ECONOMETRICS 2

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Exercises: session 3

AR processes

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Exercise # 1

Consider the AR process (x_t) defined by: $x_t = 0.2 + 0.8x_{t-1} + u_t$ where (u_t) is a WN with variance σ_u^2.

1. Is (u_t) the innovation process of (x_t) and why ?
2. Compute Ex_t.
3. Compute the variance of x_t and its autocovariance of order 1.
4. Compute the autocovariances of order h, and give the autocorrelation function of (x_t).
5. Give the MA(∞) representation of (x_t).

Exercise # 2

Suppose that (v_t) is a WN with variance σ_v^2 and consider the AR(2) process (y_t) defined by:

$$y_t = 0.5 + \frac{5}{6}y_{t-1} - \frac{1}{6}y_{t-2} + v_t$$

also denoted as

$$\phi(L)y_t = 0.5 + v_t$$

with $\phi(L) = 1 - \frac{5}{6}L + \frac{1}{6}L^2$.

1. Is (v_t) the innovation process of (y_t) and why ?
2. Compute Ey_t.
3. Compute the variance of y_t and its autocovariances of order 1 and 2.
4. i) Give the general equation which relates $\gamma_y(h)$ to the $\gamma_y(h-k)$'s, when $h > 0, k > 0$ and restate how this equation is obtained.
   ii) Show that, if $\gamma_y(h)$ is defined as $\gamma_y(h) = \alpha\frac{1}{2^h} + \beta\frac{1}{3^h}$, it is a solution of this equation: we'll admit that this is indeed the general solution of such an equation.
   What can you say of $\frac{1}{2}$ and $\frac{1}{3}$ with respect to the roots of $\phi(z)$?
   iii) Compute α and β (*Hint*: use question 3).
   iv) What can you say about $\gamma_y(h)$ when $h \to \infty$?

   Remark: all the results which have been obtained in this question can be extended to the AR(p) case.
5. How would you compute the MA(∞) representation of (y_t) ? Give the 3 first terms of this representation.

Exercise # 3

Consider the following AR process (y_t) defined by:

$$(1 - \phi L^4) y_t = u_t$$

where $0 < \phi < 1$ and u_t is a WN with variance σ_u^2.

1. Is (u_t) the innovation process of (y_t) and why?
2. Compute the variance of y_t.
3. Compute the autocovariances of order 1, 4, 5, 8.
4. Compute the partial autocorrelations of order 1, 4, 5, 8.
5. Compute the autocorrelations of order 1, 4, 5, 8.
6. Give the general formula for autocorrelation of order $4m$, where m is some integer.