases predicted CWP growth by 25.1% points i.e close to all of the net growth predicted in the model. However, as suggested in Figure 23, capital skill complementarity is the dominant channel when it comes to explaining the growth in the college wage premium, increasing predicted CWP growth by 65.5% points relative to a counterfactual model with no capital skill complementarity.

Finally, Figure 24 shows the dynamic contribution of each of the key channels in my model and reinforces the key results above.

[32]Note this calculates CWP growth as the % point difference between the average CWP in the first and last tend years of my sample period e.g $C\bar{W}P_{2007-2016} - C\bar{W}P_{1976-1985}$.

FIGURE 21. The Impact of Differences in Job Contact Rates

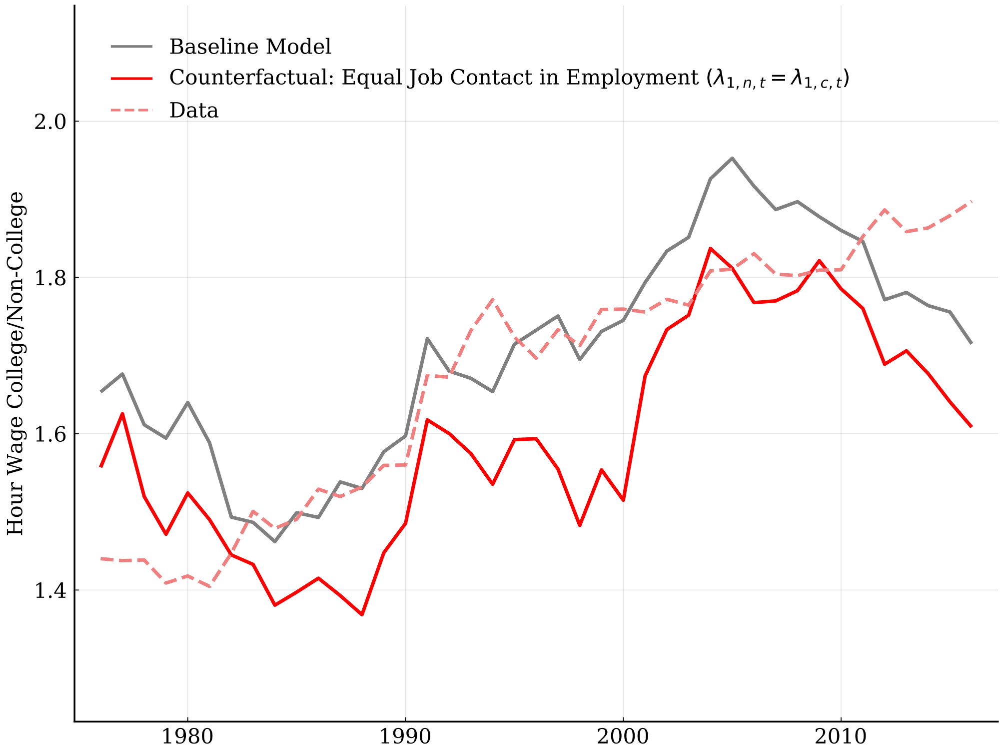


TABLE 4. Counterfactuals: Level & Growth of College Wage Premium (CWP)

	CWP level, % (mean: 1976-2016)	CWP level impact, % pt	CWP growth, % pt (1976-2016)	CWP growth impact, % pt
Data	0.673		0.398	
Baseline model	0.706		0.245	
Counterfactual scenarios:				
Equal job destruction rates	0.58	-0.126	0.218	-0.027
Equal job offer rates (employed)	0.595	-0.112	0.244	-0.001
Equal match quality distribution	0.698	-0.008	0.223	-0.022
Equal bargaining strength	0.593	-0.113	-0.006	-0.251
Equal search frictions (all)	0.407	-0.3	0.027	-0.218
No capital skill complementarity	0.343	-0.364	-0.41	-0.655
Equal Labor Supply	-0.191	-0.898	0.478	0.233
Equal Labor Efficiency	2.408	1.702	0.508	0.263

Column 2 shows the mean level of the college wage premium (CWP, defined as the percentage difference between college graduate and non-college wages) in the sample period, 1976-2016. This is shown for the CWP level in the data, the baseline model, and under the counterfactual model scenarios indicated. In each counterfactual scenario, the relevant parameter for Non-College workers (job destruction rate, job offer rate etc) is set equal to that of College Workers in each time period. Column 3 shows the difference between the mean CWP (as defined in column 2) in each counterfactual model scenario and the mean CWP in the baseline model. Column 4 shows the % point growth in the mean level of the CWP between the first and last tens of the sample period (1976-1985 and 2007-2016). Column 5 shows the difference between the CWP growth (as defined in column 3) in each counterfactual model scenario and the CWP growth in the baseline model.

6. Conclusion

This paper develops a structural model that nests a rich range of explanations for the level and growth of college wage premiums. This includes technological explanations, such as capital skill complementarity, relative labor scarcity, as well as

Figure 22. The Impact of Differences in Bargaining Parameters

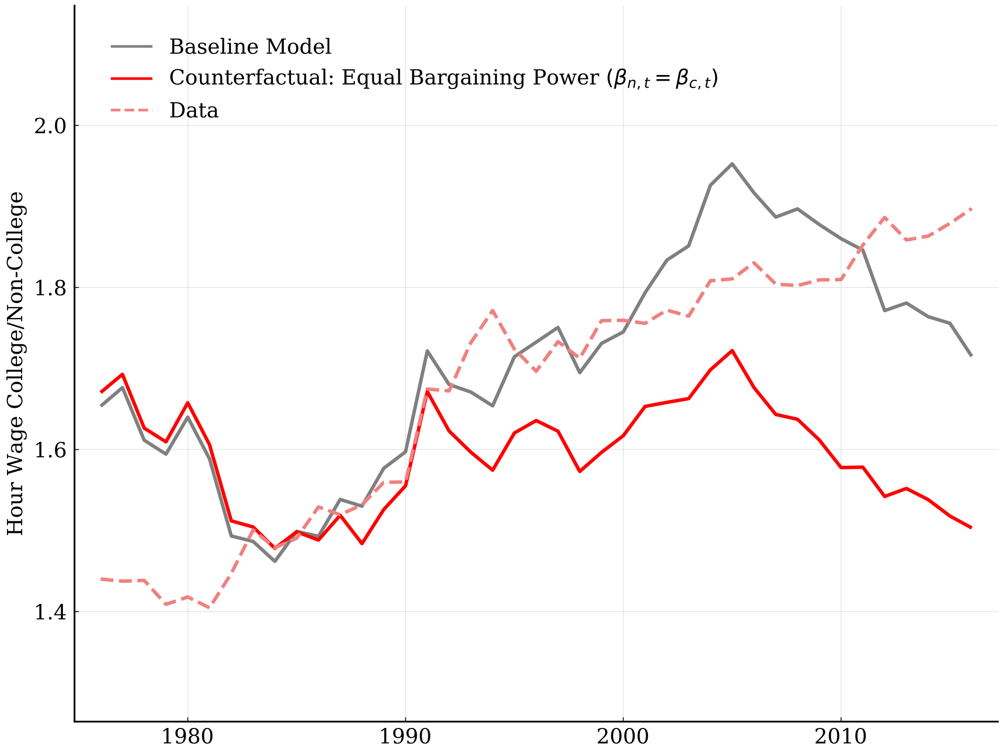


Table 5. Net and Gross Contribution: Level & Growth of college wage premium (CWP)

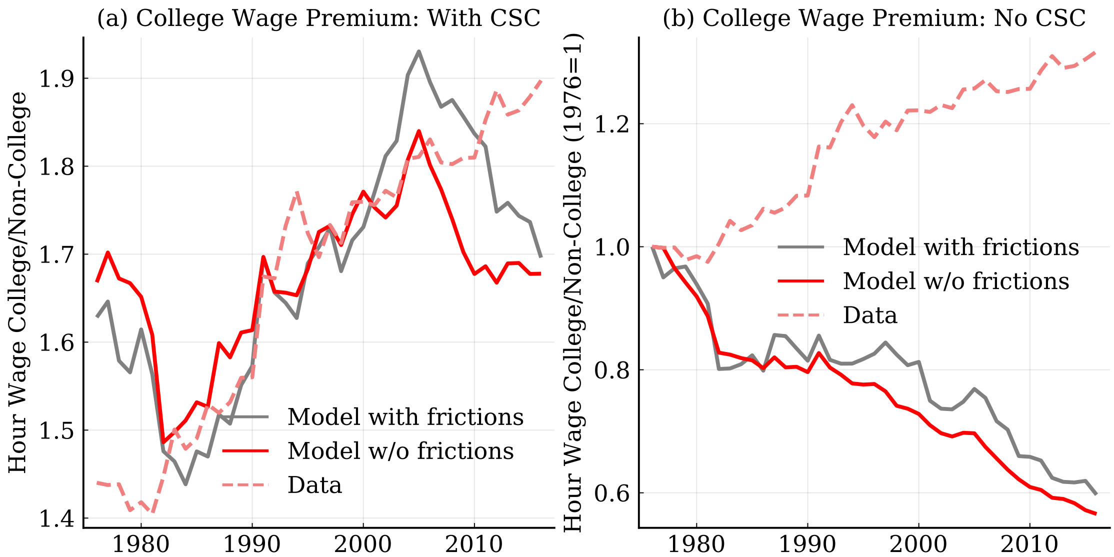
Model Channel	CWP level impact (net), %	CWP level impact (gross), %	CWP growth impact (net), %	CWP growth impact (gross), %
$\Delta_{c,n}$ Job destruction rates	0.178	0.104	0.109	0.036
$\Delta_{c,n}$ Job contact rates in employment	0.153	0.089	0.044	0.015
$\Delta_{c,n}$ Match quality distribution	-0.016	0.022	0.021	0.007
$\Delta_{c,n}$ Bargaining strength	0.109	0.064	0.717	0.24
$\Delta_{c,n}$ Capital-Labor Substitutability	0.411	0.24	2.099	0.702
$\Delta_{c,n}$ Labor Efficiency	-0.718	0.978	-0.006	0.003
$\Delta_{c,n}$ Labor Supply	0.86	0.503	-1.996	0.997

Column 2 shows the contribution of each channel to the total net college wage premium predicted by the model (average level 1976-2016). Column 3 shows the contribution of each channel to the total gross college wage premium predicted by the model (average level 1976-2016). Column 4 shows the contribution of each channel to the net increase in college wage premium predicted by the model (growth in average CWP(1976-1986) to average CWP(2006-2016)). Column 5 shows the contribution of each channel to the gross increase/decrease in college wage premium predicted by the model (growth in average CWP(1976-1986) to average CWP(2006-2016)).

monopsonistic channels arising from the presence of search frictions e.g. differences in job offer rates, job destruction rates and bargaining strength between college and non-college workers. The model can therefore provide a theoretical grounding for findings from the growing empirical literature suggesting the importance of monopsonistic forces and wage bargaining institutions (e.g. see Song, Price, Guvenen, Bloom, and von Wachter (2018), Bassier, Dube, and Naidu (2020) and Farber, Herbst, Kuziemko, and Naidu (2018)).

The quantitative application of my model suggests search frictions make a substantial contribution to the average level and growth of the college wage premium

FIGURE 23. Model Fit: No Capital Skill Complementarity (CSC)

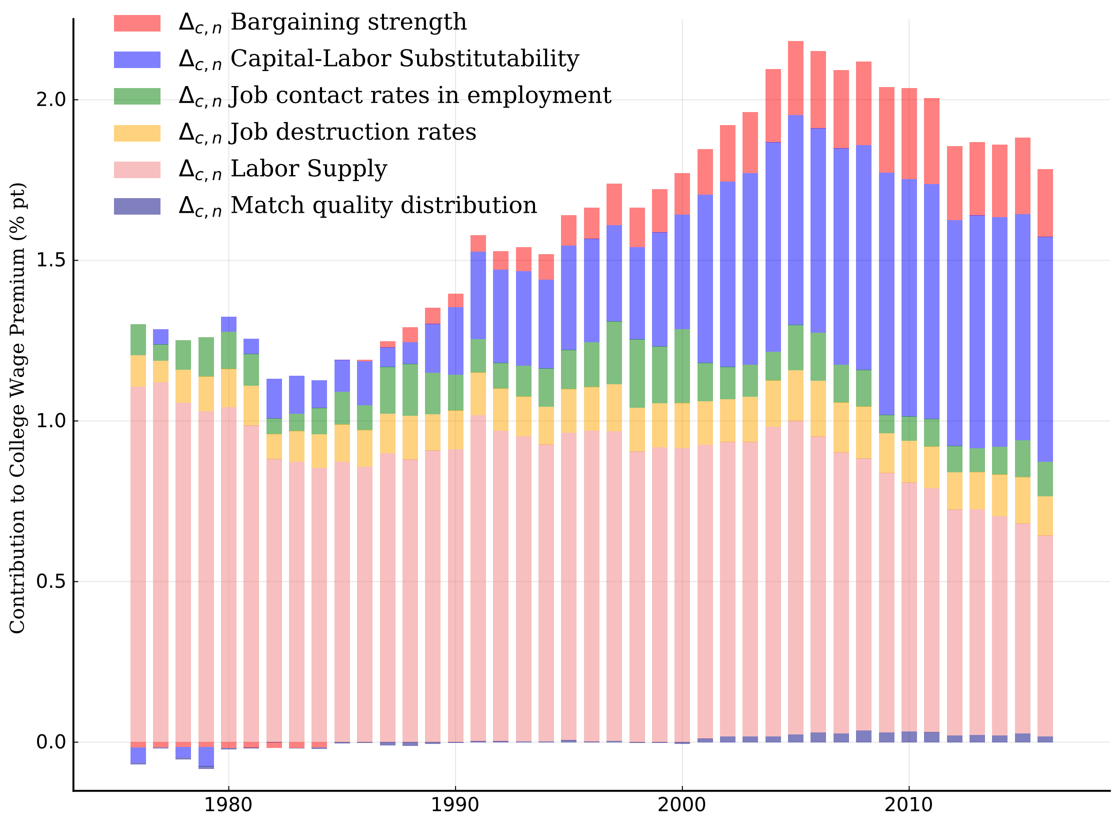


Notes: The model moments displayed in the figure are generated both from estimated parameter values using my replication of KORV's methodology i.e without frictions (the 'w/o frictions' series) and using estimates when incorporating frictions (the 'with frictions' series). Panel (a) of the figure shows model predictions for the college wage premium with all parameters at their baseline estimated values. Panel (b) of the figure shows model predictions for the college wage premium when I shut down the capital skill complementarity (CSC) channel both for the model with and without frictions, by setting $\hat{\sigma} = \hat{\gamma}$.

over my sample period. In my estimated model, college workers enjoy more stable jobs - i.e. face less job destruction - receive higher job offer rates and have a more favorable wage bargaining environment due to higher unionization rates. Graduates' advantages in terms of lower job destruction rates and higher job offer rates are large and relatively stable over my sample period, and so make a substantial contribution to the average level of the college wage premium but not its growth. In contrast, I calibrate the bargaining strength of college workers relative to non-college workers to their relative unionization rates; while both worker types see unionization rates decline, the fall is more severe for non-college workers. This force can explain a substantial part of the growth of the college wage premium in my model. It also means my estimated model suggests a somewhat weaker capital skill complementarity channel than an equivalent competitive model. However, my model still relies on this channel to offset the impact of the rising relative supply of graduate workers, which, without some form of capital skill complementarity, would lead to substantial decline in the college wage premium in my model.

Fruitful areas for future research include incorporating more firm side information, which would allow a more comprehensive approach to identification of bargaining

FIGURE 24. Dynamic Decomposition



parameters, e.g. by targeting the evolution of the labor share in sectors where non-college and college workers are concentrated. Incorporating firm side data could also allow identification of sorting channels e.g. to investigate whether there is stronger sorting of college workers to higher productivity or higher mark-up firms or sectors.

References

Bassier, I., A. Dube, and S. Naidu (2020): "Monopsony in Movers: The Elasticity of Labor Supply to Firm Wage Policies," Working Paper 27755, National Bureau of Economic Research.

Bilal, A. G., N. Engbom, S. Mongey, and G. L. Violante (2019): "Firm and Worker Dynamics in a Frictional Labor Market," Working Paper 26547, National Bureau of Economic Research.

Burdett, K., C. Carrillo-Tudela, and M. Coles (2016): "Wage inequality: A structural decomposition," *Review of Economic Dynamics*, 19, 20 – 37, special Issue in Honor of Dale Mortensen.

Burdett, K. and D. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39, 257–73.

Cahuc, P., F. Postel-Vinay, and J.-M. Robin (2006): "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, 74, 323–364.

Card, D. and J. E. DiNardo (2002): "Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles," *Journal of Labor Economics*, 20, 733–783.

Card, D., J. Heining, and P. Kline (2013): "Workplace Heterogeneity and the Rise of West German Wage Inequality*," *The Quarterly Journal of Economics*, 128, 967–1015.

Card, D. and T. Lemieux (2001): "Can Falling Supply Explain the Rising Return to College for Younger Men? A Cohort-Based Analysis," *The Quarterly Journal of Economics*, 116, 705–746.

Decker, R. A., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2016): "Declining Business Dynamism: What We Know and the Way Forward," *American Economic Review*, 106, 203–07.

DiCecio, R. (2009): "Sticky wages and sectoral labor comovement," *Journal of Economic Dynamics and Control*, 33, 538 – 553.

Farber, H. S., D. Herbst, I. Kuziemko, and S. Naidu (2018): "Unions and Inequality Over the Twentieth Century: New Evidence from Survey Data," Working Paper 24587, National Bureau of Economic Research.

Goldin, C. and L. Katz (2008): *The Race Between Education and Technology.*, Belknap Press for Harvard University Press.

Gordon, R. (1990): *The Measurement of Durable Goods Prices*, National Bureau of Economic Research, Inc.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997): "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 87, 342–362.

Katz, L. F. and K. M. Murphy (1992): "Changes in Relative Wages, 19631987: Supply and Demand Factors," *The Quarterly Journal of Economics*, 107, 35–78.

Krusell, P., L. E. Ohanian, J.-V. Rios-Rull, and G. L. Violante (2000): "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68, 1029–1054.

Laroque, G. and B. Salanie (1993): "Simulation-Based Estimation of Models with Lagged Latent Variables," *Journal of Applied Econometrics*, 8, 119–133.

Lee, D. S. (1999): "Wage Inequality in the United States During the 1980s: Rising Dispersion or Falling Minimum Wage?*," *The Quarterly Journal of Economics*, 114, 977–1023.

Molloy, R., C. L. Smith, R. Trezzi, and A. Wozniak (2016): "Understanding Declining Fluidity in the U.S. Labor Market," *Brookings Papers on Economic Activity*, 47, 183–259.

Ohanian, L., G. L. Violante, P. Krusell, and J.-V. Ros-Rull (1997): "Simulation-Based Estimation of a Nonlinear, Latent Factor AggregateProduction Function," in *in Simulation-Based Inference in Econometrics: Theory and Applications (edited by*, University Press, 359–399.

Osborne, M. and A. Rubinstein (1990): *Bargaining and markets*, Economic theory, econometrics, and mathematical economics, Academic Press.

Polgreen, L. and P. Silos (2008): "Capitalskill complementarity and inequality: A sensitivity analysis," *Review of Economic Dynamics*, 11, 302 – 313.

Song, J., D. J. Price, F. Guvenen, N. Bloom, and T. von Wachter (2018): "Firming Up Inequality*," *The Quarterly Journal of Economics*, 134, 1–50.

Stansbury, A. and L. H. Summers (2020): "The Declining Worker Power Hypothesis: An explanation for the recent evolution of the American economy," Working Paper 27193, National Bureau of Economic Research.

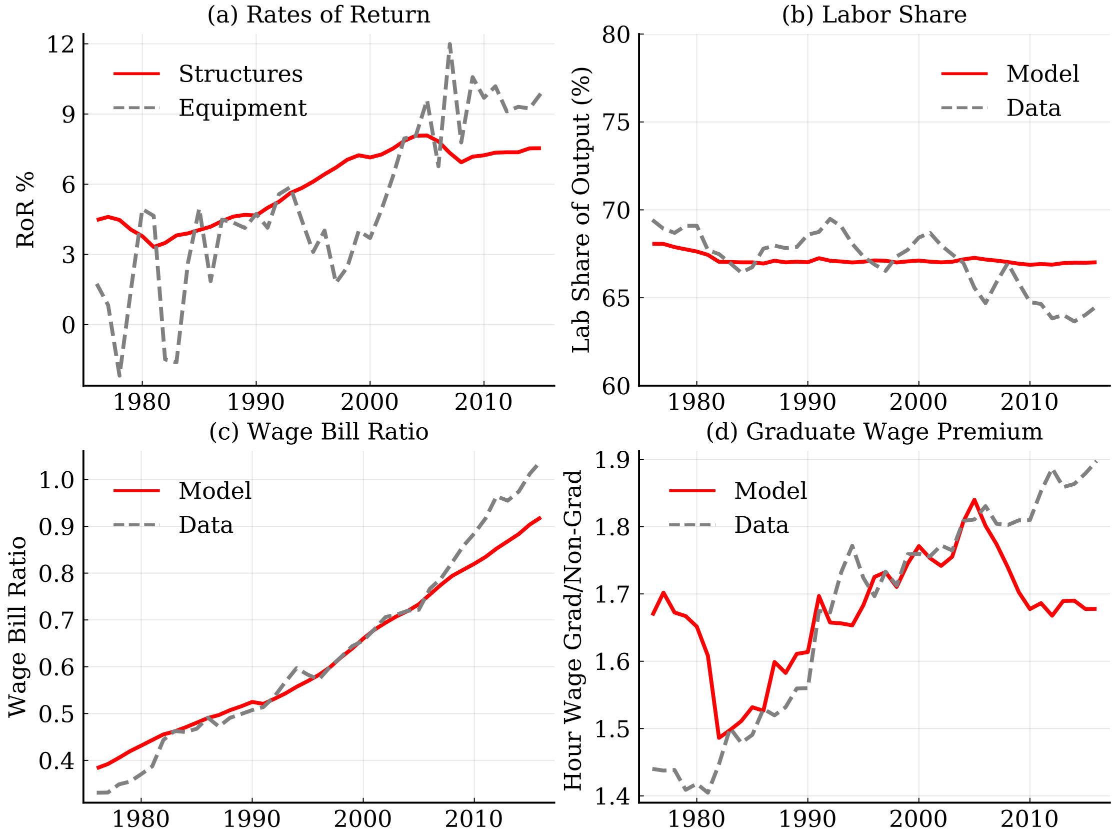
Stole, L. A. and J. Zwiebel (1996): "Organizational Design and Technology Choice under Intrafirm Bargaining," *American Economic Review*, 86, 195–222.

Appendix A. Model Results and Bargaining Power

This Appendix discusses the sensitivity of my results to the choice of the bargaining parameter. In my baseline estimation I impose a high level of bargaining power for both worker types ($\beta_u = \beta_s = 0.95$). I find that when I set the bargaining parameter at significantly lower levels, i.e. 0.75 or 0.5, and estimate the parameters of the KORV production function there is an acute tension between the model's ability to match both the rise in the college wage premium and the level of the labor share of output. The rest of this Appendix explains this tension and its quantitative impact. Overall I find that only a relatively high bargaining parameter allows the model to match the relevant trends in the data.

I first consider the intuition for why there might be a tension between matching the rise in the college wage premium and level of the labor share at lower levels of the bargaining parameter. First recall that the original, competitive, version of the KORV model is relatively successful at matching both the rise in the college wage premium and the labor share: see Figure 25. When I introduce the sequential auction model into this set-up, average wages will now be lower than the marginal product of labor if the bargaining parameter is significantly less than unity and for realistic job contact rates. In other words, the labor share will be lower in the model with frictions than in the original KORV environment for a given set of production function parameters.

Figure 25. KORV with perfect competition



Notes: The model moments displayed in the figure are generated using estimated parameter values when I replicate KORV's methodology i.e. with perfect competition and no search frictions. Panel (a) of the figure shows predicted ex-post rates of return (RoR) on capital structures and equipment. Panels (b) through (d) compare model moments to their empirical counterparts for the labor share of income, the wage bill of college workers relative to that of non-college workers, and the college wage premium respectively.

When I estimate the KORV parameters in my frictional labor market model, and have a low level of bargaining power, the estimation approach compensates for the downwards pressure this puts on the labor share by making labor more important (and capital less important) in the production of output. However, this jeopardizes the ability of the model to match the college wage premium since the increased use of capital equipment is the main channel that pushes the wage premium up.

To illustrate this quantitative impact of this tension, consider estimates of the KORV production function parameters when I set the bargaining parameter to 0.5 for both non-college and college workers - see Table 6 and Figure 26 for the corresponding model predictions. The estimate of α, the exponent of capital structures (K_{st}), hits the zero lower bound, and it also delivers lower levels of λ, the coefficient of capital equipment

Table 6. KORV parameter values: bargaining parameter impact

Parameter	No Frictions (KORV)	Baseline ($\beta = 0.95$)	$\beta = 0.5$
λ	0.521	0.462	0.219
μ	0.442	0.462	0.742
α	0.127	0.118	0.0
γ	-0.202	-0.187	-0.277
σ	0.347	0.316	0.187
$\varepsilon_{S,K_{eq}}$ $(= 1/1 - \gamma))$	0.832	0.843	0.783
$\varepsilon_{U,K_{eq}}$ $(= 1/1 - \sigma))$	1.531	1.462	1.23
CSC Strength: $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$	0.699	0.619	0.447

Notes: Rows 2-6 show estimates of the primitive parameters of the KORV production function. Row 7 shows the implied elasticity of substitution between capital equipment and skilled labor input, $\varepsilon_{S,K_{eq}}$. Row 8 shows the implied elasticity of substitution between capital equipment and unskilled labor input, $\varepsilon_{U,K_{eq}}$. Row 9 shows the implied strength of the capital skill complementarity (CSC) channel, as measured by $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$.

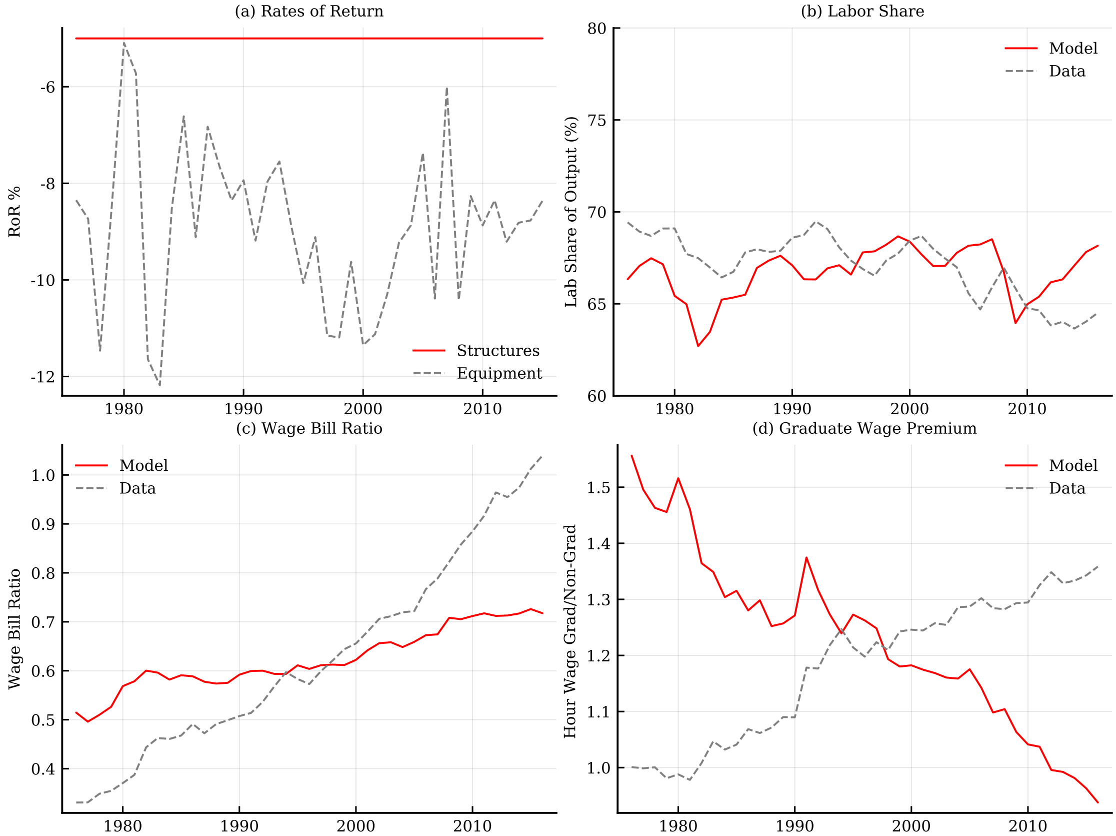
since this too increases the labor share. However a lower level of λ limits the channel of capital skill complementarity - see equation (8) - and means that although the model can fit the labor share to a reasonable approximation, it completely misses the rise in the college wage premium: see Figure 26. Indeed the fit is much worse than that of the purely competitive set-up in KORV: see Figure 25. Increasing the bargaining parameter from 0.5 to 0.95 improves the results significantly - see Figure 27. While much of the micro evidence points to much lower levels of the bargaining parameter, generally such estimates are highly model dependent.

Appendix B. Identification

There are two sets of parameters to identify in my model: the parameters of the KORV production function, and those in the sequential auction model of the labor markets in the intermediate goods sectors. While Krusell, Ohanian, Rios-Rull, and Violante (2000) do not explicitly discuss identification in their paper, they do refer to the results of a companion empirical paper Ohanian, Violante, Krusell, and Ros-Rull (1997) which shows their estimation strategy is successful at identifying the true parameters in Monte Carlo simulations. As my estimation of the parameters of the KORV production function very closely follows their method, and is done separately and subsequently to estimation of the sequential auction parameters, I do not repeat that exercise here and instead rely on their identification results.

The sequential auction structure of the labor market in my model is no different from Cahuc, Postel-Vinay, and Robin (2006), however I use employee reported data (from the CPS) to estimate the relevant parameters, whereas Cahuc, Postel-Vinay, and Robin

FIGURE 26. KORV with frictions: $\beta = 0.5$

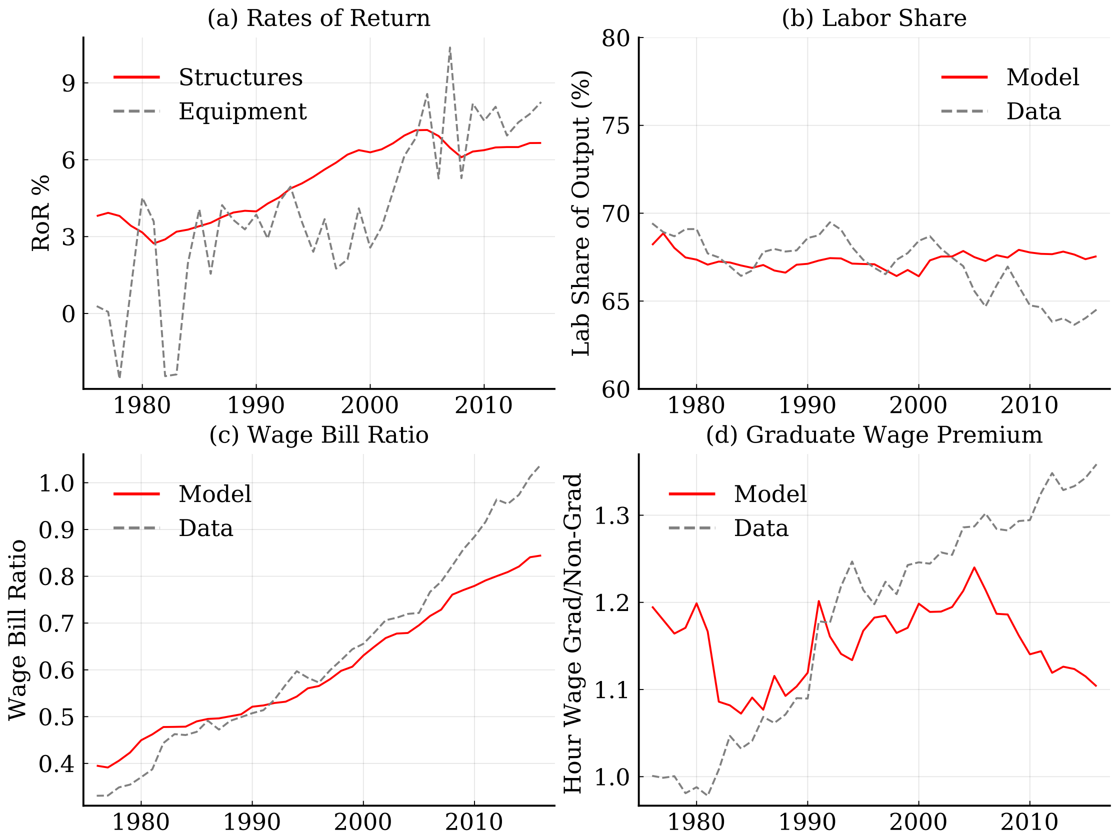


Notes: The model moments displayed in the figure are generated using estimated parameter values when I incorporate frictions but now with the bargaining parameter set uniformly at 0.5 (rather than 0.95). Panel (a) of the figure shows predicted ex-post rates of return (RoR) on capital structures and equipment. Panels (b) through (d) compare model moments to their empirical counterparts for the labor share of income, the wage bill of college workers relative to that of non-college workers, and the college wage premium respectively.

(2006) used matched-employee-employer (MEE) data. I chose to use CPS data because a key motivation for this paper is to test the robustness of findings in Krusell, Ohanian, Rios-Rull, and Violante (2000) to incorporating frictions; I therefore sought to maintain as much consistency as possible to their estimation approach which used CPS data for wages and labor input. However, the MEE data that Cahuc, Postel-Vinay, and Robin (2006) use plays a key role in their identification strategy so it is worth considering whether the parameters I wish to identify in the sequential auction model are identified when using employee data only.

First bargaining parameters by worker skill level are much more difficult, if not impossible, to identify without some form of employer information. In the absence of such data, neither match output nor firm fixed effects are observable or estimable and hence reliable estimates of bargaining parameters are not readily available. This is why I choose to

FIGURE 27. KORV with frictions - baseline bargaining power: $\beta = 0.95$



Notes: The model moments displayed in the figure are generated using estimated parameter values when I incorporate frictions and set the bargaining parameter at its baseline value of 0.95. Panel (a) of the figure shows predicted ex-post rates of return (RoR) on capital structures and equipment. Panels (b) through (d) compare model moments to their empirical counterparts for the labor share of income, the wage bill of college workers relative to that of non-college workers, and the college wage premium respectively.

set bargaining parameters by assumption.[33] The remaining objects of interest in the sequential auction model are job contact rates, (note job destruction rates come straight from the data) and the distribution of match quality, where I will consider the possibility of both non-parametric and parametric identification.

B.1. **Job Contact Rates.** There are two job contact rates in the sequential auction model for each worker type: those for the unemployed and employed: $\lambda_{0,i}$ and $\lambda_{1,i}$ respectively. $\lambda_{0,i}$ determines the unemployment rate and, because it influences the outside option of workers, the minimum match quality of firm that a worker will accept an offer

[33]The analysis of Appendix A suggests the labor share of income is informative about the average bargaining power of all workers, however it would not help to estimate bargaining parameters by skill level.

at. However, the unemployment rate does not play a role in the estimation of the KORV parameters (since labor input is total hours worked by workers and is taken straight from the data) or in the estimation of any other parameters in the sequential auction model, and I will estimate the lower bound of acceptable match quality directly, as described in the next section. I therefore have no need to estimate $\lambda_{0,i}$.

I instead focus on estimation of $\lambda_{1,i}$, which is key for determining both average match quality, and average wages of worker of a given worker type. Both variables play a role in estimating the parameters of the KORV production function, as described in Section 5.4.

I estimate $\lambda_{1,i}$ using SMM and targeting the proportion of continuous employed workers in a given year who have moved employers at least once (the multiple employer rate). I denote this proportion τ_i. In the model, the expression for this moment is given in equation 31, which is obtained by substituting the expression for the cross section distribution of match quality in equation (19) into equation (27).

$$\tau_i = 1 - \int_{\nu_{inf_i}}^{\nu_{max}} (1 - \lambda_{1,i,}\bar{F}_i(\nu))^{12} \frac{1 + \kappa_{1,i}}{[1 + \kappa_{1,i}\bar{F}_i(\nu)]^2} f_i(\nu) d\nu \tag{31}$$

As I am estimating $\lambda_{1,i}$ separately, and prior to, the estimation of the match quality distribution F, I require that equation (31) is independent of F. This can be proven by integrating by change of variable i.e. if I let $r = \bar{F}_i(\nu)$ so that $\frac{dr}{d\nu} = -f(\nu)$ the expression for τ becomes as shown in equation (32), which is independent of F.

$$\tau_i = 1 - \int_0^1 (1 - \lambda_{1,i,t} r)^{12} \frac{1 + \kappa_{1,i}}{[1 + \kappa_{1,i} r]^2} dr \tag{32}$$

The presence of the twelfth order polynomial in equation (32) hinders an analytical proof of identification, however it is easy to verify with a symbolic equation solver that this expression is a positive monotonic function of $\lambda_{1,i}$ which given the quadratic objective function in SMM proves identification of $\lambda_{1,i}$. This result is not surprising given it is possible to prove (analytically) that the monthly steady state job mobility rate is increasing in $\lambda_{1,i}$.

B.2. **Distribution of Match Quality.** There are two key considerations in the identification of the distribution of match quality. The first consideration is whether the distribution can be non-parametrically identified or not. I will argue that it can be, but only by relying heavily on the structure of the model. Therefore when it comes to estimation I prefer to assume a log normal distribution of match quality. The second consideration is then whether the parameters of this distribution are identified. I will estimate the parameters of the match quality distribution by targeting moments of the empirical wage distribution. However, higher order moments of the wage distribution in the model are not tractable, hindering an analytical proof of identification. I instead present evidence from Monte Carlo simulations that my estimation strategy can identify the 'true' parameters of the match quality distribution.

I start by showing that, in theory, the match distribution could be identified non-parametrically. A worker's wage depends both on their current employer's match quality ν^+ and their outside option match quality (the second highest quality match they've had contact with, denoted ν^-) as shown in equation (33). I therefore can't simply invert the wage equation to back out the quality of the current match, ν^+. Note I assume that the other parameter values in the equation are known due to the identification arguments presented above for job contact rates, and because other parameters either come straight from the data, like job destruction rates, or are set by assumption, like the bargaining parameter and discount factor.

$$\phi(p_i, \nu^-, \nu^+) = p_i\left(\nu^+ - (1-\beta)\int_{\nu^-}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx\right) \tag{33}$$

While the general wage equation for workers is not immediately helpful for identification, workers who were unemployed in the previous period and then get a job ('entrant workers') have a common level of ν^-, which equals ν_{inf_i}, the lower bound of the match quality distribution. Entrants will therefore be paid the wage shown in equation (34).

$$\phi(p_i, \nu_{inf_i}, \nu^+) = p_i\left(\nu^+ - (1-\beta)\int_{\nu_{inf_i}}^{\nu^+} \frac{\rho + \delta + \lambda_1 \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx\right) \tag{34}$$

I argued previously that if the bargaining parameter is high enough to guarantee that wages are an increasing function of the employer's match quality (which is the case in my baseline), then ν_{inf} is identified as the lower bound of wages in the empirical wage distribution. Therefore, in principle, I could identify the distribution of ν by inverting equation (**??**) for each wage in the empirical distribution of entrants' wages. This inversion can be done as follows: I start by letting $w = \phi(p_i, \nu_{inf_i}, \nu^+)$ and differentiating w with respect to ν^+ to get:

$$\begin{aligned}
\frac{dw}{d\nu+} &= p_i\left[1 - (1-\beta)\frac{\rho + \delta_i + \lambda_{1,i}\bar{F}_i(\nu^+)}{\rho + \delta_i + \lambda_{1,i}\beta\bar{F}_i(\nu^+)}\right] \\
&= p_i\left[\frac{\beta(\rho + \delta_i) + (2\beta - 1)\lambda_{1,i}\bar{F}_i(\nu^+)}{\rho + \delta_i + \lambda_{1,i}\beta\bar{F}_i(\nu^+)}\right] \\
\implies \frac{d\nu+}{dw} &= \frac{1}{p_i}\frac{\rho + \delta_i + \lambda_{1,i}\beta\bar{F}_i(\nu^+)}{\beta(\rho + \delta_i) + (2\beta - 1)\lambda_{1,i}\bar{F}_i(\nu^+)}
\end{aligned} \tag{35}$$

Further note that under the assumption I have made about the bargaining parameter, a worker's wage is an increasing function of the match quality of their employer (ν^+), which implies that $\bar{F}_i(\nu^+) = \bar{F}_i^w(w(\nu^+))$. This is helpful since, while $\bar{F}_i(\nu^+)$ is not observable in the data, $\bar{F}_i^w(w(\nu^+))$ is. Substituting $\bar{F}_i(\nu^+) = \bar{F}_i^w(w(\nu^+))$ into equation

(35) I can then derive an expression for $\nu+$ in terms of w by solving this differential equation.

However, this relies heavily on the structure of the model and, moreover, on part of the structure - the entrant wage distribution - that was not a particular focus of Cahuc, Postel-Vinay, and Robin (2006). I therefore choose to make a parametric assumption for the distribution of match quality, and assume it is log normal.

I must now show that I can identify the parameters of this log normal distribution i.e. the mean parameter, ζ_i^ν, the variance parameter, η_i^ν, and the lower bound, ν_{inf_i}. Recall that my estimation of these parameters is based on a SMM approach as summarized in equation (36), where $\underline{w_i}$ is the lowest wage in the wage distribution, $Q^{50}_{w_i}$ is the median wage and $\mathbb{E}^{F_{i,t}}(\nu)$ is the mean of the match quality sampling distribution, which will be targeted at a fixed value (I impose $\mathbb{E}^{F_{i,t}}(\nu) = 1$).

$$\begin{aligned}
(\,\zeta^{\nu*}_{i,t}\,, \eta^{\nu*}_{i,t}\,, \nu^{*}_{inf_{i,t}}) &= \underset{\zeta^{\nu}_{i,t},\eta^{\nu}_{i,t},\nu_{inf_{i,t}}}{\operatorname{argmin}} \; (m_t - \hat{m}_t)^T W (m_t - \hat{m}_t) \qquad (36)\\
m_t \equiv \big(& var_{\log(w_{i,t})}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}), \underline{w}_{i,t}(\nu_{inf_{i,t}})/Q^{50}_{w_{i,t}}(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}}),\\
&\mathbb{E}^{F_{i,t}}(\nu)(\zeta^{\nu}_{i,t}, \eta^{\nu}_{i,t}, \nu_{inf_{i,t}})\big)
\end{aligned}$$

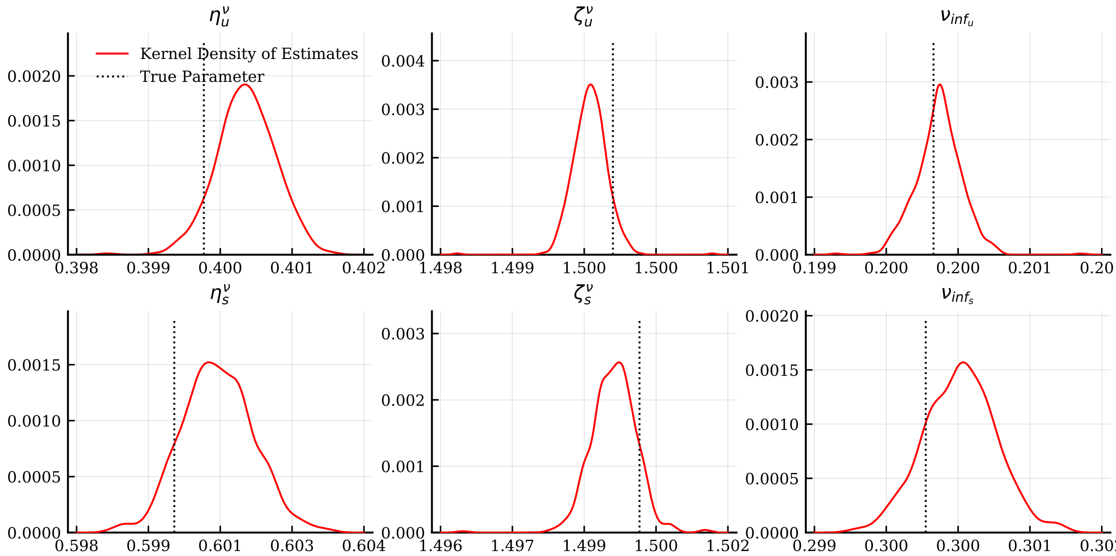
Proof of identification is hindered by the lack of tractability of the higher order moments of the wage distribution so I test whether my estimation procedure correctly identifies the true parameters of the model using Monte Carlo methods. That is I simulate a cross-section sample of wages for 50,000 workers (i.e. slightly less than the 60,000 that feature in the CPS) from the model with an arbitrary choice of parameters (the 'true' parameters). I then estimate the model using this simulated data to see if I recover the true parameters. Job contact rates, which also affect the cross-sectional distribution of workers wages, are set at their estimated values for the first year of my sample (1976) though the results of this exercise are not sensitive to their level.

Recall that I estimate the lower bound of the match quality distribution by targeting the ratio of the lower bound of wages in my sample relative to the median. As argued above this gives exact identification of the $\nu_{inf,i}$. I therefore feed the true parameter for the lower bound of the match quality into my estimation procedure directly, since it is exactly identified, rather than the minimum of simulated wages i.e. I set $\underline{w_{i,t}}/Q^{50}_{w_{i,t}}$ - the empirical moment I am targeting - to $\nu_{inf_{i,t}}/Q^{50}_{w^{sim}_{i,t}}$, where the superscript sim denotes simulated wage data.[34]

I estimate 500 sets of parameters corresponding to 500 simulations of data from the true model, producing the results shown in Figure 28. My estimation strategy is reasonably successful in recovering the true parameters, though not perfect: while there are some biases in the estimates, in each case they are very small in size.

[34]As in my actual empirical estimation of the sequential auction parameters, I normalize the price of the intermediate good, p_i, to one when performing the Monte Carlo test of identification.

Figure 28. Monte Carlo Analysis of Identification



Notes: The dotted line in each panel represents the true parameter value used to simulate data for estimation. 500 sets of data were simulated to generate 500 estimates for each parameter. The red line in each panel represents the kernel density of estimated parameter values. The top (bottom) row shows the variance parameter (η_i^ν), mean parameter (ζ_i^ν) and lower bound parameter $\nu_{inf,i}$ of the log normal match quality distribution for non-college (college) workers.

Appendix C. Robustness

This section tests the robustness of parameter estimates of the KORV production function in my model to changes to my empirical strategy for estimating the parameters of the sequential auction model of the intermediate goods markets. In particular, I consider the impact of: (i) estimating the lower bound of the match quality distribution by targeting the average wage of workers in the first percentile of the wage distribution (rather than average wage of the bottom five percentiles) - see column 4 of Table 7, (ii) estimating the lower bound of the match quality distribution by targeting the average wage of workers in bottom two percentiles - see column 5, (iii) estimating the lower bound of the match quality distribution by targeting the minimum of the empirical wage distribution (after trimming, as is done in my baseline specification) - see column 6, (iv) estimating the variance parameter of the (log normal) sampling distribution of match quality by targeting residual wage variance, where I now control for age as well as race, sex and years of education in calculating this residual variance - see column 7, (v) estimating the variance parameter of the sampling distribution by targeting the interquartile range of residual log wages, rather than the variance - see column 8. None of these changes to my empirical strategy make a significant difference to my results, as illustrated in Table 7.

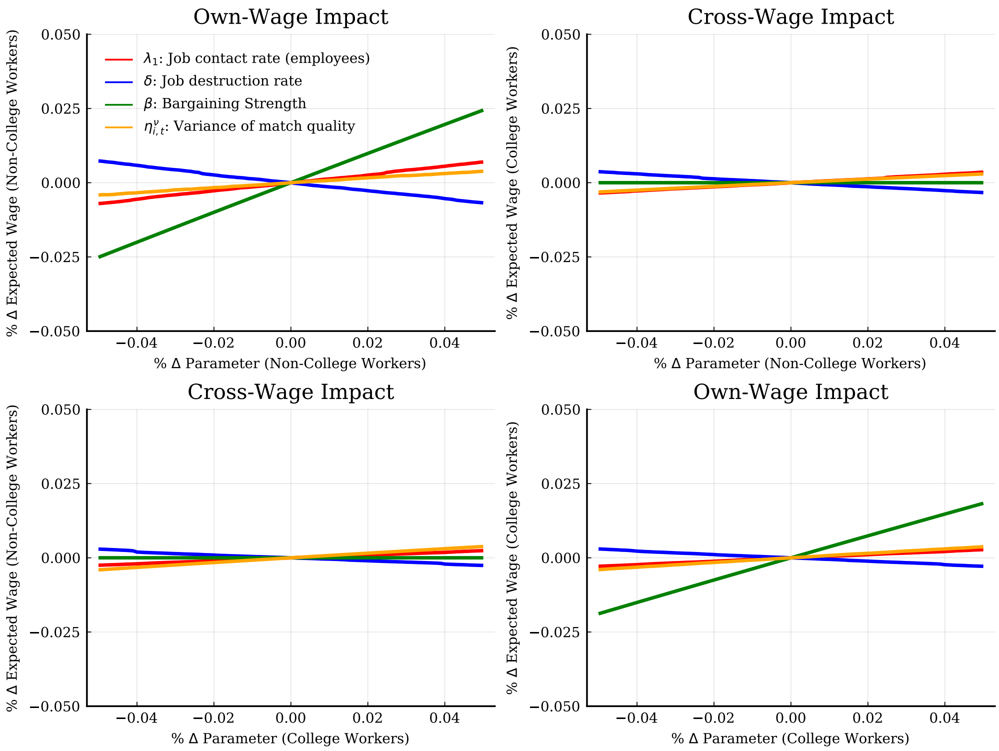
Table 7. KORV parameter values with frictions: robustness

(1) Parameter	(2) Without Frictions	(3) With Frictions: Baseline	(4)	(5)	(6)	(7)	(8)
λ	0.521	0.462	0.461	0.461	0.461	0.464	0.462
μ	0.442	0.462	0.451	0.533	0.512	0.508	0.594
α	0.127	0.118	0.118	0.118	0.118	0.118	0.118
γ	-0.202	-0.187	-0.185	-0.185	-0.184	-0.189	-0.185
σ	0.347	0.316	0.315	0.315	0.315	0.317	0.311
$\varepsilon_{S,K_{eq}}$ $(= 1/1-\gamma))$	0.832	0.843	0.844	0.844	0.844	0.841	0.844
$\varepsilon_{U,K_{eq}}$ $(= 1/1-\sigma))$	1.531	1.462	1.46	1.46	1.46	1.465	1.451
CSC Strength: $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$	0.699	0.619	0.616	0.616	0.616	0.624	0.607

Notes: Rows 2-6 show estimates of the primitive parameters of the KORV production function. Row 7 shows the implied elasticity of substitution between capital equipment and skilled labor input, $\varepsilon_{S,K_{eq}}$. Row 8 shows the implied elasticity of substitution between capital equipment and unskilled labor input, $\varepsilon_{U,K_{eq}}$. Row 9 shows the implied strength of the capital skill complementarity (CSC) channel, as measured by $\varepsilon_{U,K_{eq}} - \varepsilon_{S,K_{eq}}$.

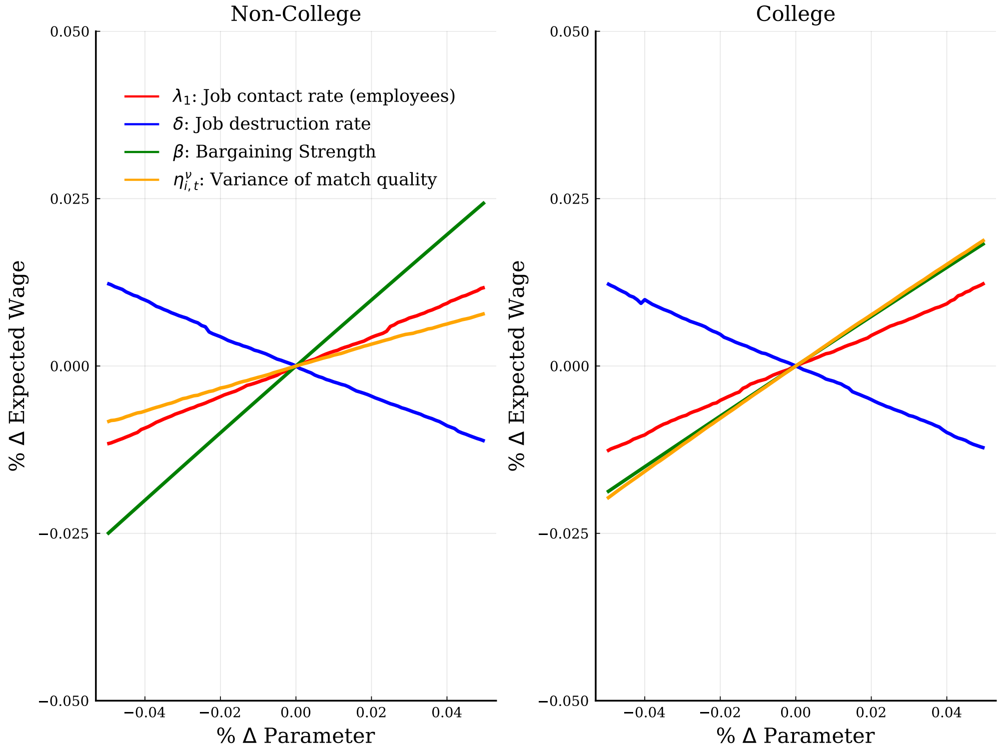
Appendix D. Parameter Impacts

FIGURE 29. Parameter Impacts on Expected Wages



Notes:The average expected wages shown on the y-axis are wages averaged over both the match quality distribution in my model in each time period and averaged over all time periods. Deviations (of wages and parameters) are shown with respect to a baseline where all parameters are set a constant level in each time period equal to the average level of each parameter over the sample period. I then seperately vary each parameter by ±5% in each time period, and simulate the impact on average expected wages.

Figure 30. Parameter Impacts on Expected Wages



Notes: The average expected wages shown on the y-axis are wages averaged over both the match quality distribution in my model in each time period and averaged over all time periods. The wages are calculated with the price of the intermediate good normalised to one so wage impacts reflect the search mechanisms in my model only and not the final good production function: this is why there are now no cross-wage impacts. Deviations (of wages and parameters) are shown with respect to a baseline where all parameters are set a constant level in each time period equal to the average level of each parameter over the sample period. I then seperately vary each parameter by ±5% in each time period, and simulate the impact on average expected wages.