Contents

General Notes:

Terminology:

- Stochastic Process = Stochastic Model

Basics:

GDP

Real GDP increasing since WWII due to

- Capital
- Productivity
- Population
  - Both more labour and more consumers

Recession = Drop in the level of production (Real GDP)

The process of Recessions and expansions after recessions is known as the Business Cycle

We're going to focus on GDP Growth Rate.

Unemployment Rate

Unlike Real GDP, there is no trend.

- This is not a production function but a ratio

Great proxy for the US business cycle

- Very flexible labour market in the US

Interest Rate

Short term vs Long Term.

- The central bank decides the short term rate.
- The market decides the long term rate.

The Sovereign Rate

The rate at which Governments are able to borrow money.

- Usually look at the 10 year rate
  - The average period of maturity for government issued bonds

Definitions

Stochastic Process:

$(X_t)_t \in Z$is a sequence of random variables, taking real values, indexed by $t \in Z$

- Our (very ambitious) objective:

  Find the best process (X_t) that generated x_t
  - Start with results, end with model

Simulation: When you create a model and generate outcomes. (Start with model, end with results)

Process

The main characteristic of a trajectory $(x_1, ..., x_T)$ steming from a stochastic process is the non-independence of the variables.

- The first i of the usual i.i.d. hypothesis is no more valid!
- Standard tool for measuring dependence: **linear correlation coefficient**

Autocorrelation

Partial Autocorrelation

But what if we want to find the autocorrelation between two particular points in the trend and not the trend as a whole?

- Partial Autocorrelation

Identifying a candidate process

ACF provides a measure of the persistence of the process or its ***memory***

Starting from this information, we will search for a type of process able to fit this persistence.

3 types:

- No Memory
- Short Memory
- Long Memory

No Memory - White Noise Process

No autocorrelation (no dependent).

The value of tomorrow is completely uncorrelated with value of today.

- **Careful:** This does not mean they are independent

EXAMPLE: Exchange Rates

Weak white noise:

$$E(XY) = 0 \text{ given } \mu(X, Y) = 0$$

but maybe: There is a correlation between their variances

$$E(X^2Y^2) \neq 0$$

Strong White Noise:

Not only **non-correlated** but also **fully independent** from one another.

Short Memory

A stochastic process is said to be short-memory is its ACF is such that:

$$\rho(k) \leq C_{r^k}, k \to \infty$$

, where $C > 0, 0 < r < 1$ and $k = 1, 2, ...$

Examples:

AR = Autoregressive.

Long Memory

When the ACF is non null for large k, a.k.a strongly persistent

———— Not covered in this course ————

Stationarity

While time series are non-independent, are they identically distributed?

Strong Stationarity

Too strict and complicated to work out in real life

Weak Stationarity

- Mean is constant over time
- The covariance is constant over time

These properties allow us to estimate the mean by using the empirical mean.

$$\hat{\mu} = \bar{X_T}$$

Therefort he natural predictor of a stationary process is the mean.

Wold Theorem

Defining a *good* predictor

- Unbias
- Minimum variance
  - Otherwise you get large standard errors and uncertainty

Forcasting Error Process

$$e_{t+h} = X_{t+h} - \hat{X}_t(h)$$

Criteria to measure the forcasting accuracy

$$Mean\ Error = E(e_{T+h})$$

- Might not be bias but can hide inaccuracy

$$Mean\ Absolute\ Error = E(|e_{T+h}|)$$

- Cannot differentiate at Zero

$$Mean\ Squared\ Error = E(e_{T+h}^2)$$

- Allows us to differentiate!

$$Root\ Mean\ Squared\ Error = \sqrt{E(e_{T+h}^2)}$$

- Allows us to differentiate!
- Values are similar to those observed

Linear Process

$$X_t = \sum_{i=0}^{\infty} a_i \varepsilon_i$$

where:

- coefficients a_i is absolutely summable (finite)
- $(\varepsilon_t)_t$ is strict white noise

Best Predictor

The predictor $\hat{X}_T(h)$ that minimises the MSE is the least squares predictor.

$$\hat{X}_T(h) = E(X_{T+h}|I_T)$$

Distribution of the Predictor

For any linear process, we get $E(e_{T+h}) = 0$, and;

$$E(e_{T+h}^2) = \sigma_\varepsilon^2 \sum_{i=0}^{h-1} a_i^2$$

, where $a_0 = 1$.

Normal Distribution:

Example

ϕ represents the strength of the dependence between X_t and X_{t-1}

$$X_{t+1} = \phi X_t + \varepsilon t + 1$$

$$E(X_{t+1}|I_t) = E(\phi X_t + \varepsilon t + 1|I_t)$$

$$E(X_{t+1}|I_t) = E(\phi X_t|I_t) + (\varepsilon t + 1|I_t)$$

$$E(X_{t+1}|I_t) = \phi X_t + 0$$

$$\hat{X}_{t+1} = \phi X_t$$

Example 2:

$$X_t(2) = \phi X_{t+1} + \varepsilon t + 2$$

$$E(X_{t+2}|I_t) = E(\phi X_{t+1} + \varepsilon t + 2|I_t)$$

$$E(X_{t+1}|I_t) = E(\phi X_{t+1}|I_t) + (\varepsilon t + 2|I_t)$$

$$E(X_{t+1}|I_t) = \phi X_{t+1} + 0$$

$$\hat{X}_{t+1} = \phi(\phi X_t)$$

$$\hat{X}_{t+1} = \phi^2 X_t$$

Impulse Response Function (IRF)

Objective: Assess the impact of a shock at a given date on the process dynamics.

General Definition:

$$GIRF = E(X_{t+h}|I_t, \varepsilon_t = \delta, \varepsilon_s = 0, s > t) - E(X_{t+h}|I_t, \varepsilon_s = 0, s \geq t)$$

GIRF = Whats going on with impulse - What would be going on without impulse

ARMA Models