ECONOMETRICS 2

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Exercises: session 5

ARMA processes.

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Exercise # 1

Consider the ARMA(1,1) process (x_t) defined by

$$(1 - 0.8L)x_t = 0.4 + (1 - 0.4L)u_t$$

with $u_t \sim WN(0, \sigma_u^2)$.

1. Compute Ex_t.
2. Is (u_t) the innovation process of (x_t) and why ?
3. Compute the AR(∞) representation of (x_t).
4. Compute the MA(∞) representation of (x_t).
5. Compute $Cov(u_t, x_t)$ and $Cov(x_t, u_{t-1})$.
   HINT: This can be done using the MA(∞) representation of (x_t), but this can be also directly obtained from the ARMA representation of (x_t): do it this way.
6. Compute $\gamma_x(0)$ and $\gamma_x(1)$.
7. i) Show that, for $h \geq 2$, $\gamma_x(h) = 0.8\gamma_x(h-1)$.
   ii) Compute $\gamma_x(h)$ for any h.
   iii) What happens to $\gamma_x(h)$ when $h \longrightarrow +\infty$?

Exercise # 2

(*Extracted from 2017's exam*) Let (x_t) the MA(2) process defined by the following equation:

$$x_t = m + \left(1 - \frac{1}{2}L\right)\left(1 - \frac{1}{3}L\right)\varepsilon_t$$

where ε_t is a WN(0,σ^2).

1. Compute Ex_t.
2. Is (ε_t) the innovation process of (x_t) and why ?
3. How would you compute the $AR(\infty)$ representation of (x_t) ?
   Compute this representation entirely if you can (*bonus*) or give its general form and just compute the 3 first terms.
4. i) Give the best linear forecast $x^*_{t+1|t}$ of x_{t+1} at time t as a function of the x_{t-k}'s and give the forecast error variance.
   ii) Give the best linear forecast $x^*_{t+2|t}$ of x_{t+2} at time t as a function of the x_{t-k}'s. Compute the forecast error variance.
   iii) In practise, the x_t's are observed only for $t \geq 1$ and the corresponding truncation of $x^*_{t+1|t}$ is taken to get an approximate forecast of x_{t+1} at time t. If this approximation is denoted as $\hat{x}_{t+1|t}$ what is the variance of the associated forecast error $x_{t+1} - \hat{x}_{t+1|t}$ (give its formula, but don't compute it) ? What can you say of this variance when $t \to \infty$ and why ?
   iv) Give the best linear forecast $x^*_{t+h|t}$ of x_{t+h} at time t, for $h > 2$. Compute the forecast error variance.