Assignment 4

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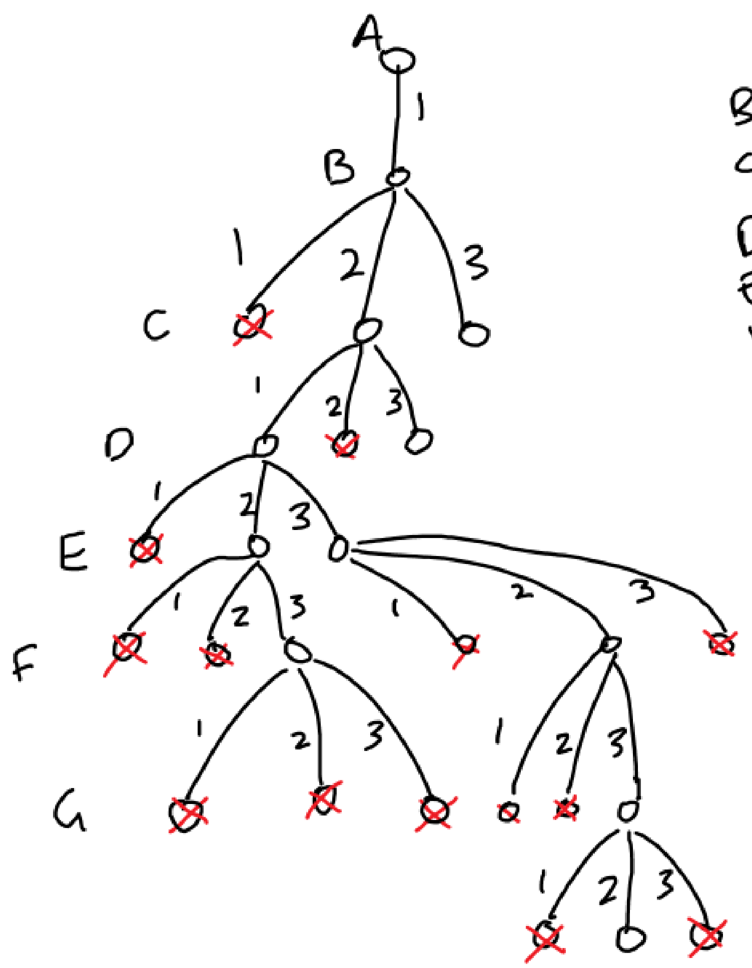
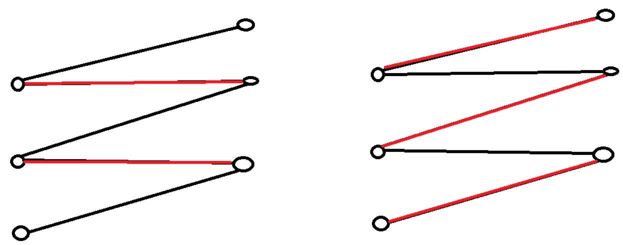
Problem 1. *Backtracking: Vertex Colouring.*

Solution. The colouring I found was A=1, B=2, C=1, D=3, E=2, F=3, G=2



Problem 2. *Local Search: Matching*

Solution. (a) If the algorithm chooses the one of the two horizontal red edges on the left, then it will not find the maximum matching which is the three diagonal edges on the right.



(b) Let $A = M \cup M'$. There are now some k number of connected components $C_1, ...C_k \subseteq A$. Every $e'_i \in M'$ can share end points with at most 2 edges $e_i, e_j \in M$ and form a C_i which gives the constraint $|M'| \leq 2|M|$, but it is given that $|M'| > 2|M|$, so there must exist some C_j such that $C_j \cap M = \emptyset$ where C_j is an $e' \in M'$ that can be added to M that maintains the bipartite property of M.

(c) If M were the maximum matching of $G = V, E$ and M' the gradient ascent algorithm matching, then we know that there is no $e \in E$ and so, no $e \in M$ that can be added to M' by the algorithm definition. Thus, by the constraint found in (b), we have $|M'| \geq \frac{1}{2}|M|$.

Problem 3. *Local Search: Machine Scheduling.*

Solution. (a) At the conclusion of the algorithm there might be a machine with a smaller total time M_s and a machine with larger total time M_l. There must be at least 2 jobs on M_l. This is because it is given that the largest job t_j does not take more than half the total time of all jobs, i.e. $t_j \leq \frac{1}{2}(T_l + T_s)$. The smallest job t_i on M_l was not moved to the other machine M_s so $t_i \geq T_l - T_s$. If this is the case, then,

$T_l - T_s \leq t_i \leq \frac{1}{2}(T_l + T_s)$

$T_l \leq t_i + T_s \leq \frac{1}{2}(T_l) + T_s$

$2 \cdot T_l \leq 2(t_i + T_s) \leq T_l + 2 \cdot T_s$

And if we ignore the middle inequality and subtract T_l from both sides,

$T_l \leq 2 \cdot T_s$

Which is the given upper bound for the machine with the larger total time.

(b) For some job t_j, it will only move if $t_j < |T_1 - T_2|$. Since $|T_1 - T_2|$ is always decreasing, after some number of moves, t_j will no longer move. Once the new rule for the choice of job is introduced, then t_j will only be moved when it is the largest job. If t_j was the largest job then moving t_j will reduce $|T_1 - T_2| \leq t_j$. Thus t_j is only moved once. Since all jobs are only moved once, the maximum number of move operations is n, the number of jobs.

(c) If M_1 started with two jobs that are $t = 3$ each and M_2 had four jobs with $t = 1$ each, then the algorithm will stop because no single job moved will not decrease $|T_1 - T_2|$. This result is not the globally optimal solution which is giving each of M_1 and M_2 one of the $t = 3$ jobs and two of the $t = 1$ jobs.