Assignment 1

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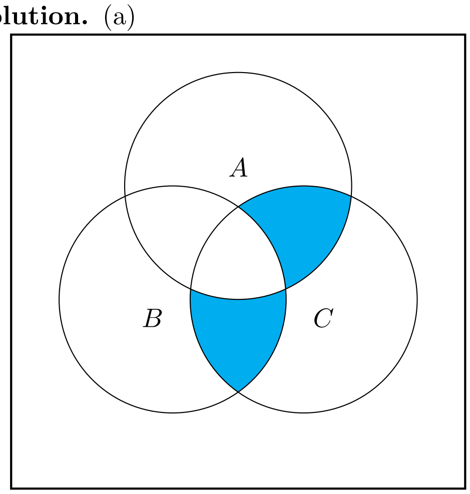
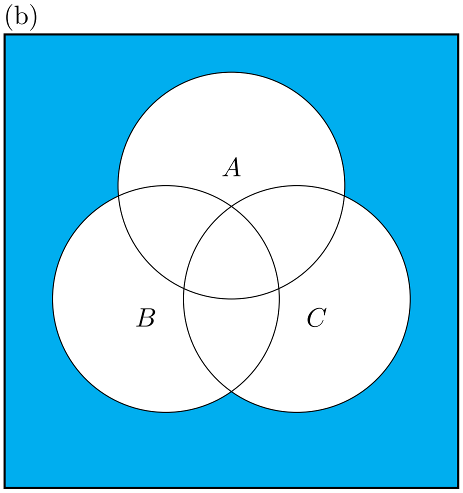
Problem 1. *Venn Diagrams. Draw the Venn diagrams for*
(a) $(A \oplus B) \cap C$
(b) $\bar{A} \cap \bar{B} \cap \bar{C}$

Solution. (a)



(b)



Problem 2. *Set Identities. Prove the following*
(a) $\neg(A \cap B) = \neg A \cup \neg B$
(b) $(B - A) \cup (C - A) = (B \cup C) - A$

Solution. (a)

Proof. I will show that $\neg(A \cap B) \subset \neg A \cup \neg B$ in i) then $\neg A \cup \neg B \subset \neg(A \cap B)$ in ii)

i)
Take any $x \in \neg(A \cap B)$
So, $x \in U - A \cap B$
So, $x \in U$ and $x \notin A \cap B$
If $x \notin A \cap B$, then $x \in U - A$ and $x \in U - B$
Therefore, $x \in \neg A \cup \neg B$

ii)
Take any $y \in \neg A \cup \neg B$
Either $y \in \neg A \subset \neg(A \cap B)$ or $y \in \neg B \subset \neg(A \cap B)$
Therefore, $y \in \neg(A \cap B)$

□

(b)

Proof. I will show that $(B - A) \cup (C - A) \subset (B \cup C) - A$ in i) then $(B \cup C) - A \subset (B - A) \cup (C - A)$ in ii)

i)
Take any $x \in (B - A) \cup (C - A)$
So, either $x \in (B - A)$ or $x \in (C - A)$
Since, $x \in (B - A) \subset (B \cup C) - A$ because $B \subset B \cup C$
and $x \in (C - A) \subset (B \cup C) - A$ because $C \subset B \cup C$
and both $B - A$ and $C - A$ are without A
Therefore, $x \in (B \cup C) - A$

ii)
Take any $y \in (B \cup C) - A$
So, $y \in B \cup C$ but $y \notin A$
If either $y \in B$ or $y \in C$ and $y \notin A$ then $y \in (B - A) \cup (C - A)$

□

Problem 3. *Propositions. Which of the following sentences are statements?*
(a) Montral is an island

(b) $6+5=10$
(c) $x+5=10$

Solution. All of the sentences are statements except (c) because (a) and (b) are true and false respectively whereas c depends upon x and cannot be evaluated to be true or false. A statement or proposition must be either determinately true or false.

Problem 4. *Conditional Statements. Which of the following implications are true?*
(a) If $1+1=2$ *then pigs can fly.*
(b) If pigs can fly then $1+1=2$.
(c) If $1+1=3$ *then pigs can fly.*
(d) If pigs can fly then $1+1=3$.

Solution. Given an implication of the form "if p then q" or $p => q$, we can assess the validity of each implication by assuming that the statements "pigs can fly" and $1+1=3$ are false while $1+1=2$ is true.
The implications by letter are:
(a) $p = true$ $q = false$ therefore, FALSE.
(b) $p = false$ $q = true$ therefore, TRUE.
(c) $p = false$ $q = false$ therefore, TRUE.
(d) $p = false$ $q = false$ therefore, TRUE.

Problem 5. *Tautologies. Which of the following are tautologies? If the statement is a tautology give a proof using the appropriate rules of logic at each step of the proof. If not, then justify your answer by giving a counter-example or using a proof table.*
(a) $p \Rightarrow (p \vee q)$
(b) $\neg(p \Rightarrow q) \equiv \neg q$
(c) $\neg(p \oplus q) \equiv (p \Leftrightarrow q)$
(d) $((p \Rightarrow q) \Rightarrow r) \equiv (p \Rightarrow (q \Rightarrow r))$
(e) $(\neg p \wedge (p \Rightarrow q)) \equiv \neg q$

Solution. (a) IS a tautology:

Proof. $p \Rightarrow (p \vee q)$
$\equiv \neg p \vee (p \vee q)$
$\equiv (\neg p \vee p) \vee q$
$\equiv 1 \vee q$
$\equiv 1$

□

(b) IS NOT a tautology:
When p and q are both false, $\neg(p \Rightarrow q)$ evaluates to false while $\neg q$ evaluates to true.

(c) IS a tautology:

Proof. I will show that $p \oplus q$ is equivalent to $p \Leftrightarrow q$:

$p \oplus q$
$\equiv \neg((p \vee q) \wedge \neg(p \wedge q))$
$\equiv \neg(p \vee q) \vee (p \wedge q)$
$\equiv p \Leftrightarrow q$

$\square$

(d) IS NOT a tautology:

When p, q and r are all false, $(p \Rightarrow q) \Rightarrow r$ evaluates to false while $p \Rightarrow (q \Rightarrow r)$ evaluates to true.

(e) IS NOT a tautology:

When p is false and q is true, $\neg p \wedge (p \Rightarrow q)$ evaluates to true while $\neg q$ evaluates to false.

Problem 6. *Circuits. Show how NAND gates can be used to simulate OR, AND and NOT gates.*

Solution. I will represent a NAND gate with this pattern: $\neg(p \wedge q)$ where the inputs are represented as statements p and q.

OR Gate
$a \vee b \equiv \neg(\neg(a \wedge a) \wedge \neg(b \wedge b))$

AND Gate
$a \wedge b \equiv \neg(\neg(a \wedge b) \wedge \neg(a \wedge b))$

NOT Gate
$\neg a \equiv \neg(a \wedge a)$