Concepts, Definitions, and Inheritance

Interpreting the atoms of lexical decomposition

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Decompositional Semantics

"But, we can know the Markerese translation of an English sentence without knowing the first thing about the meaning of the English sentence... Translation into Markerese is at best a substitute for real semantics, relying either on our tacit competence (at some future date) as speakers of Markerese or on our ability to do real semantics at least for the one language Markerese."

-David Lewis, "General semantics" (1970)

Outline

1. Decompositional Semantics
2. Concepts
3. Inheritance Networks
4. The Genus-differentia Inheritance Theorem (GDIT)
5. Practical Consequences for Cognitive Science
6. Steps for future research

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Real Semantics for Markerese

Some guiding intuitions about real semantics:

1. It must explain/capture the sense of aboutness.
2. It must admit of empirical inquiry.
3. It cannot be a reduction to some other formal language, which we will take to mean:
4. It will take the form of interpreting Markerese as some real-world phenomenon.

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Proposal

Interpret semantic atoms as concepts.

The Classical View of Concepts: Genus-Differentia Definitions

1. A full analysis of a concept is given by a genus-differentia definition.

   Example: human := rational animal

   - genus: animal
   - differentia: rational

The Classical View of Concepts: Genus-Differentia Definitions

What constitutes a "full analysis"?

(I) Definitions specify what is *intrinsic* to a concept, by virtue of their stipulation of necessary and sufficient conditions for category membership, and

(R) Definitions specify a hierarchy relation that holds between concepts, which *relates* a concept to others within a hierarchy.

The Classical View of Concepts: Its Downfall

1. Wittgenstein argued for family resemblances, rather than definitions, determine category membership.

2. Beginning with Rosch's experiments, empirical research has validated Wittgenstein's result.

   Example: Brain-dead humans are not rational.

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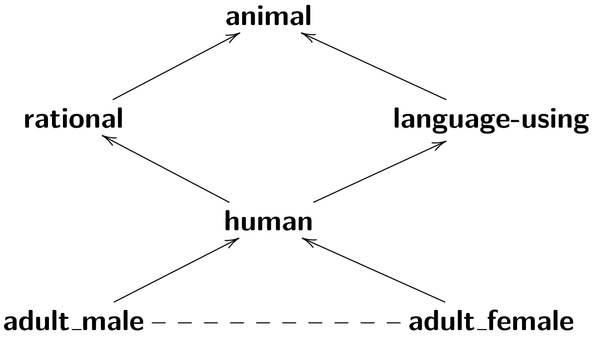
   Example: Brain-dead humans are not rational.

   Important: This only refutes (I), and there is reason to believe that (R) is true.

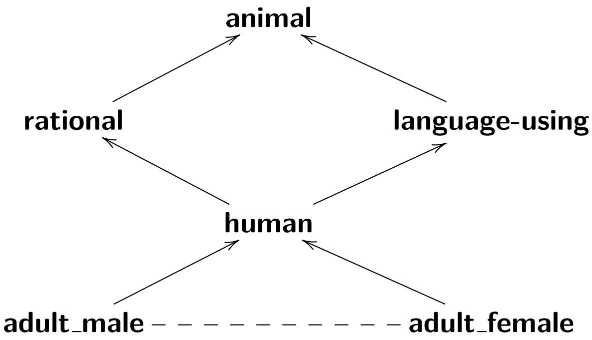
Concepts: Hierarchical Organization

1. Rosch's experiments also showed that concepts are organized hierarchically.
2. Genus-differentia definitions do serve as a set of coordinates that locate a concept within the hierarchy.

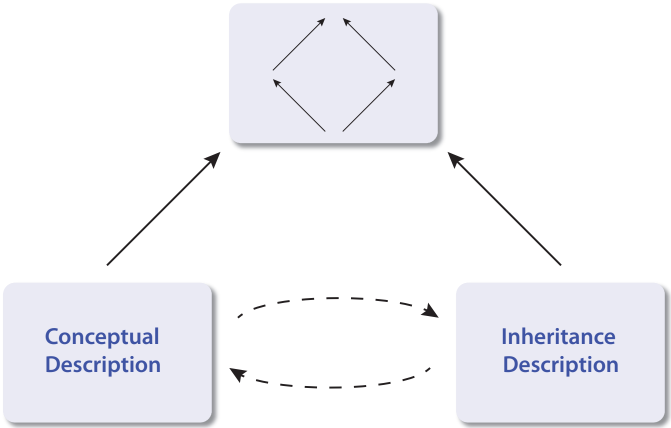
Conceptual Hierarchy



Inheritance Networks



Inheritance Networks vs. Conceptual Hierarchy



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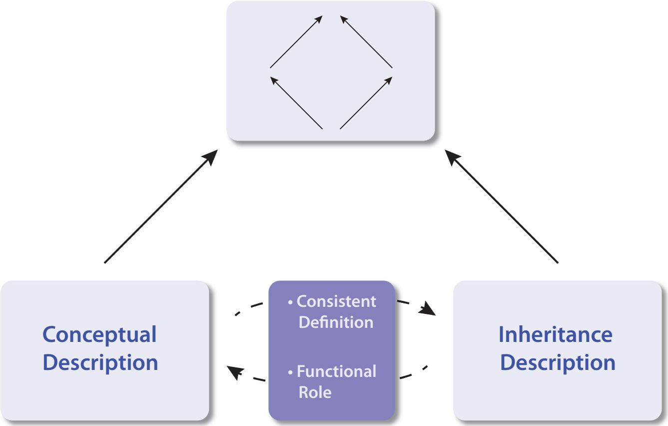
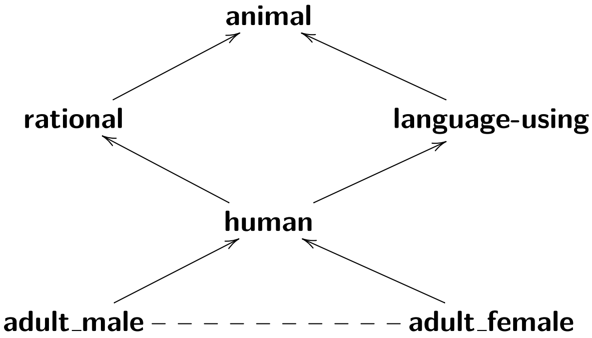


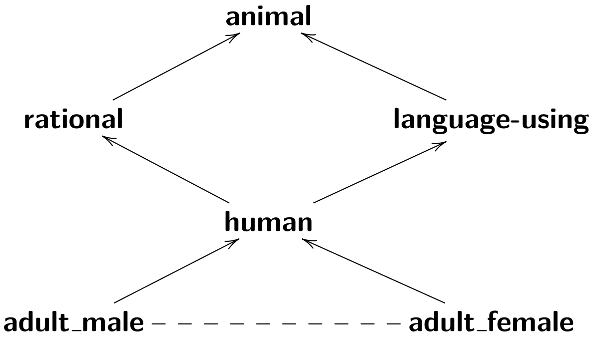
Inheritance Networks - Consistent Definition



$$D_{\textbf{human},1} = \{\textbf{human}, \textbf{rational}, \textbf{animal}\}$$

$$D_{\textbf{human},2} = \{\textbf{human}, \textbf{language-using}, \textbf{animal}\}$$

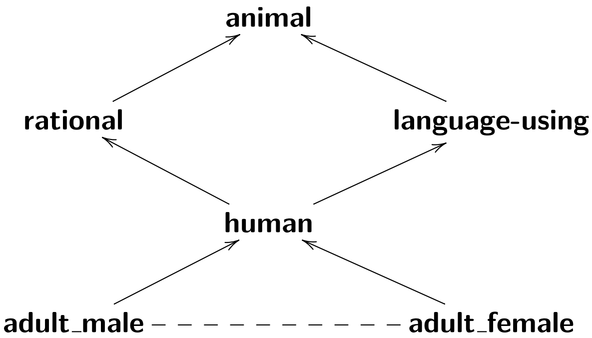
Inheritance Networks - Consistent Definition



$$D_{\textbf{man},1} = \{\textbf{adult\_male}, \textbf{human}, \textbf{rational}, \textbf{animal}\}$$

$$D_{\textbf{man},2} = \{\textbf{adult\_male}, \textbf{human}, \textbf{language-using}, \textbf{animal}\}$$

Inheritance Networks - Functional Role



$$i(\textbf{human}) = \{D_{\textbf{human},1}, D_{\textbf{human},2}, D_{\textbf{man},1}, D_{\textbf{man},2}, D_{\textbf{woman},1}, D_{\textbf{woman},2}\}.$$

$$i(\textbf{adult\_male}) = \{D_{\textbf{man},1}, D_{\textbf{man},2}\}.$$

The GDIT

Genus-differentia Inheritance Theorem

$i(\alpha) \subseteq i(\beta) \Leftrightarrow \alpha \sqsubseteq \beta$

- Inheritance relation: $\sqsubseteq$
- Semantic atoms/concepts: α, β
- Functional roles: $i(\alpha), i(\beta)$

The GDIT

Genus-differentia Inheritance Theorem

Inheritance networks, semantic content, and conceptual hierarchy are isomorphic to each other.

- The functional role is a partial set of identity conditions for semantic content and concepts.
- If α and β are the same concept, then they are located at the same position in the hierarchy.

Practical Consequences for Cognitive Science: A puzzle about concepts

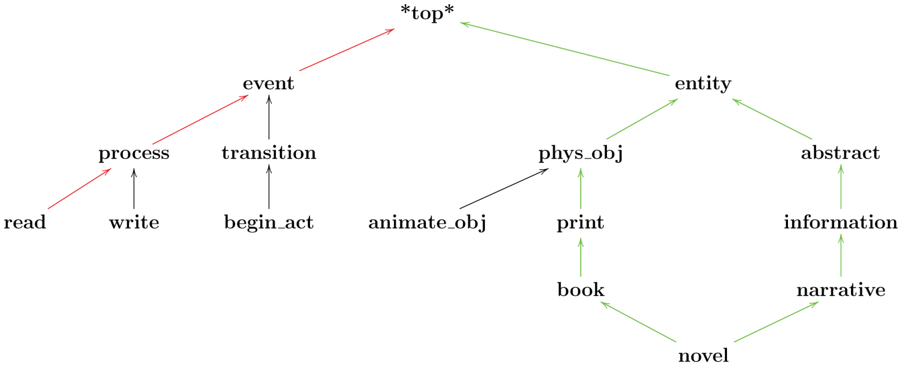
Test Sentence: "He began the novel."

Practical Consequences for Cognitive Science

$$\begin{bmatrix} \textbf{novel} & \\ \mathrm{ARG}_1 = & \textbf{x:book} \\ \mathrm{QUALIA} = & \begin{bmatrix} \mathrm{CONST} = \textbf{narrative(x)} \\ \mathrm{FORMAL} = \textbf{book(x)} \\ \mathrm{TELIC} = \textbf{read(x)} \\ \mathrm{AGENT} = \textbf{write(x)} \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} \textbf{begin} & \\ \mathcal{A} = & \begin{bmatrix} \mathrm{ARG}_1 = \textbf{animate\_obj} \\ \mathrm{ARG}_2 = \textbf{event}_1 \end{bmatrix} \\ \mathcal{E} = & \begin{bmatrix} \mathrm{E}_1 = \textbf{transition} \\ \ldots \end{bmatrix} \\ \mathcal{Q} = & \begin{bmatrix} \mathrm{FORMAL} = \textbf{P(event}_1\textbf{,x)} \\ \mathrm{AGENT} = \textbf{begin\_act(event}_1\textbf{,x)} \end{bmatrix} \end{bmatrix}$$

Practical Consequences for Cognitive Science



Areas for Future Research

1. Can we learn about inheritance by looking at linguistics and/or concepts?
2. Adding relations to the partial order
3. Can we arrive at a sufficient condition for identity between concepts?

Thank you!

Formal definitions

(Inheritance Network) An inheritance network $\mathcal{I}$ is a triple $\langle \mathcal{B}, \sqsubseteq, \# \rangle$ where:

- $\mathcal{B}$ is a finite set of basic elements
- $\sqsubseteq \subseteq \mathcal{B} \times \mathcal{B}$ is the basic *inheritance* relation
- $\# \subseteq \mathcal{B} \times \mathcal{B}$ is the basic *disjointness* relation

(Inheritance/Disjointness) The *inheritance* relation $\sqsubseteq^* \subseteq \mathcal{B} \times \mathcal{B}$ is the smallest such that:

- $P \sqsubseteq^* P$ (Reflexivity)
- if $P \sqsubseteq Q$ and $Q \sqsubseteq^* R$ then $P \sqsubseteq^* R$ (Transitivity)

The *disjointness* relation $\#^* \subseteq \mathcal{B} \times \mathcal{B}$ is the smallest such that:

- if $P \# Q$ or $Q \# P$ then $P \#^* Q$ (Symmetry)
- if $P \sqsubseteq^* Q$ and $Q \#^* R$ then $P \#^* R$ (Chaining)

Formal definitions

(Consistent Definition) A set $D \subseteq \mathcal{B}$ is a *consistent definition* for $\mathcal{B}$ iff:

1. For all $x, y \in D$, it is not the case that $x \# y$.
2. For all $x \in D$, $y_1, \ldots, y_n \in \mathcal{B}$, if $x \sqsubseteq y_i$, then $y_i \in D$ iff there is no $y_j \in D$ such that neither $y_i \sqsubseteq y_j$ nor $y_j \sqsubseteq y_i$.
3. There exists an α_b (called the *base atom* or *base* of D), such that for all $y_1, \ldots, y_n \in \mathcal{B}$, if $y_i \sqsubseteq \alpha_b$, then $y_i \notin D$, i.e. D has a minimal element.

(Funtional Role). The *functional role* of an an atom is a function $i : \mathcal{B} \to \mathcal{F}$) such that

$$i(x) = \{D \in \mathcal{F} | x \in D\},$$

where $\mathcal{F}$ is the set of all consistent definitions D on $\mathcal{B}$.

Formal definitions

Genus-differentia Inheritance Theorem

Given $\sqsubseteq$ and $\#$, such that $\sqsubseteq$ is inclusion and $\#$ is disjointness over $\mathcal{B}$, a consistently definable set of atoms, there exists a non-empty functional role i such that $a \sqsubseteq b \Leftrightarrow i(a) \subseteq i(b)$ and $a \# b \Leftrightarrow i(a) \cap i(b) = \emptyset$.

Proof of the GDIT - Lemma 3.1

Lemma 3.1

For $a, b \in \mathcal{B}$ such that it is not the case that $a\#b$, let $D_a = \{c | a \sqsubseteq c\}$ and $D_b = \{c | b \sqsubseteq c\}$. $D^* = D_a \cup D_b$ is a consistent definition on $\mathcal{B}$.

Proof. Because of the reflexivity of $\sqsubseteq$, it is obvious that D^* meets condition (2) above. Suppose (1) does not hold of D^*, i.e. there exists $x, y \in D^*$ such that $x\#y$. Then, by chaining we know that $a\#b$, since for all $x \in D^*$, either $a \sqsubseteq x$ or $b \sqsubseteq x$. But we have already said that it is not the case that $a\#b$, so (1) must hold of D^*. Therefore D^* is a consistent definition on $\mathcal{B}$. $\square$

Proof of the GDIT - Lemma 3.2

Lemma 3.2

Let $a, b \in \mathcal{B}$ be such that it is not the case that $a \sqsubseteq b$, and let $\mathcal{F} = \{D | D \text{ is a consistent definition}\}$. If a is consistently definable, then there exists some $D^a_{\neg b} \in \mathcal{F}$ such that $a \in D^a_{\neg b}$ and $b \notin D^a_{\neg b}$.

Proof. Assume a is consistently definable. Then there exists some $D^a \in \mathcal{F}$ such that $a \in D^a$. Either $a \# b$ or not. Suppose $a \# b$. Then $b \notin D^a$, by condition (1) for consistent definition. Suppose it is not the case that $a \# b$. Then there is no relation between a and b, which means that if D^a is a consistent definition, then $D^a_{\neg b} = D^a \setminus \{b\}$ is also a consistent definition. By the definition of $D^a_{\neg b}$, $b \notin D^a_{\neg b}$ and $a \in D^a_{\neg b}$. □

Proof of the GDIT

Let $\mathcal{F} = \{D | D \text{ is a consistent definition on } \mathcal{B}\}$, and let $i : \mathcal{B} \rightarrow \mathcal{P}(\mathcal{F})$ be a function such that $i(x) = \{D \in \mathcal{F} | x \in D\}$.

Assume $a \sqsubseteq b$. Suppose $D \in i(a)$. Then $a \in D$, by the definition of i, which means that $b \in D$ by (2). So $D \in i(b)$, by the definition of i. We therefore have $a \sqsubseteq b \Rightarrow i(a) \subseteq i(b)$.

Assume that it is not the case that $a \sqsubseteq b$. Then, by (3.2) there exists some $D^a_{\neg b} \in \mathcal{F}$ such that $a \in D^a_{\neg b}$ and $b \notin D^a_{\neg b}$. Then $D^a_{\neg b} \in i(a)$, but $D^a_{\neg b} \notin i(b)$, which means that $i(a) \not\subseteq i(b)$. So we have: If it is not the case that $a \sqsubseteq b$, then it is not the case that $i(a) \subseteq i(b)$, which contraposits to $i(a) \subseteq i(b) \Rightarrow a \sqsubseteq b$.

We have therefore shown that $a \sqsubseteq b \Leftrightarrow i(a) \subseteq i(b)$, and we now turn to proving that $a \# b \Leftrightarrow i(a) \cap i(b) = \emptyset$.

Proof of the GDIT

Assume $a\#b$. Now suppose $i(a) \cap i(b) \neq \emptyset$. Then there exists some $D \in \mathcal{F}$ such that $a \in D$ and $b \in D$. But, by (1), this cannot be the case. So, $a\#b \Rightarrow i(a) \cap i(b) = \emptyset$.

Assume $i(a) \cap i(b) = \emptyset$. Then there exists no $D \in \mathcal{F}$ such that $a \in D$ and $b \in D$. Now suppose it is not the case that $a\#b$. Let D_a, D_b, and D^* be defined as in (3.1). Then D^* is a consistent definition on $\mathcal{B}$. But, by the reflexivity of $\sqsubseteq$, $a \in D_a$ and $b \in D_b$, which means that $a, b \in D^*$. So there is some $D \in \mathcal{F}$ such that $a \in D$ and $b \in D$, a contradiction. Therefore, $i(a) \cap i(b) = \emptyset \Rightarrow a\#b$.

We have therefore shown that $i(a) \cap i(b) = \emptyset \Leftrightarrow a\#b$.