Assignment 4

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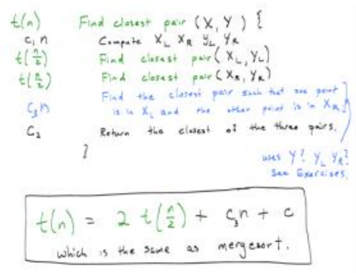
Question 2



$t(n)$	Find closest pair (X, Y) {
$c_1 n$	Compute X_L X_R Y_L Y_R
$t(\frac{n}{2})$	Find closest pair(X_L, Y_L)
$t(\frac{n}{2})$	Find closest pair(X_R, Y_R)
$c_3 n$	Find the closest pair such that one point is in X_L and the other point is in X_R.
c_2	Return the closest of the three pairs.
	}

uses Y? Y_L Y_R? See Exercises.

$$\boxed{t(n) = 2t\left(\tfrac{n}{2}\right) + c_3 n + c}$$

which is the same as mergesort.

$k=0$ $\quad t(n) = 2t\left(\frac{n}{2}\right) + c_3 n + c + n\log n$

$k=1$ $\quad t(n) = 2\left(2t\left(\frac{\frac{n}{2}}{2}\right) + c_3\frac{n}{2} + c + \frac{n}{2}\log\left(\frac{n}{2}\right)\right) + c_3 n + c + n\log n$

$\quad t(n) = 4t\left(\frac{n}{4}\right) + 2c_3\frac{n}{2} + 2c + 2\left(\frac{n}{2}\log\left(\frac{n}{2}\right)\right) + c_3 n + c + n\log n$

$k=2$ $\quad t(n) = 4\left(2t\left(\frac{\frac{n}{4}}{2}\right) + c_3\frac{n}{4} + c + \frac{n}{4}\log\left(\frac{n}{4}\right)\right) + 2c_3\frac{n}{2} + 2c + 2\left(\frac{n}{2}\log\left(\frac{n}{2}\right)\right) + c_3 n + c + n\log n$

$\quad t(n) = 8t\left(\frac{n}{8}\right) + 4c_3\frac{n}{4} + 4c + 4\left(\frac{n}{4}\log\left(\frac{n}{4}\right)\right) + 2c_3\frac{n}{2} + 2c + 2\left(\frac{n}{2}\log\left(\frac{n}{2}\right)\right) + c_3 n + c + n\log n$

$k=3$ $\quad t(n) = 8\left(2t\left(\frac{\frac{n}{8}}{2}\right) + c_3\frac{n}{8} + c + \frac{n}{8}\log\left(\frac{n}{8}\right)\right) + 4c_3\frac{n}{4} + 4c + 4\left(\frac{n}{4}\log\left(\frac{n}{4}\right)\right) + 2c_3\frac{n}{2} + 2c + 2\left(\frac{n}{2}\log\left(\frac{n}{2}\right)\right) + c_3 n + c + n\log n$

$\quad t(n) = 16t\left(\frac{n}{16}\right) + 8c_3\frac{n}{8} + 8c + 8\left(\frac{n}{8}\log\left(\frac{n}{8}\right)\right) + 4c_3\frac{n}{4} + 4c + 4\left(\frac{n}{4}\log\left(\frac{n}{4}\right)\right) + 2c_3\frac{n}{2} + 2c + 2\left(\frac{n}{2}\log\left(\frac{n}{2}\right)\right) + c_3 n + c + n\log n$

$$t(n) = 16t\left(\tfrac{n}{16}\right) \quad \begin{array}{l} + n\log n \\ + \cancel{2}\left(\frac{n}{\cancel{2}}\log\left(\frac{n}{2}\right)\right) \\ + \cancel{4}\left(\frac{n}{\cancel{4}}\log\left(\frac{n}{4}\right)\right) \\ + \cancel{8}\left(\frac{n}{\cancel{8}}\log\left(\frac{n}{8}\right)\right) \end{array} \quad \begin{array}{l} + c \\ + 2c \\ + 4c \\ + 8c \end{array} \quad \begin{array}{l} + c_3 n \\ + \cancel{2}c_3\frac{n}{\cancel{2}} \\ + \cancel{4}c_3\frac{n}{\cancel{4}} \\ + \cancel{8}c_3\frac{n}{\cancel{8}} \end{array}$$

$$= 2^{k+1}\left(t\left(\frac{n}{2^{k+1}}\right)\right) + n\left(\sum_{i=0}^{k}\log\left(\frac{n}{2^i}\right)\right) + c\left(\sum_{i=0}^{k}2^i\right) + (k+1)c_3 n$$

$$= 2^{k+1}\left(t\left(\frac{n}{2^{k+1}}\right)\right) + n\left(\sum_{i=0}^{k}\log(n) - \sum_{i=0}^{k} i\cancel{\log(2)}\right) + c\left(2^{k+1} - 1\right) + (k+1)c_3 n$$

$$= 2^{k+1}\left(t\left(\frac{n}{2^{k+1}}\right)\right) + n\left(k\log(n) - \frac{k^2+k}{2}\right) + c\left(2^{k+1} - 1\right) + (k+1)c_3 n$$

algorithm recurrence stops when $|x| \leq 3$, so this recurrence stops when $\frac{n}{2^{k+1}} = 3$

$$\frac{n}{3} = 2^{k+1}$$

$$\log\left(\frac{n}{3}\right) = (k+1)\log(2)$$

$$k = \log\left(\frac{n}{3}\right) - 1$$

$$= 2^{k+1}\left(t\left(\frac{n}{2^{k+1}}\right)\right) + n\left(k\log(n) - \frac{k(k+1)}{2}\right) + c\left(2^{k+1} - 1\right) + (k+1)c_3 n$$

$$= 2^{\log\left(\frac{n}{3}\right)} + n\left(\log\left(\frac{n}{3}\right) - 1\right)\left(\log(n)\right) - \frac{n\left(\log\left(\frac{n}{3}\right) - 1\right)\left(\log\left(\frac{n}{3}\right)\right)}{2} + c\left(2^{\log\left(\frac{n}{3}\right)} - 1\right) + \log\left(\frac{n}{3}\right)c_3 n$$