Question 1

1. Convert 11001001 from binary to decimal and hexadecimal.

$11001001_2 = 1*128_{10} + 1*64_{10} + 0*32_{10} + 0*16_{10} + 1*8_{10} + 0*4_{10} + 0*2_{10} + 1*1_{10} = 201_{10}$

$1100_2 = C_{16}$, $1001_2 = 9_{16}$, $1100\ 1001\ _2 = C\ 9_{16}$

To verify:

201/16 = 12 remainder $9_{10} = 9_{16}$
12/16 = 0 remainder $12_{10} = C_{16}$

<u>Thus, $11001001_2 = 201_{10} = C9_{16}$</u>

2. Convert 0.111101 from binary to decimal and hexadecimal.

$0.111101_2 = 1*1/2_{10} + 1*1/4_{10} + 1*1/8_{10} + 1*1/16_{10} + 0*1/32_{10} + 1*1/64_{10} = 61/64_{10}$

$1111_2 = F_{16}$, $0100_2 = 4_{16}$, $0.\ 1111\ 0100\ _2 = F4_{16}$

<u>Thus, $0.111101_2 = 61/64_{10} = F4_{16}$</u>

3. Convert BAD from hexadecimal to decimal and binary.

$BAD_{16} = 11*256_{10} + 10*16_{10} + 13*1_{10} = 2989_{10}$

$B_{16} = 1011_2$, $A_{16} = 1010_2$, $D_{16} = 1101_2$, $B\ A\ D\ _{16} = 1011\ 1010\ 1101_2$

To verify:

2989/2 = 1494 remainder 1
1494/2 = 747 remainder 0
747/2 = 373 remainder 1
373/2 = 186 remainder 1

186/2 = 93 remainder 0
93/2 = 46 remainder 1
46/2 = 23 remainder 0
23/2 = 11 remainder 1

11/2 = 5 remainder 1
5/2 = 2 remainder 1
2/2 = 1 remainder 0
1/2 = 0 remainder 1

<u>Thus, $BAD_{16} = 101110101101_2 = 2989_{10}$</u>

4. Convert 0.AF from hexadecimal to decimal and binary.

$0.AF_{16} = 10*1/16_{10} + 15*1/256_{10} = 175/256_{10}$

$A_{16} = 1010_2$, $F_{16} = 1111_2$, $0.AF_{16} = 0.\ 1010\ 1111\ _2$

<u>Thus, $0.AF_{16} = 0.\ 10101111_2 = 175/256_{10}$</u>

5. Convert 12648430 from decimal to binary and hexadecimal.

12648430/2 = 6324215 remainder 0
6324215/2 = 3162107 remainder 1
3162107/2 = 1581053 remainder 1
1581053/2 = 790526 remainder 1
790526/2 = 395263 remainder 0
395263/2 = 197631 remainder 1
197631/2 = 98815 remainder 1
98815/2 = 49407 remainder 1
49407/2 = 24703 remainder 1
24703/2 = 12351 remainder 1
12351/2 = 6175 remainder 1
6175/2 = 3087 remainder 1
3087/2 = 1543 remainder 1
1543/2 = 771 remainder 1
771/2 = 385 remainder 1
385/2 = 192 remainder 1
192/2 = 96 remainder 0
96/2 = 48 remainder 0
48/2 = 24 remainder 0
24/2 = 12 remainder 0
12/2 = 6 remainder 0
6/2 = 3 remainder 0
3/2 = 1 remainder 1
1/2 = 0 remainder 1
0/2 = 0 remainder 0

<u>So, $12648430_{10} = 110000001111111111101110_2$</u>

12648430/2 = 790526 remainder 14
790526/2 = 49407 remainder 14
49407/2 = 3087 remainder 15
3087/2 = 192 remainder 15
192/2 = 12 remainder 0
12/2 = 0 remainder 12
0/2 = 0 remainder 0

<u>So, $12648430_{10} = C0FFEE_{16}$</u>

6. Convert 3.140625 from decimal to binary and hexadecimal.

0.140625 * 2 = 0.28125
0.28125 * 2 = 0.5625
0.5625 * 2 = 1.125
0.125 * 2 = 0.25
0.25 * 2 = 0.5
0.5 * 2 = 1
0 * 2 = 0

$3_{10} = 0011_2$

<u>So, $3.140625_{10} = 11.001001_2$</u>

$0011_2 = 3_{16}$, $0010_2 = 2_{16}$, $0100_2 = 4_{16}$, $0011 . 0010\ 0100\ _2 = 3.24_{16}$

<u>Thus, $3.140625_{10} = 11.001001_2 = 3.24_{16}$</u>

7. Represent -123 as an 8 bit signed integer using two's complement format. Write your answer in both binary and hexadecimal.

Invert bits of 123, change leading bit to a 1, and add 1:

123/2 = 61 remainder 1
61/2 = 30 remainder 1
30/2 = 15 remainder 0
15/2 = 7 remainder 1
7/2 = 3 remainder 1
3/2 = 1 remainder 1
1/2 = 0 remainder 1
0/2 = 0 remainder 0

$123\ _{10} = 0111\ 1011\ _2$
$-123\ _{10} = 10000100_2 + 1_2 = 1000\ 0101\ _2$

$1000_2 = 8_{16}$, $0101_2 = 5_{16}$, $1000\ 0101\ _2 = 85_{16}$

<u>Thus, as an 8-bit signed integer, $-123_{10} = 1000\ 0101_2 = 85_{16}$</u>

Question 2

1. Represent 3.141592653 as an IEEE 754 single precision floating point number. Is the representation exact? Show your work. Give your answer in both binary and hexadecimal.

Sign: The number is positive, sign bit = 0.

Exponent: 1 is the largest exponent of 2 still smaller than 3.14…, so 1 is my exponent part. The bias is 127, so the exponent component is $127 + 1 = 128\ _{10} = 1000\ 0000\ _2$.

Mantissa: My exponent is 1, so the mantissa is equal to 3.14… / 2^1 = 1.5707963265_{10}; $0.5707963265_{10} \approx 0.10010010000111111011011_2$. It is approximately equal because there were insufficient bits of precision for the exact binary representation of the fractional part. I followed the rounding rules per IEEE 754 as the rounded bit was closer to 1 than 0 in decimal form. The rounded bit is highlighted in the attached calculation.

Sign	Exponent	Mantissa
0	100 0000 0	100 1001 0000 1111 1101 1011

Using 4-bit groups, I determined the hexadecimal representation to be:

0100 0000 0100 1001 0000 1111 1101 1011 $_2$ = 0x 4 0 4 9 0 F D B $_{16}$

2. Represent 1.0 x 10^{-15} as an IEEE single precision floating point number. Hint: use log to convert 10^{-15} to an integer power of two multiplied by a factor. Show your work and give your answer in both binary and hexadecimal.

Sign: The number is positive, sign bit = 0.

Exponent: Using a calculator, I found that 1.0 x $10^{-15} \approx$ 1.0 x $2^{-49.8289\ldots}$. As such, -50 will be the largest exponent of 2 still smaller than 1.0 x 10^{-15}. The bias is 127, so the exponent component is 127 + (-50) = 77 $_{10}$ = (0)100 1101 $_2$.

Mantissa: My exponent is -50, so the mantissa is equal to 1.0 x 10^{-15} / 2^{-50} = 1.125899906842624_{10}; $0.125899906842624_{10} \approx 0.00100000001110101111101_2$. It is approximately equal because there were insufficient bits of precision for the exact binary representation of the fractional part. I followed the rounding rules per IEEE 754 as the rounded bit was closer to 1 than 0 in decimal form. The rounded bit is highlighted in the attached calculation. For this part, I stopped once I reached 23 bits of precision.

Sign	Exponent	Mantissa
0	010 0110 1	001 0000 0001 1101 0111 1101

Using 4-bit groups, I determined the hexadecimal representation to be:

0010 0110 1001 0000 0001 1101 0111 1101 $_2$ = 0x 2 6 9 0 1 D 7 D $_{16}$

Question 3

1. Prove that the function $NOR(A,B) = \overline{A+B}$ is universal by showing how to express the functions $AND(A,B), OR(A,B), and\ NOT(A)$ using only $NOR(A,B)$.

$$AND(A,B) = NOR\big(NOR(A,A), NOR(B,B)\big)$$

$$OR(A,B) = NOR(C,C)\ where\ C = NOR(A,B)$$

$$NOT(A) = NOR(A,A)$$

2. Convert the function $A \cdot B \cdot \bar{C} + A \cdot \bar{B} \cdot C + \bar{A} \cdot B \cdot C$ from its sum of products representation to a product of sums.

$$
\begin{aligned}
& (A \cdot B \cdot \bar{C}) + (A \cdot \bar{B} \cdot C) + (\bar{A} \cdot B \cdot C) \\
&= \overline{\overline{(A \cdot B \cdot \bar{C}) + (A \cdot \bar{B} \cdot C) + (\bar{A} \cdot B \cdot C)}} \\
&= \overline{\overline{(A \cdot B \cdot \bar{C})} \cdot \overline{(A \cdot \bar{B} \cdot C)} \cdot \overline{(\bar{A} \cdot B \cdot C)}} \\
&= \overline{(\bar{A} + \bar{B} + C) \cdot (\bar{A} + B + \bar{C}) \cdot (A + \bar{B} + \bar{C})}
\end{aligned}
$$

Question 4

See other file attached (Question 4.circ)

Attachment:

Question 2 Part 1:

0.5707963265 * 2 = 1.141592653
0.141592653 * 2 = 0.283185306
0.283185306 * 2 = 0.566370612
0.566370612 * 2 = 1.132741224
0.132741224 * 2 = 0.265482448
0.265482448 * 2 = 0.530964896
0.530964896 * 2 = 1.061929792
0.0619297920000008 * 2 = 0.123859584000002
0.123859584000002 * 2 = 0.247719168000003
0.247719168000003 * 2 = 0.495438336000007
0.495438336000007 * 2 = 0.990876672000013
0.990876672000013 * 2 = 1.98175334400003
0.981753344000026 * 2 = 1.96350668800005
0.963506688000052 * 2 = 1.9270133760001
0.927013376000104 * 2 = 1.85402675200021
0.854026752000209 * 2 = 1.70805350400042
0.708053504000418 * 2 = 1.41610700800083
0.416107008000836 * 2 = 0.832214016001671
0.832214016001671 * 2 = 1.66442803200334
0.664428032003343 * 2 = 1.32885606400668
0.328856064006686 * 2 = 0.657712128013372
0.657712128013372 * 2 = 1.31542425602674
0.315424256026745 * 2 = 0.630848512053489
0.630848512053489 * 2 = 1.26169702410697
0.261697024106979 * 2 = 0.523394048213958
0.523394048213958 * 2 = 1.04678809642791
0.0467880964279175 * 2 = 0.093576192855835
0.093576192855835 * 2 = 0.18715238571167
0.18715238571167 * 2 = 0.37430477142334
0.37430477142334 * 2 = 0.748609542846679
0.748609542846679 * 2 = 1.49721908569335
0.497219085693359 * 2 = 0.994438171386718
0.994438171386718 * 2 = 1.98887634277343
0.988876342773437 * 2 = 1.97775268554687
0.977752685546875 * 2 = 1.95550537109375
0.95550537109375 * 2 = 1.9110107421875
0.9110107421875 * 2 = 1.822021484375
0.822021484375 * 2 = 1.64404296875
0.64404296875 * 2 = 1.2880859375
0.2880859375 * 2 = 0.576171875
0.576171875 * 2 = 1.15234375

0.15234375 * 2 = 0.3046875
0.3046875 * 2 = 0.609375
0.609375 * 2 = 1.21875
0.21875 * 2 = 0.4375
0.4375 * 2 = 0.875
0.875 * 2 = 1.75
0.75 * 2 = 1.5
0.5 * 2 = 1
0 * 2 = 0

Question 2 part 2

0.125899906842624 x 2 = 0.251799813685248
0.251799813685248 x 2 = 0.503599627370496
0.503599627370496 x 2 = 1.00719925474099
0.00719925474099203 x 2 = 0.0143985094819841
0.0143985094819841 x 2 = 0.0287970189639681
0.0287970189639681 x 2 = 0.0575940379279363
0.0575940379279363 x 2 = 0.115188075855873
0.115188075855873 x 2 = 0.230376151711745
0.230376151711745 x 2 = 0.46075230342349
0.46075230342349 x 2 = 0.92150460684698
0.92150460684698 x 2 = 1.84300921369396
0.843009213693961 x 2 = 1.68601842738792
0.686018427387921 x 2 = 1.37203685477584
0.372036854775843 x 2 = 0.744073709551685
0.744073709551685 x 2 = 1.48814741910337
0.48814741910337 x 2 = 0.976294838206741
0.976294838206741 x 2 = 1.95258967641348
0.952589676413481 x 2 = 1.90517935282696
0.905179352826963 x 2 = 1.81035870565393
0.810358705653925 x 2 = 1.62071741130785
0.620717411307851 x 2 = 1.2414348226157
0.241434822615702 x 2 = 0.482869645231403
0.482869645231403 x 2 = 0.965739290462806