Asset Allocation

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Resume

Quant Portfolio Manager – ASA Investments (2021-2023)

- Managed a book of strategies (daily rebalanced) that trades brazilian futures (e.g. DI1, WDO, CMDTS) with OOS Sharpe Ratio above 1.
- Developed the OMS to send orders to Bloomberg's EMSX API.

Quant Developer – Itaú BBA – Treasury (2016-2021)

- Developed and improved HFT strategies run by the trading desk (e.g. on-off arbitrage, market making of DOL/WDO options, BCB intraday interventions).
- Implemented models to evaluate and assess the risk of different products (e.g. exotic options, swaps, interest rate).

Agenda

1) Portfolio Theory
2) Methodology
3) Model Selection
4) Client's Preferences
5) Factors
6) Tweaks to the Model
7) Backtesting
8) One Last Thing
9) Next Steps

Equal Weight Allocation & Inverse-Variance Porfolio

Splits the weights (w_i) **equally** between the assets (i) to build the portfolio.

$$\boxed{w_i = {}^{1}/_{N}}$$

The allocator assumes that it is not possible to estimate anything with confidance. It is the naive approach.

Splits the weights (w_i) in **inverse proportion** of the the variance (σ_i^2) of the returns.

$$iv_i = {}^{1}/_{\sigma_i^2}$$

$$\boxed{w_i = \frac{iv_i}{\sum_{i=1}^{N} iv_i}}$$

The allocator assumes that it is possible to estimate the expected variance of the assets.

Research has documented that volatility is persistent (clustering) [1]. Periods of low volatility tend to follow periods of low volatility, and vice-versa.

[1] Harvey, C., Rattray, S., Hemert, O. (2021): Strategic Risk Management, 106-107

Minimum Variance Portfolio

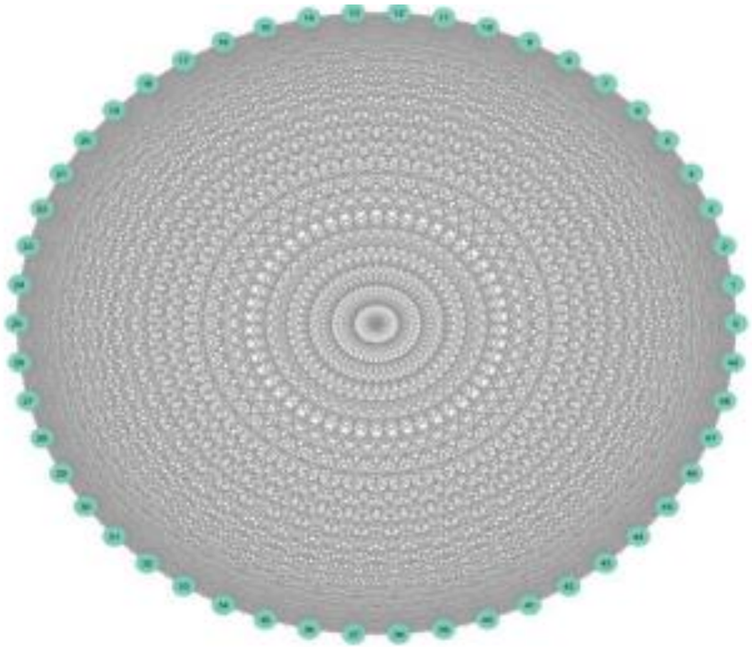
Splits the weights such as to achive the minimum variance portfolio.

$\sigma_p^2 = w^T C w$

$$\boxed{\begin{array}{c}\min_{w} w^T C w \\ \text{st.}\, w^T \mathbb{1}n = 1 \\ w \geq= 0\end{array}}$$

$\mathbb{1}n$: *Array of* $1s$ $(mx1)$ | *w: Array of weights (mx1)*
C: *Covariance Matrix* (mxm)

The allocator assumes that it is possible to estimate the **covariance** between the assets. The solution is invariant to the expected returns.



The optimization becomes unstable for ill-conditioned C. That`s because the solution depends on the inversion of the C [(2)], and as we add correlated assets to the portfolio, the instability grows.

One reason for the instability is that the space is modeled as a **complete graph**, where each node is a potential candidate to substitute another.

[(2)] López de Prado, M. (2018): AFML, 222-223 | Proof at the end of the presentation

Hierarchical Risk Parity

The HRP method uses the information contained in the Covariance Matrix without requiring its inversion [3].

1) Tree clustering

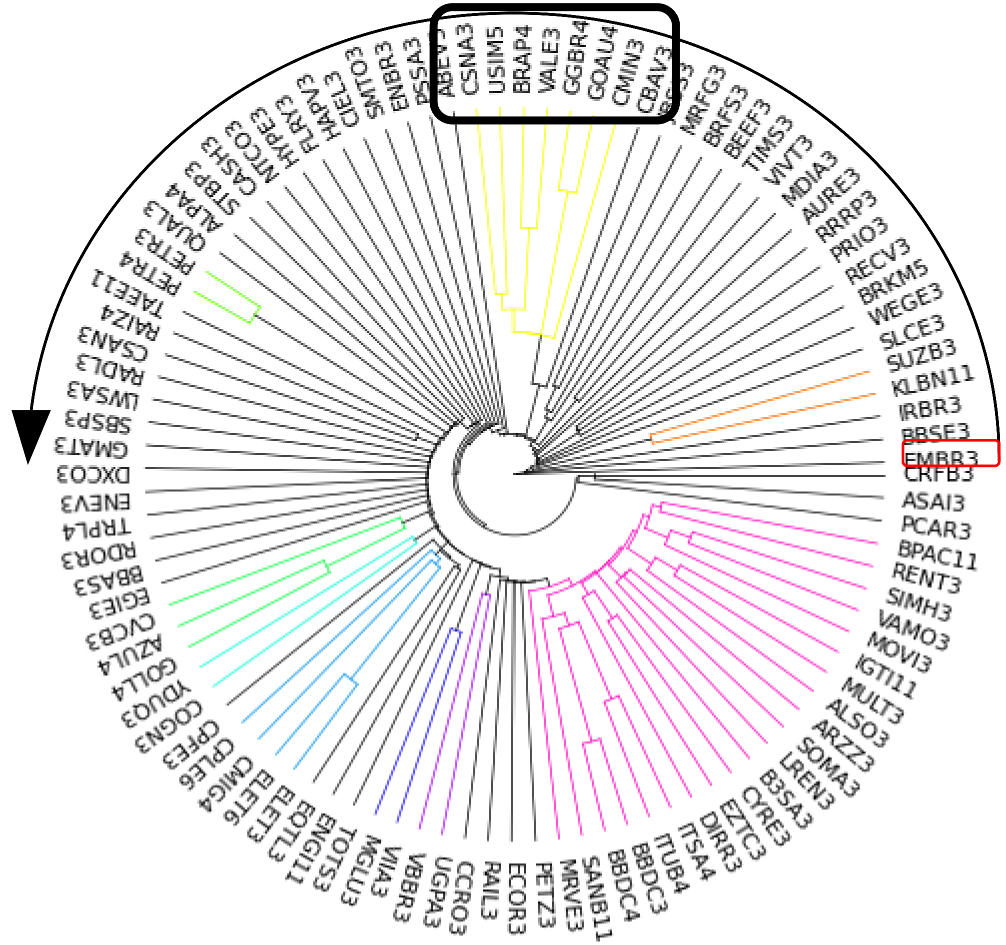
Group similar assets into clusters, based on a distance metric. The result is a hierarchical clustering tree.

2) Quase-Diagonalization

Reorganize the correlation matrix, so that the largest values lie along the diagonal. Similar assets are placed together, and dissimilar investments are placed far apart.

3) Recursive Bisection

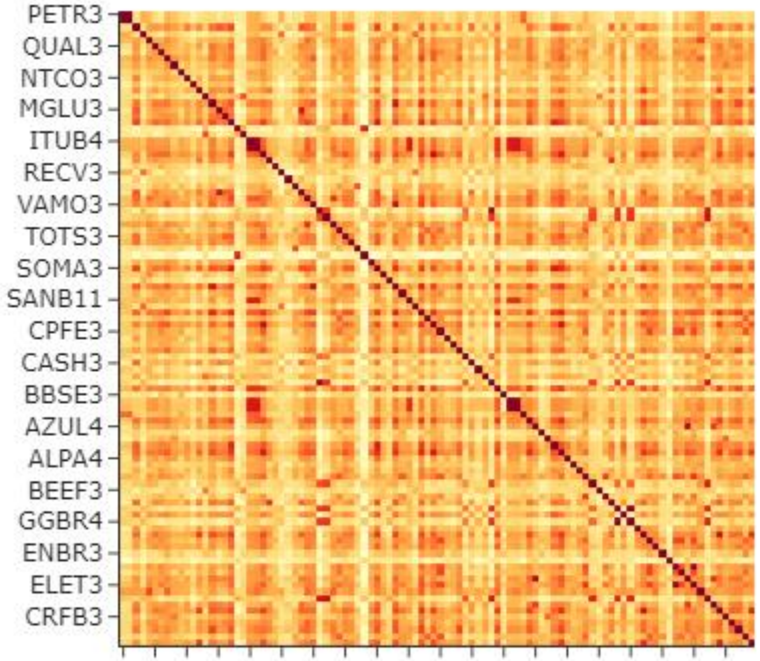
Split allocations by recursively dividing the reordered list of assets into smaller subsets. By recursively dividing the assets into subsets, it aims to capture different risk profiles and diversify the portfolio across various asset groups.



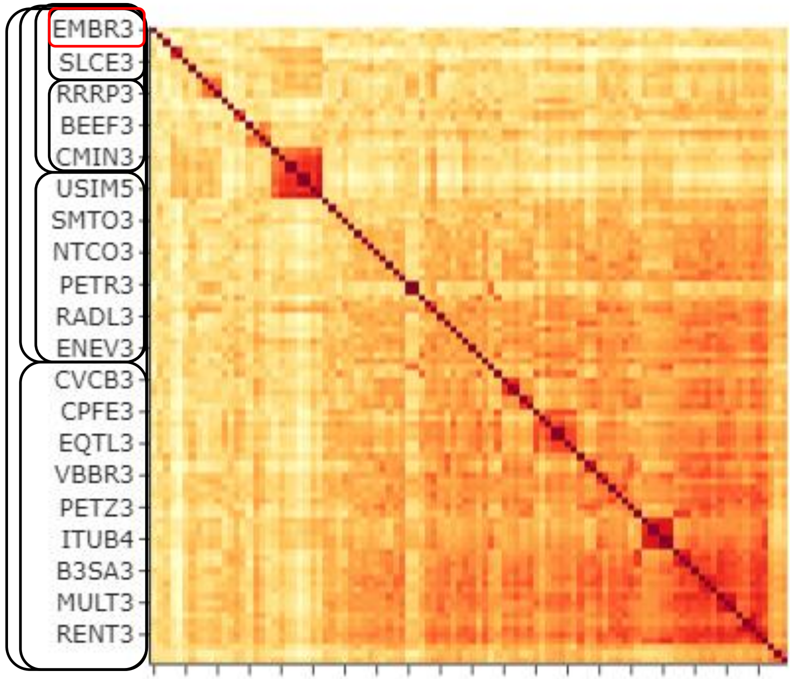
[3] López de Prado, M. (2018): AFML, 221-224

HRP Algorithmic

Correlation Matrix
IBX Index (06/30/2023)



Reindexed Correlation Matrix



Recursive Bisection [4]

1)Assign a unit weight to all items.
2)Create a list of clusters. Each element is a subset of tickers. Initiate the list with one cluster that contains all the ordered tickers.
3)**Bisect each element of the clusters list** (if the size of the cluster is above 1 ticker). Replace the clusters list to the new list.
4)**For each pair of cluster** in the clusters list, compute the variance of the cluster using the IVP method. Then re-scale the weights of the tickers in each cluster: α for the first cluster and (1- α) for the second.

$$\boxed{\alpha = 1 - \frac{Vi1}{Vi1 + Vi2}}$$

6) While is still possible to bisect the cluster list, go to step (3).

[4] López de Prado, M. (2018): AFML, 224-231

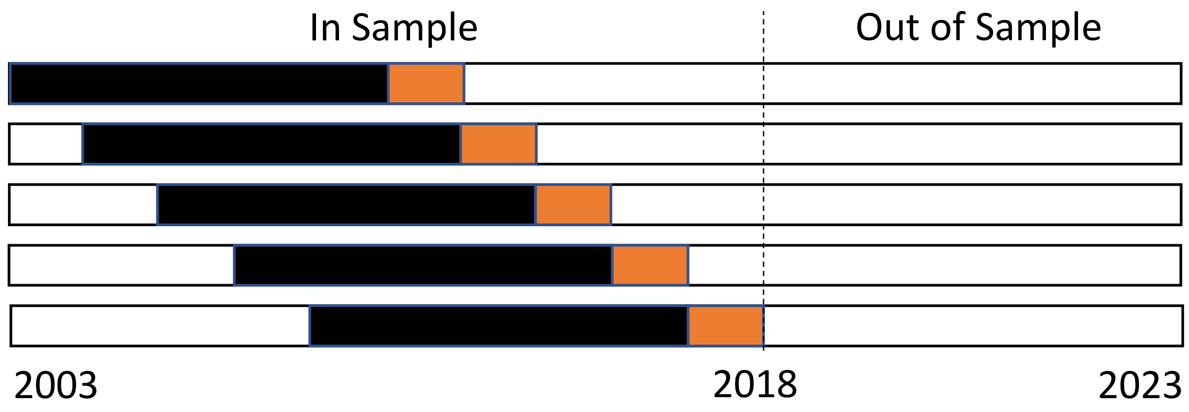
Methodology (Walk Forward)

1) Download data from Bloomberg (e.g. historical index composition, closing prices, volume) for the IBX Index for each date. Thus, **without the suvivorship bias** [5].

2) Treat the data and compute the return for each asset.

3) For each date in [rebalancing dates]:

<u>Retrieve:</u>
a) index composition of previous date
b) returns for the tickers in (a)
<u>Filter:</u>
c) Filter valid tickers for the date (e.g. volume)
<u>Calculate:</u>
d) weights using past data without look-ahead bias
e) portfolio return ($\sum_{i=1}^{N} wi * Ri$)



Example:
Rebalancing every 5d || Window of 756d
Previous Date: 06/23/2023 || Current Date: 06/30/2023

Index members: 06/23/2023
Data for weights: 06/12/2020 – 06/22/2023
Data for returns: 06/26/2023 – 06/30/2023

Assumptions:
No transaction costs or fees

(5) Problem of the suvivorship bias detailed at the end of the presentation

Model Selection

In Sample		Sharpe Ratio (Rf = 0)			MAR (CAGR/MDD)			Max Drawdown Duration (days)		
		Daily	Weekly	Montly	Daily	Weekly	Montly	Daily	Weekly	Montly
Benchmark		0.72			0.31			1552		
EWA		0.84	0.83	0.79	0.35	0.37	0.38	937	520	924
IVP	1M	0.98	0.98	0.77	0.42	0.47	0.35	717	700	2058
	2M	0.94	0.92	0.81	0.40	0.42	0.42	585	920	1533
	3M	0.89	0.88	0.78	0.36	0.38	0.36	587	920	1995
MVP	1Y	0.96	0.96	0.86	0.35	0.39	0.37	740	735	1617
	2Y	0.90	0.87	0.82	0.36	0.37	0.37	724	735	1260
	3Y	0.97	0.93	0.91	0.39	0.39	0.46	724	660	1239
HRP	1Y	1.03	1.04	0.99	0.37	0.41	0.45	513	495	504
	2Y	1.04	1.04	0.97	0.41	0.44	0.47	462	475	483
	3Y	1.06	1.05	1.00	0.43	0.46	0.52	462	460	336

As expected, the HRP model performed consistently and better than the other models. It tends to have **higher sharpe** ratio and **lower max drawdown duration**. The model even seems to do well with montly rebalancing periods.

Client's Preferences

Each of us is different! Some of us prefer more risk, others prefer only some types of risk. Maybe the client wants a portfolio without exposure to the systematic risk of the market (βm) or maybe it is the client's preference to have like X% of target volatility.

This can be expressed as constraints to the portfolio optimization model.

Adding the risk free rate (w_{rf}) as an option to the the risk portfolio (w_{rp}), may provide the condition to reach a target volatility.

$$\boxed{\begin{gathered} R_p = w_{rf} * R_{rf} + \ wp_{rp} * R_{rp} \\ w_{rf} + wp_{rp} = 1 \end{gathered}}$$

For example: the client is a pension fund with the risk mandate to carry only 5% of annualized volatility.

You estimate that the volatility ex-ante $^{(*)}$ of the portfolio position of a given date is 16% p.a.. One solution is to scale down the weights of the portfolio by a factor of $^5/_{16}$. The rest of the capital could be allocated to the risk free rate.

Other approach that would reduce the volatility of the strategy is to hedge the stock market systematic risk. Thus, isolating the alpha of the allocator.

*We can estimate the ex-ante volatility using the weights applied to the returns of the last N days (N=21). The theory is the same of the IVP portfolio. It is a good proxy of the future volatility, because volatility tends to cluster (short term).

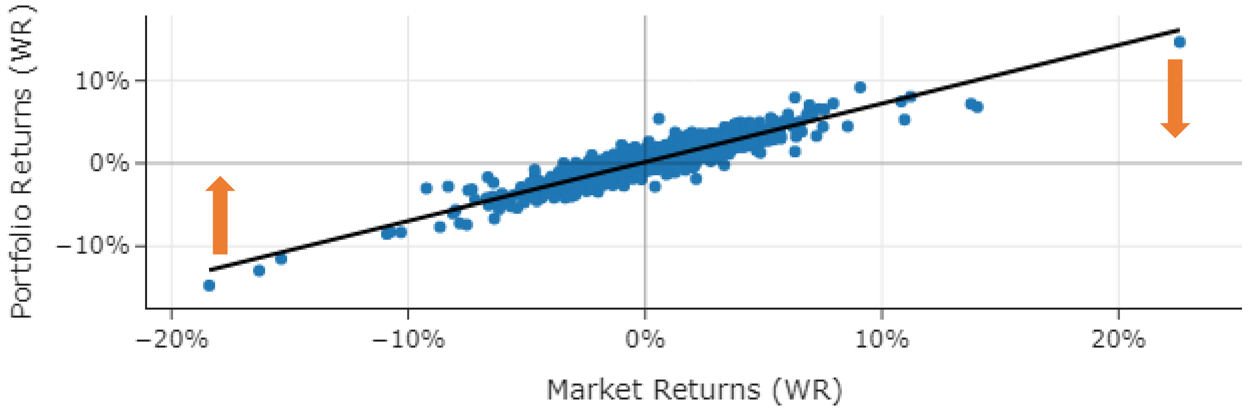
Factors

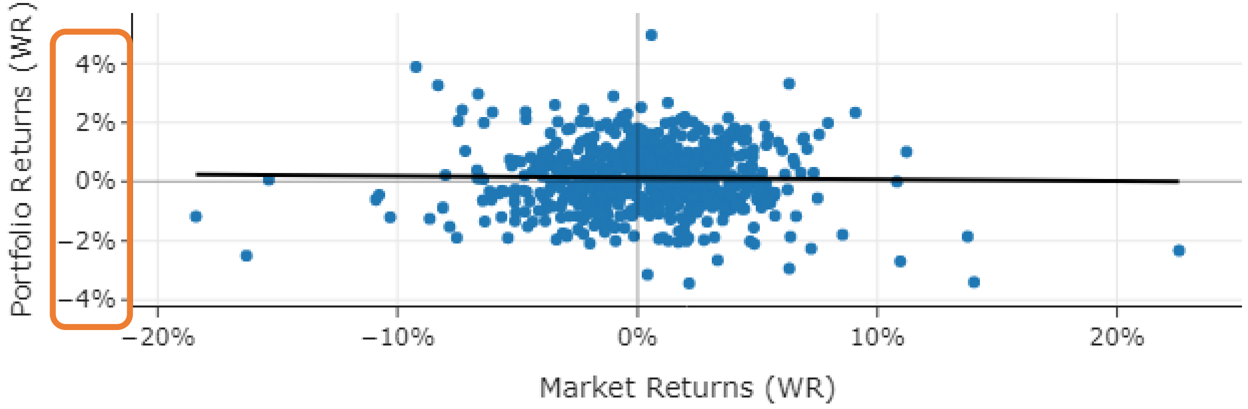
The client`s preference can even be regarding the type of **risk exposure** that the strategy may have.

For example, consider that the client has already an investment portfolio and is evaluating if your strategy may improve the performance of the overall. If both are highly correlated to some factor (e.g. Stock market, oil) the benefit of the diversification will be low [6]. It is even cheaper for the client to get the exposure with ETFs.

$$\boxed{R_p = \alpha_p + \beta_m * R_m + \varepsilon \Rightarrow \widehat{\beta_m} = \frac{COV(R_p, R_m)}{\sigma_m^2}}$$

We can open a short position of β weight on the etf/future to net the factor exposure (neutralize).





Strategy	SR (Rf = 0)	Volatility	Correlation
HRP 3Y WR	1.05	19%	92%
Dollar Neutral	0.18	10%	-70%
Hedge Beta 3Y	0.87	7%	-2%

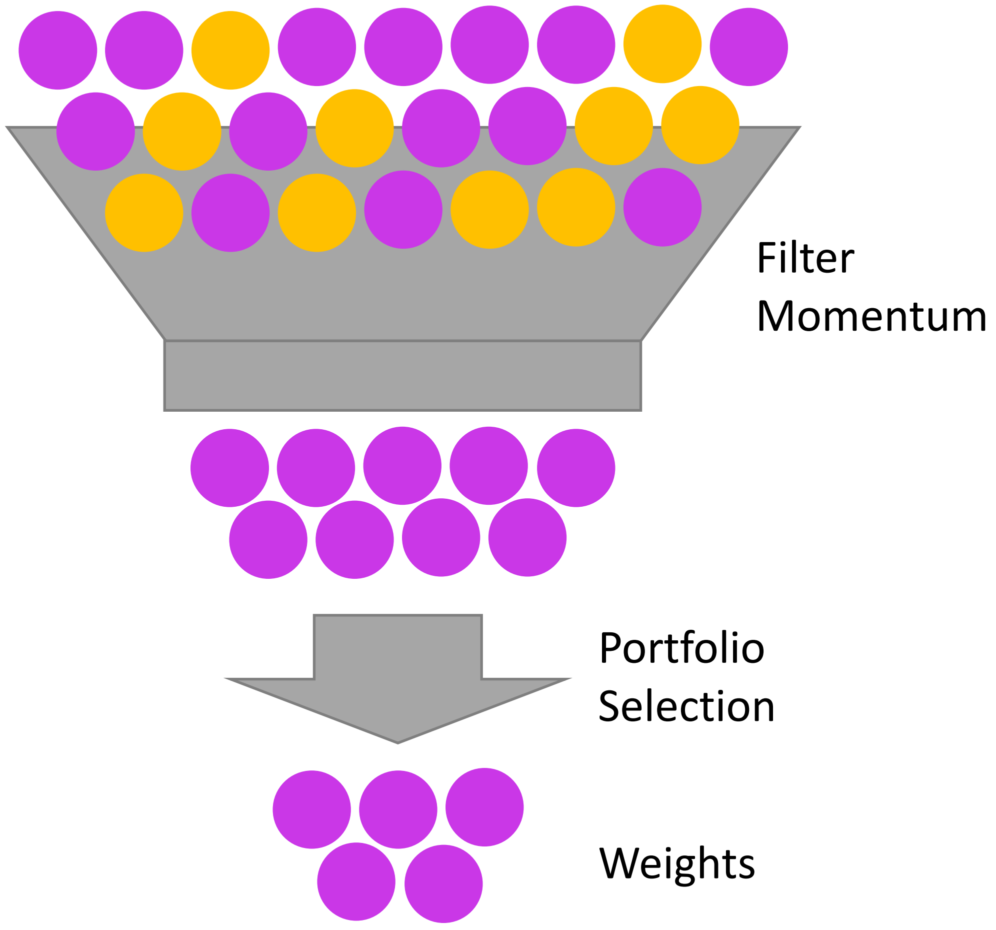
(6) Proof at the end of the presentation

Tweaks to the Model (Momentum)

It is hard to estimate with confidence the expected returns of each asset. But maybe we can improve our asset allocation with some tweaks.

"I kind of ran across this weird result that stocks returns have strong momentum (measured over the last 12M, leaving out the most recent month)", Cliff Asness [7]

The proposed solution is to create a **filter** $\mathbb{F}$ to select the assets with the desired properties (e.g. momentum) before applying the optimization method.



[7] Pedersen, L. (2015): Efficiently Inefficient, 164-170

Model Selection (Part 2)

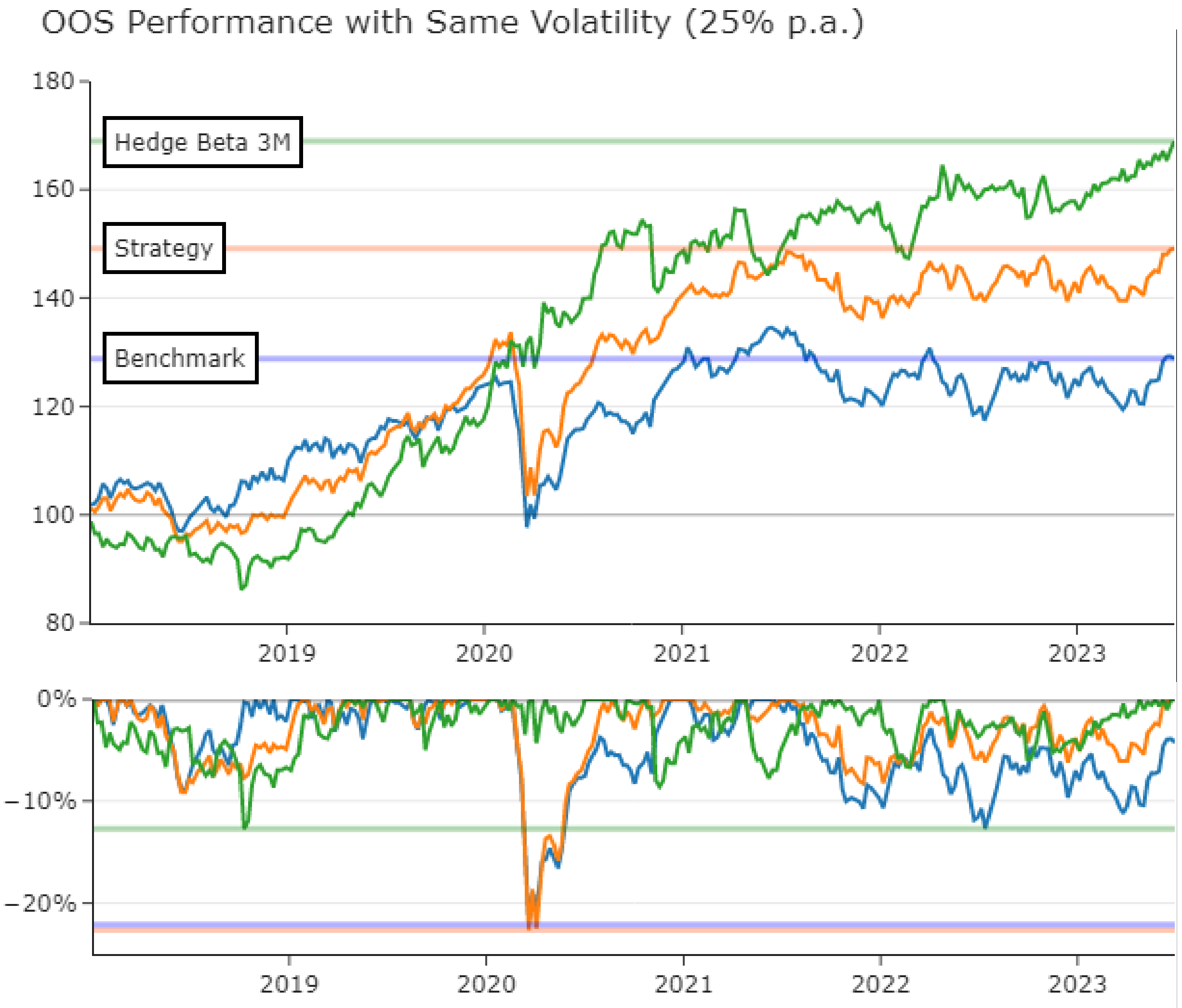
In Sample		Sharpe Ratio (Rf = 0)			MAR (CAGR/MDD)			Max Drawdown Duration (days)		
		Daily	Weekly	Montly	Daily	Weekly	Montly	Daily	Weekly	Montly
Benchmark			0.72			0.31			1552	
HRP	1Y	1.03	1.04	0.99	0.37	0.41	0.45	513	495	504
	2Y	1.04	1.04	0.97	0.41	0.44	0.47	462	475	483
	3Y	1.06	1.05	1.00	0.43	0.46	0.52	462	460	336
F25%	1Y	1.12	1.16	1.08	0.44	0.47	0.50	383	390	567
	2Y	1.13	1.15	1.06	0.45	0.49	0.50	378	455	525
	3Y	1.15	1.16	1.08	0.46	0.50	0.51	396	455	525
F50%	1Y	1.22	1.21	1.12	0.48	0.50	0.53	396	395	714
	2Y	1.21	1.20	1.13	0.51	0.51	0.59	380	530	378
	3Y	1.22	1.19	1.13	0.49	0.51	0.56	584	575	525
F75%	1Y	1.25	1.21	1.08	0.52	0.53	0.56	551	735	588
	2Y	1.20	1.13	1.06	0.48	0.45	0.53	582	895	693
	3Y	1.21	1.13	1.01	0.47	0.44	0.48	881	950	903

The filter approach is very simple. It doesn`t require any kind of prediction on the expected return for each asset. It just pre-select the assets that will go to the portfolio optimization model.

The momentum factor is very consistent in the literature, and the **data seems to support it**. We can observe that both the SR and MAR improves by removing the “bad apples”.

@_gncosta

Backtesting (HRP 3Y WR 75%)



	SR (Rf = 0)		MDD (Normalized VOL)	
	In Sample	Out of Sample	In Sample	Out of Sample
Benchmark	0.72	0.47	-29%	-22%
Strategy	1.13 57%	0.72 51%	-33%	-23%
Hedge Beta (3M)	0.90	0.92	-24%	-13%

Weights of 2023-30-06

Benchmark = {VALE3: 13,2%, ITUB4: 6,3%, PETR4: 6,2%...}

Strategy = {BBSE3: 8,0%, BBAS3: 7,5%, RADL: 7,3%, WEGE3: 6,9%...}

Strategy Hedge = {BBSE3: 8,0%, BBAS3: 7,5%, ..., ETF* : -82% }

Considerations:

- Sharpe Ratio decayed in the OOS sample, but the proportion between the benchmark and the strategy was **consistent**.
- The COVID event was responsable for the max drawdown of the benchmark and the strategy. **The hedged strategy against the systematic risk (β_m) performed well during the stress.**
- Not found any evidence to disqualify the strategy at the moment, futher tests should be conduced (e.g. other assets) to verify if the allocation model is truly consistent.

One more thing... (descorrelated alpha)

It`s very challenge to estimate with confidence the expected returns for a lot of different assets, but maybe we can focus on one: **DI1 Future**.

The hypothesis well known in the brazilian quant industry is that there is **momentum** in the price series.

To synthesize the price series of a fixed period (e.g. 18 months) we need to interpolate the DI1 contracts for each date.

I'll present the backtest for a real strategy that was constructed with that hypothesis. The strategy was created on the beginning of the 2022 (truly OOS after).

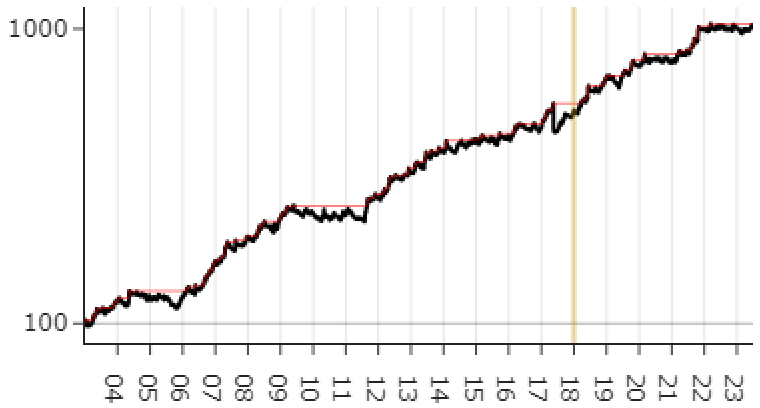
The signal is a function that maps the data to [-1,1]. There is also an ex-ante volatility adjustment to the position size.

The CDI rate was used to measure the excess return. Thus, the sharpe ratio is calculated correctly. Transaction costs are included.

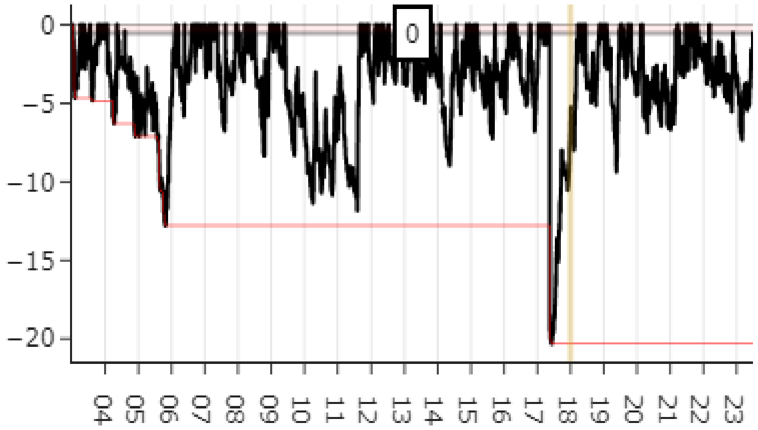
Trend Following (DI1)

```
SharpeRatio: 1.2
Sharpe(OOS): 1.4
Alpha/MaxDD: 0.6
Alpha/AvgDD: 3.3
BreakevenRt: 2.5
LowerTailRt: 1.7
UpperTailRt: 2.3

Exc.Return:    5%
Volatility:    4%
Costs&Fees:  0.4%

Signal Statistics
Buy:   43% SR: 0.9
Sell:  57% SR: 1.5
Out:    0% CG:   5%
```

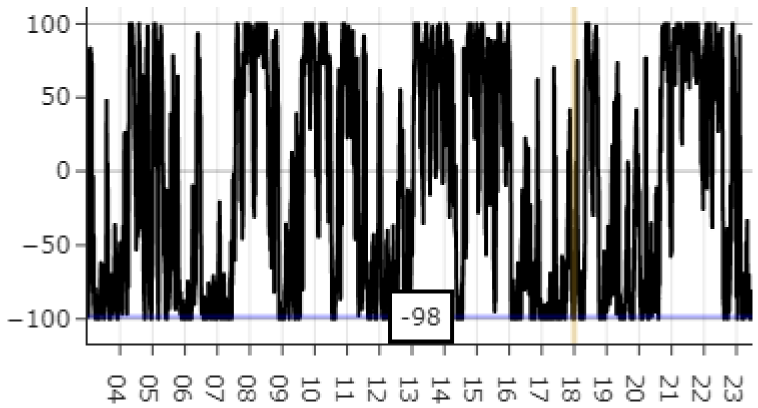
Performance



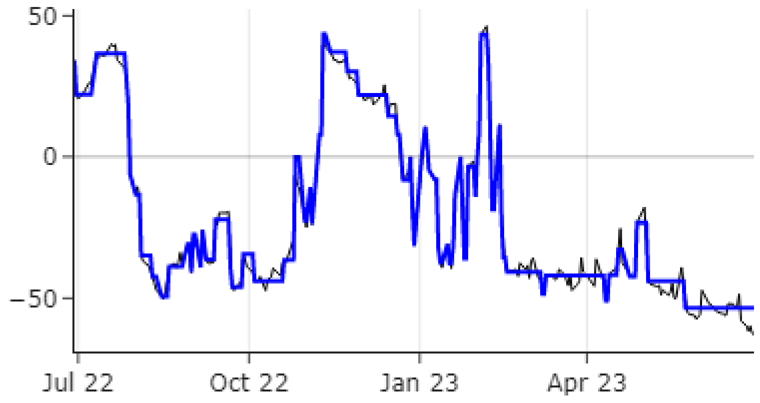
Drawdown



Signal



Position



Next Steps

- Test the model with **different indexes/stocks** (e.g. Small caps) to truly validate OOS.
- Expand the asset universe to **different asset classes** (e.g. Interest Rates, Cmdtys, FX).
- Explore the concepts of factors (e.g. Momentum, Value, Growth) to add value to the algo.
- Explore more advanced portfolio **optimization methods** (e.g. HERC, NCO, DRL).
- Refine the **constraints** (e.g. Turnover, Costs, Short Selling, Cost of Carry).

Bibliography

López de Prado, M. (2018): Advances in Financial Machine Learning (AFML)

López de Prado, M. (2020): Machine Learning for Asset Managers

Harvey, C., Rattray, S., Hemert, O. (2021): Strategic Risk Management

Isichenko, M. (2021): Quantitative Portfolio Management

Pedersen, L. (2015): Efficiently Inefficient

Carver, R. (2023): Advanced Futures Trading Strategies

Thanks!

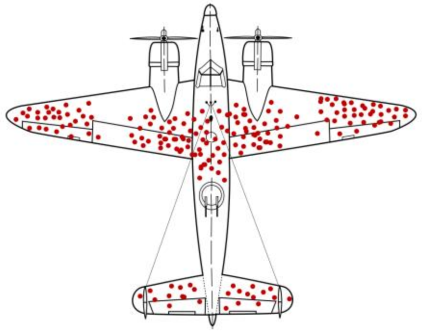
Any questions?

Survivorship Bias (e.g. IBOV Index)

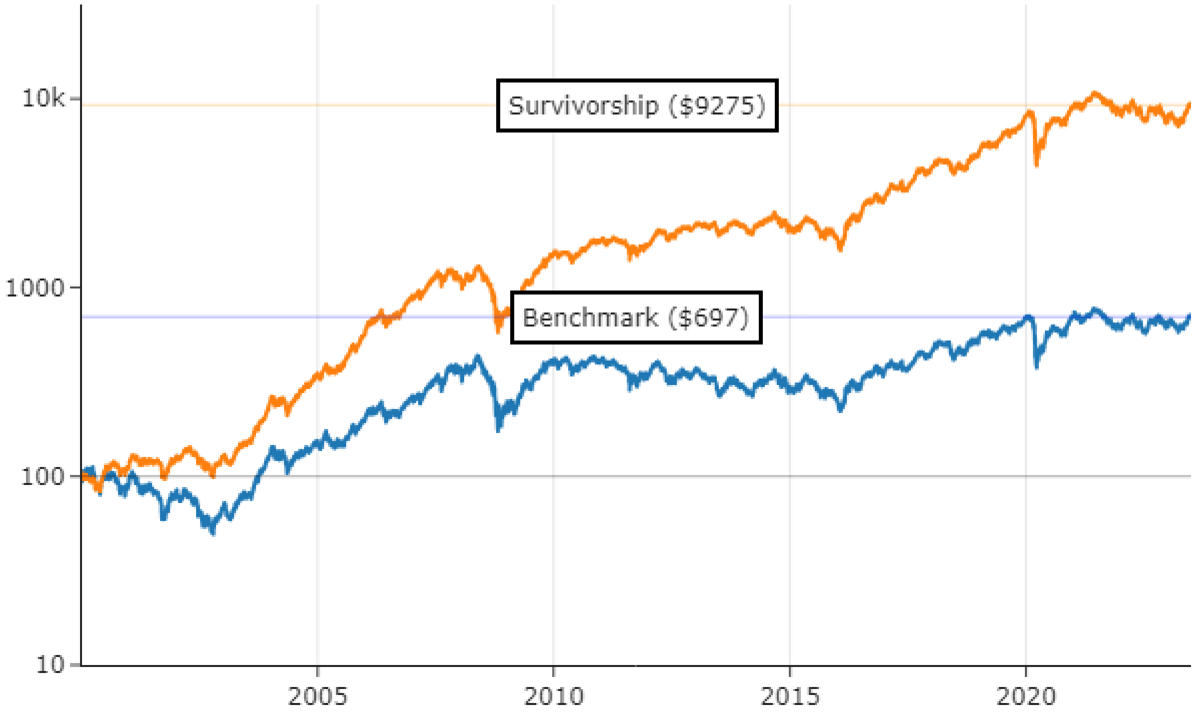
If only we knew which stocks are going to fail... Only companies that were successful enough to survive until the end of the period...

Remember OGXP3?

AMER3?



*Current weights of IBOV Index (2023-06-30) applied since 2000-01-03. For each date, the weights were rescaled to sum 100% (not all tickers from today traded in 2000). Thus, for each date, the stocks that suvivor had the same w.



Daily Rebalance	2000-01-03	2023-06-30
# Assets	36	85
Top Weight	Telebras	Vale
	~45%	~12%

Modern Portfolio Theory (MPT)

2 assets (a,b):

$\mu_p = \mathbb{E}(w_a * R_a + w_b * R_b) = w_a * \mu_a + w_b * \mu_b$

$\sigma_p^2 = VAR(w_a * R_a + w_b * R_b) = w_a^2 * VAR(R_a) + w_b^2 * VAR(R_b) + 2 * w_a * w_b * \mathrm{COV}(R_a, R_b)$

N assets (i): $\sum_{i=1}^{N} w_i = 1$

$$\mu_p = \sum_{i=1}^{N} w_i * \mu_i = \boldsymbol{w^T} * \boldsymbol{\mu}$$

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i * w_j * \mathrm{COV}(R_i, R_j) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i * w_j * \rho_{i,j} * \sigma_i * \sigma_j = \boldsymbol{w^T} * \boldsymbol{\Sigma} * \boldsymbol{w}$$

Closed solution:

$\min_{s.t.: w^T * a = 1} \frac{1}{2} * \boldsymbol{w^T} * \boldsymbol{\Sigma} * \boldsymbol{w}$, where a is the portfolio`s constraints. Use lagrangian we obtain:

$$\boxed{w^* = \frac{\Sigma^{-1} * a}{a^T * \Sigma^{-1} * q}}$$

Equal Weights: If $a = \mathbb{1}n$ and $\Sigma = \sigma * \mathrm{I}n$

Inverse Variance Portfolio: If $a = \mathbb{1}n$ and Σ is a diagonal matrix with $c_{i,j} = 0$ (for i <> j)

Minimum Variance Portfolio: Only condition is that the weights must sum 1 ($a = \mathbb{1}n$)

Quantifying Diversification

Consider:

$$\mu i = \mu, \qquad \sigma i = \sigma, \qquad \rho i = \rho, \qquad wi = \frac{1}{N}, \qquad SRi = \frac{\mu i}{\sigma i} = \frac{\mu}{\sigma}$$

$$\mu p = w^T * \mu = \begin{pmatrix} \frac{1}{N} \cdots \frac{1}{N} \end{pmatrix} * \begin{pmatrix} \mu \\ \vdots \\ \mu \end{pmatrix} = \mu$$

$$\sigma p^2 = w^T * \Sigma * w = \begin{pmatrix} \frac{1}{N} \cdots \frac{1}{N} \end{pmatrix} * \begin{pmatrix} \sigma^2 & \cdots & \rho * \sigma^2 \\ \vdots & \ddots & \vdots \\ \rho * \sigma^2 & \cdots & \sigma^2 \end{pmatrix} * \begin{pmatrix} \frac{1}{N} \\ \vdots \\ \frac{1}{N} \end{pmatrix} = \begin{pmatrix} \frac{1}{N} * \sigma^2 + \frac{1}{N} * \rho * \sigma^2 * (N-1) & \cdots \end{pmatrix} * \begin{pmatrix} \frac{1}{N} \\ \vdots \\ \frac{1}{N} \end{pmatrix}$$

$$= \left(\frac{1}{N} * \sigma^2 + \frac{1}{N} * \rho * \sigma^2 * (N-1)\right) * \left(\frac{1}{N}\right) * (N) = \left(\frac{1}{N} * \sigma^2 + \frac{1}{N} * \rho * \sigma^2 * (N-1)\right) = \left(\sigma^2 * \left(\frac{1 + \rho N - \rho)}{N}\right)\right) = \left(\sigma^2 * \left(\rho * \frac{(1-\rho)}{N}\right)\right)$$

$$SRp = \frac{\mu p}{\sigma p} = \frac{\mu}{\sqrt{\left(\sigma^2 * \left(\rho * \frac{(1-\rho)}{N}\right)\right)}} = \frac{\mu}{\sigma * \sqrt{\left(\rho * \frac{(1-\rho)}{N}\right)}} = SR * {}^{1}\Big/ \sqrt{\left(\rho * \frac{(1-\rho)}{N}\right)}$$

$$\boxed{\text{If N} \rightarrow \infty, \ \text{SRp} = \text{SR} * {}^{1}/_{\sqrt{\rho}}}$$

$$\boxed{\text{If } \rho \rightarrow 0, \qquad \text{SRp} = \text{SR} * \sqrt{N}}$$

Mission Impossible: The Flawless Backtest

Common backtesting errors include:

- Survivorship bias: Using as investment universe the current one, hence ignoring that some companies went bankrupt and securities were delisted along the way.
- Look-ahead bias: Using information that was not public at the moment the simulated decision would have been made. Be certain about the timestamp for each data point. Take into account release dates, distribution delays, and backfill corrections.
- Storytelling: Making up a story ex-post to justify some random pattern.
- Data mining and data snooping: Training the model on the testing set.
- Transaction costs: Simulating transaction costs is hard because the only way to be certain about that cost would have been to interact with the trading book (i.e., to do the actual trade).
- Shorting: Taking a short position on cash products requires finding a lender. The cost of lending and the amount available is generally unknown, and depends on relations, inventory, relative demand, etc.

(*) Slide extracted from presentation (López de Prado, M.) at: https://ssrn.com/abstract=3257497