WEBSITE PROBLEMS: **DYNAMICS**

Level 6

Exercise 1: A particle with mass m travelling at speed u collides with a stationary particle of the same mass. What is the angle between the velocities of the two masses after this collision?

a) 26.6°

b) 45°

c) 53.2°

d) 90°

e) 180°

[*Another old question from multiple places*]

Exercise 2: A police spaceship, of mass $m = 10,000$ kg, travelling at a speed $u = 2$ km s^{-1} needs to arrest another ship travelling ahead of it at 2.5 km s^{-1}. The police spaceship is capable of splitting itself into two equal parts and supplying them with kinetic energy from a single reserve of 2 GJ. Is the reserve sufficient? Bonus: If the reserve is sufficient; how long would it take for the police spaceship to catch the other craft, given that it was initially 3000 km behind? [*Cambridge University Tripos 2011*]

Exercise 3: A car of mass 1500 kg was travelling down a motorway when a larger car of mass 2500 kg moved in from another lane at an angle of $\alpha = 15$ ° to the direction of the road and collided with it. After the collision these two cars stuck to each other, moving at an angle $\theta = 8$ ° to the direction of the road. They came to a stop 46 m from the point of collision. Assuming that the the coefficient of friction between the road and the combined two cars after the collision is 0.7 and no air resistance, find the speeds of the cars just before the collision. [*Created by MC for the Rutherford Schools Physics Project*]

Exercise 4: A table top is in the form of a smooth horizontal rectangle ABCD with AB = a, BC = b where $a > b$. A small circular disc P of mass m is at rest at the centre of the table. An identical disk Q is projected with speed u along the table top in a direction parallel to AB and collides with P. After the collision P and Q move directly towards B and C respectively.

a) Draw diagrams illustrating the situation before and after the collision both the frame of the table and in the zero momentum frame (ZMF).

b) The loss of kinetic energy in the collision is:

$$\frac{(a^2 - b^2)mu^2}{4a^2}$$

Show that the discs reach B and C simultaneously, and find the time from the collision until Q reaches C.

Hint: The zero momentum frame (ZMF) is a set of axes moving such that the total momentum of the objects being considered is zero. To transform into it, subtract the speed of the centre of mass of the bodies. It is defined in terms of vectors, but here we only need the 1D form: $v_{\text{ZMF}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$ in the direction of v_1 and v_2. The ZMF is sometimes called the Centre of Mass frame, since in it the Centre of Mass is fixed.

[*Adapted with permission from UCLES, A Level Further Mathematics, June 1989, Syllabus C, Special Paper, Question 8.*]

Exercise 5:

a) A collision occurs between two (non-relativistic) bodies of equal mass m and velocity vectors $\underline{v}_1$ and $\underline{v}_2$; how much kinetic energy is available for conversion to other forms of energy?

b) A particle of mass m travelling with speed V along the $+x$ direction collides elastically with a stationary particle of mass $2m$.The particle of mass m is deflected through an angle of 30°. What are the final velocity vectors of the two particles in the Laboratory frame?

Your answer should be illustrated by appropriate diagrams in both the laboratory and zero momentum frames. [*Cambridge University Tripos 1996*]

Exercise 6: A cyclist travels along a straight road at speed v on two separate occasions; on Day 1 there is no wind, whereas on Day 2 there is a steady wind blowing at a speed w in a direction perpendicular to the road (i.e. a cross-wind). The force on the cyclist due to air resistance is proportional to the square of the speed of the air relative to the cyclist; as a consequence, to maintain the same constant speed v on Day 2 as on Day 1, the cyclist has to produce a greater power output. In what follows, assume that $v = 20$ km h^{-1} and $w = 40$ km h^{-1}

a) Draw diagrams showing the velocities of the cyclist and air in both the road frame and the cyclist's frame of reference (i) on Day 1 when there is no wind and (ii) on Day 2 when there is the cross-wind. Hence find the speed of the air and the direction in which it is moving relative to the cyclist when the steady cross-wind is blowing.

b) Draw additional diagrams showing the velocity of the cyclist and the force due to air resistance on Day 1 and on Day 2. Hence deduce the factor by which the cyclist must increase power to maintain a constant speed in the cross-wind.

[*From Julia Riley's 1A Dynamics Question Sheet*]

Exercise 7: A particle of mass m_1 and velocity v makes an elastic, head-on collision with a stationary particle of mass m_2. Find the velocity of the zero momentum frame relative to the laboratory frame. By considering the collision in the zero momentum frame, show that in the laboratory frame the fraction of the initial kinetic energy transferred to m_2 is given by

$$\frac{4m_1m_2}{(m_1+m_2)^2}. \tag{1}$$

Three balls of masses m_1, m_2 and m_3 are suspended in a horizontal line by light wires and are almost touching. The mass m_1 is given a horizontal velocity v so that it collides head-on with the mass m_2. Find an expression for the final kinetic energy of m_3, and sketch it as a function of m_2. What value of m_2 results in the maximum energy transfer to the mass m_3? [*Cambridge University Tripos 2004*]

Exercise 8: A particle of mass m_1 collides with a stationary particle of mass m_2 ($m_1 > m_2$). What is the maximum angle of deflection of mass m_1? [*Seen in many places, including 1A question sheet*]

Exercise 9: A basketball of mass m_1 and radius r_1 and a tennis ball of mass m_2 and radius r_2 are held with the bottom of the tennis ball touching the top of the basketball, and the bottom of the basketball is a height h above the ground. The balls are then dropped together. What is the maximum height that the centre of mass the tennis ball reaches after the balls bounce off the floor?

Assume that the mass of the tennis ball is much smaller than the mass of the basketball, and that all collisions are elastic. [*Found in Harvard questions of the week, but also seen in many different places*]

Exercise 10: A uniform heavy inextensible chain AD of length $\left(4+\frac{\pi}{2}\right)l$ and mass ρ per unit length is at rest in the position shown in Figure 1 , in which the points A, B, C and D lie in a vertical plane. The section AB of length $2l$ rests on a smooth horizontal table. The next section of the chain is within a fixed smooth tube BC in the form of a quadrant of a circle, radius l, whose centre O is vertically below B. The section CD of the chain hangs vertically.

The chain is released from rest at $t = 0$ s. At time t the length of the vertical section CD is $2l + x$, where $x \leq 2l$, and the speed of the chain is v.

a) Find v^2 in terms of l, x and g.

b) Show that the momentum of the section of chain within the tube has horizontal component $\rho l v$.

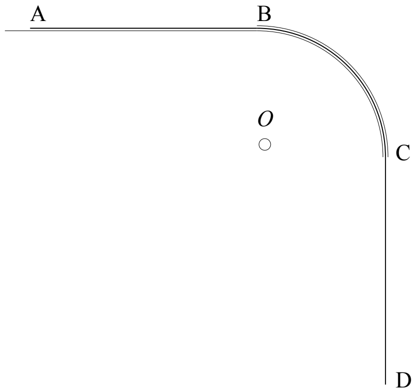

Figure 1

c) Show further that the horizontal component of the force on the chain is:

$$\frac{2g\rho}{l(8+\pi)}\left(9l^2 - 6lx - 2x^2\right)$$

Hint:

a) It is easiest to consider equating the kinetic energy gained after moving a distance x to the gravitational potential energy (GPE) lost in the same distance.

b) Consider a small element, and integrate to find the total horizontal momentum of the chain.

c) The rate of change of horizontal momentum in the tube is equal to the sum of the two horizontal forces acting on the chain plus the momentum flux into the tube.

[*Used with permission from UCLES, A Level Further Mathematics, Syllabus A, June 1987, Special Paper, Question 10.*]