Reading seminar: From surface topology to Fermat's Last Theorem

Fall 2025

Information

Organizers: David Zhu (zzhu19@sas.upenn.edu), Fangji Liu (fjliu@sas.upenn.edu).
Meeting time: Friday 3:30pm – 5:00pm
Classroom: Fisher Bennett Room 138
Zoom link: https://upenn.zoom.us/j/6497776520

Seminar description

This is a seminar in order to understand McMullen's paper From dynamics on surfaces to rational points on curves, which combines techniques from geometric topology, dynamics and complex algebraic geometry to reinterpret celebrated results in number theory. We will go through basics in surface topology, including mapping class group theory, monodromy representations, Nielsen-Thurston classification and so on. At the end we want to compare McMullen's result back to algebra, which is Faltings' theorem on finiteness of rational points on genus > 1 curves. A corollary of this theorem is a weaker form of Fermat's Last Theorem.

References

- Farb–Margalit's book: A primer on mapping class groups. We will mainly use Part 1 of this book to cover background knowledge on surface topology.
- McMullen's paper: From dynamics on surfaces to rational points on curves.
- McMullen's lecture notes: Riemann surfaces, dynamics and geometry. (Chapter 10 for example)

Schedule

Lecture 0: Organization meeting.

Lecture 1: Basic surface topology: surfaces, simple closed curves, the change of coordinates principle. (Chapter 1 of Farb–Margalit)

Lecture 2: Mapping class groups: definitions, basic properties, the Alexander method. (Chapter 2 of Farb–Margalit)

Lecture 3: Dehn twists: definitions, examples, properties, operations. (Chapter 3 of Farb–Margalit)

Lecture 4: Generators and relations of mapping class groups: complex of curves, generators, the lantern relation. (Chapter 4 & 5.1, 5.2 of Farb–Margalit but focus on Chapter 4)

Lecture 5: The symplectic representation: intersection number, symplectic automorphisms, the Torelli group. (6.1-6.5 of Farb–Margalit but focus on 6.1–6.3)

Lecture 6: The $84(g-1)$ **theorem**: finiteness of automorphism groups, torsion in mapping class groups. (Chapter 7 of Farb–Margalit, 7.5 optional)

Lecture 7: Teichmuller space and the moduli space of complex curves: A brief introduction to Teichmuller theory and moduli space theory on Riemannian surfaces. (Section 2 of McMullen's paper & Part 2 of Farb–Margalit).

Lecture 8: Nielsen–Thurston classification: The classification theorem, 3 types of mapping classes. (Section 2 of McMullen's paper & Chapter 13 of Farb–Margalit)

Lecture 9: The monodromy determination theorem and McMullen's theorem: the monodromy representation, determination of bundle, finiteness, Parshin's trick to deduce McMullen's theorem. (Section 3 & 4 of McMullen's paper)

Lecture 10: Faltings' theorem: sketch of proof, comparing to McMullen's theorem. (Section 5 & 6 of McMullen's paper)