Lesson 9 Lecture Example

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Lesson 9 - Install packages

Install necessary packages using library()

Perform data housekeeping - upload, name columns, display to make sure it reads properly, etc.

```
knitr::opts_chunk$set(echo = TRUE)

library(e1071)
library(xtable)
library("xlsx") # Needed to read data
```

```
## Warning: package 'xlsx' was built under R version 4.0.3
```

```
library(psych) # For geometric mean in Example 5.3
```

```
## Warning: package 'psych' was built under R version 4.0.3
```

```
library(MASS) # Needed for ginv() function

rm(list = ls())
```

Read data file (data-table-B09.xlsx)

```
exL <- read.xlsx("data-table-B09.xlsx",
                 sheetIndex = 1,
                 colIndex = c(1,2,3,4,5),
                 as.data.frame = TRUE,
                 header = TRUE)
```

Assign labels to data columns using names() and attach() commands

```
names(exL) <- c("fluid_vel", "viscosity","mesh_open","fluid_gas_vel","pressure_drop")
attach(exL)
```

Output data to make sure it reads properly

```
out <- as.data.frame(c(exL))
colnames(out) <- c("fluid_vel", "viscosity","mesh_open","fluid_gas_vel","pressure_drop")
tab <- (xtable(out, digits=c(0,2,1,2,3,1)))
print(tab, type="html")
```

	fluid_vel	viscosity	mesh_open	fluid_gas_vel	pressure_drop
1	2.14	10.0	0.34	1.000	28.9
2	4.14	10.0	0.34	1.000	31.0
3	8.15	10.0	0.34	1.000	26.4
4	2.14	10.0	0.34	0.246	27.2
5	4.14	10.0	0.34	0.379	26.1
6	8.15	10.0	0.34	0.474	23.2
7	2.14	10.0	0.34	0.141	19.7
8	4.14	10.0	0.34	0.234	22.1
9	8.15	10.0	0.34	0.311	22.8
10	2.14	10.0	0.34	0.076	29.2
11	4.14	10.0	0.34	0.132	23.6
12	8.15	10.0	0.34	0.184	23.6
13	2.14	2.6	0.34	0.679	24.2
14	4.14	2.6	0.34	0.804	22.1
15	8.15	2.6	0.34	0.890	20.9
16	2.14	2.6	0.34	0.514	17.6
17	4.14	2.6	0.34	0.672	15.7
18	8.15	2.6	0.34	0.801	15.8
19	2.14	2.6	0.34	0.346	14.0
20	4.14	2.6	0.34	0.506	17.1
21	8.15	2.6	0.34	0.669	18.3
22	2.14	2.6	0.34	1.000	33.8
23	4.14	2.6	0.34	1.000	31.7
24	8.15	2.6	0.34	1.000	28.1
25	5.60	1.2	0.34	0.848	18.1
26	5.60	1.2	0.34	0.737	16.5
27	5.60	1.2	0.34	0.651	15.4
28	5.60	1.2	0.34	0.554	15.0
29	4.30	2.6	0.34	0.748	19.1
30	4.30	2.6	0.34	0.682	16.2
31	4.30	2.6	0.34	0.524	16.3
32	4.30	2.6	0.34	0.472	15.8
33	4.30	2.6	0.34	0.398	15.4
34	5.60	10.1	0.25	0.789	19.2
35	5.60	10.1	0.25	0.677	8.4
36	5.60	10.1	0.25	0.590	15.0
37	5.60	10.1	0.25	0.523	12.0
38	5.60	10.1	0.34	0.789	21.9
39	5.60	10.1	0.34	0.677	21.3
40	5.60	10.1	0.34	0.590	21.6
41	5.60	10.1	0.34	0.523	19.8
42	4.30	10.1	0.34	0.741	21.6
43	4.30	10.1	0.34	0.617	17.3
44	4.30	10.1	0.34	0.524	20.0
45	4.30	10.1	0.34	0.457	18.6

46	2.40	10.1	0.34	0.615	22.1
47	2.40	10.1	0.34	0.473	14.7
48	2.40	10.1	0.34	0.381	15.8
49	2.40	10.1	0.34	0.320	13.2
50	5.60	10.1	0.55	0.789	30.8
51	5.60	10.1	0.55	0.677	27.5
52	5.60	10.1	0.55	0.590	25.2
53	5.60	10.1	0.55	0.523	22.8
54	2.14	112.0	0.34	0.680	41.7
55	4.14	112.0	0.34	0.803	33.7
56	8.15	112.0	0.34	0.889	29.7
57	2.14	112.0	0.34	0.514	41.8
58	4.14	112.0	0.34	0.672	37.1
59	8.15	112.0	0.34	0.801	40.1
60	2.14	112.0	0.34	0.306	42.7
61	4.14	112.0	0.34	0.506	48.6
62	8.15	112.0	0.34	0.668	42.4

```
# Output data structure and dimensions
str(exL)
```

‘data.frame’: 62 obs. of 5 variables: $ fluid_vel : num 2.14 4.14 8.15 2.14 4.14 8.15 2.14 4.14 8.15 2.14 … $ viscosity : num 10 10 10 10 10 10 10 10 10 10 … $ mesh_open : num 0.34 0.34 0.34 0.34 0.34 0.34 0.34 0.34 0.34 0.34 … $ fluid_gas_vel: num 1 1 1 0.246 0.379 0.474 0.141 0.234 0.311 0.076 … $ pressure_drop: num 28.9 31 26.4 27.2 26.1 23.2 19.7 22.1 22.8 29.2 …

```
dim(exL)
```

[1] 62 5

create multiple least squares model

```
model <- lm(pressure_drop ~ fluid_vel + viscosity + mesh_open + fluid_gas_vel)

summary(model)
```

Call: lm(formula = pressure_drop ~ fluid_vel + viscosity + mesh_open + fluid_gas_vel)

Residuals: Min 1Q Median 3Q Max -9.9958 -3.3092 -0.2419 3.3924 10.5668

Coefficients: Estimate Std. Error t value Pr(>|t|)
(Intercept) 5.89453 4.32508 1.363 0.17828
fluid_vel -0.47790 0.34002 -1.406 0.16530
viscosity 0.18271 0.01718 10.633 3.78e-15 ***mesh_open 35.40284 11.09960 3.190 0.00232*** *fluid_gas_vel 5.84391 2.90978 2.008 0.04935*
— Signif. codes: 0 ‘***’ 0.001*** *’’ 0.01 ”* 0.05 ‘.’ 0.1 ’ ’ 1

Residual standard error: 5.014 on 57 degrees of freedom Multiple R-squared: 0.6914, Adjusted R-squared: 0.6697 F-statistic: 31.92 on 4 and 57 DF, p-value: 5.818e-14

```
xtable(summary(model))
```

	Estimate	Std. Error	t value	Pr(>|t|)
	<dbl>	<dbl>	<dbl>	<dbl>
(Intercept)	5.8945253	4.32507771	1.362872	1.782831e-01
fluid_vel	-0.4779013	0.34001900	-1.405514	1.652960e-01
viscosity	0.1827137	0.01718375	10.632936	3.779436e-15
mesh_open	35.4028387	11.09960045	3.189560	2.316482e-03
fluid_gas_vel	5.8439137	2.90977841	2.008371	4.935047e-02

5 rows

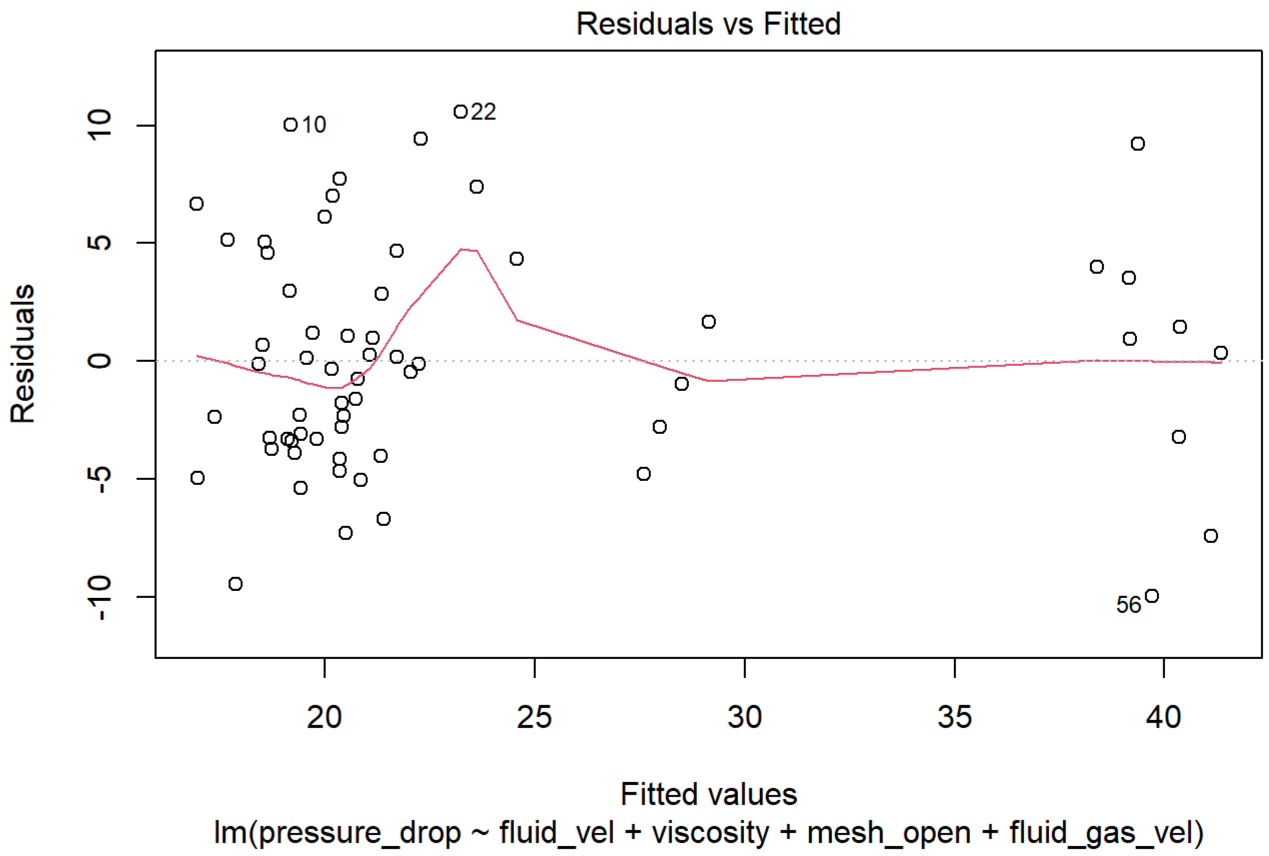
```
xtable(anova(model))
```

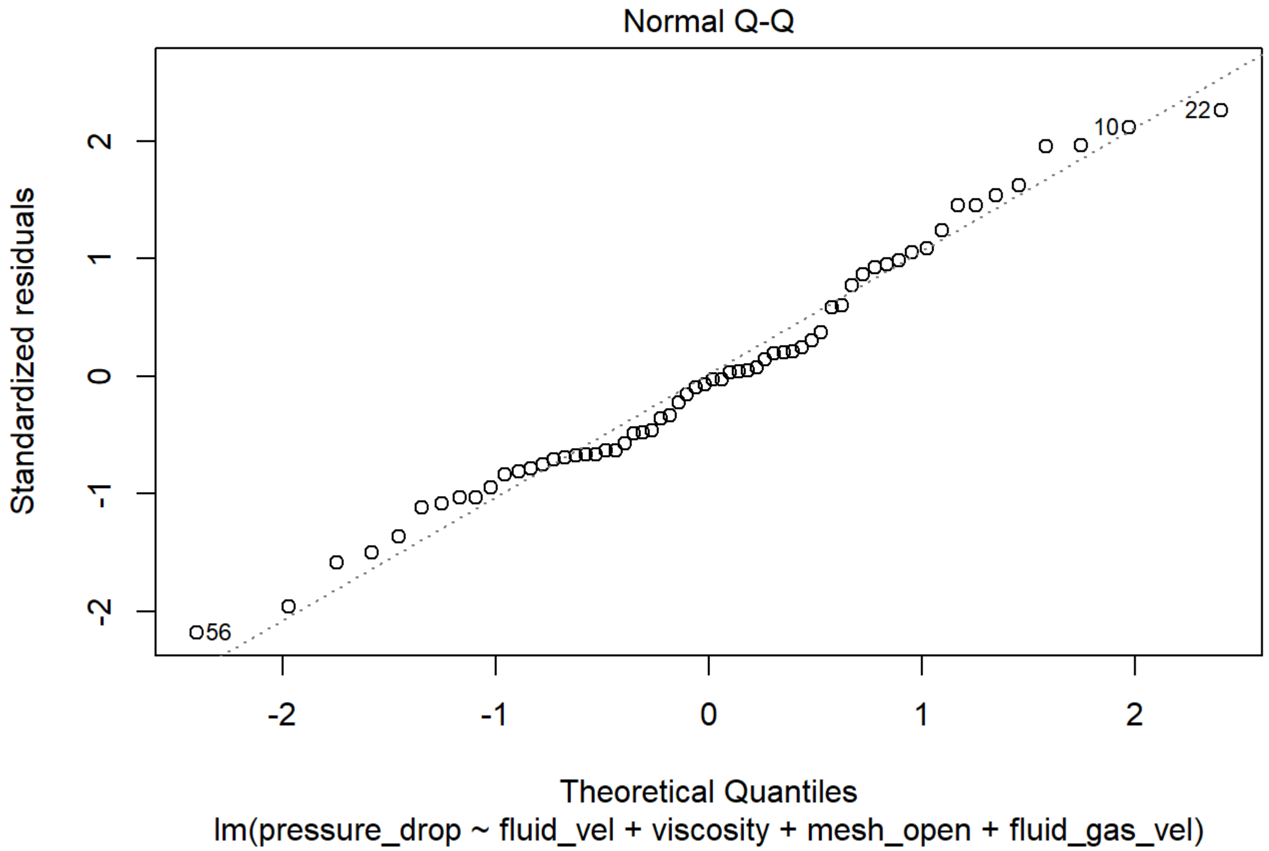
	Df	Sum Sq	Mean Sq	F value	Pr(>F)
	<int>	<dbl>	<dbl>	<dbl>	<dbl>
fluid_vel	1	9.595968	9.595968	0.3817523	5.391278e-01
viscosity	1	2839.780203	2839.780203	112.9737594	3.834506e-15
mesh_open	1	258.951631	258.951631	10.3017618	2.183922e-03
fluid_gas_vel	1	101.389947	101.389947	4.0335528	4.935047e-02
Residuals	57	1432.788219	25.136635	*NA*	*NA*

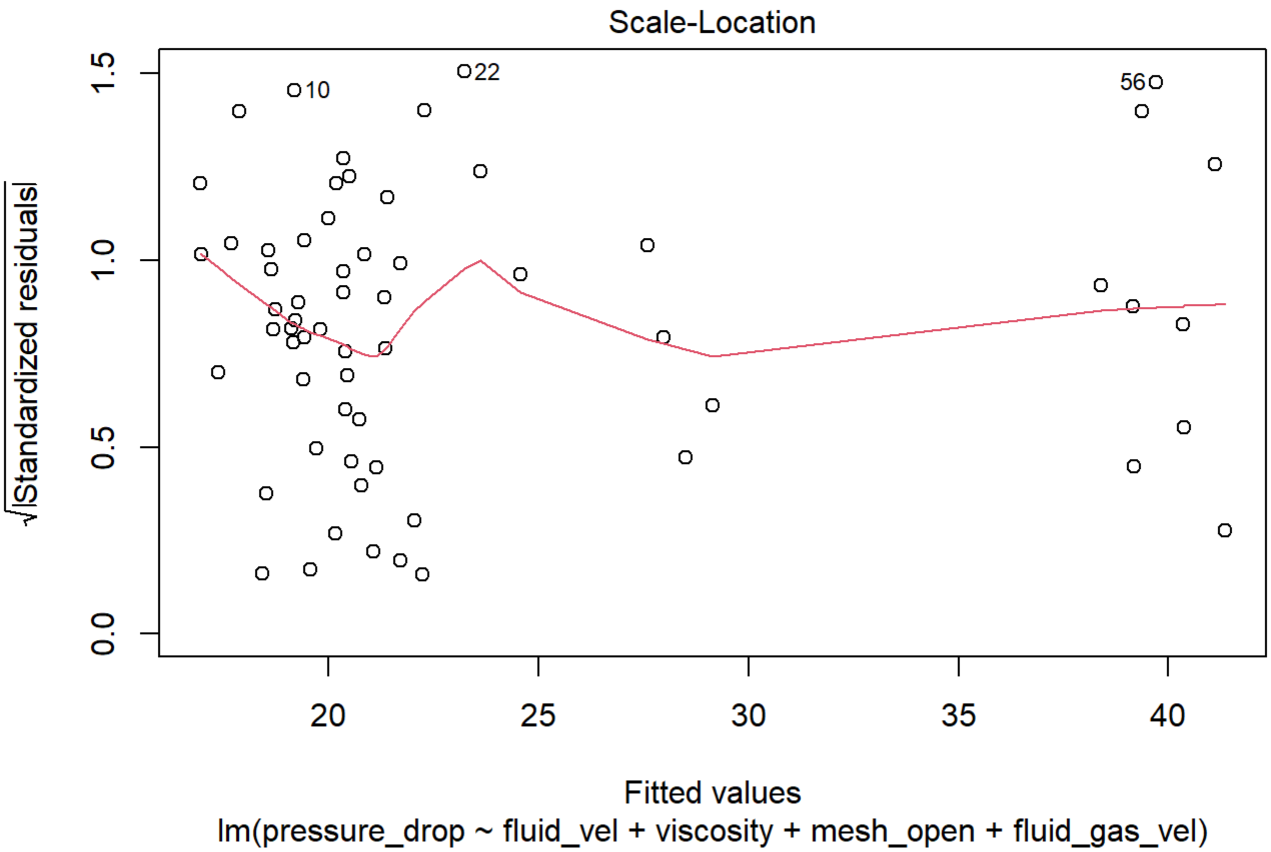
5 rows

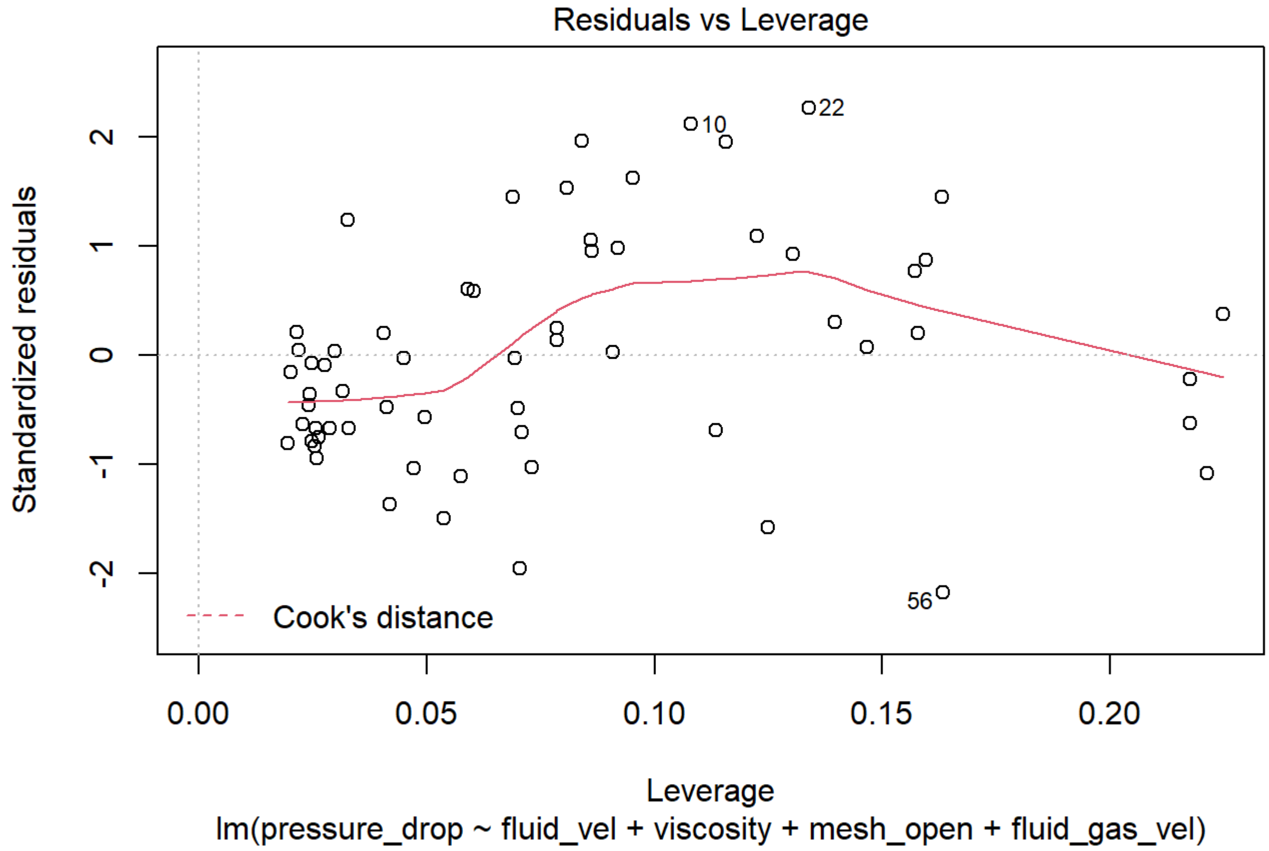
make a plot of the multiple least squares model

```
plot(model)
```

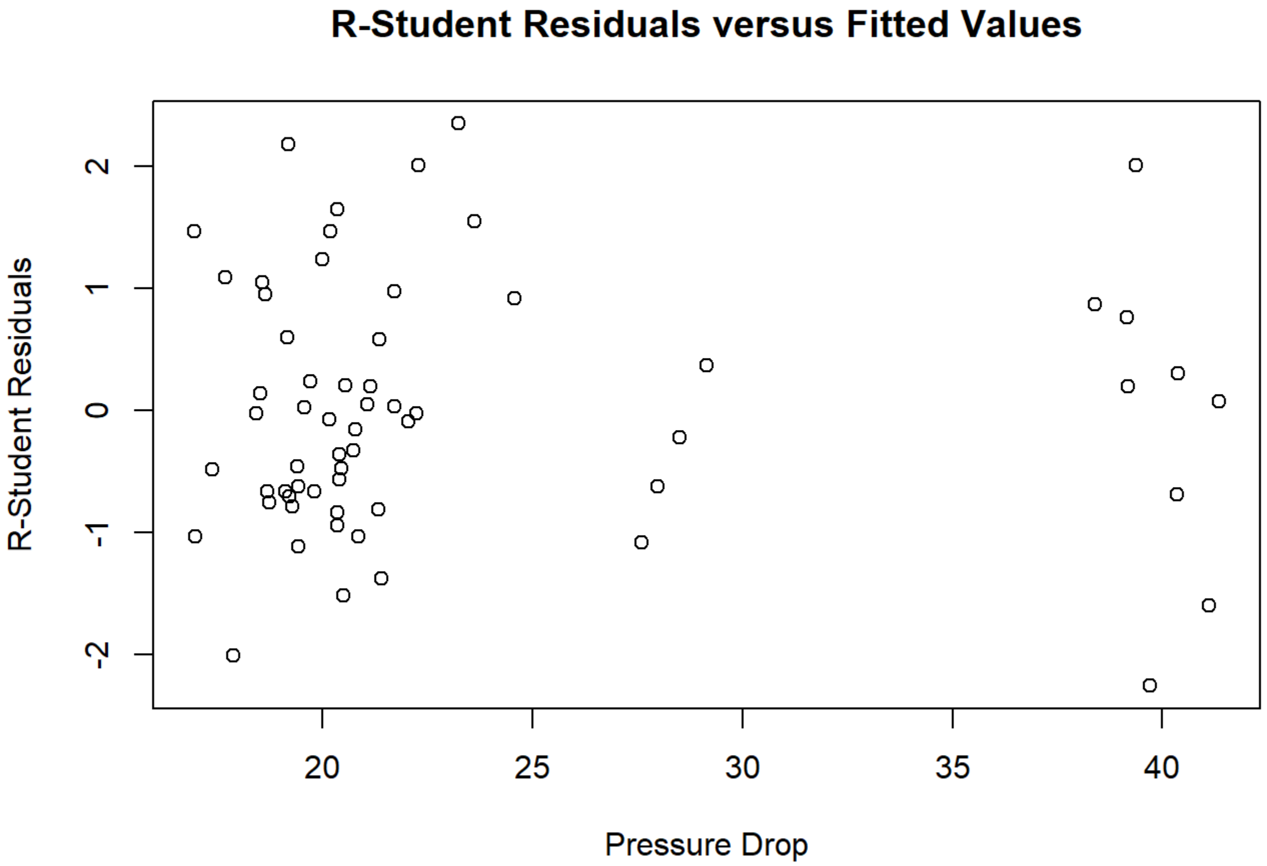

Residuals vs Fitted
Residuals
10
5
0
-5
-10
10
22
56
20
25
30
35
40
Fitted values
lm(pressure_drop ~ fluid_vel + viscosity + mesh_open + fluid_gas_vel)


Normal Q-Q
Standardized residuals
2
1
0
-1
-2
22
10
56
-2
-1
0
1
2
Theoretical Quantiles
lm(pressure_drop ~ fluid_vel + viscosity + mesh_open + fluid_gas_vel)

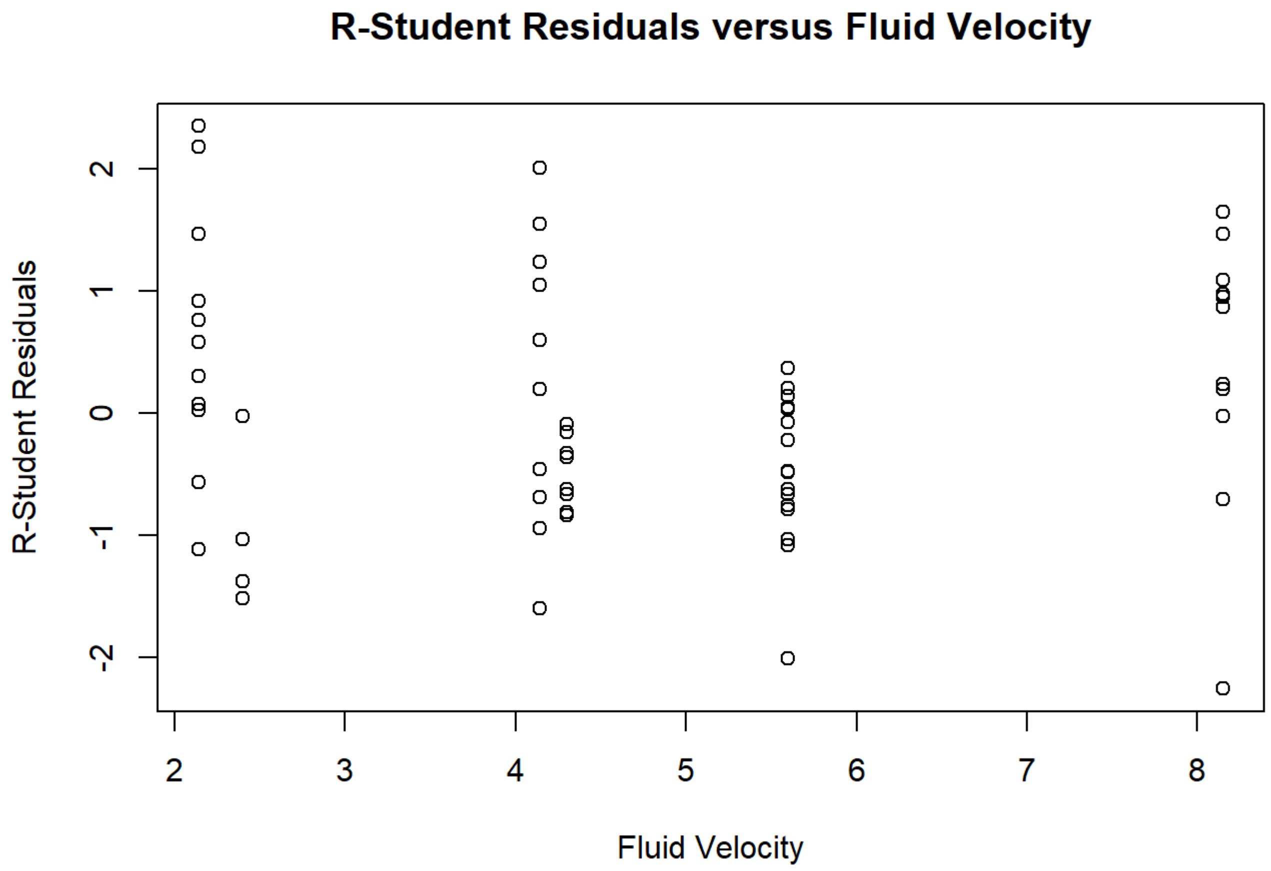

Scale-Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
20
25
30
35
40
10
22
56
Fitted values
lm(pressure_drop ~ fluid_vel + viscosity + mesh_open + fluid_gas_vel)


Residuals vs Leverage
Standardized residuals
-2
-1
0
1
2
0.00
0.05
0.10
0.15
0.20
10
22
56
Cook's distance
Leverage
lm(pressure_drop ~ fluid_vel + viscosity + mesh_open + fluid_gas_vel)

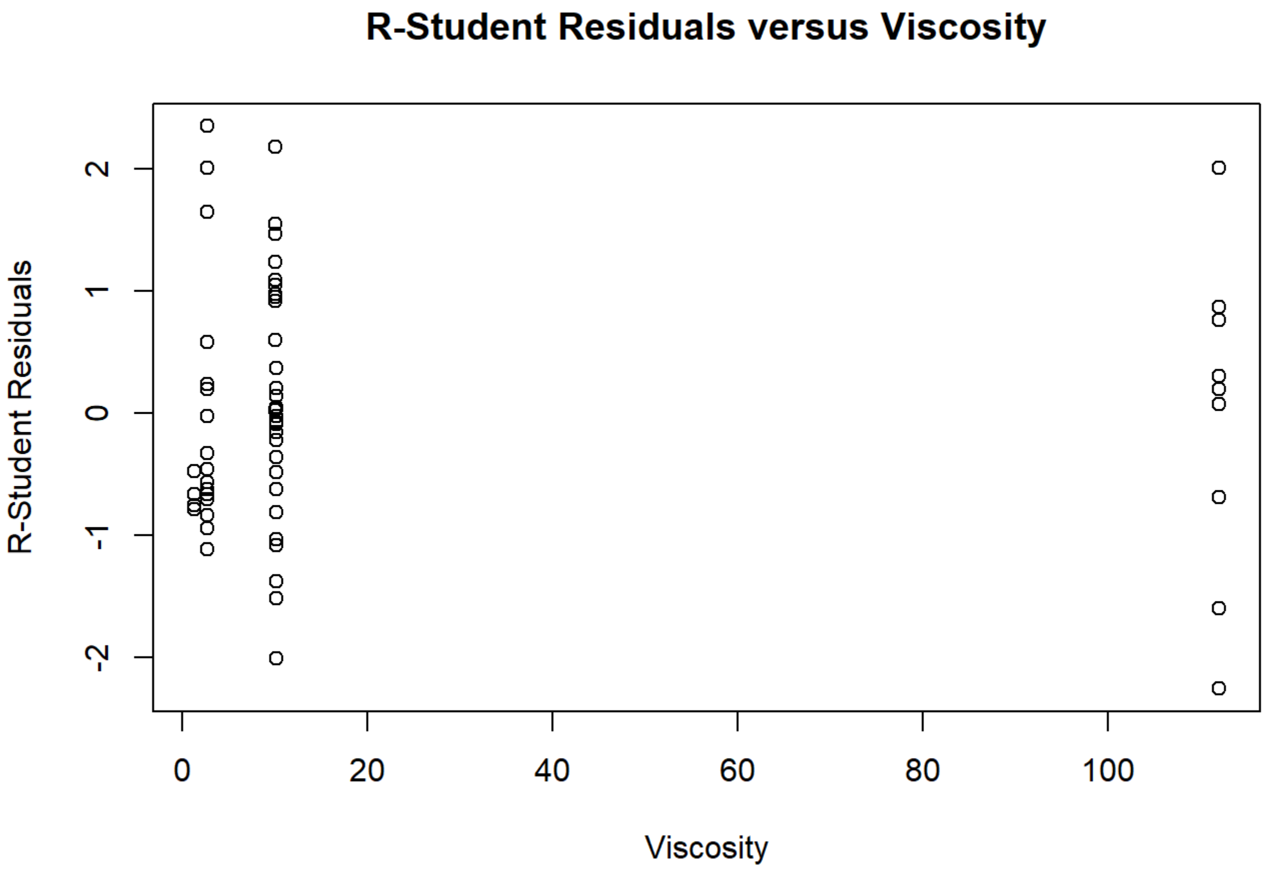
```
# Let’s look at a plot of the R-student residuals versus the fitted values

R_Student_Residuals <- rstudent(model)
y_hat <- model$fitted.values
plot(y_hat, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Pressure Drop", main = "R
-Student Residuals versus Fitted Values")
```



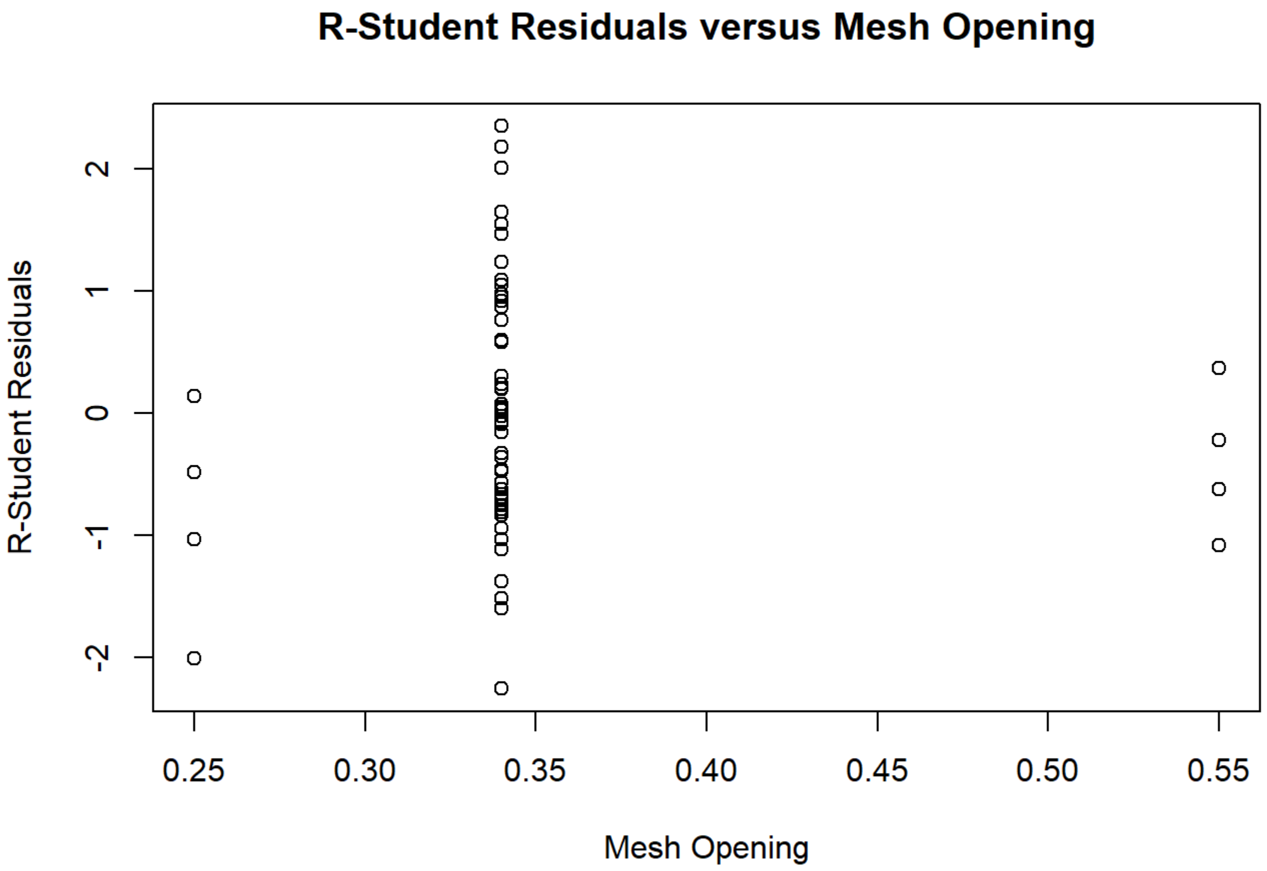
```
plot(fluid_vel, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Fluid Velocity", main
= "R-Student Residuals versus Fluid Velocity")
```



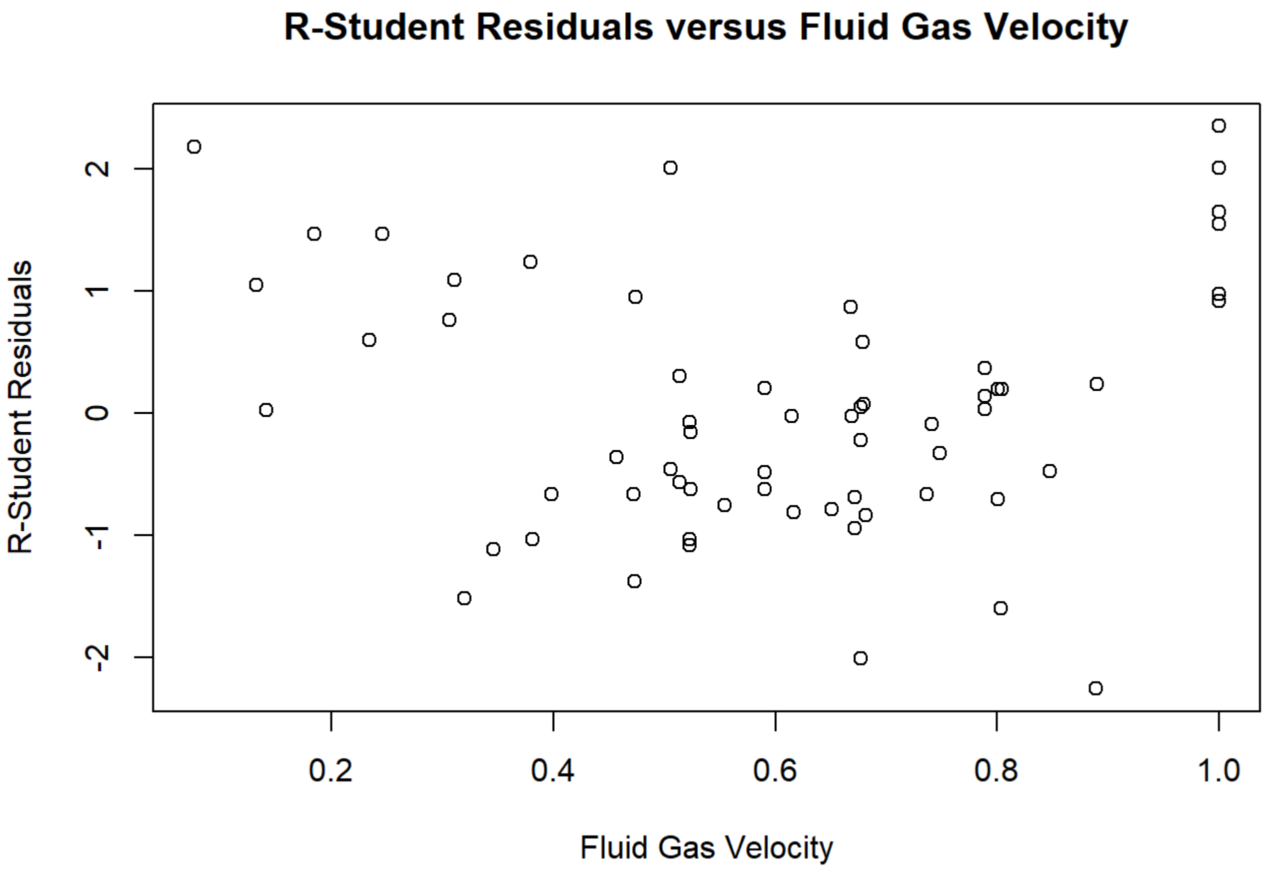
```
plot(viscosity, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Viscosity", main = "R
-Student Residuals versus Viscosity")
```



```
plot(mesh_open, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Mesh Opening", main =
"R-Student Residuals versus Mesh Opening")
```



```
plot(fluid_gas_vel, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Fluid Gas Velocit
y", main = "R-Student Residuals versus Fluid Gas Velocity")
```



Since there’s a huge gap in the residuals versus viscosity data, we’ll try a natural log on the regressor. Redo the exercise above, but, with the lm() using log of vicosity.

```
model <- lm(pressure_drop ~ fluid_vel + log(viscosity) + mesh_open + fluid_gas_vel)

summary(model)
```

Call: lm(formula = pressure_drop ~ fluid_vel + log(viscosity) + mesh_open + fluid_gas_vel)

Residuals: Min 1Q Median 3Q Max -13.4818 -3.5114 -0.2417 2.9716 12.8961

Coefficients: Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.8207 4.9007 -0.167 0.86760
fluid_vel -0.5492 0.3739 -1.469 0.14739
log(viscosity) 5.0960 0.5574 9.143 9.03e-13 ***mesh_open 28.0374 12.1975 2.299 0.02521***
fluid_gas_vel 10.3162 3.2096 3.214 0.00215 — Signif. codes: 0 ‘***’ 0.001 ’’** *0.01* ’ 0.05 ‘.’ 0.1 ’ ’ 1

Residual standard error: 5.514 on 57 degrees of freedom Multiple R-squared: 0.6267, Adjusted R-squared: 0.6005 F-statistic: 23.92 on 4 and 57 DF, p-value: 1.201e-11

```
xtable(summary(model))
```

	Estimate <dbl>	Std. Error <dbl>	t value <dbl>	Pr(>|t|) <dbl>
(Intercept)	-0.8206807	4.9007332	-0.1674608	8.676002e-01
fluid_vel	-0.5492321	0.3739394	-1.4687728	1.473920e-01
log(viscosity)	5.0960394	0.5573821	9.1428120	9.031359e-13
mesh_open	28.0374015	12.1974870	2.2986211	2.521404e-02
fluid_gas_vel	10.3162044	3.2095719	3.2141995	2.154800e-03

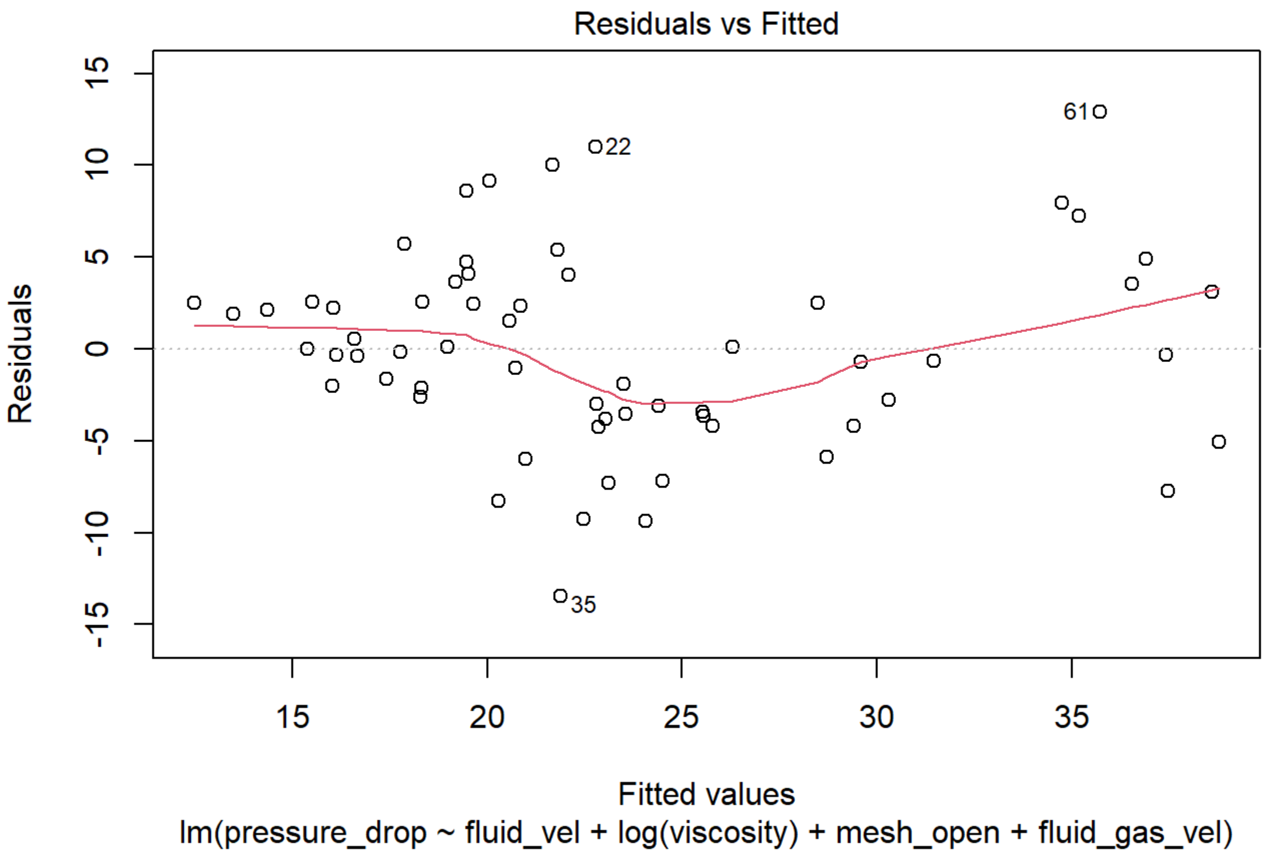
5 rows

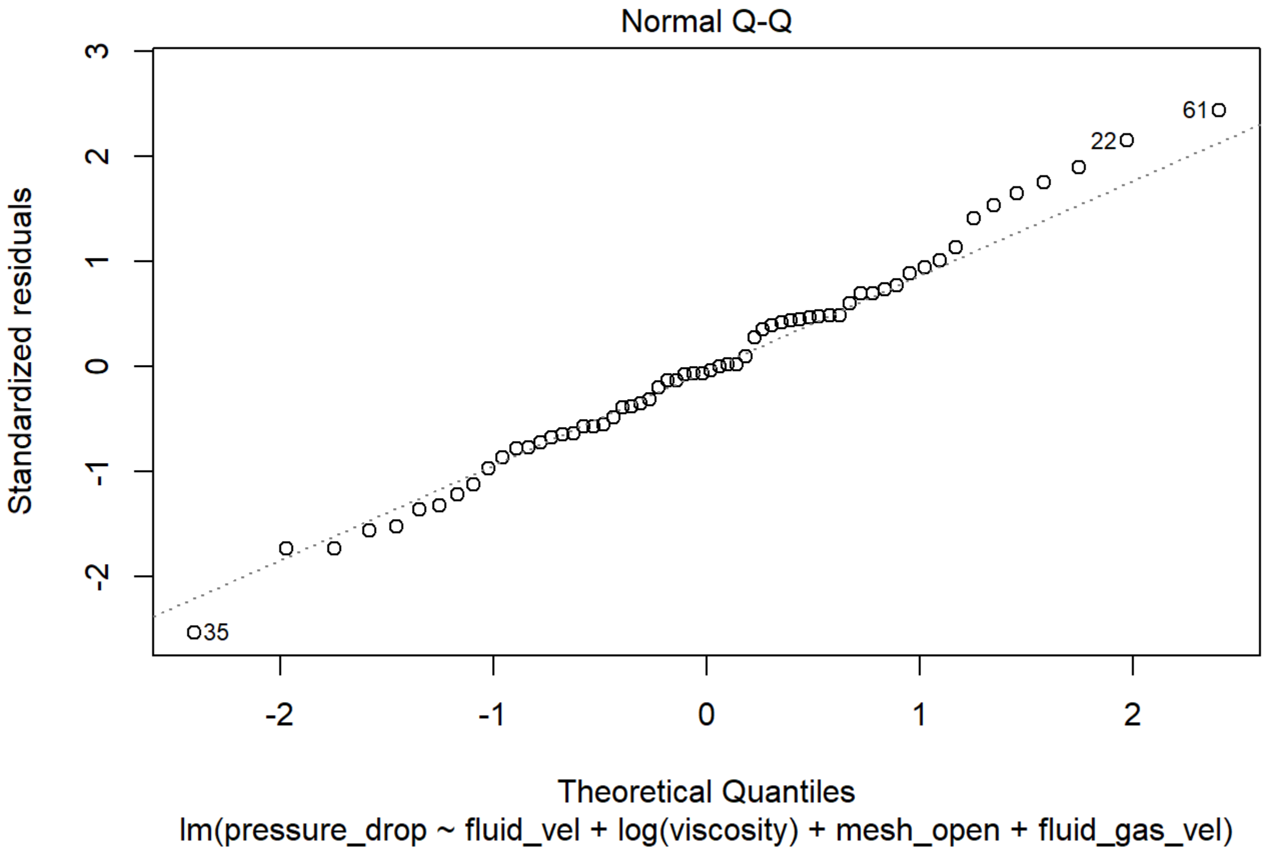
```
xtable(anova(model))
```

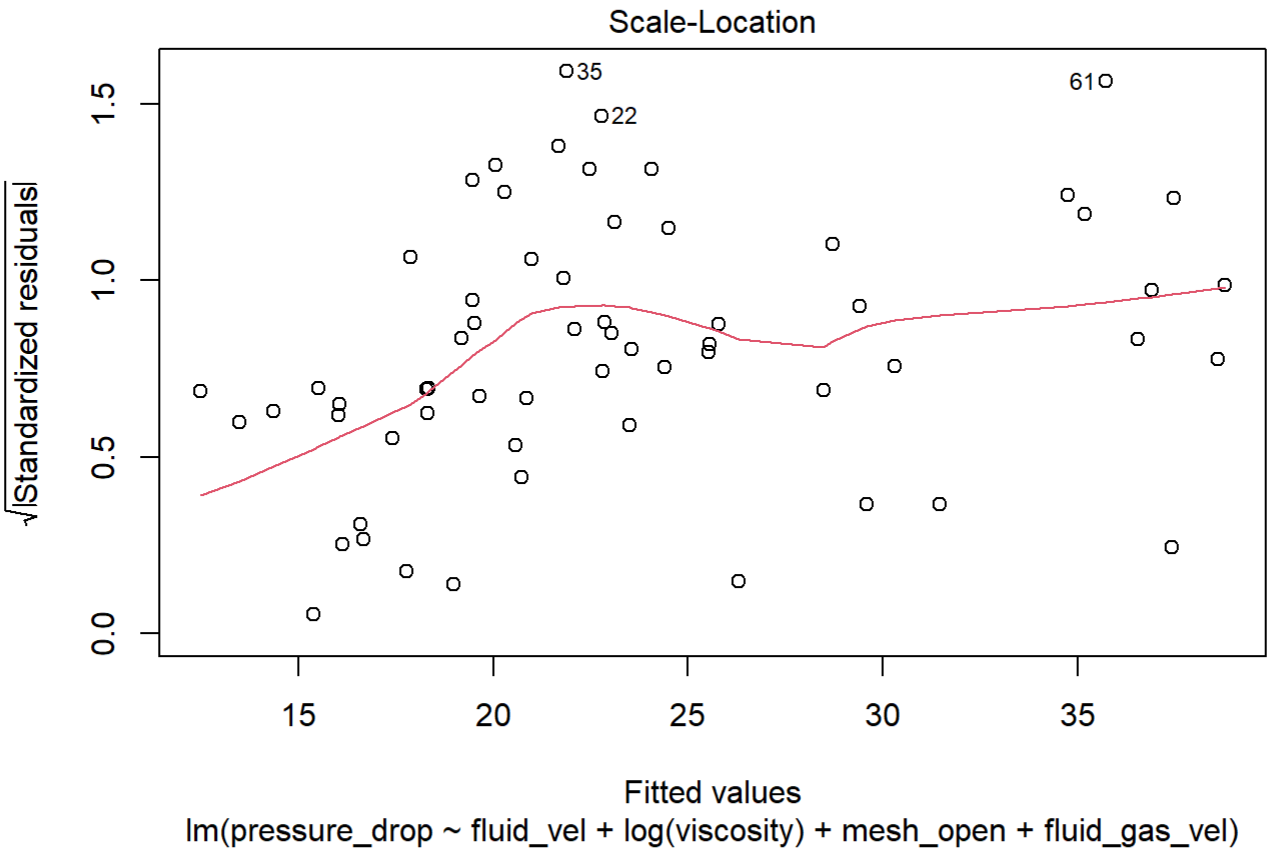
	Df <int>	Sum Sq <dbl>	Mean Sq <dbl>	F value <dbl>	Pr(>F) <dbl>
fluid_vel	1	9.595968	9.595968	0.3156013	5.764656e-01
log(viscosity)	1	2420.863436	2420.863436	79.6196421	2.063142e-12
mesh_open	1	164.821263	164.821263	5.4207973	2.346952e-02
fluid_gas_vel	1	314.120098	314.120098	10.3310783	2.154800e-03
Residuals	57	1733.105202	30.405354	NA	NA

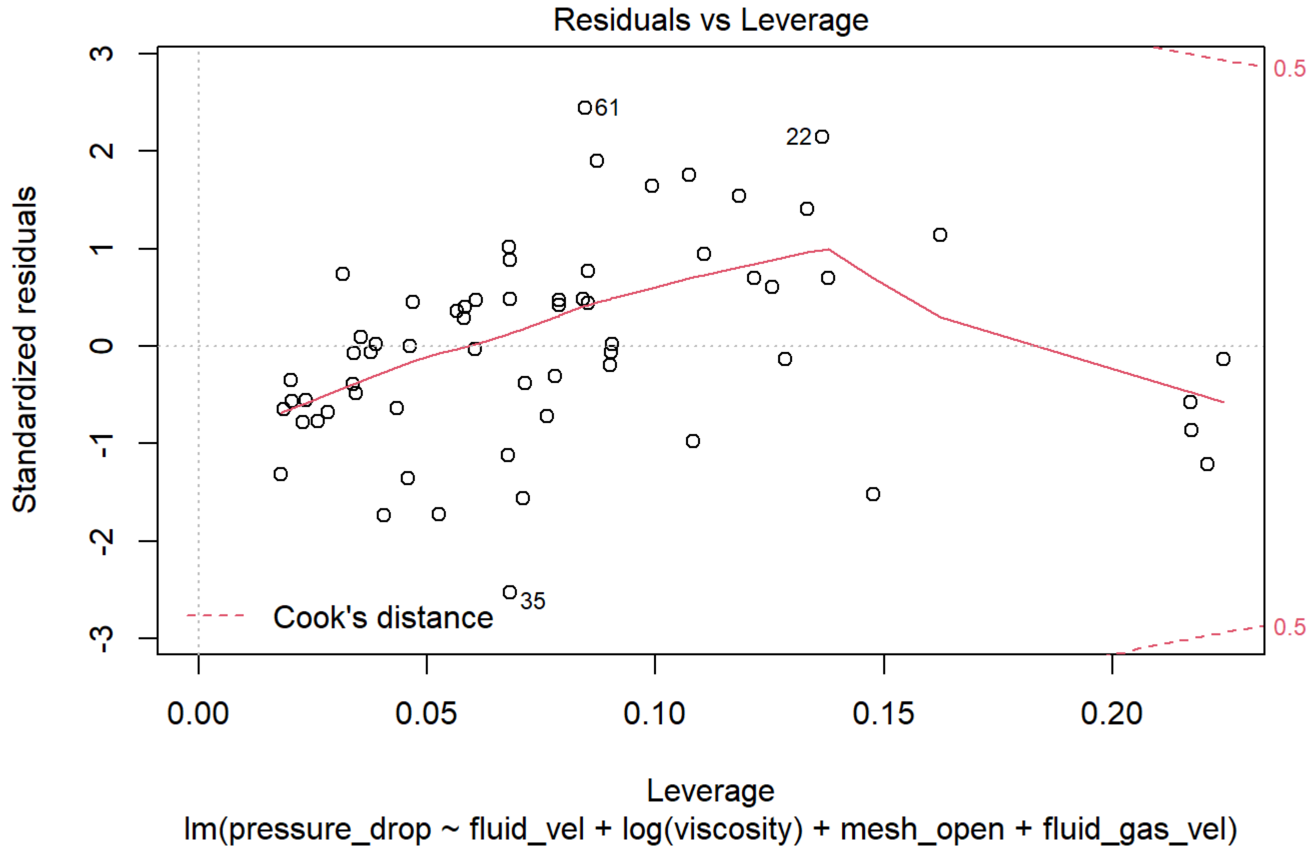
5 rows

```
plot(model)
```

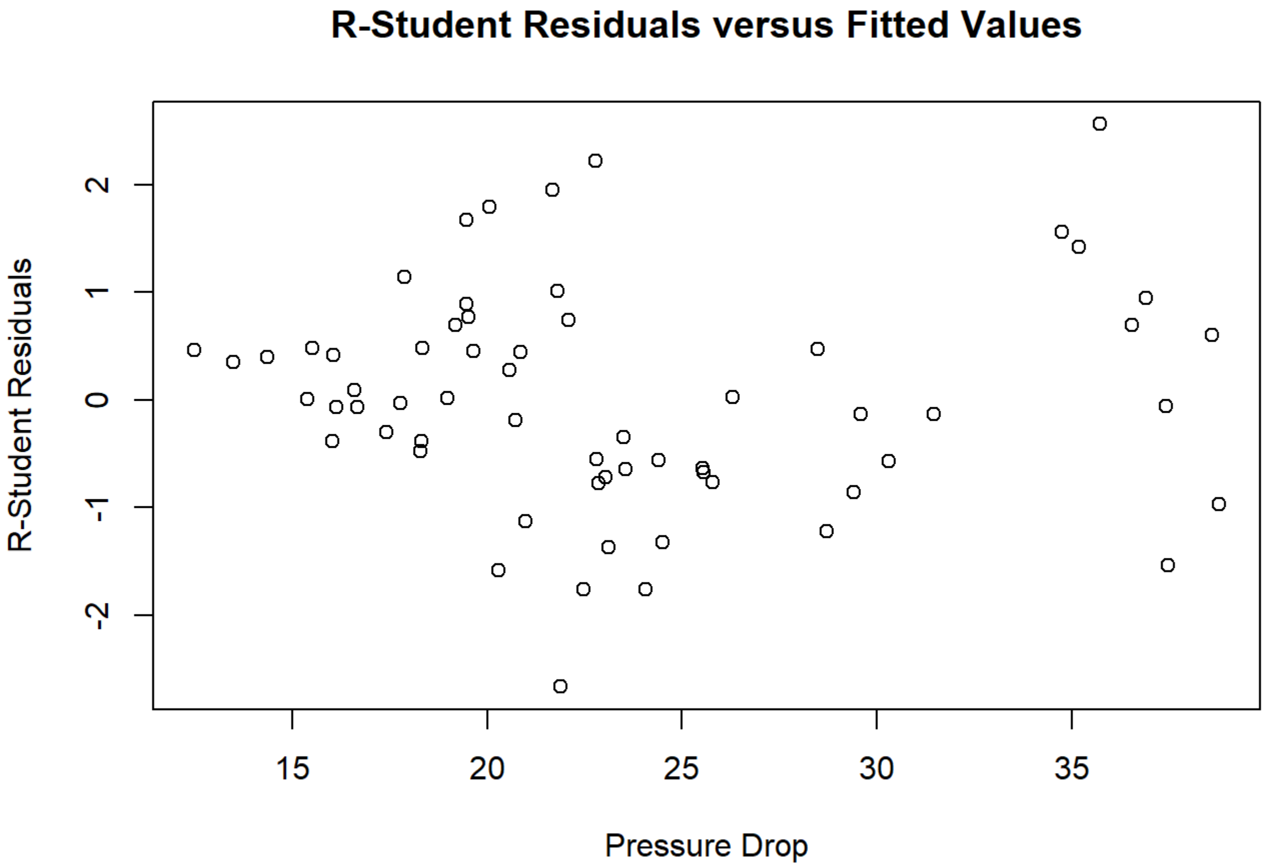

Residuals vs Fitted
Residuals
15
10
5
0
-5
-10
-15
15
20
25
30
35
61
22
35
Fitted values
lm(pressure_drop ~ fluid_vel + log(viscosity) + mesh_open + fluid_gas_vel)


Normal Q-Q
Standardized residuals
3
2
1
0
-1
-2
-2
-1
0
1
2
61
22
35
Theoretical Quantiles
lm(pressure_drop ~ fluid_vel + log(viscosity) + mesh_open + fluid_gas_vel)

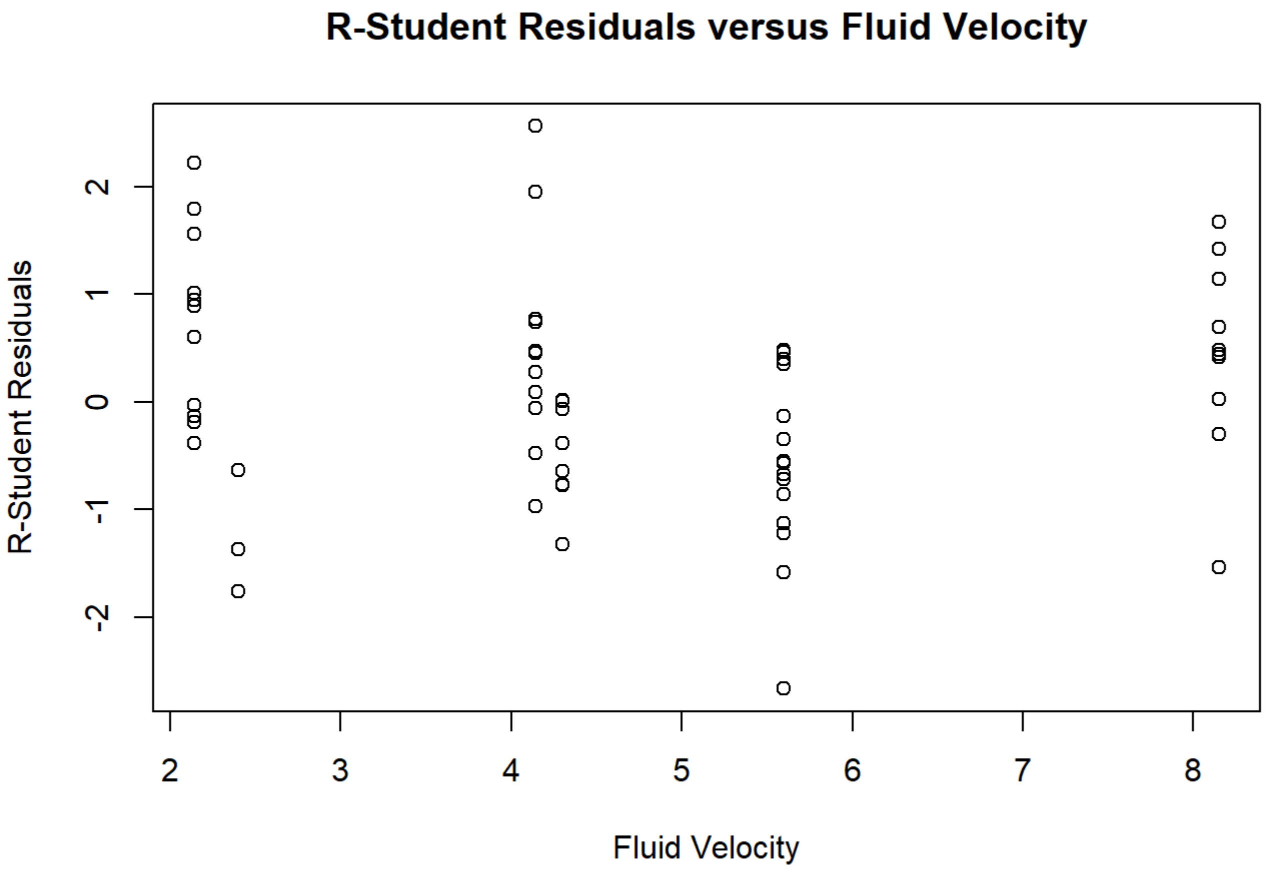

Scale-Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
15
20
25
30
35
35
22
61
Fitted values
lm(pressure_drop ~ fluid_vel + log(viscosity) + mesh_open + fluid_gas_vel)


Residuals vs Leverage
Standardized residuals
-3
-2
-1
0
1
2
3
0.00
0.05
0.10
0.15
0.20
61
22
35
0.5
0.5
Cook's distance
Leverage
lm(pressure_drop ~ fluid_vel + log(viscosity) + mesh_open + fluid_gas_vel)

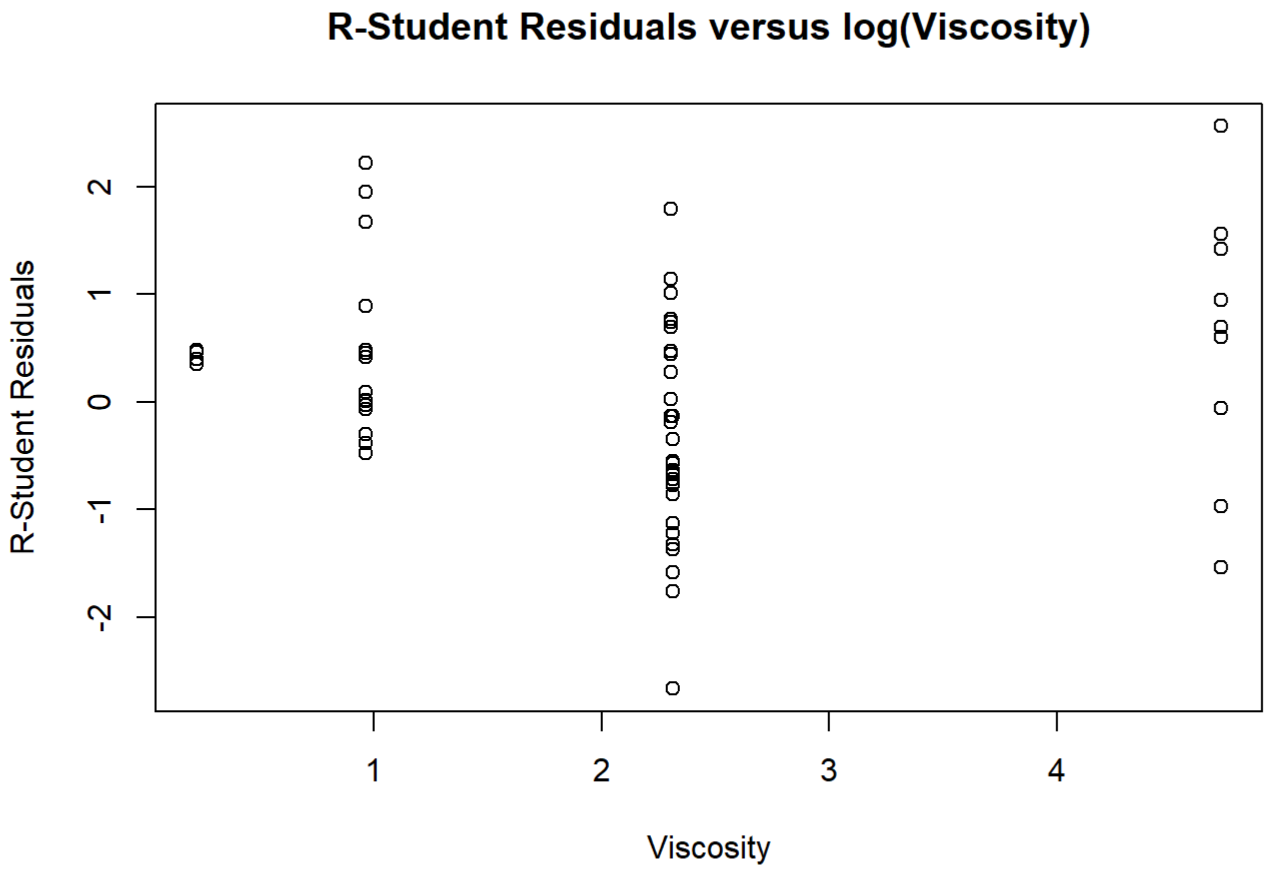
```
R_Student_Residuals <- rstudent(model)
y_hat <- model$fitted.values
plot(y_hat, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Pressure Drop", main = "R
-Student Residuals versus Fitted Values")
```



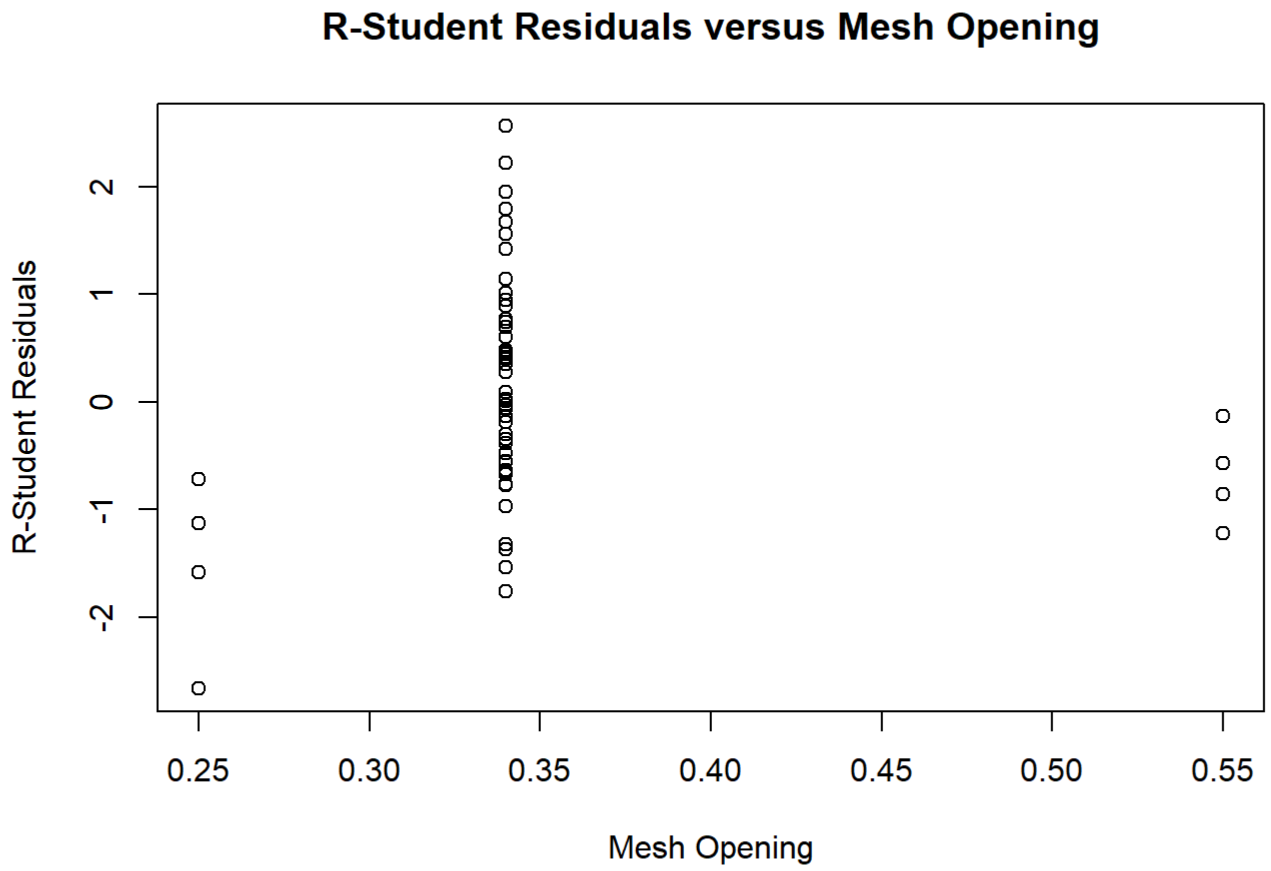
```
plot(fluid_vel, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Fluid Velocity", main
= "R-Student Residuals versus Fluid Velocity")
```



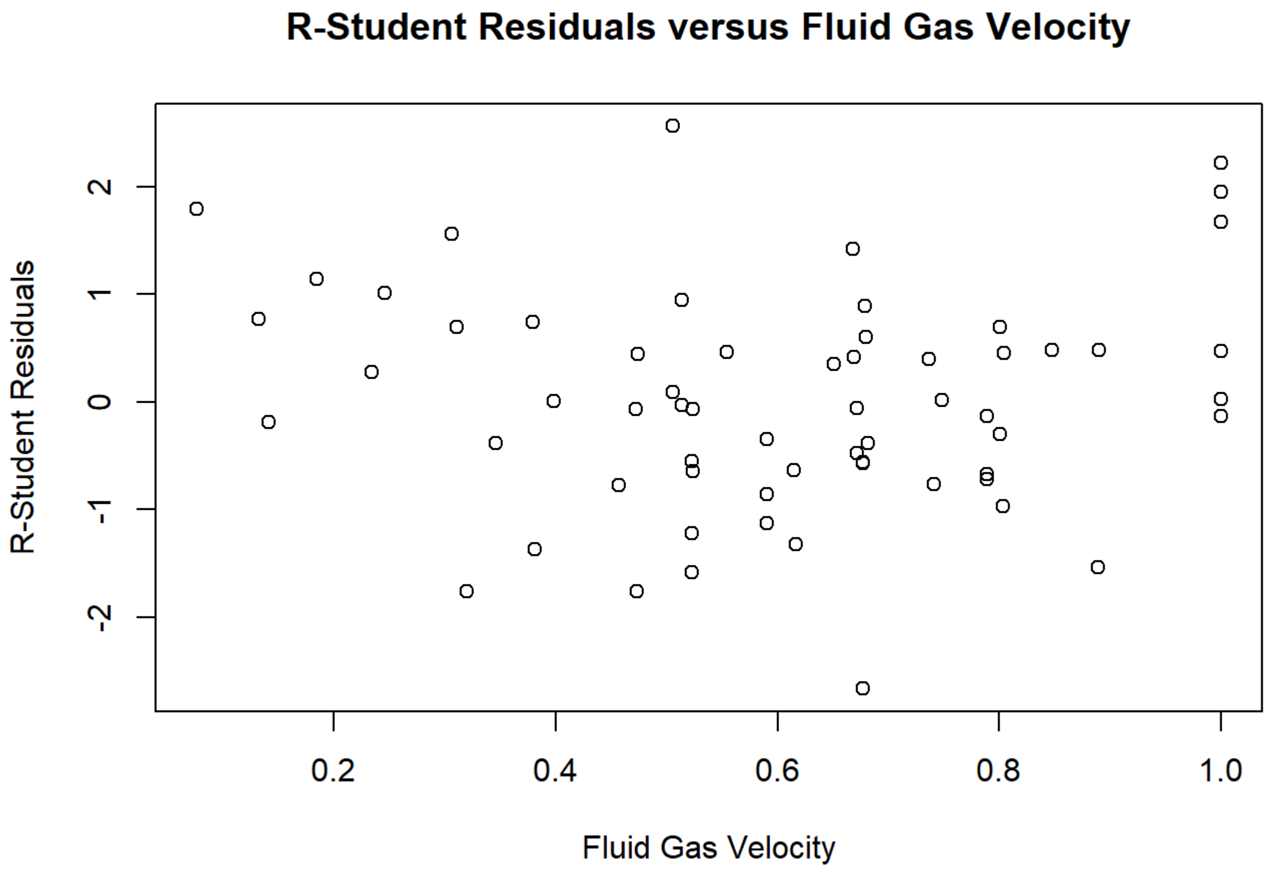
```
plot(log(viscosity), R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Viscosity", main
= "R-Student Residuals versus log(Viscosity)")
```



```
plot(mesh_open, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Mesh Opening", main =
"R-Student Residuals versus Mesh Opening")
```



```
plot(fluid_gas_vel, R_Student_Residuals, ylab = "R-Student Residuals", xlab = "Fluid Gas Velocity", main = "R-Student Residuals versus Fluid Gas Velocity")
```



Lesson 9 additions

```
exL2 <- read.xlsx("data-table-B13.xlsx",
                  sheetIndex = 1,
                  colIndex = c(2,3,4,5,6,7,8),
                  as.data.frame = TRUE,
                  header = TRUE)

names(exL2) <- c("thrust",
                 "primary",
                 "secondary",
                 "fuel",
                 "press",
                 "exhaust",
                 "ambient")
attach(exL2)

# Output data structure and dimensions
str(exL2)
```

```
## 'data.frame':    40 obs. of  7 variables:
##  $ thrust   : num  4540 4315 4095 3650 3200 ...
##  $ primary  : num  2140 2016 1905 1675 1474 ...
##  $ secondary: num  20640 20280 19860 18980 18100 ...
##  $ fuel     : num  30250 30010 29780 29330 28960 ...
##  $ press    : num  205 195 184 164 144 216 206 196 171 149 ...
##  $ exhaust  : num  1732 1697 1662 1598 1541 ...
##  $ ambient  : num  99 100 97 97 97 87 87 87 85 85 ...
```

```
dim(exL2)
```

```
## [1] 40  7
```

```
out <- as.data.frame(c(exL2))
colnames(out) <- c("thrust", "primary","secondary","fuel","press","exhaust","ambient")
tab <- xtable(out, digits=c(0,0,0,0,0,0,0,0))
print(tab, type="html")
```

	thrust	primary	secondary	fuel	press	exhaust	ambient
1	4540	2140	20640	30250	205	1732	99
2	4315	2016	20280	30010	195	1697	100
3	4095	1905	19860	29780	184	1662	97
4	3650	1675	18980	29330	164	1598	97
5	3200	1474	18100	28960	144	1541	97
6	4833	2239	20740	30083	216	1709	87
7	4617	2120	20305	29831	206	1669	87
8	4340	1990	19961	29604	196	1640	87
9	3820	1702	18916	29088	171	1572	85
10	3368	1487	18012	28675	149	1522	85
11	4445	2107	20520	30120	195	1740	101
12	4188	1973	20130	29920	190	1711	100
13	3981	1864	19780	29720	180	1682	100
14	3622	1674	19020	29370	161	1630	100
15	3125	1440	18030	28940	139	1572	101
16	4560	2165	20680	30160	208	1704	98
17	4340	2048	20340	29960	199	1679	96
18	4115	1916	19860	29710	187	1642	94
19	3630	1658	18950	29250	164	1576	94
20	3210	1489	18700	28890	145	1528	94
21	4330	2062	20500	30190	193	1748	101
22	4119	1929	20050	29960	183	1713	100
23	3891	1815	19680	29770	173	1684	100
24	3467	1595	18890	29360	153	1624	99
25	3045	1400	17870	28960	134	1569	100
26	4411	2047	20540	30160	193	1746	99
27	4203	1935	20160	29940	184	1714	99

28	3968	1807	19750	29760	173	1679	99
29	3531	1591	18890	29350	153	1621	99
30	3074	1388	17870	28910	133	1561	99
31	4350	2071	20460	30180	198	1729	102
32	4128	1944	20010	29940	186	1692	101
33	3940	1831	19640	29750	178	1667	101
34	3480	1612	18710	29360	156	1609	101
35	3064	1410	17780	28900	136	1552	101
36	4402	2066	20520	30170	197	1758	100
37	4180	1954	20150	29950	188	1729	99
38	3973	1835	19750	29740	178	1690	99
39	3530	1616	18850	29320	156	1616	99
40	3080	1407	17910	28910	137	1569	100

```
# Perform preliminary calculations that we might use at some point
N <- length(thrust)

X <- matrix(c(rep(1, N),
              exL2[, "primary"],
              exL2[, "secondary"],
              exL2[, "fuel"],
              exL2[, "press"],
              exL2[, "exhaust"],
              exL2[, "ambient"]), ncol = 7, byrow = FALSE) # Define X matrix of regressor observ
ations
xTx <- t(X) %*% X # Calulate the matrix product of X_Transpose and X
C <- ginv(xTx, tol=.Machine$double.eps) # Define C matrix
```

Jet Turbine Ex – lm() fit

```
model.jet <- lm(thrust ~ primary + secondary + fuel + press + exhaust + ambient)

model.jet
```

```
##
## Call:
## lm(formula = thrust ~ primary + secondary + fuel + press + exhaust +
##     ambient)
##
## Coefficients:
## (Intercept)      primary    secondary         fuel        press      exhaust
##  -4.738e+03    1.119e+00   -3.018e-02    2.306e-01    3.850e+00    8.219e-01
##     ambient
##  -1.695e+01
```

```
summary(model.jet)
```

```
##
## Call:
## lm(formula = thrust ~ primary + secondary + fuel + press + exhaust +
##     ambient)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -49.949 -19.028  -1.572  17.139  49.606
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.738e+03  2.445e+03  -1.938 0.061213 .
## primary      1.119e+00  2.865e-01   3.904 0.000441 ***
## secondary   -3.018e-02  3.823e-02  -0.789 0.435478
## fuel         2.306e-01  1.180e-01   1.954 0.059231 .
## press        3.850e+00  2.686e+00   1.433 0.161246
## exhaust      8.219e-01  3.508e-01   2.343 0.025298 *
## ambient     -1.695e+01  2.620e+00  -6.468 2.45e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.51 on 33 degrees of freedom
## Multiple R-squared:  0.9977, Adjusted R-squared:  0.9972
## F-statistic:  2350 on 6 and 33 DF,  p-value: < 2.2e-16
```

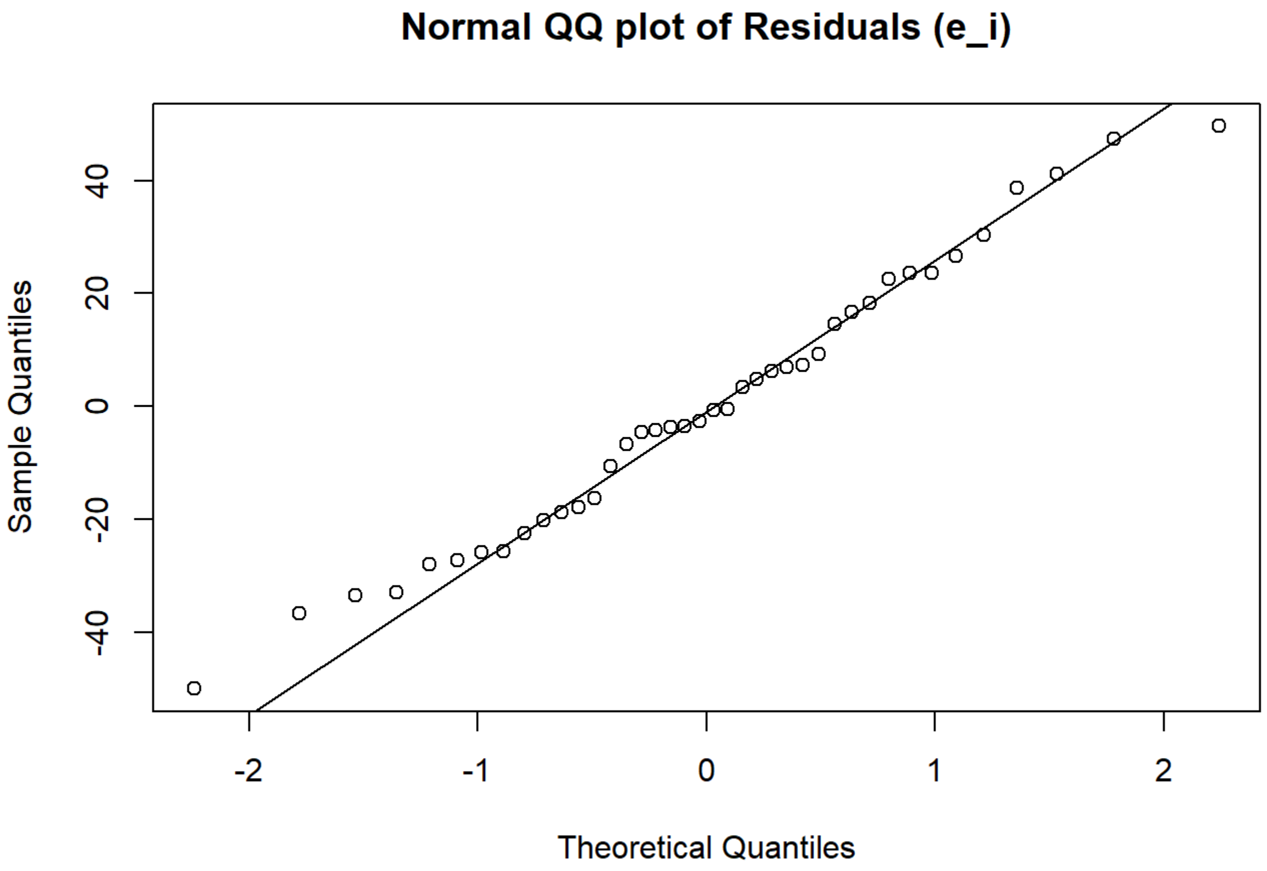
```
anova(model.jet)
```

	Df <int>	Sum Sq <dbl>	Mean Sq <dbl>	F value <dbl>	Pr(>F) <dbl>
primary	1	9833159.525	9833159.5248	13993.072038	5.900797e-45
secondary	1	5992.318	5992.3175	8.527364	6.262953e-03
fuel	1	25818.683	25818.6826	36.741261	8.052267e-07
press	1	4706.945	4706.9453	6.698216	1.424044e-02
exhaust	1	9771.674	9771.6745	13.905576	7.209539e-04
ambient	1	29397.218	29397.2181	41.833694	2.450406e-07
Residuals	33	23189.637	702.7163	NA	NA

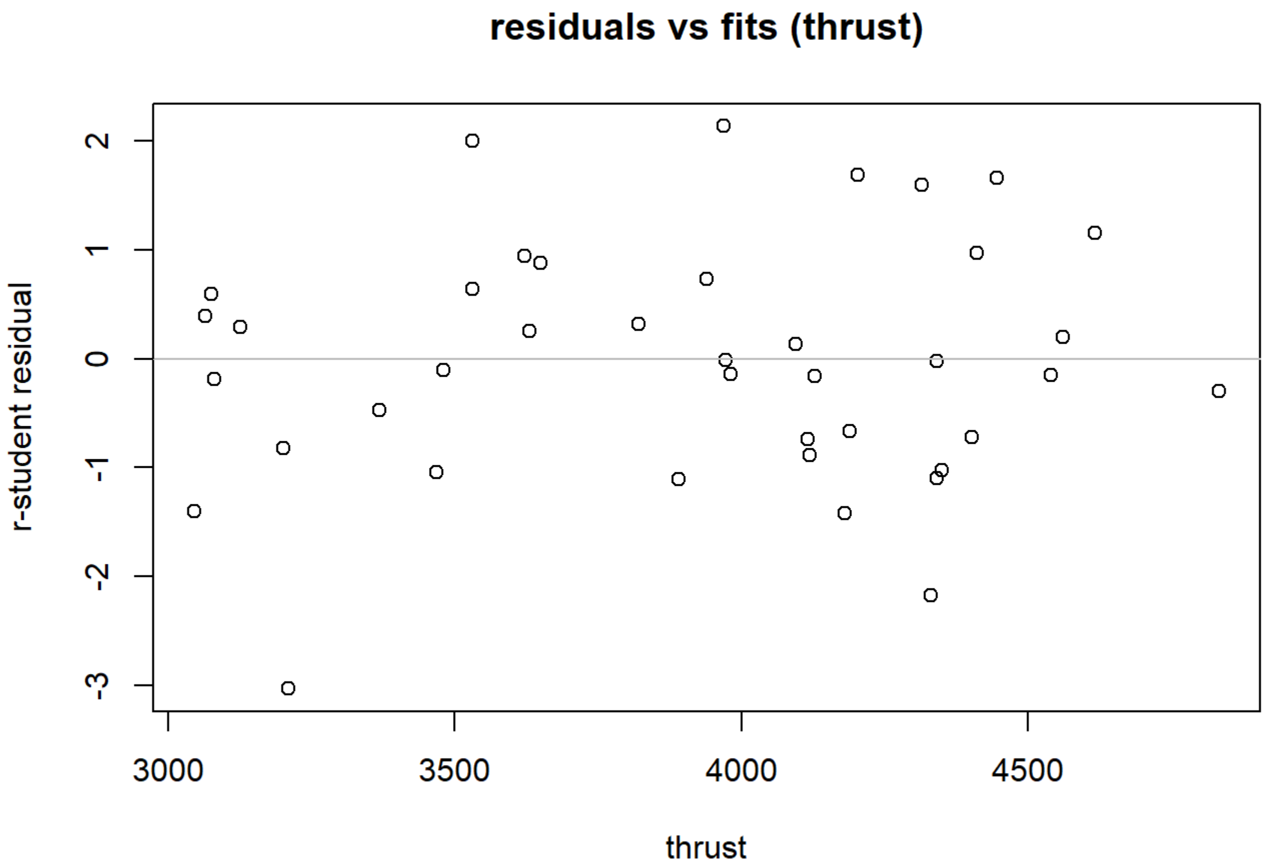
7 rows

```
df_T <- N - 1
df_Regression <- length(model.jet$coefficients) - 1 # Number of predictor variables, two in this case
k <- df_Regression # Sometimes helpful to use the variable
p <- k + 1 # Same reasoning
df_Residual <- df_T - df_Regression

y_observed <- exL2[, "thrust"] # Create vector of dependent variable observations
y_predicted <- (X %*% ginv(t(X)%*%X,tol=.Machine$double.eps) %*% t(X)) %*% y_observed # Hat matrix approach, just for grins.
y_bar <- mean(y_observed)

xTy <- t(X) %*% y_observed # Define rhs of least squares normal equations in matrix form
BetaCoefficients <- C %*% xTy # Calculate using ginv()
ResidualVector <- y_observed - X%*%BetaCoefficients # Define vector of residuals for each observation
SS_Residuals <- t(ResidualVector) %*% ResidualVector # Dot product of residual vector and its transpose

SST_dt <- sum((y_observed - y_bar)^2)

SS_Regression_dt <- sum((y_predicted - y_bar)^2)
SS_Residual_dt <- SST_dt - SS_Regression_dt

MS_Regression <- SS_Regression_dt / df_Regression
MS_Residual <- SS_Residual_dt / df_Residual

F_test <- MS_Regression / MS_Residual
t_test <- BetaCoefficients / sqrt(MS_Residual * diag(C)) # Formula on p.88

significanceLevel <- 0.05
F_critical <- qf(1-significanceLevel/2, df_Regression, df_Residual)
t_critical <- qt(1-significanceLevel/2, df_Residual)
#p_val <- 2*(1 - pt(t_test, df_Residual)) # Multiply by two since it is a two-tailed hypothesis test

# compare with my calculations
alpha = 0.05

tcritical_value = abs(qt(alpha/2, df = df_Residual))
p_values = 2*pt(-abs(t_test), df = df_Residual)
```

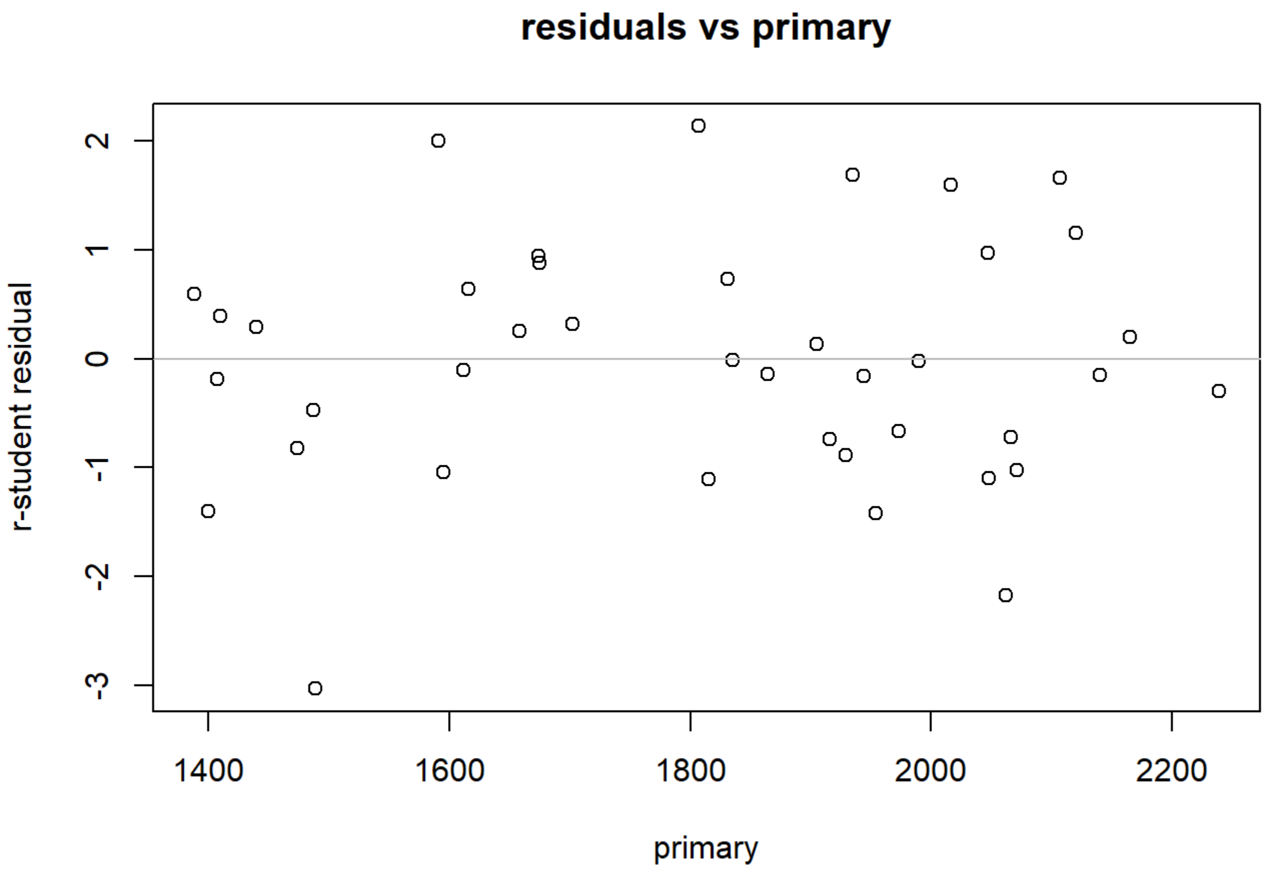
```
qqnorm(model.jet$residuals,main="Normal QQ plot of Residuals (e_i)")
qqline(model.jet$residuals)
```



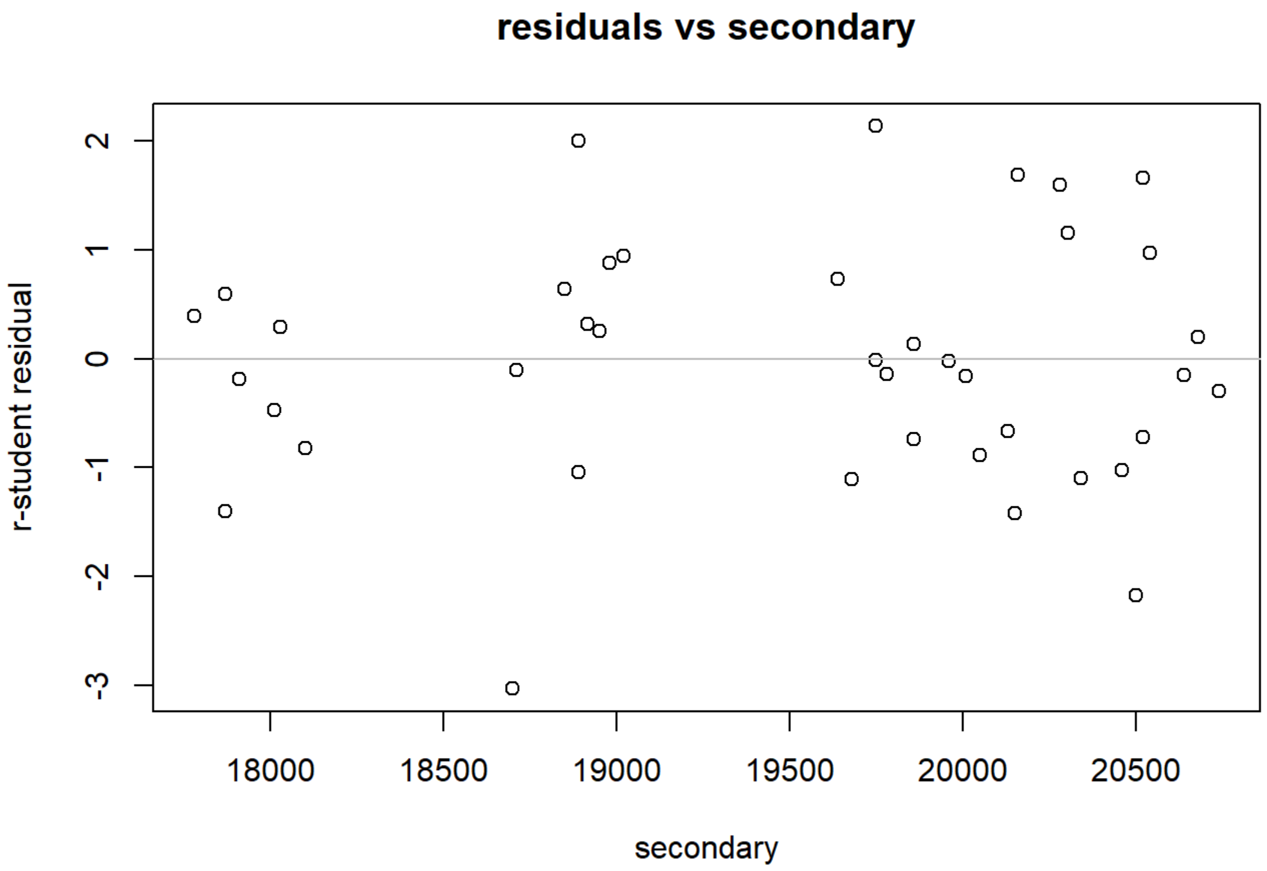
```
plot(thrust,rstudent(model.jet), main="residuals vs fits (thrust)",
     ylab="r-student residual")
abline(0, 0, col="gray")
```



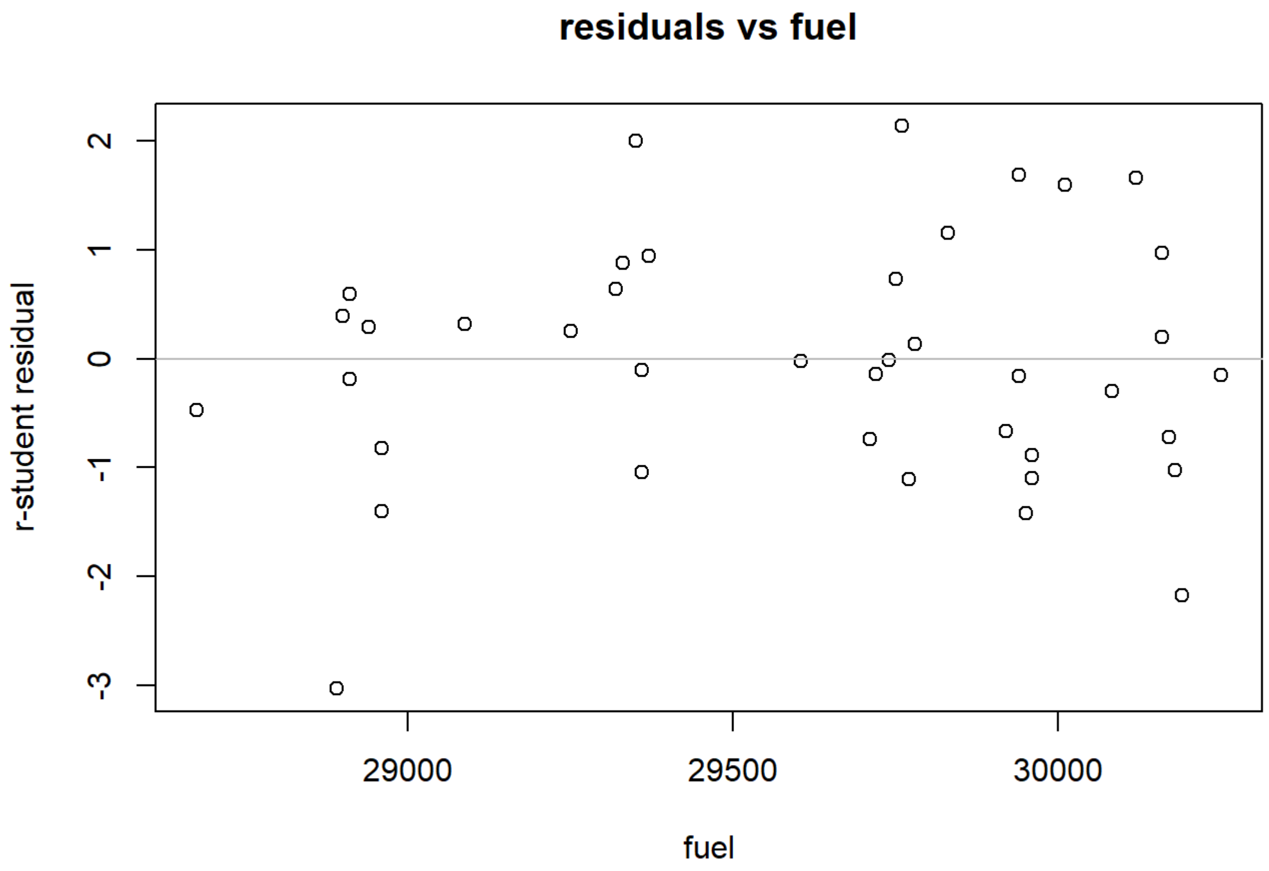
```
plot(primary,rstudent(model.jet), main="residuals vs primary",
     ylab="r-student residual")
abline(0, 0, col="gray")
```



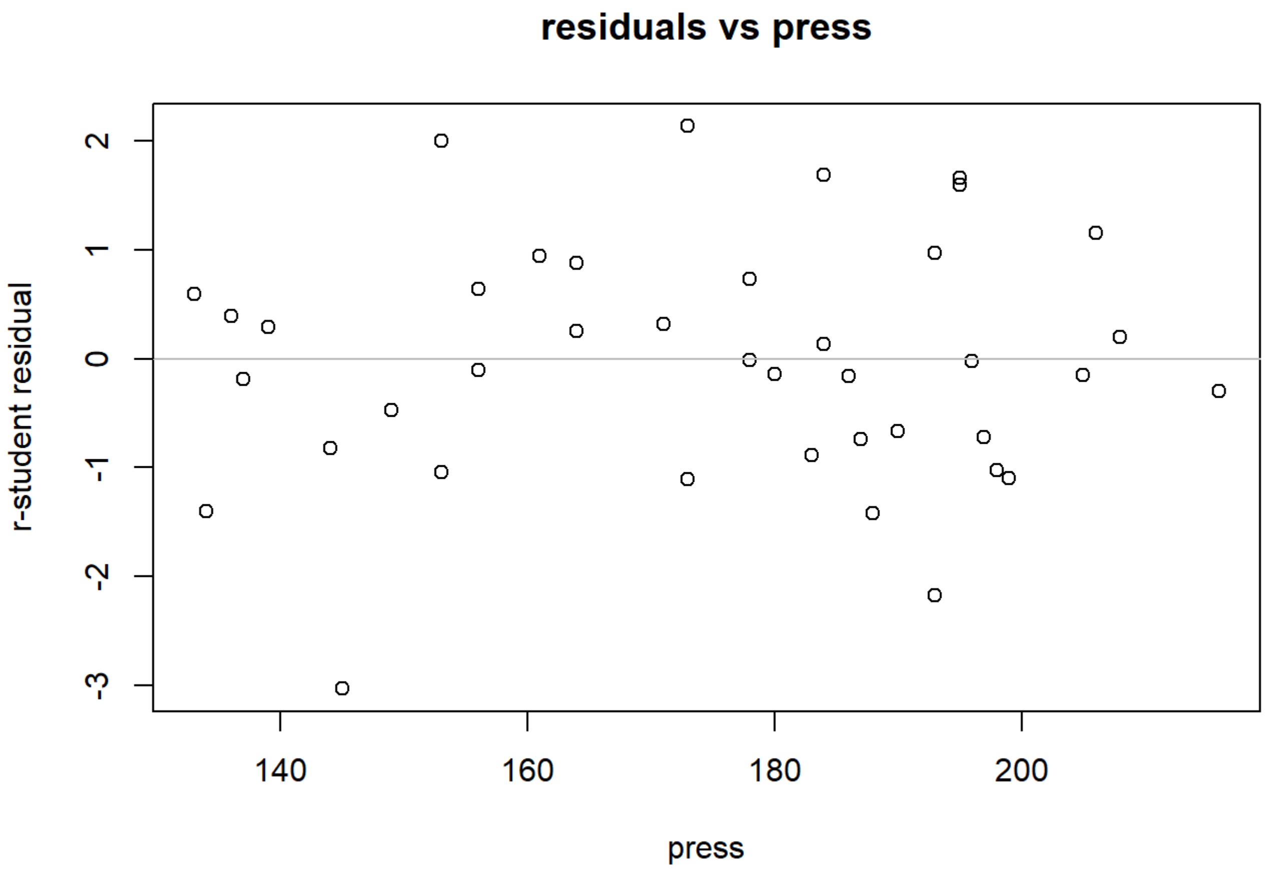
```
plot(secondary,rstudent(model.jet), main="residuals vs secondary",
     ylab="r-student residual")
abline(0, 0, col="gray")
```



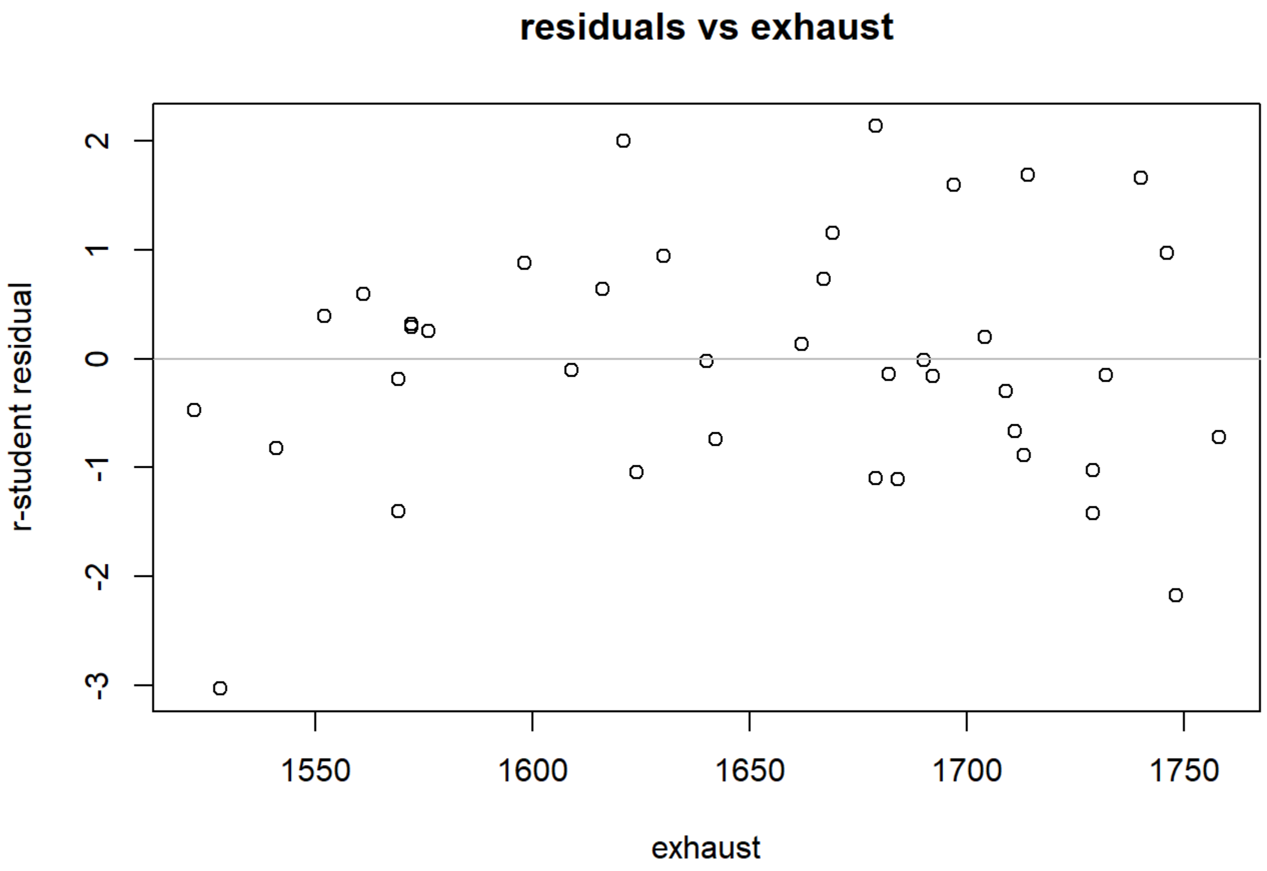
```
plot(fuel,rstudent(model.jet), main="residuals vs fuel",
     ylab="r-student residual")
abline(0, 0, col="gray")
```



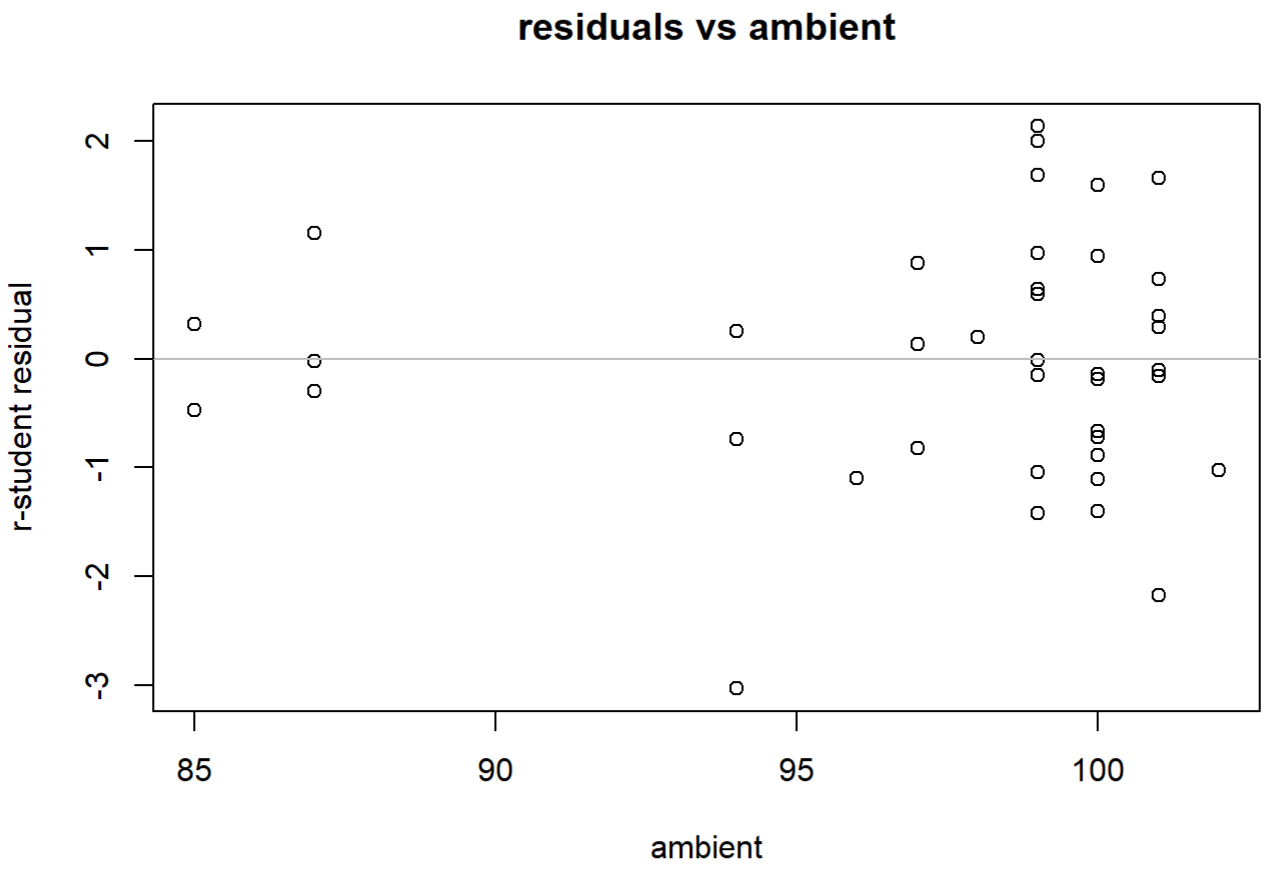
```
plot(press,rstudent(model.jet), main="residuals vs press",
     ylab="r-student residual")
abline(0, 0, col="gray")
```



```
plot(exhaust,rstudent(model.jet), main="residuals vs exhaust",
     ylab="r-student residual")
abline(0, 0, col="gray")
```



```
plot(ambient,rstudent(model.jet), main="residuals vs ambient",
     ylab="r-student residual")
abline(0, 0, col="gray")
```


residuals vs ambient
r-student residual
2
1
0
-1
-2
-3
85
90
95
100
ambient