Assignment 6

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Problem 1. *Cycles and Circuits.*

Solution. (a) I will use the theorem proven in class that a connected simple graph is Eulerian iff it has no vertices of odd degree.

Take a complete graph G' with $|V'| = 11$. It has $|E'| = \frac{|V'|(|V'|-1)}{2}$. So, $|E'| = 55$. This complete graph is Eulerian because every vertex is connected to 10 other vertices so every vertex has an even degree. Since G' is a complete graph, I can transform G' to G by removing 2 edges.

So I remove the 1st edge, say the edge was (u, v). Now both u and v have an odd degree of 9. I must remove a 2nd edge. I either remove an edge that is adjacent to u or v, or I remove an edge not adjacent to u or v to achieve G.

In the case where I must remove an edge (v, w) that is adjacent to v, I now have vertices u and w that have an odd degree of 9. $|E'| = 53$ but the graph is not Eulerian. The case for an edge adjacent to u is symmetrical.

In the case where I must remove an edge w_i, w_j that is not adjacent to either u or v, I now have 4 vertices, u, v, w_i, and w_j that have an odd degree of 9. Again, $|E'| = 53$ but the graph is not Eulerian.

Thus G cannot be Eulerian.

(b) The idea here is similar to that in (a). A complete graph G' with $|V'| = 11$ has a Hamiltonian Cycle because starting from any vertex, there is always an edge to any other unvisited vertex.

To get from G' to G, I must remove 2 edges. Either I remove 2 non-cycle edges or I remove at most 2 cycle edges.

In the case where I must remove 2 non-cycle edges, the original Hamiltonian Cycle from G' is still a Hamiltonian Cycle in G so G is a Hamiltonian Graph.

In the case where I must remove 2 cycle edges, there are two options. Either I removed two edges that share an endpoint vertex or I removed two edges that do not.

If I removed two edges (u_i, v) and (v, u_j) then I can rebuild the Hamiltonian cycle by removing some non-adjacent edge (u_k, u_l) and adding (u_i, u_j), (v, u_k) and $(v, u_l$, all of which exist because they were not the edges removed from the complete graph G'. This graph contains a Hamiltonian cycle.

If I removed two edges (u_i, v_i) and (u_j, v_i) such that a path still exists on the cycle from v_i to u_j then I can rebuild the Hamiltonian cycle by adding (u_i, u_j) and (v_i, v_j). This graph contains a Hamiltonian cycle.

Thus G must be Hamiltonian.

Problem 2. *Trees.*

Solution. For this problem I assume that a path consists of edges and the vertices they connect so two paths intersect if they share at least one vertex, but not necessarily any edges.

Proof by induction.

In a tree of consisting of 1 node, the longest "path" is the single node. So the two longest paths must intersect the single node.

I assume that the two longest paths must intersect on a tree T' of n nodes.

I can take a tree T with $n + 1$ nodes and remove a leaf node to create T'.

I assumed that in T' the two longest paths must intersect.

In order to get back to T, I must add back the node I removed. So I either add a child node to a leaf node or I add a leaf node to a parent node.

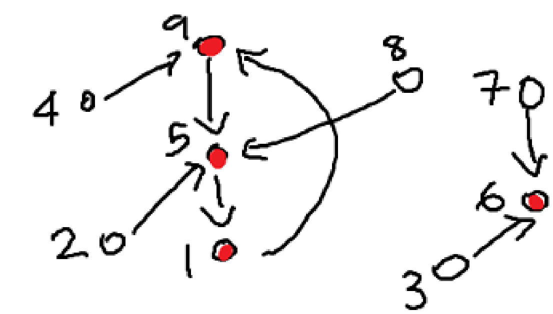
If I add another leaf node v to an existing parent node u, the new set of maximum length paths might include a path that begins at v if the children of u were leaf nodes that were in the set of maximum length paths. All the previous maximum length paths passed through u and the new maximum length path that begins at v also passes through u. So all the maximum length paths intersect at u. If the new set of maximum length paths do not include a path through v then the maximum length paths in T are identical as in T'.

If I add a child node v to an existing leaf node u, the diameter of T may or may not increase from that of T'. If it does increase, then all the new maximum length paths must include v so all new maximum length paths must intersect at least at v and u. If the diameter does not increase then the maximum length paths in T are identical as in T'.

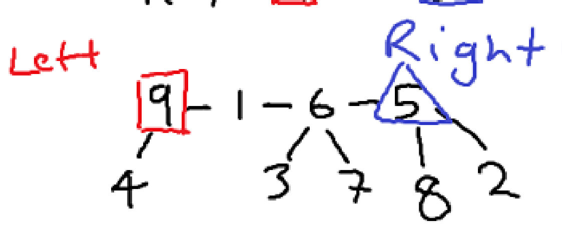
Problem 3. *Joyal Codes.*

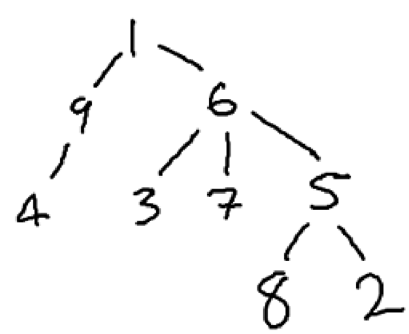
Solution. (a)

$f(x)=$	$\boxed{9}$	5	6	9	$\boxed{1}$	$\boxed{6}$	6	5	$\boxed{5}$
$x=$	$\boxed{1}$	2	3	4	$\boxed{5}$	$\boxed{6}$	7	8	$\boxed{9}$



$x=$	1	5	6	9
$f(x)=$	$\boxed{9}$	1	6	5





(b) The path from left to right is 4852 which I add to the table in order,

f(x)	4	8	5	2
x	2	4	5	8

And then I add the remaining nodes,

f(x)	8	4	5	8	5	4	2	2	4
x	1	2	3	4	5	6	7	8	9

So the code is 845854224.