Math 223 Assignment 1

Due in class: January 20, 2015

Instructions: Submit a hard copy of your solution with your name and student number. Late assignments will not be graded and will receive a grade of zero.

1. Prove the following

   (a) If a triangle has sides a, b and c and if θ is the acute angle opposite c (Figure 1) then

   $$c^2 = a^2 + b^2 - 2ab\cos(\theta).$$

   This result is known as Law of Cosines.

   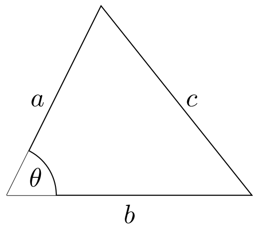

   Figure 1

   Hint: use Pythagoras' theorem.

   (b) Let v and w be nonzero vectors. If θ is the angle between v and w then

   $$v \cdot w = \|v\| \, \|w\| \cos(\theta).$$

   Hint: compute $\|v - w\|^2$ and use the Law of Cosines.

2. Show that for any complex number z and w:

   (a) $\mathrm{Re}(z) = \frac{1}{2}(z + \bar{z})$.

   (b) $\mathrm{Im} z = \frac{1}{2i}(z - \bar{z})$.

   (c) $zw = 0$ implies $z = 0$ or $w = 0$.

   Recall that the real part of the complex number $z = a + bi$ is given by a and the imaginary part is given by b. We will denote the real and complex part of z by $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ respectively.

3. Let

   $$A = \begin{bmatrix} 1 & 0 & -i \\ -1 & i & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} i & 0 \\ 0 & -i \\ 1 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 1 & 2 \\ i & 2i & i \end{bmatrix}.$$

   Compute each product in the following list that is well defined:

   - ABC
   - BAC
   - CBA
   - BCA
   - ACB

- CAB

4. Reduce each of the following matrices to row reduced echelon form:

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 3 & 0 & 1 \\ 1 & 1 & -3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

5. Find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 1 & 3 & 0 \\ 0 & 0 & 3 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 3 & 1 \end{bmatrix}.$$

6. If a subspace of $M_{2,2}$ contains

$$A = \begin{bmatrix} 2 & 10 \\ 10 & 12 \end{bmatrix} \text{ and } \quad B = \begin{bmatrix} 1 & 4 \\ 4 & 5 \end{bmatrix}$$

does it contain I?.

7. Determine whether or not W is a subspace of $\mathbb{R}^3$ where W consists of all vectors $(a, b, c) \in \mathbb{R}^3$ such that

   (a) $a = 3b$,

   (b) $a \leq b \leq c$,

   (c) $ab = 0$,

   (d) $b = a^2$.

8. Suppose U and W are subspaces of V for which $U \cup W$ is also a subspace. Show that $U \subseteq W$ or $W \subseteq U$.