Complex AG HW

David Zhu

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Homework 1

Problem 1

Show $\mathbb{P}^1 \cong S^2$ and that $\mathbb{P}61$ is simply connected.

Problem 2

Compute the algebraic dimension of $\mathbb{P}^2$ and of $\mathbb{C}/(\mathbb{Z} + i\mathbb{Z})$

Problem 3

Prove that holomorphic maps $\mathbb{P}^1 \to$ complex tori are constant.

Problem 4

For the Hopf manifold in $n = 1$, identify it as an elliptic curve and determine its lattice.

Problem 5

Explain how to recover the bundle E corresponding to a locally free sheaf F. Define the sheaf $O(k)$ on projective space and check that it corresponds to the correct bundle.

Problem 6

Show that the Grassmannian $\mathrm{Gr}(2, 4)$ is a 4-dimensional quadric. (Use tthe Plücker embedding

$$\mathrm{Gr}(2, 4) \to \mathbb{P}^5$$

Problem 7

Explain why the flag varieties are projective varieties.