# Math 223 Assignment 6

Due in class: April 9, 2015

**Instructions:** Submit a hard copy of your solution with your name and student number. Late assignments will not be graded and will receive a grade of zero.

1. Let $A \in M_{n\times n}$ and $v$ be an eigenvector of $A$ with eigenvalue $\lambda$.
   (a) (20 points) Show that if $p(t)$ is a polynomial, then $p(A)v = p(\lambda)v$.
   (b) (10 points) Show that $\lambda$ is a root of the minimal polynomial of $A$.

2. Consider the matrix
$$A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$
   (a) (20 points) Find the minimal polynomial of $A$.
   (b) (20 points) Find the Jordan Canonical form of $A$.

3. (30 points) Let $A \in M_{n\times n}$. Suppose that for $v \in \mathbb{R}^n$ we have $A^k v = 0$ and $A^{k-1}v \neq 0$. Show that $\{v, Av, A^2v, \ldots, A^{k-1}v\}$ is linearly independent.