Davie Truong
CMPS 130
dtruong8@ucsc.edu

# HomeWork 1

0.1 Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

a) $\{1, 3, 5, 7, \dots\}$
the set of all odd natural numbers

b) $\{\dots, -4, -2, 0, 2, 4, \dots\}$
the set of all even integers

c) $\{n \mid n = 2m \text{ for some } m \text{ in } \mathcal{N}\}$
the set of natural numbers that are multiple of 2

d) $\{n \mid n = 2m \text{ for some } m \text{ in } \mathcal{N}, \text{ and } n = 3k \text{ for some } k \text{ in } \mathcal{N}\}$
the set of natural numbers that are multiples of 2 and 3

e) $\{w \mid w$ is a string of 0s and 1s and $w$ equals the reverse of $w\}$
the set of 0s and 1s, in which it is also a palendrome

f) $\{n \mid n \text{ is an integer and } n = n+1\}$
the set is empty because $n \neq n+1$; unless this is coders format then the set contains integers that have been incremented by 1

0.2 Write formal descriptions of the following sets

a) The set containing the numbers 1, 10, and 100
$\{1, 10, 100\}$

b) The set containing all integers that are greater than 5
$\{5, 6, 7, 8, 9, \dots\}$

c) The set containing all natural numbers that are less than 5
$\{\dots -1, 0, 1, 2, 3, 4, 5\}$

d) The set containing the string aba
$\{aba\}$

e) The set containing the empty string
$\{\Sigma\}$

f) The set containing nothing at all

$\{\emptyset\}$

0.3 Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$

a) Is A a subset of B?

no

b) Is B a subset of A?

yes

c) What is $A \cup B$

$\}x, x, y, y, z\}$

d) What is $A \cap B$

$\{x, y\}$

e) What is $A \times B$

$\{(x,x),(x,y),(y,x),(y,y),(z,x),(z,y)\}$

f) What is the power set of B

$\{\emptyset, \{x\}, \{y\}, \{x,y\}\}$

0.4 If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.

the number of elements woud be $a \cdot b$, by definition of cross product, each element of A must pair with each element of B.

0.5 If C is a set with c elements, how many elements are in the power set of C? Explain your answer.

the number of elements would be $2^c$, each additional element to the set expands the number of combinational sets that can be made

# HomeWork 1 - continue

0.6 Let X be the set {1,2,3,4,5} and Y be the set {6,7,8,9,10}. The unary function f: X → Y and the binary function g: X×Y → Y are described in the following tables.

| n | f(n) |
|---|---|
| 1 | 6 |
| 2 | 7 |
| 3 | 6 |
| 4 | 7 |
| 5 | 6 |

| g | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| 1 | 10 | 10 | 10 | 10 | 10 |
| 2 | 7 | 8 | 9 | 10 | 6 |
| 3 | 7 | 7 | 8 | 8 | 9 |
| 4 | 9 | 8 | 7 | 6 | 10 |
| 5 | 6 | 6 | 6 | 6 | 6 |

a) What is the value f(2)?
7

b) What are the range and domain of f?
domain = (1,2,3,4,5)
range = (6,7)

c) What is the value of g(2,10)?
6

d) What are the range and domain of g?
domain: (1,2,3,4,5)
range: (6,7,8,9,10)

e) What is the value of g(4,f(4))?
g(4,7) = 8

0.7 For each part, give a relation that satisfies the condition.

a) Reflexive and symmetric but not transitive
{A,B,C,D)
(A,A),(B,B),(C,C),(D,D)
(A,B),(B,A),(C,D),(D,C)

b) Reflexive and transitive but not symetric
{A,B,C,D)
(A,A),(B,B),(C,C),(D,D)
(A,B),(B,C),(C,D)

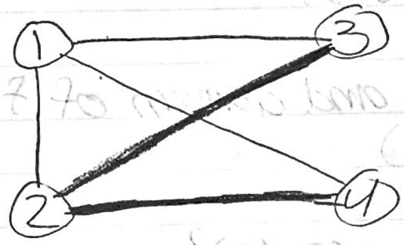
c) Symetric and Transitive but not reflexive

$\{A, B, C, D\}$

$(A,B),(B,A),(C,D),(D,C)$

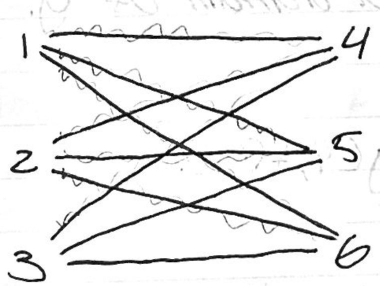
$(B,C)$

0.8 Consider the undirected graph $G=(V,E)$ where $V$, the set of nodes, is $\{1,2,3,4\}$ and $E$, the set of edges, is $\{\{1,2\},\{2,3\},\{1,3\},\{2,4\},\{1,4\}\}$. Draw the graph $G$. What are the degrees of each node? Indicate a path from node 3 to node 4 on your drawing of $G$.




degrees: 1 - 3
2 - 3
3 - 2
4 - 2

0.9 Write a formal description of the following graph




$(\{1,2,3,4,5,6\},\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),$
$(3,5),(3,6)\}$

# HomeWork 1 - continue

0.10 Find the error in the following proof that 2=1.
Consider the equation $a=b$. Multiply both sides by $a$ to obtain $a^2=ab$. Subtract $b^2$ from both sides to get $a^2-b^2=ab-b^2$. Now factor each side, $(a+b)(a-b) = b(a-b)$, and divide each side by $(a-b)$ to get $a+b=b$. Finally, let $a$ and $b$ equal 1, which shows that $2=1$.

| | |
|---|---|
| $a=b$ | $a=b$, let $a=1, b=1$ i.e. |
| $a^2=ab$ | $1^2=1$ |
| $a^2-b^2=ab-b^2$ | $1^2-1^2=1\cdot 1-1^2$ |
| error $(a+b)(a-b)=b(a-b)$ | $(1+1)(1-1)=1(1-1)$ |
| $(a+b) = b$ | if the values are the same, when we subtract $(a-b)$, the result is 0, |

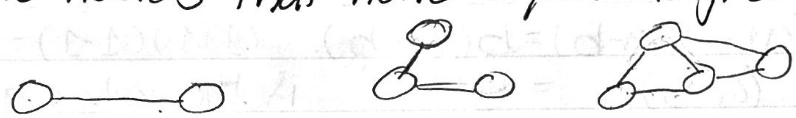
thus a division is impossible making the statement

$$\frac{(a+b)(a-b)}{(a-b)} = \frac{b(a-b)}{(a-b)} \rightarrow (a+b)=b \text{ false.}$$

0.12 Find the error in the following proof that all horses are the same color.
Claim: In any set of $h$ horses, all horses are the same color
Proof: By induction on $h$.
Basis: For $h=1$. In any set containing just one horse, all horses clearly are the same color
Induction Step: For $k \geq 1$, assume that the claim is true for $h=k$ and prove that it is true for $h=k+1$. Take any set $H$ of $k+1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set $H_1$ with just $k$ horses. By the induction hypothesis, all the horses in $H_1$ are the same color. Now replaced the removed horses and remove a different one to obtain the set $H_2$. By the same argument, all the horses in $H_2$ are the same color. There for all the horses in $H$ must be the

same color, and the proof is complete.
When a horse is removed from $H_1$ and replaced to form the new set $H_2$, it is assumed that the new horse has the same color. This was not proved, but rather the prevous set $H_1$ was proved. Therefore it can't be said "By the same argument, all horses in $H_2$ are the same color", because the new replaced horse was not identified.

0.13 Show that every graph with two or more nodes contains two nodes that have equal degrees.



Proof: By definition, a graph containing two nodes must be connected with each other. Thus this connection by each node results in the degree of 1.
Adding a node can result in two scenerios:
1) The graph becomes a tree in which the children have the same numeber of degrees
2) The graph contains a cycle in which two nodes contains the same number of degrees to complete the cycle

Prove from the definitions of set union, intersection, complement and equality that

$$\overline{(A \cap B)} = (\overline{A} \cup \overline{B})$$

$A\{1,2,3\}$ $\overline{\{2,3\}} = 1,3$
$B\{2,3\}$ $\overline{A} = 3$, $\overline{B} = 1$

Proof: Suppose x is an element of $\overline{(A \cap B)}$. Then x is not in $A \cap B$ from the definition of a complement set. Therefore, x is in not A and x is in not B, from the definition of the union of two sets show x to be in $(\overline{A} \cup \overline{B})$

# Homework 1 - continue

Show the set of odd numbers is countable.

$f: X \to Y$

$f(n) = 2n-1$

since there is a one to one mapping of odd numbers, it is countable

Proof By induction on $n$.

For all $n \in N$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Basis: $\sum_{i=1}^{1} i^2 = 1 = \frac{1(2)(3)}{6} = 1$

Induction: let $n \geq 1$ and assume that the sumation is true.

$$\sum_{i=1}^{n+1} i^2 = \sum_{i=1}^{n} i^2 + (n+1)^2$$

$$= \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$(n+1)(n+1)$

$$= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6}$$

$$= \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6}$$