Predictive Simulation of Reaching to Moving Targets using Nonlinear Model Predictive Control

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Outline

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- Dynamic Modeling
  - Planar arm model
- Nonlinear Model Predictive Control
  - NMPC framework
  - Dynamic optimization
- Simulation Results
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  - Moving targets
- Conclusion and Future Work

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Introduction

Forward Dynamics Simulations: A forward dynamics simulation is the solution (integration) of the differential equations that define the dynamics of a musculoskeletal model.

1. **Dynamic Optimization:** Dynamic optimizations find the optimal muscle activation patterns that result in an optimal motion by minimizing or maximizing a performance criterion.

- Input Parameterization e.g., Fourier function approximation - Sharif Shouijeh et al. (2015)
- Optimal Control Problem e.g., Dynamic Programing

2. **Optimal Feedback Control:**

- Computed Muscle Control (CMC) – Thelen et al. (2003)
- Iterative Linear Quadratic Gaussian (iLQG) – Todorov et.al (2007)
- Nonlinear Model Predictive Control (NMPC) – Mehrabi et al. (2016)

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Planar Musculoskeletal Arm Model

Multibody Dynamic Model:
- 2 segment model (2-DOF):
  - Upper-arm
  - Fore-arm

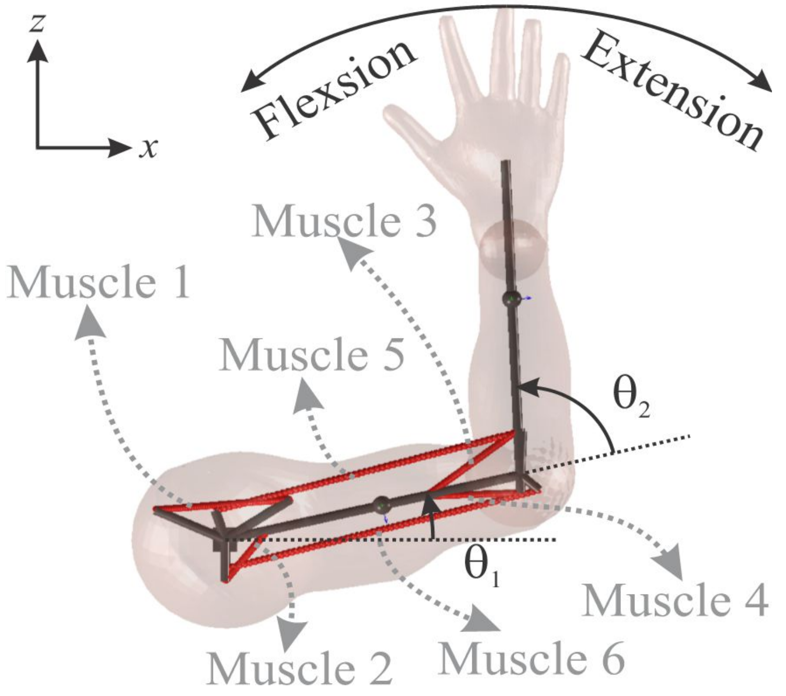
Actuation:
- 6 skeletal muscles (Hill-type muscle model)

  $0 \leq a_i \leq 1$

Limits:
- Joint Limits

$0 \leq \theta_{\text{shoulder}}, \theta_{\text{elbow}} \leq 180^{\text{o}}$

Planar Arm Model



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Planar Musculoskeletal Arm Model (Cont'd)

❖ Hill Muscle Model [1]:

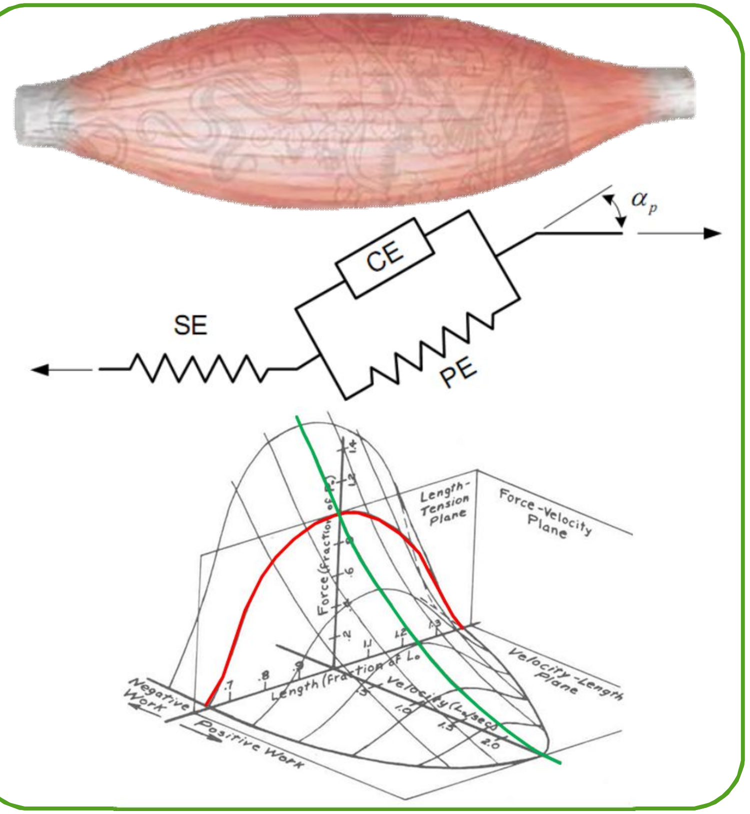
Hill muscle model consists of:

1. Contractile Element (CE)
2. Parallel Element (PE)
3. ~~Series Element (SE)~~

Total muscle force:

$$F_{TM}(t) = a(t)F_0^{\max}\{F_{PE}(L_M) + F_{CE}(L_M, V_M)\}\cos(\alpha_p)$$

where $\boldsymbol{F_{CE}}$ is the contractile element force and $\boldsymbol{F_{PE}}$ is the parallel element force. F_0^{max} is the maximum isometric muscle force and α_p is the muscle pennation angle.



[1] - A. Hill. The Heat of Shortening and the Dynamic Constants of Muscle. Proceedings of the Royal Society of London. Series B, Biological Sciences, 126(843):pp. 136|-195, 1938.

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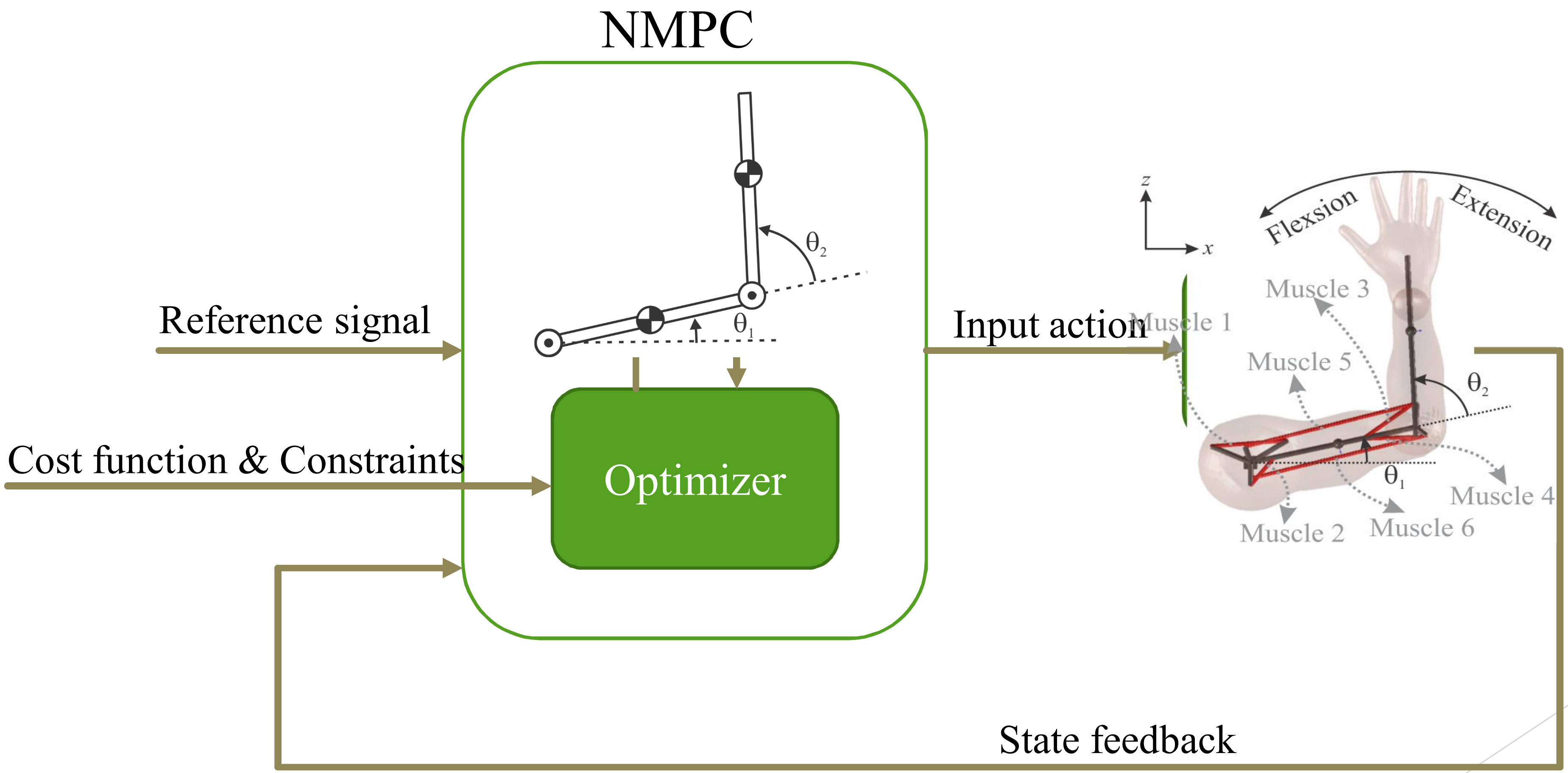
Nonlinear Model Predictive Control



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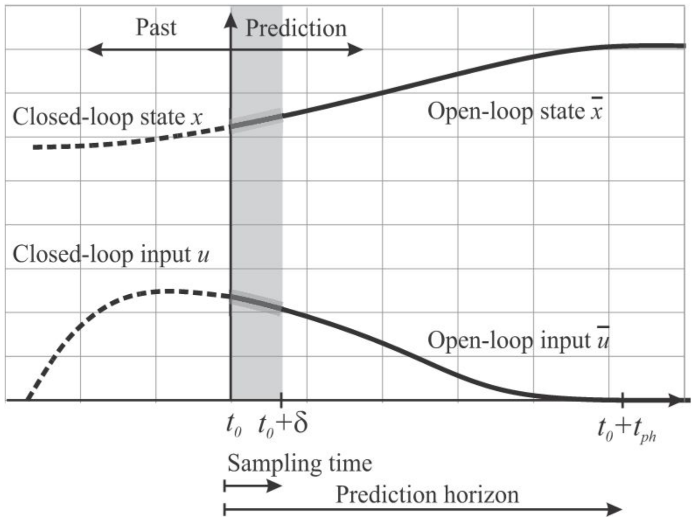
Principles of NMPC

Procedure:

1. Find the optimal dynamics based on the initial condition for a predefined prediction horizon
2. Apply the optimal inputs until the next sampling period
3. Measure the new states
4. Go to step 1



Objective function:

$$J = \Psi\left(t_0 + t_{ph}\right) + \int_{t_0}^{t_0+t_{ph}} \psi\left(x(t), a(t)\right) dt$$

subject to: $0 < a(t) < 1$

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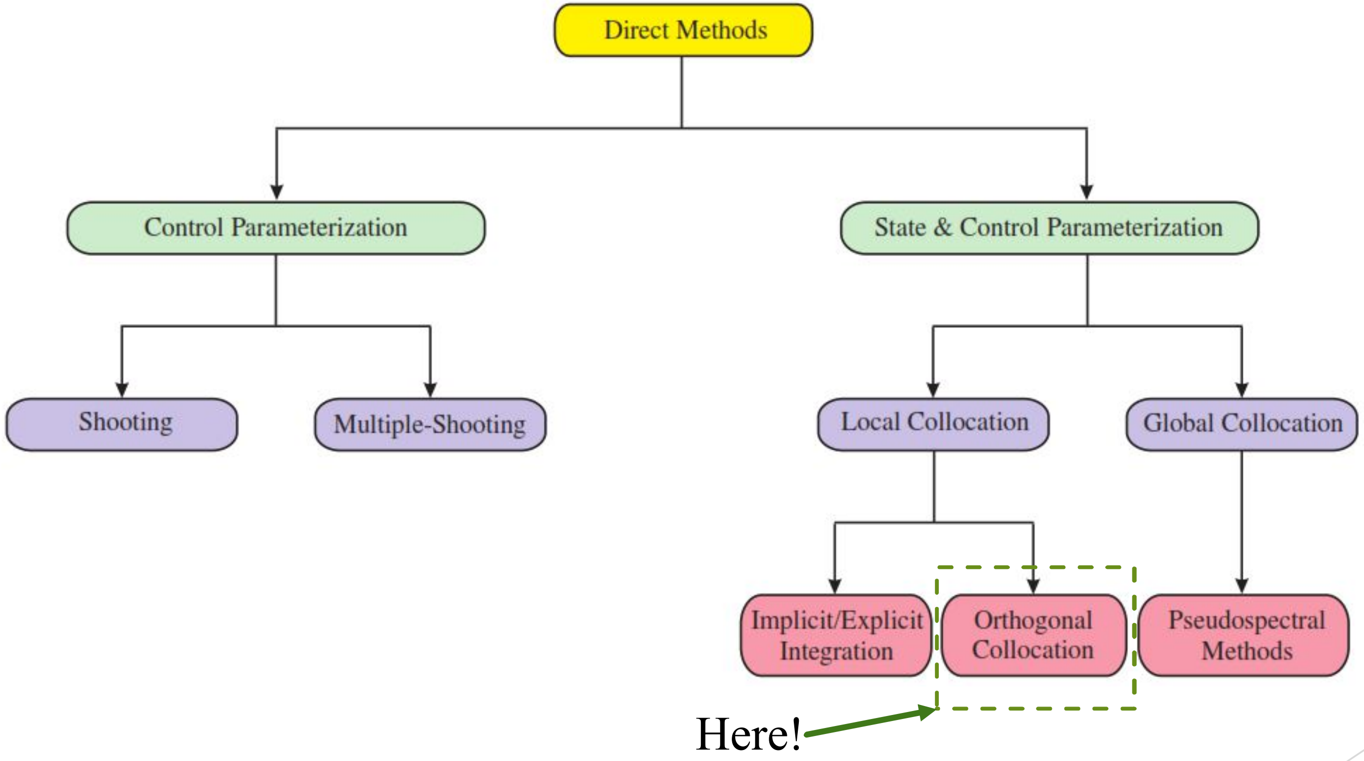
Optimal Control Problem

Optimal Control Problem: Find the joint torque functions of time that maximize performance

Different direct methods for solving an optimal control problem[1] :



[1] - Rao, Anil V. "A survey of numerical methods for optimal control." *Advances in the Astronautical Sciences* 135.1 (2009): 497-528.

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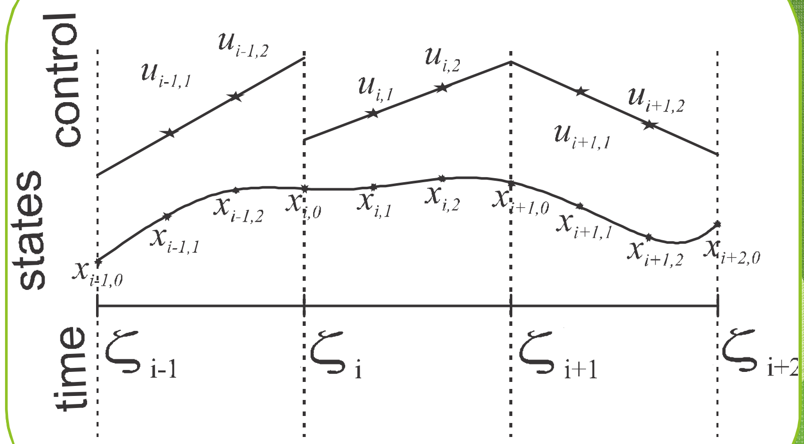
Direct Collocation Method

Nonlinear constraints

Equations of motion are satisfied at all collocation points

$$\dot{x} = f(x,u,t), \qquad x(t_0) = x_0$$
$$g(x,u,t) = 0$$



Objective function

$$\text{Objective function} := \Phi\big(x(t_f), t_f\big) + \int_{t_0}^{t_f} \Psi(x(t), u(t), t)\, dt$$

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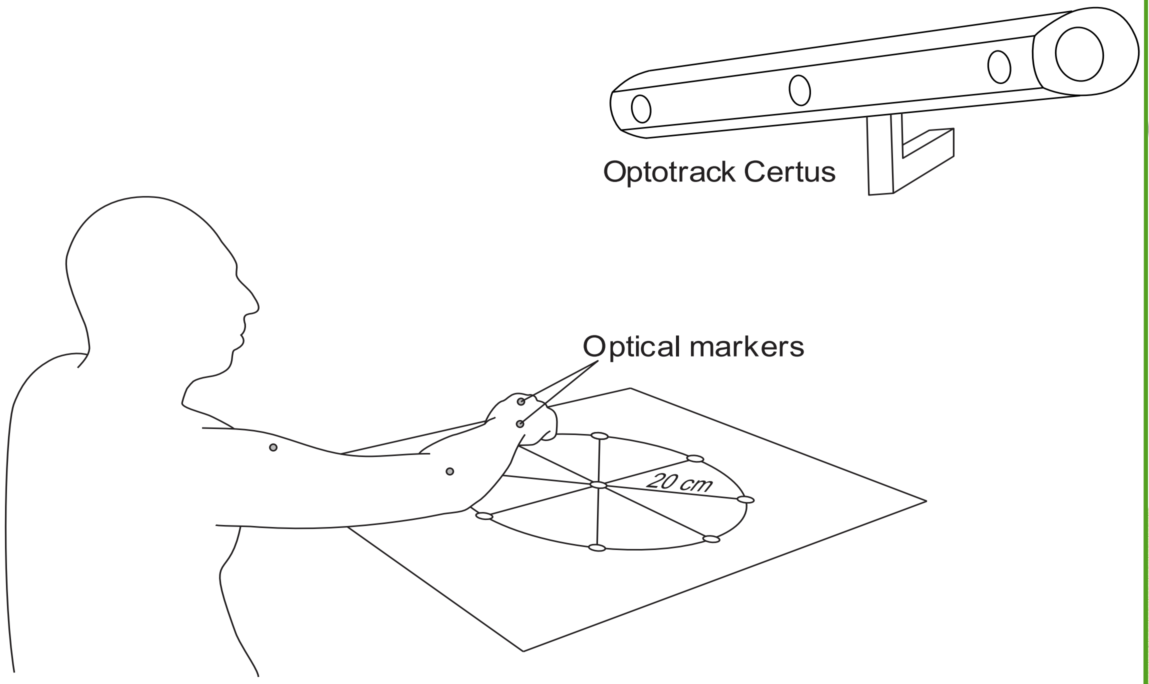
Experiments

Optotrak Certus motion capture system (Northern Digital Inc.) was used to capture the reaching motion:

- Four active markers
- Sampling rate (30 Hz)

Experimental Protocols:

- Reaching 8 points spread evenly on a circle with radius of 20 cm
- Reaching to moving targets



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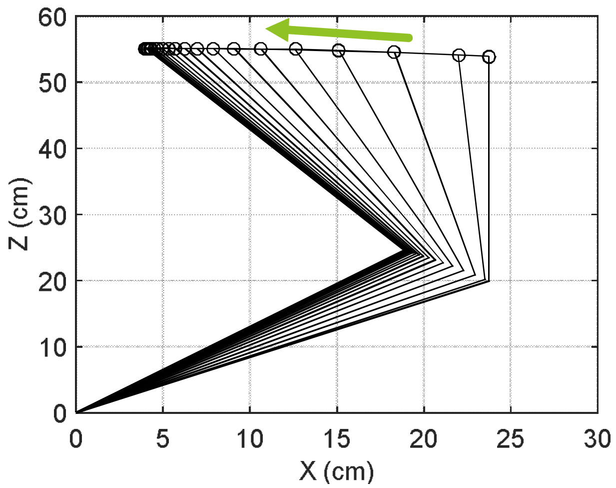
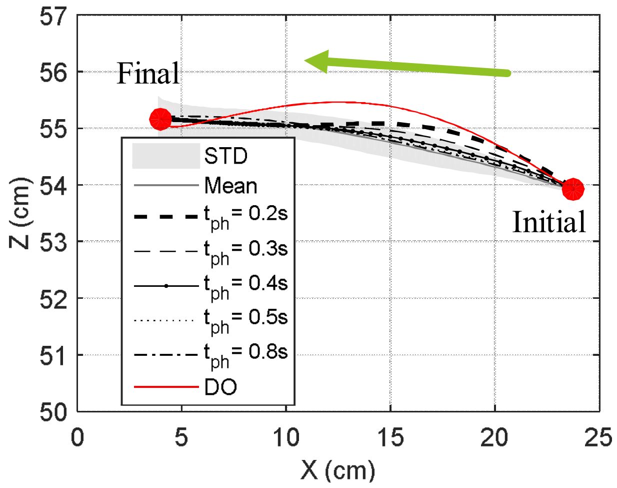
Simulation Results

Reaching Objective Function:

$$J = \sum_{i=1}^{K} \left(q(\zeta(t_j) - \zeta_{des})^2 + r\, G^M \left(a\,(t_j) \right) \right), \text{where } G^M = \frac{d}{dt} a^2$$



Reaching task on a plane. The hand moves from its original position to the marked position on its left



Hand position trajectory

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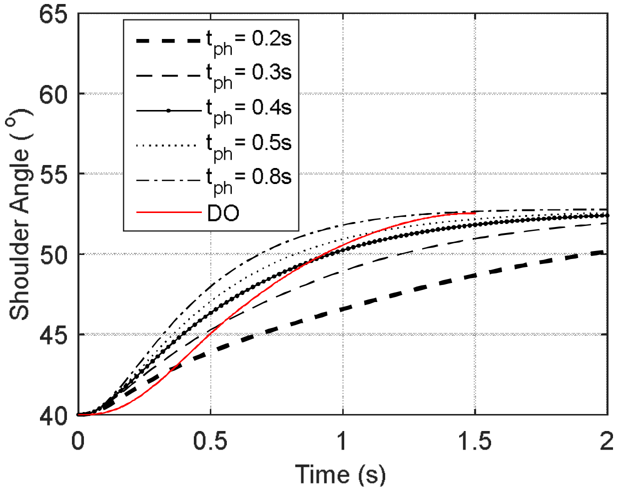
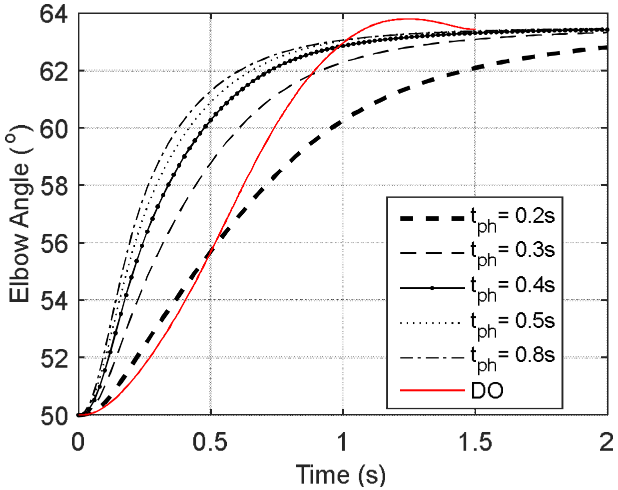
Prediction Horizon Variation

- The effect of prediction horizon on the elbow and shoulder angle variation in the reaching motion



Shoulder angle variation during the reaching task



Elbow angle variation during the reaching task

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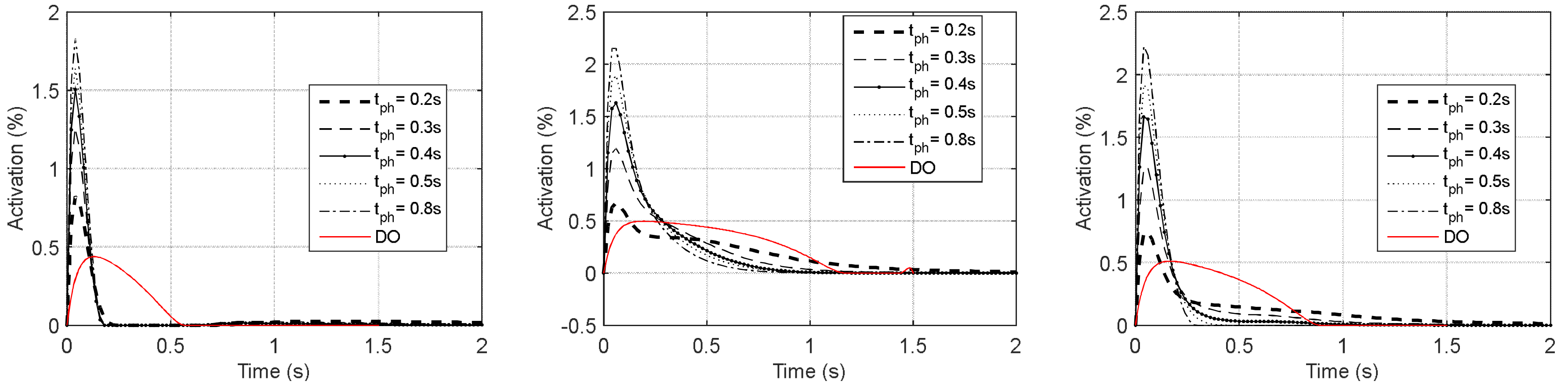
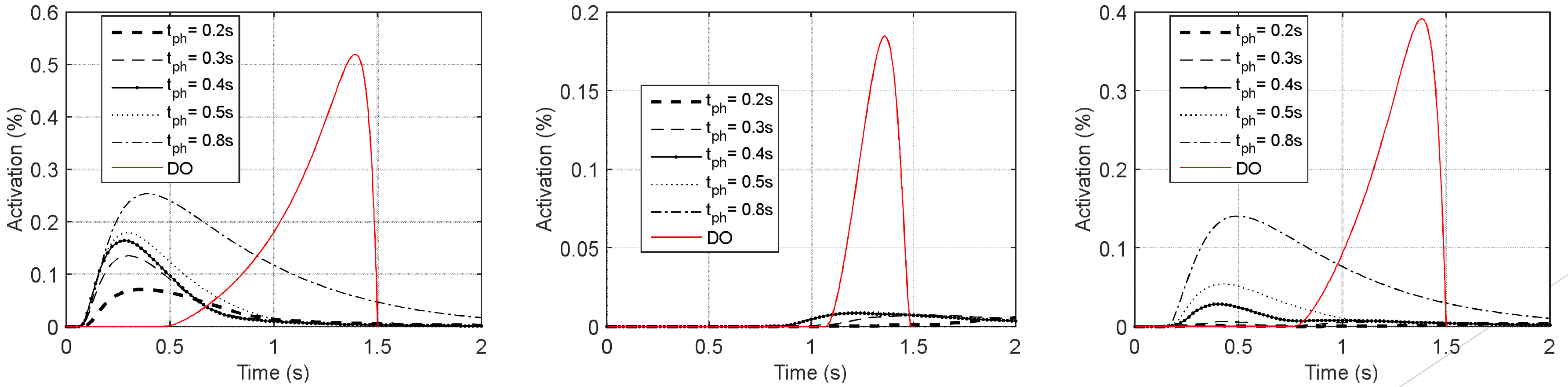
Prediction Horizon Variation



Optimal activations of flexor muscles



Optimal activations of flexor muscles

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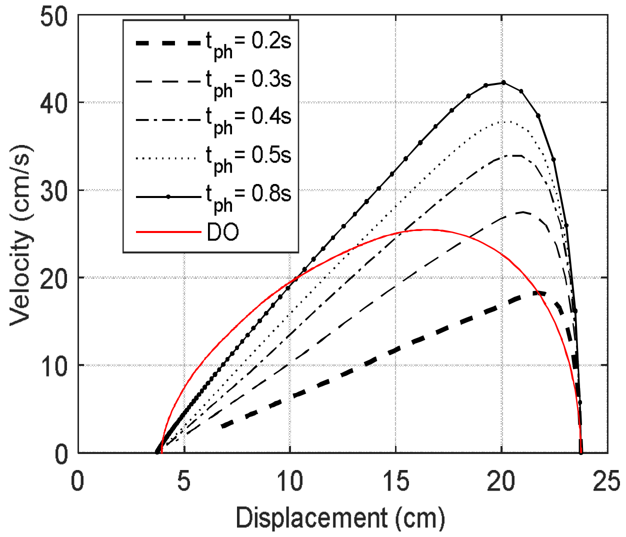
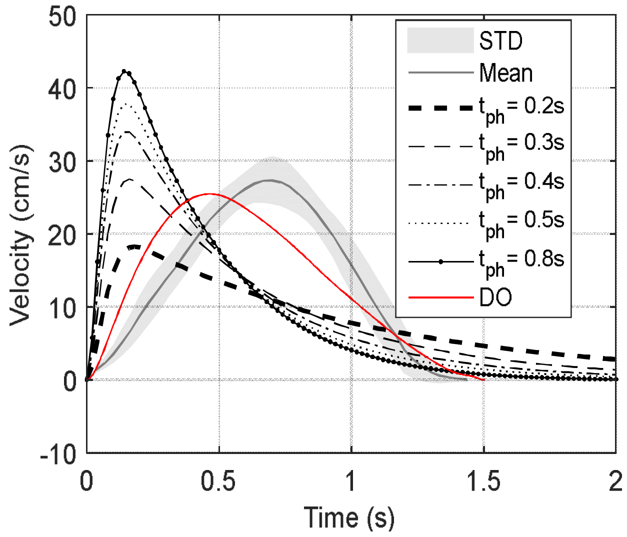
Prediction Horizon Variation

- The effect of prediction horizon on the hand speed in the reaching motion



Hand speed versus displacement



Hand speed versus time

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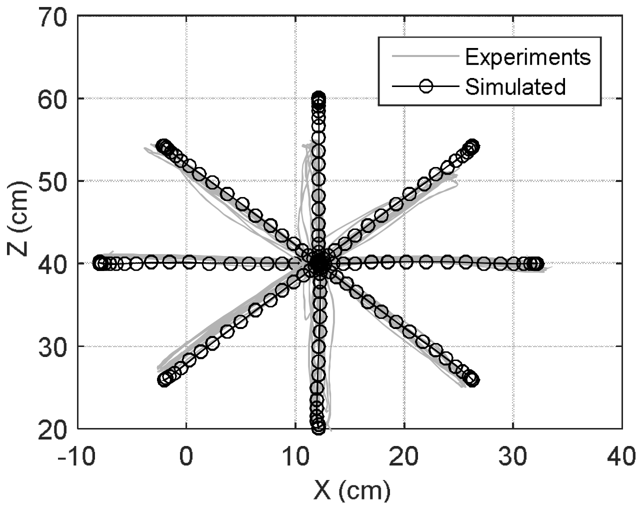
Trajectory Tracking

- Target positions of the hand are located on a circle centered at the initial position of the hand and radius of 20 cm.



Hand speed versus displacement

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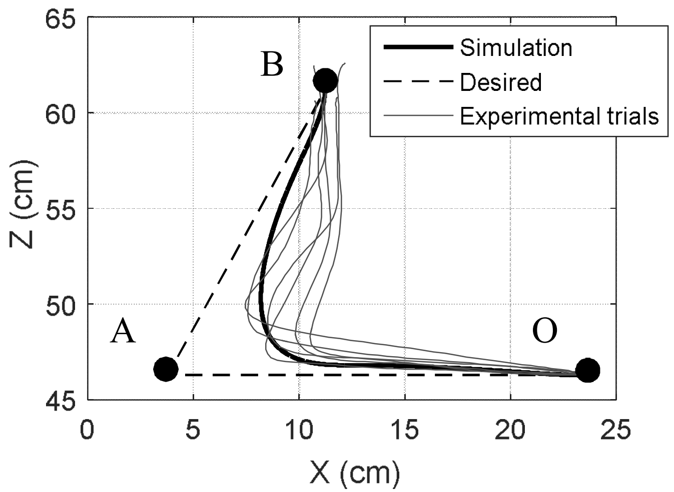
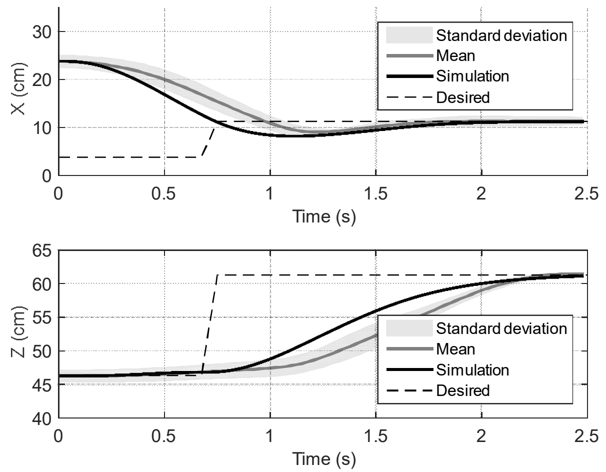
Reaching to a Moving Target

- The hand is initially at rest at point O, and moves towards the target at point A. Then one second later, the target position suddenly moves to the point B.



The hand trajectory when the target is suddenly moved from A to B



Displacement of hand in X and Z directions when the target id suddenly moved from A to B

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Conclusion

- A planar musculoskeletal arm model was developed.
- A nonlinear model predictive controller was developed to simulate goal-oriented and predefined reaching motions.
- Predictive simulation of reaching to moving target has been performed.
- This purely predictive/mathematical simulation seems to capture reality.

Future Work

- Developing a real-time nonlinear model predictive control (NMPC) algorithm
- Developing upper-limb rehabilitation robot controllers based on the proposed controller
- Incorporating muscle synergy theory to NMPC to develop real-time simulations of more complicated motions such as gait

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Direct Collocation Method

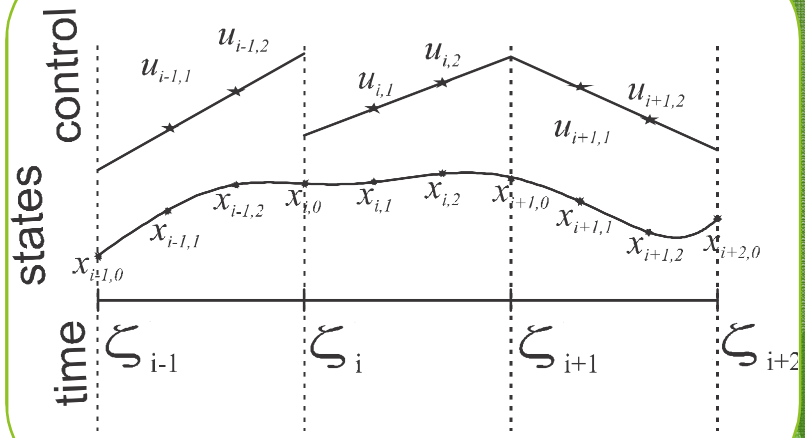
let's consider Lagrange polynomial approximation for states and control profiles.

State: K_x+1 degree polynomial with K_x collocation point

$$x^{K_x+1}(t) = \sum_{j=0}^{K_x} x_{ij}\,\phi_j(\tau), \quad \text{where} \quad \phi_j(\tau) = \prod_{\substack{k=0\\k\neq j}}^{K_x} \frac{\tau-\tau_k}{\tau_j-\tau_k}, \begin{cases} t\in[\zeta_{i-1},\zeta_i\,] \\ \tau\in[0,1] \end{cases}$$

Control profile: K_u degree polynomial with K_u collocation point

$$u^{K_u}(t) = \sum_{j=1}^{K_u} u_{ij}\,\theta_j(\tau), \quad \text{where} \quad \theta_j(\tau) = \prod_{\substack{k=1\\k\neq j}}^{K_u} \frac{\tau-\tau_k}{\tau_j-\tau_k}, \begin{cases} t\in[\zeta_{i-1},\zeta_i\,] \\ \tau\in[0,1] \end{cases}$$



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Direct Collocation Method (Cont'd)

Nonlinear constraints

Equations of motion are satisfied at all collocation points

$$M\,\dot{x}(t) = f(x(t), u(t), t), \qquad x(t_0) = x_0$$
$$g(x(t), u(t), t) = 0$$

At each collocation point:

$$\text{Res} \coloneqq M \sum_{j=0}^{K_x} x_{ij}\,\dot{\phi}_j(t_k) - \Delta\zeta_i\, f(x_{ik}, u_{ik}, t_{ik}), \quad \begin{cases} i = 1, \dots, NE \\ j = 0, \dots, K \\ k = 1, \dots, K \end{cases}$$

Objective function

$$\text{Objective function} \coloneqq \Phi\big(x(t_f), t_f\big) + \int_{t_0}^{T} \Psi(x(t), u(t), t)\,dt$$