L15Ex_JetTurbine_Rick_Davila

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comment out – use in the case where “html” is used

```{r setup, include=TRUE, results=“asis”}

Perform data housekeeping - upload, name columns, display to make sure it reads properly, etc.

```
knitr::opts_chunk$set(echo = TRUE)

#Sys.setenv(JAVA_HOME='C:\\Program Files\\Java\\jdk-14.0.1') # for 64-bit version
#library(rJava)

library("xlsx") # Needed to read data
```

```
## Warning: package 'xlsx' was built under R version 4.0.4
```

```
library(MuMIn)
```

```
## Warning: package 'MuMIn' was built under R version 4.0.3
```

```
library(MASS)   # Needed for ginv() function


jt_data <- read.xlsx("data-table-B13.xlsx", sheetIndex = 1, sheetName=NULL, rowIndex=NULL, startRow=NULL, endRow=NULL, colIn
dex= NULL, as.data.frame=TRUE, header=TRUE, colClasses=NA, keepFormulas=FALSE, encoding="unknown")

# Give labels to data columns
names(jt_data) <- c("thrust", "primary", "secondary", "fuel", "press", "exhaust", "ambient")
attach(jt_data)

# Output data to make sure it reads properly
jt_data # Note: On the spreadsheet provided on the course website, Observation #7 is 580 but in Table 8.1 and subsequent tex
tbook results tables it is 680.
```

thrust <dbl>	primary <dbl>	secondary <dbl>	fuel <dbl>	press <dbl>	exhaust <dbl>	ambient <dbl>
4540	2140	20640	30250	205	1732	99
4315	2016	20280	30010	195	1697	100
4095	1905	19860	29780	184	1662	97
3650	1675	18980	29330	164	1598	97
3200	1474	18100	28960	144	1541	97
4833	2239	20740	30083	216	1709	87
4617	2120	20305	29831	206	1669	87
4340	1990	19961	29604	196	1640	87
3820	1702	18916	29088	171	1572	85
3368	1487	18012	28675	149	1522	85

1-10 of 40 rows Previous **1** 2 3 4 Next

```
# Output data structure and dimensions
str(jt_data)
```

```
## 'data.frame':    40 obs. of  7 variables:
##  $ thrust   : num  4540 4315 4095 3650 3200 ...
##  $ primary  : num  2140 2016 1905 1675 1474 ...
##  $ secondary: num  20640 20280 19860 18980 18100 ...
##  $ fuel     : num  30250 30010 29780 29330 28960 ...
##  $ press    : num  205 195 184 164 144 216 206 196 171 149 ...
##  $ exhaust  : num  1732 1697 1662 1598 1541 ...
##  $ ambient  : num  99 100 97 97 97 87 87 87 85 85 ...
```

```
dim(jt_data)
```
```

```
## [1] 40  7
```

```
jt_lm <- lm(thrust~primary+secondary+fuel+press+exhaust+ambient, data=jt_data, na.action = "na.fail") # Linear model of raw
 data

# Use dredge() function to automatically perform all regressors regression
combinations <- dredge(jt_lm, extra = c(R_Sq = function(x) summary(x)$r.squared,R_Sq_Adj = function(x) summary(x)$adj.r.squa
red, MS_Res = function(x) summary(x)$sigma^2,Cp, MallowCp = function(x) summary(x)$sigma^2*df.residual(x)/summary(jt_lm)$sig
ma^2-dim(jt_data)[1]+2*length(x$coefficients)))
```

```
## Fixed term is "(Intercept)"
```

```
print(combinations)
```

```
## Global model call: lm(formula = thrust ~ primary + secondary + fuel + press + exhaust +
##     ambient, data = jt_data, na.action = "na.fail")
## ---
## Model selection table
##      (Intrc)    ambnt    exhst     fuel     press  prmry     scndr    R_Sq
## 24  -4280.00 -17.5100  0.64670  0.20980           1.4420           0.99750
## 32  -3982.00 -16.2800  0.83430  0.18430  3.746000 1.0960           0.99760
## 28     37.62 -12.9900  1.26000           4.608000 1.2410           0.99740
## 22  -7018.00 -17.8400           0.34430           1.3630           0.99720
## 20    366.80 -13.9800  1.09700                    1.7060           0.99720
## 56  -4976.00 -18.1500  0.63070  0.25270           1.4710 -0.027500 0.99750
## 64  -4738.00 -16.9500  0.82190  0.23060  3.850000 1.1190 -0.030180 0.99770
## 54  -7763.00 -18.5900           0.39120           1.4000 -0.032610 0.99730
## 30  -7190.00 -17.4500           0.34900  1.296000 1.2360           0.99720
## 60    -21.22 -13.0300  1.23800           4.534000 1.2280  0.006968 0.99740
## 52    233.90 -14.0200  1.05700                    1.6620  0.014440 0.99720
## 62  -7993.00 -18.1900           0.39870  1.455000 1.2580 -0.034210 0.99730
## 16  -7205.00 -17.3000  1.11100  0.29530 12.700000                  0.99660
## 48  -7630.00 -17.6600  1.10700  0.32040 12.850000        -0.015610 0.99660
## 50    417.20  -9.1780                             1.7520  0.062050 0.99600
## 18   1103.00  -8.0730                             1.9820           0.99570
## 12   -947.30 -11.8200  1.91600          16.260000                  0.99590
## 14 -12230.00 -19.1100           0.54380 10.880000                  0.99580
## 44  -1224.00 -12.1000  1.74900          15.110000         0.040110 0.99600
## 26   1106.00  -8.0950                   -0.055790 1.9870           0.99570
## 58    417.00  -9.1760                    0.004801 1.7520  0.062050 0.99600
## 46 -12720.00 -19.5300           0.57300 11.060000        -0.018710 0.99580
## 31   6870.00           1.29100 -0.31060 10.020000 1.2970           0.99460
## 63   7381.00           1.28100 -0.35350  9.420000 1.2480  0.049720 0.99470
## 25    218.70                            10.930000 0.9826           0.99310
## 29   2961.00                   -0.10270  6.796000 1.5450           0.99360
## 23   8473.00           0.82290 -0.34710           2.4020           0.99340
## 21   5281.00                   -0.18860           2.3250           0.99300
## 55   9079.00           0.84830 -0.40640           2.2340  0.072520 0.99370
## 61   3536.00                   -0.14990  6.193000 1.4900  0.052690 0.99370
## 53   5764.00                   -0.24040           2.1620  0.068900 0.99320
## 57    350.80                            10.680000 1.0430 -0.010150 0.99320
## 27    138.00           0.06337          11.500000 0.9145           0.99320
## 15   3829.00           1.65700 -0.21490 21.210000                  0.99310
## 47   4712.00           1.62200 -0.28030 19.770000         0.069820 0.99340
## 11   -612.30           0.63980          19.820000                  0.99230
## 59    300.60           0.13330          11.650000 0.9545 -0.019340 0.99320
## 42  -1354.00  -4.8180                   15.080000         0.158800 0.99270
## 43   -668.10           0.59880          19.600000         0.008317 0.99230
## 41   -931.10                            18.060000         0.086400 0.99170
## 19   1086.00          -0.65240                    2.1520           0.99160
## 13  -2259.00                    0.09179 19.750000                  0.99130
## 45  -1118.00                    0.01063 18.150000         0.079010 0.99170
## 51   1117.00          -0.64180                    2.1620 -0.003383 0.99160
## 9     165.00                            21.430000                  0.99020
## 49   1283.00                                      2.2760 -0.076800 0.99060
## 17    296.90                                      1.9930           0.99000
## 10    106.00   0.5735                   21.440000                  0.99030
## 6  -24590.00 -39.1200           1.09100                            0.99010
## 38 -21610.00 -35.8900           0.92750                   0.079720 0.99040
## 8  -25320.00 -38.7400 -0.27400  1.13000                            0.99010
## 40 -22200.00 -35.7700 -0.17780  0.95890                   0.076670 0.99040
## 36  -4496.00 -24.8300  1.73800                            0.407700 0.98300
## 34  -4619.00 -17.5700                                     0.525000 0.97960
## 35  -5740.00          -1.70500                            0.639200 0.96030
## 37   1800.00                   -0.40070                   0.716500 0.96020
## 39  -1870.00          -0.97730 -0.20330                   0.687700 0.96090
## 33  -6301.00                                              0.523500 0.95270
## 4   -3585.00 -48.3100  7.38300                                     0.94520
## 7  -39690.00          -5.28600  1.76800                            0.88760
## 5  -25860.00                    1.00500                            0.86270
## 3   -6643.00           6.38500                                     0.75980
## 1    3904.00                                                       0.00000
## 2    5441.00 -15.7700                                              0.02174
##     R_Sq_Adj   MS_Res       Cp  MallowCp df   logLik  AICc  delta weight
## 24  0.997200    714.2    32140     5.572  6 -185.510 385.6   0.00  0.239
## 32  0.997300    694.9    31970     5.623  7 -184.383 386.3   0.70  0.169
## 28  0.997100    740.1    33300     6.862  6 -186.223 387.0   1.42  0.117
## 22  0.997000    772.3    33980     7.566  5 -187.639 387.0   1.48  0.114
## 20  0.996900    779.0    34280     7.909  5 -187.811 387.4   1.82  0.096
## 56  0.997200    724.5    33330     7.054  7 -185.217 387.9   2.37  0.073
## 64  0.997200    702.7    33030     7.000  8 -184.009 388.7   3.10  0.051
## 54  0.996900    779.7    35090     8.834  6 -187.265 389.1   3.51  0.041
## 30  0.996900    788.9    35500     9.293  6 -187.500 389.5   3.98  0.033
## 60  0.997000    761.0    35000     8.818  7 -186.199 389.9   4.33  0.027
## 52  0.996900    797.4    35880     9.716  6 -187.714 390.0   4.41  0.026
```

```
## 62  0.996900    795.5    36590    10.490  7 -187.087 391.7   6.11  0.011
## 16  0.996200    972.0    43740    18.410  6 -191.674 397.9  12.33  0.001
## 48  0.996100    997.1    45870    20.240  7 -191.605 400.7  15.14  0.000
## 50  0.995700   1105.0    48600    24.590  5 -194.795 401.4  15.79  0.000
## 18  0.995500   1152.0    49530    26.650  4 -196.183 401.5  15.94  0.000
## 12  0.995600   1121.0    49340    25.450  5 -195.097 402.0  16.39  0.000
## 14  0.995500   1150.0    50590    26.900  5 -195.597 403.0  17.39  0.000
## 44  0.995600   1122.0    50510    25.910  6 -194.553 403.7  18.08  0.000
## 26  0.995400   1184.0    52090    28.650  5 -196.183 404.1  18.56  0.000
## 58  0.995500   1136.0    51130    26.590  6 -194.795 404.1  18.57  0.000
## 46  0.995400   1178.0    53000    28.660  6 -195.514 405.6  20.01  0.000
## 31  0.994000   1540.0    69320    46.720  6 -200.883 416.3  30.74  0.000
## 63  0.993900   1547.0    71150    46.830  7 -200.384 418.3  32.70  0.000
## 25  0.992800   1840.0    79130    62.900  4 -205.553 420.2  34.68  0.000
## 29  0.993000   1774.0    78040    58.860  5 -204.266 420.3  34.73  0.000
## 23  0.992900   1809.0    79600    60.680  5 -204.661 421.1  35.52  0.000
## 21  0.992600   1884.0    80990    65.180  4 -206.017 421.2  35.61  0.000
## 55  0.993000   1778.0    80000    58.540  6 -203.748 422.0  36.48  0.000
## 61  0.993000   1782.0    80180    58.740  6 -203.793 422.1  36.57  0.000
## 53  0.992700   1863.0    81970    63.440  5 -205.248 422.3  36.70  0.000
## 57  0.992600   1889.0    83120    64.770  5 -205.526 422.8  37.25  0.000
## 27  0.992600   1890.0    83140    64.810  5 -205.533 422.8  37.26  0.000
## 15  0.992500   1907.0    83890    65.680  5 -205.712 423.2  37.62  0.000
## 47  0.992600   1885.0    84830    63.890  6 -204.922 424.4  38.82  0.000
## 11  0.991900   2056.0    88410    74.260  4 -207.769 424.7  39.11  0.000
## 59  0.992400   1937.0    87150    66.460  6 -205.462 425.5  39.90  0.000
## 42  0.992000   2025.0    89110    71.760  5 -206.920 425.6  40.04  0.000
## 43  0.991700   2112.0    92920    76.190  5 -207.756 427.3  41.71  0.000
## 41  0.991200   2231.0    95940    83.470  4 -209.403 427.9  42.38  0.000
## 19  0.991200   2245.0    96530    84.200  4 -209.527 428.2  42.63  0.000
## 13  0.990900   2322.0    99860    88.270  4 -210.204 429.6  43.99  0.000
## 45  0.991000   2292.0   100800    85.410  5 -209.393 430.6  44.98  0.000
## 51  0.990900   2307.0   101500    86.190  5 -209.525 430.8  45.25  0.000
## 9   0.990000   2549.0   107000   101.800  3 -212.599 431.9  46.30  0.000
## 49  0.990100   2510.0   107900    98.180  4 -211.762 432.7  47.10  0.000
## 17  0.989800   2602.0   109300   104.700  3 -213.012 432.7  47.13  0.000
## 10  0.989800   2610.0   112200   103.400  4 -212.542 434.2  48.66  0.000
## 6   0.989500   2665.0   114600   106.300  4 -212.955 435.1  49.49  0.000
## 38  0.989600   2645.0   116400   103.500  5 -212.257 436.3  50.71  0.000
## 8   0.989300   2724.0   119800   107.500  5 -212.844 437.5  51.89  0.000
## 40  0.989300   2714.0   122100   105.200  6 -212.210 439.0  53.40  0.000
## 36  0.981600   4698.0   206700   208.700  5 -223.750 459.3  73.70  0.000
## 34  0.978500   5469.0   235200   253.900  4 -227.334 463.8  78.25  0.000
## 35  0.958200  10650.0   458000   526.800  4 -240.668 490.5 104.91  0.000
## 37  0.958000  10700.0   459900   529.100  4 -240.749 490.6 105.08  0.000
## 39  0.957600  10800.0   475200   521.300  5 -240.397 492.6 106.99  0.000
## 33  0.951400  12370.0   519600   633.000  3 -244.193 495.1 109.49  0.000
## 4   0.942300  14700.0   632200   740.100  4 -247.114 503.4 117.80  0.000
## 7   0.881500  30170.0  1297000  1555.000  4 -261.490 532.1 146.56  0.000
## 5   0.859000  35900.0  1508000  1905.000  3 -265.500 537.7 152.10  0.000
## 3   0.753500  62770.0  2636000  3358.000  3 -276.676 560.0 174.45  0.000
## 1   0.000000 254700.0 10440000 14100.000  2 -305.205 614.7 229.17  0.000
## 2  -0.004005 255700.0 10740000 13790.000  3 -304.766 616.2 230.63  0.000
## Models ranked by AICc(x)
```

```
# Note - the book calculates Cp very differently than R does. The "Cp" column is done automatically through R and the 'Mallo
wCp' column calculates it manually using the formula in the book.

# Reproduce the analysis for the candidate models in the video.
```

Influence analysis from the full model

```
# full model lm()
model <- lm(thrust~primary+secondary+fuel+press+exhaust+ambient)
summary(model)
```

```
##
## Call:
## lm(formula = thrust ~ primary + secondary + fuel + press + exhaust +
##     ambient)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -49.949 -19.028  -1.572  17.139  49.606
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.738e+03  2.445e+03  -1.938 0.061213 .
## primary      1.119e+00  2.865e-01   3.904 0.000441 ***
## secondary   -3.018e-02  3.823e-02  -0.789 0.435478
## fuel         2.306e-01  1.180e-01   1.954 0.059231 .
## press        3.850e+00  2.686e+00   1.433 0.161246
## exhaust      8.219e-01  3.508e-01   2.343 0.025298 *
## ambient     -1.695e+01  2.620e+00  -6.468 2.45e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.51 on 33 degrees of freedom
## Multiple R-squared:  0.9977, Adjusted R-squared:  0.9972
## F-statistic:  2350 on 6 and 33 DF,  p-value: < 2.2e-16
```

```
anova(model)
```

	Df <int>	Sum Sq <dbl>	Mean Sq <dbl>	F value <dbl>	Pr(>F) <dbl>
primary	1	9833159.525	9833159.5248	13993.072038	5.900797e-45
secondary	1	5992.318	5992.3175	8.527364	6.262953e-03
fuel	1	25818.683	25818.6826	36.741261	8.052267e-07
press	1	4706.945	4706.9453	6.698216	1.424044e-02
exhaust	1	9771.674	9771.6745	13.905576	7.209539e-04
ambient	1	29397.218	29397.2181	41.833694	2.450406e-07
Residuals	33	23189.637	702.7163	NA	NA

7 rows

```
# sequence of observations
Obs <- seq(1, length(thrust))

influence_stats <- data.frame(cbind(Obs))

# R-student
r_student <- rstudent(model)
influence_stats$r_student <- data.frame(cbind(r_student))

# calculate hat matrix automatically
hat_diags <- lm.influence(model)$hat
influence_stats$hat_ii <- data.frame(cbind(hat_diags))

#### this section as a check on observation 14
X <- cbind(matrix(1,length(thrust),1),
           as.matrix(primary),
           as.matrix(secondary),
           as.matrix(fuel),
           as.matrix(press),
           as.matrix(exhaust),
           as.matrix(ambient))
y <- as.matrix(thrust)

xTx <- t(X) %*% X
H_matrix <- X %*% ginv(xTx, tol=.Machine$double.eps) %*% t(X)

# get the diagonal
diag(H_matrix)
```

```
##  [1] 0.12994307 0.12250028 0.06338237 0.07278295 0.15639595 0.30013879
##  [7] 0.22509487 0.17833859 0.26643906 0.31359530 0.49868576 0.15367090
## [13] 0.16480427 0.10883324 0.21108009 0.21532500 0.11930778 0.10651238
## [19] 0.13896439 0.74092500 0.16172386 0.08827135 0.09072730 0.12909248
## [25] 0.18765157 0.15834909 0.10074719 0.14878392 0.12499829 0.16160608
## [31] 0.10937081 0.07117872 0.12171270 0.12504443 0.22537912 0.15126268
## [37] 0.19029046 0.15810527 0.04571739 0.16326465
```

```
#### this section as a check on observation 14

# Cooks D
D_i_auto <- cooks.distance(model)
influence_stats$Cooks_D <- c(D_i_auto)

#### as a check on observation 14
# Calculate studentized residuals, r_i (eqn 4.8)
e_i <- model$residuals
MS_Res <- anova(model)$'Mean Sq'[8]
r_i <- e_i/sqrt(MS_Res * (1-hat_diags))

p <- sum(hat_diags)

D_i <- ((r_i)^2/p) * (hat_diags/(1-hat_diags))

# Calculate DFFITS and DFBETAS
influence_stats$DFFITS <- c(dffits(model))
dfbetas.col <- dfbetas(model)
influence_stats$DFBETAS_0 <- c(dfbetas.col[,1])
influence_stats$DFBETAS_1 <- c(dfbetas.col[,2])
influence_stats$DFBETAS_2 <- c(dfbetas.col[,3])
influence_stats$DFBETAS_3 <- c(dfbetas.col[,4])
influence_stats$DFBETAS_4 <- c(dfbetas.col[,5])
influence_stats$DFBETAS_5 <- c(dfbetas.col[,6])
influence_stats$DFBETAS_6 <- c(dfbetas.col[,7])

# Calculate Covariance Ratio
influence_stats$covratio <- c(covratio(model))
```

print out table

```
library(e1071)
library(xtable)

out <- influence_stats
colnames(out) <- c("Obs $i$",
                   "$rStudent$",
                   "$h_{ii}$",
                   "$D_i$",
                   "$DFFITS_i$",
                   "$DFBETAS_{0i}$",
                   "$DFBETAS_{1i}$",
                   "$DFBETAS_{2i}$",
                   "$DFBETAS_{3i}$",
                   "$DFBETAS_{4i}$",
                   "$DFBETAS_{5i}$",
                   "$DFBETAS_{6i}$",
                   "$COVRATIO_i$")

tab <- (xtable(out, digits=c(0,0,5,5,5,4,4,4,4,4,4,4,4)))
print(tab, type="html")
```

	Obs i	$rStudent$	h_{ii}	D_i	$DFFITS_i$	$DFBETAS_{0i}$	$DFBETAS_{1i}$	$DFBETAS_{2i}$	$DFBETAS_{3i}$	$DFBETAS_{4i}$	$DFBETAS_{5i}$	$DFBETAS_{6i}$	C
1	1	-0.14904	0.12994	0.00049	-0.0576	0.0276	0.0026	0.0282	-0.0296	-0.0043	0.0232	0.0082	
2	2	1.59195	0.12250	0.04830	0.5948	0.0285	-0.2107	0.0467	-0.0441	0.3132	-0.1162	0.3150	
3	3	0.13183	0.06338	0.00017	0.0343	-0.0189	-0.0044	-0.0019	0.0189	-0.0012	-0.0232	-0.0079	
4	4	0.87830	0.07278	0.00871	0.2461	-0.0268	-0.0797	0.0158	0.0272	0.0905	-0.1001	0.0644	
5	5	-0.82054	0.15640	0.01801	-0.3533	0.1109	-0.0733	0.1386	-0.1426	0.0652	0.2129	0.0292	
6	6	-0.29977	0.30014	0.00566	-0.1963	0.0230	-0.0748	0.0782	-0.0389	0.0654	0.0107	0.0902	
7	7	1.15052	0.22509	0.05440	0.6201	0.0508	0.2387	-0.1781	0.0039	-0.1521	-0.0299	-0.1535	
8	8	-0.02378	0.17834	0.00002	-0.0111	-0.0034	0.0007	-0.0010	0.0032	-0.0023	-0.0024	0.0001	
9	9	0.31449	0.26644	0.00528	0.1895	0.0112	-0.0721	-0.0004	-0.0127	0.0705	0.0610	-0.0524	
10	10	-0.47417	0.31360	0.01503	-0.3205	-0.0597	-0.0098	0.0481	0.0454	-0.0051	-0.1141	0.0959	
11	11	1.65891	0.49869	0.37137	1.6546	0.8567	1.4492	0.2146	-0.7530	-1.1539	0.0787	0.4801	
12	12	-0.66232	0.15367	0.01158	-0.2822	-0.1856	0.0671	-0.0641	0.1980	-0.1510	-0.1834	-0.1906	
13	13	-0.14169	0.16480	0.00058	-0.0629	-0.0458	0.0153	-0.0271	0.0495	-0.0314	-0.0387	-0.0463	
14	14	0.94501	0.10883	0.01563	0.3302	0.2615	0.0549	0.1174	-0.2635	0.0346	0.1547	0.2373	

15	15	0.29342	0.21108	0.00338	0.1518	0.1039	0.0393	-0.0016	-0.0969	0.0116	0.0612	0.0969
16	16	0.20042	0.21533	0.00162	0.1050	0.0140	0.0308	-0.0021	-0.0081	-0.0054	-0.0520	0.0489
17	17	-1.09662	0.11931	0.02313	-0.4036	0.0415	0.0807	-0.0382	-0.0413	-0.1171	0.1930	-0.1100
18	18	-0.74026	0.10651	0.00946	-0.2556	0.1191	0.0619	-0.0054	-0.1190	-0.0298	0.1682	0.0500
19	19	0.25237	0.13896	0.00151	0.1014	-0.0389	-0.0428	0.0164	0.0366	0.0271	-0.0566	-0.0106
20	20	-3.03095	0.74093	3.00723	-5.1257	-1.5096	-0.8850	-4.4122	1.7614	1.6256	1.4121	-1.1783
21	21	-2.17062	0.16172	0.11673	-0.9534	0.1289	-0.5363	0.0713	-0.1458	0.6561	0.0642	0.2852
22	22	-0.88251	0.08827	0.01084	-0.2746	0.1299	-0.0104	0.0301	-0.1198	0.0915	0.0028	0.1516
23	23	-1.10852	0.09073	0.01740	-0.3502	0.2027	0.0628	-0.0084	-0.1796	0.0748	0.0296	0.2064
24	24	-1.04381	0.12909	0.02301	-0.4019	0.2270	0.0743	-0.0442	-0.2030	0.1009	0.0635	0.2503
25	25	-1.39796	0.18765	0.06268	-0.6719	0.1978	-0.1986	0.3825	-0.2491	0.2290	0.0645	0.2310
26	26	0.97401	0.15835	0.02554	0.4225	-0.1227	0.0809	0.0476	0.1042	-0.2086	0.0255	-0.2270
27	27	1.68350	0.10075	0.04297	0.5635	-0.0572	-0.0332	0.2267	0.0059	-0.1351	0.1358	-0.1842
28	28	2.13478	0.14878	0.10272	0.8925	-0.5331	-0.3119	0.2402	0.4453	-0.1068	-0.1434	-0.5300
29	29	1.99887	0.12500	0.07475	0.7555	-0.4158	-0.1982	0.1369	0.3622	-0.1334	-0.1290	-0.4290
30	30	0.59159	0.16161	0.00983	0.2597	-0.0580	0.0930	-0.1001	0.0752	-0.1167	-0.0282	-0.0907
31	31	-1.02414	0.10937	0.01837	-0.3589	0.0392	0.0411	0.0861	-0.0410	-0.0989	0.0668	-0.1219
32	32	-0.15920	0.07118	0.00029	-0.0441	0.0145	-0.0000	0.0095	-0.0151	-0.0010	0.0211	-0.0061
33	33	0.72959	0.12171	0.01069	0.2716	-0.0578	-0.1863	-0.0203	0.0432	0.1981	-0.0172	0.0777
34	34	-0.10180	0.12504	0.00022	-0.0385	0.0206	0.0069	0.0216	-0.0223	-0.0062	0.0163	0.0043
35	35	0.39035	0.22538	0.00650	0.2106	0.0354	0.0801	-0.1074	-0.0126	-0.0224	-0.0157	0.0618
36	36	-0.72233	0.15126	0.01348	-0.3049	-0.0666	0.0771	0.0055	0.0847	-0.0943	-0.2241	-0.0142
37	37	-1.42149	0.19029	0.06580	-0.6891	-0.1907	0.3316	-0.0950	0.2503	-0.3529	-0.5666	-0.0662
38	38	-0.01572	0.15811	0.00001	-0.0068	-0.0011	0.0048	-0.0017	0.0019	-0.0046	-0.0046	-0.0009
39	39	0.64170	0.04572	0.00287	0.1405	-0.0134	-0.0250	0.0187	0.0082	0.0031	0.0075	-0.0094
40	40	-0.18561	0.16326	0.00099	-0.0820	-0.0322	0.0040	0.0204	0.0294	-0.0239	-0.0398	-0.0290

Influence thresholds

```
# Identify observations that exceed limits of 1 +/- 3p/n for COVRATIO
n <- length(thrust)
limit_plus <- (1 + 3*p/n)
limit_minus <- (1 - 3*p/n)
points <- which(influence_stats$covratio > limit_plus | influence_stats$covratio < limit_minus)

# hat matrix cutoff
hm_cutoff <- 2*p/n
pts_hm_cutoff <- which(influence_stats$hat_ii > hm_cutoff)

# Cook's cutoff
Di_cutoff <- 1.0
pts_Di_cutoff <- which(influence_stats$Cooks_D > Di_cutoff)

# DFFITS cutoff
DFFITS_cutoff <- 2 * sqrt(p/n)
pts_DFFITS_cutoff <- which(abs(influence_stats$DFFITS) > DFFITS_cutoff)

# DFBETAS cutoff
DFBETAS_cutoff <- 2/(sqrt(n))

# loop through subset of matrix containing DFBETAS values
count <- 0
pts_DFBETAS_cutoff <- vector() # initialize an empty vector
for (row in 1:n) {
  for (col in 6:12) {

    if (abs(out[row,col]) > DFBETAS_cutoff) {
        count <- count + 1
        pts_DFBETAS_cutoff[count] <- row    # get obs from out matrix
      }
  }
}

if (count > 0) {
  pts_DFBETAS <- unique(pts_DFBETAS_cutoff)
  }
```

Flagged Observations

1. Point(s) $6, 9, 10, 15, 16, 35$ exceed the cutoff $COVRATIO_i$ limits of $1 - \frac{3p}{n} = 0.475$ and $1 + \frac{3p}{n} = 1.525$.
2. Point(s) $11, 20$ exceed h_{ii} cutoff of $\frac{2p}{n} = 0.35$.
3. Point(s) 20 exceed D_i cutoff of 1.

4. Point(s) $11, 20, 21, 28$ where $|DFFITS_i|$ exceed cutoff of $2\sqrt{\frac{p}{n}} = 0.83666$.
5. Point(s) $11, 20, 21, 25, 28, 29, 37$ where $|DFBETAS_{j,i}|$ exceed cutoff of $\frac{2}{\sqrt{n}} = 0.3162278$.

This is consistent with the video where “problem children” observations 11, 20 and 28 are flagged.

Model A: per video

```
library(car) # Needed for vif() function
```

```
## Loading required package: carData
```

```
# linear regression model - Model A
model.A <- lm(thrust~primary+fuel+exhaust+ambient)
vif(model.A)
```

```
##    primary       fuel    exhaust    ambient
##  70.470116 122.452277  27.587699   6.630369
```

```
summary(model.A)
```

```
##
## Call:
## lm(formula = thrust ~ primary + fuel + exhaust + ambient)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -60.883 -18.004   2.588  14.675  45.911
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4280.1645  2257.5088  -1.896   0.0662 .
## primary         1.4420     0.1426  10.114 6.30e-12 ***
## fuel            0.2098     0.1016   2.066   0.0463 *
## exhaust         0.6467     0.3262   1.982   0.0553 .
## ambient       -17.5103     2.3360  -7.496 8.86e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.72 on 35 degrees of freedom
## Multiple R-squared:  0.9975, Adjusted R-squared:  0.9972
## F-statistic:  3468 on 4 and 35 DF,  p-value: < 2.2e-16
```

```
anova(model.A)
```

	Df <int>	Sum Sq <dbl>	Mean Sq <dbl>	F value <dbl>	Pr(>F) <dbl>
primary	1	9833159.525	9833159.5248	13767.981136	5.000241e-47
fuel	1	29183.216	29183.2157	40.861125	2.357520e-07
exhaust	1	4567.425	4567.4254	6.395119	1.610481e-02
ambient	1	40128.663	40128.6628	56.186485	8.857655e-09
Residuals	35	24997.171	714.2049	NA	NA

5 rows

```
pr <- resid(model.A)/(1 - lm.influence(model.A)$hat)
press_stat <- sum(pr^2)

# Press Statistic
press_stat
```

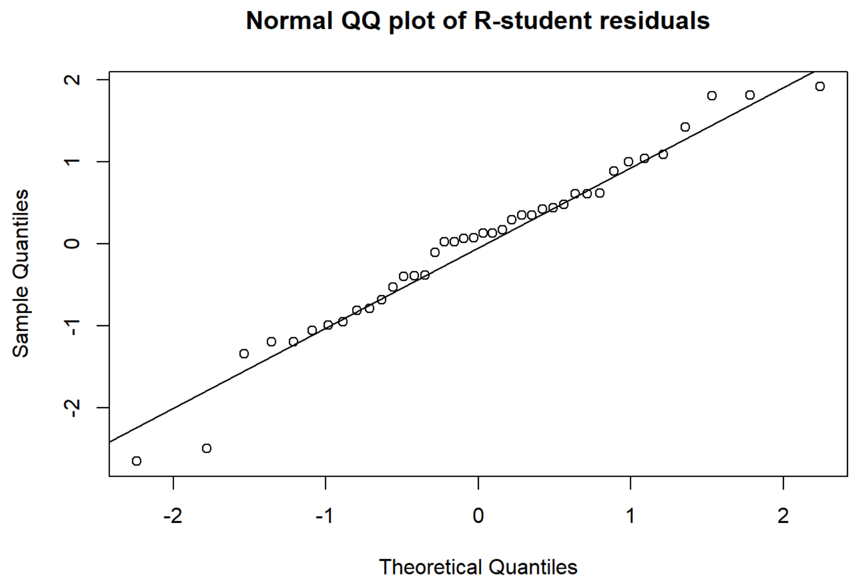
```
## [1] 31685.4
```

Model A Results: There’s a relationship (overall F and p values) Calculated PRESS Statistic: 3.1685398×10^4 Cp: 5.6 (video)

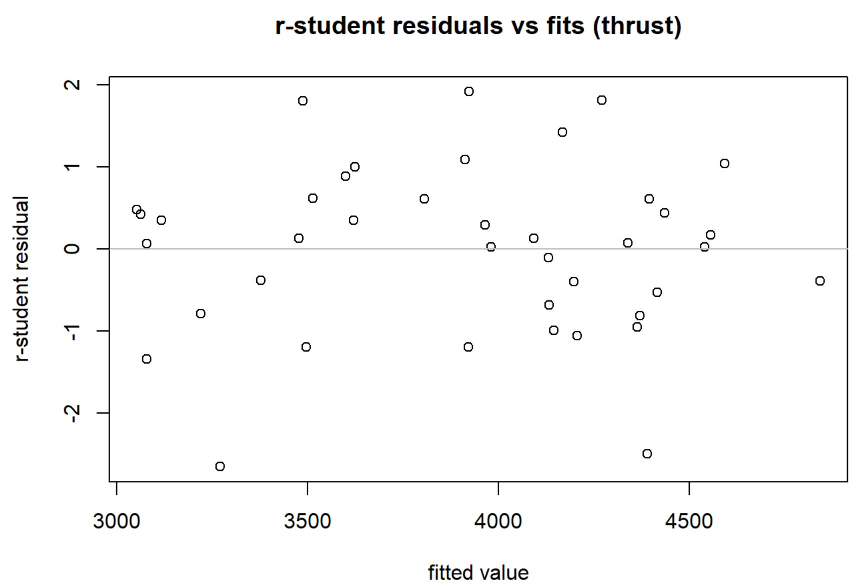
VIF’s – all above 5 – collinarity is present

Perform full analysis on Model A

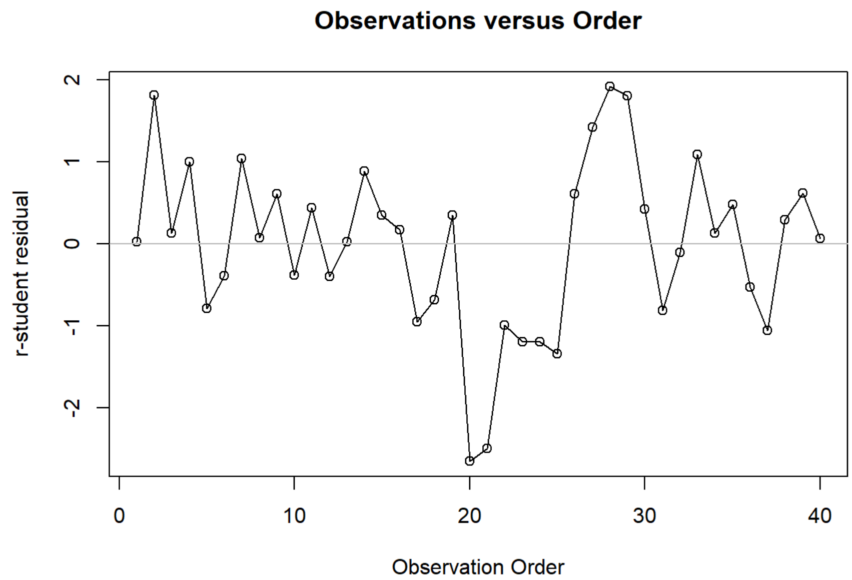
```
qqnorm(rstudent(model.A),main="Normal QQ plot of R-student residuals")
qqline(rstudent(model.A))
```



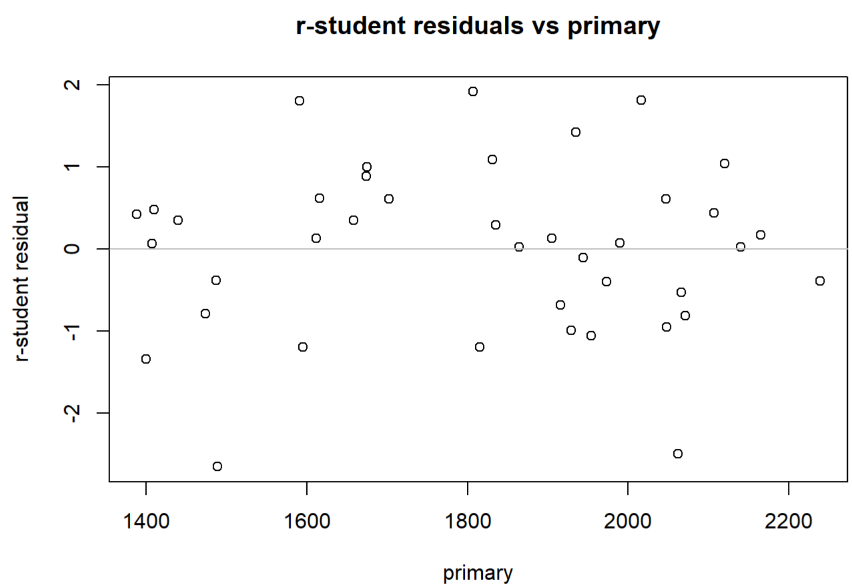

```
plot(model.A$fitted.values,rstudent(model.A), main="r-student residuals vs fits (thrust)",
     ylab="r-student residual",
     xlab="fitted value")
abline(0, 0, col="gray")
```




```
Obs <- seq(1, length(thrust))
plot(Obs, rstudent(model.A),  main = "Observations versus Order", xlab = "Observation Order", ylab = "r-student residual")
lines(Obs, rstudent(model.A))
abline(0, 0, col="gray")
```

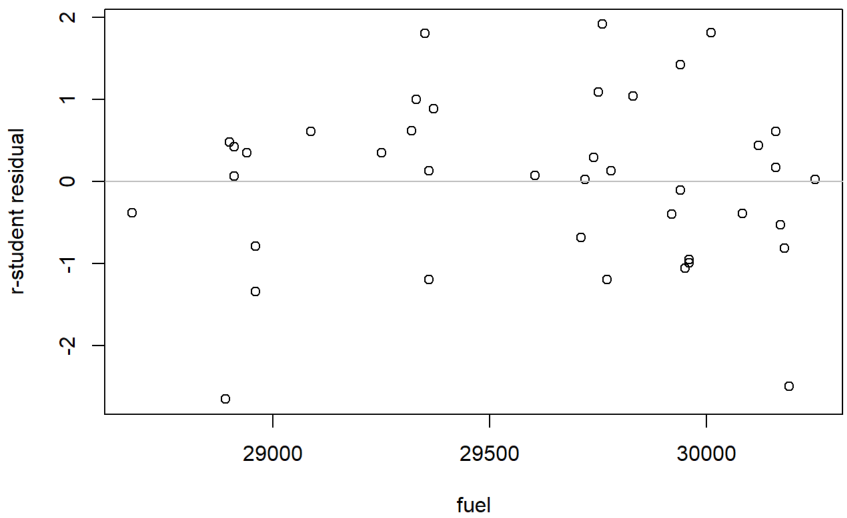



```
plot(primary,rstudent(model.A), main="r-student residuals vs primary",
     ylab="r-student residual",
     xlab="primary")
abline(0, 0, col="gray")
```



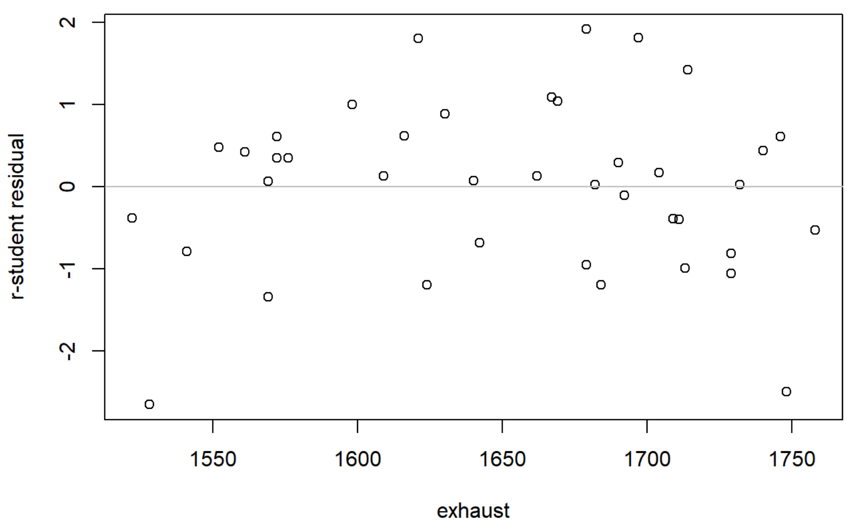

```
plot(fuel,rstudent(model.A), main="r-student residuals vs fuel",
     ylab="r-student residual",
     xlab="fuel")
abline(0, 0, col="gray")
```

r-student residuals vs fuel

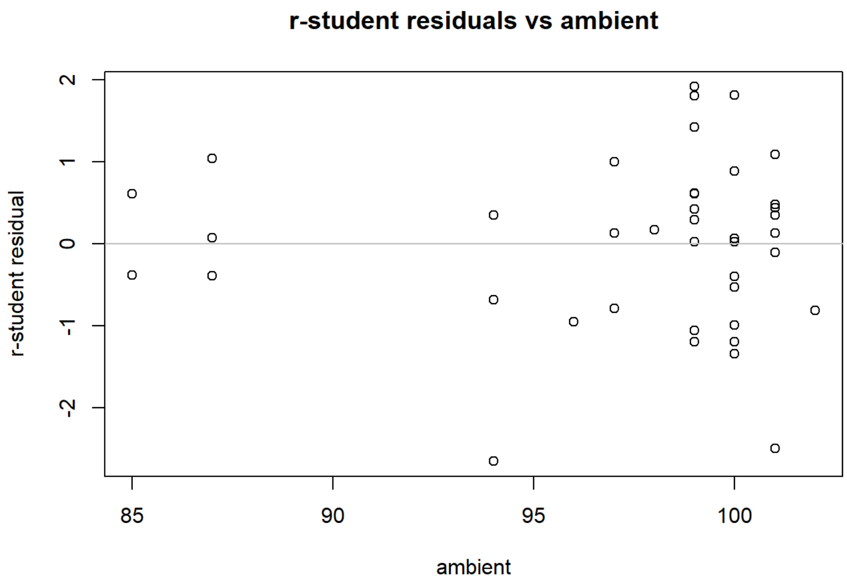



```
plot(exhaust,rstudent(model.A), main="r-student residuals vs exhaust",
     ylab="r-student residual",
     xlab="exhaust")
abline(0, 0, col="gray")
```

r-student residuals vs exhaust




```
plot(ambient,rstudent(model.A), main="r-student residuals vs ambient",
     ylab="r-student residual",
     xlab="ambient")
abline(0, 0, col="gray")
```



Model A Normality Plot vs residuals – no issues Residuals vs fits – random scatter vs fits – no issues Residuals vs Order – no major issues Residuals vs primary, fuel, exhaust, ambient – good random scatter

Influence analysis of model A

```
# sequence of observations
Obs <- seq(1, length(thrust))

influence_stats <- data.frame(cbind(Obs))

# R-student
r_student <- rstudent(model.A)
influence_stats$r_student <- data.frame(cbind(r_student))

# calculate hat matrix automatically
hat_diags <- lm.influence(model.A)$hat
influence_stats$hat_ii <- data.frame(cbind(hat_diags))

#### this section as a check on observation 14
X <- cbind(matrix(1,length(thrust),1),
           as.matrix(primary),
           as.matrix(fuel),
           as.matrix(exhaust),
           as.matrix(ambient))
y <- as.matrix(thrust)

xTx <- t(X) %*% X
H_matrix <- X %*% ginv(xTx, tol=.Machine$double.eps) %*% t(X)

# get the diagonal
diag(H_matrix)
```

```
##  [1] 0.09851206 0.08720474 0.06310057 0.06244372 0.12583910 0.21502829
##  [7] 0.19133808 0.16895931 0.22948414 0.30650956 0.25157241 0.09978607
## [13] 0.08977048 0.09345752 0.20983836 0.21464388 0.10784381 0.10498505
## [19] 0.12477611 0.13577220 0.08322384 0.07707565 0.08656640 0.11971784
## [25] 0.10126321 0.11850837 0.07954760 0.13631186 0.11737097 0.10209431
## [31] 0.09544737 0.06788580 0.05678206 0.08361876 0.16282205 0.13678220
## [37] 0.13530589 0.07351461 0.04486463 0.14043341
```

```
#### this section as a check on observation 14

# Cooks D
D_i_auto <- cooks.distance(model.A)
influence_stats$Cooks_D <- c(D_i_auto)

#### as a check on observation 14
# Calculate studentized residuals, r_i (eqn 4.8)
e_i <- model.A$residuals
MS_Res <- anova(model.A)$'Mean Sq'[8]
r_i <- e_i/sqrt(MS_Res * (1-hat_diags))

p <- sum(hat_diags)

D_i <- ((r_i)^2/p) * (hat_diags/(1-hat_diags))

# Calculate DFFITS and DFBETAS
influence_stats$DFFITS <- c(dffits(model.A))
dfbetas.col <- dfbetas(model.A)
influence_stats$DFBETAS_0 <- c(dfbetas.col[,1])
influence_stats$DFBETAS_1 <- c(dfbetas.col[,2])
influence_stats$DFBETAS_2 <- c(dfbetas.col[,3])
influence_stats$DFBETAS_3 <- c(dfbetas.col[,4])
influence_stats$DFBETAS_4 <- c(dfbetas.col[,5])

# Calculate Covariance Ratio
influence_stats$covratio <- c(covratio(model.A))

# Identify observations that exceed limits of 1 +/- 3p/n for COVRATIO
n <- length(thrust)
limit_plus <- (1 + 3*p/n)
limit_minus <- (1 - 3*p/n)
points <- which(influence_stats$covratio > limit_plus | influence_stats$covratio < limit_minus)

# hat matrix cutoff
hm_cutoff <- 2*p/n
pts_hm_cutoff <- which(influence_stats$hat_ii > hm_cutoff)

# Cook's cutoff
Di_cutoff <- 1.0
pts_Di_cutoff <- which(influence_stats$Cooks_D > Di_cutoff)

# DFFITS cutoff
DFFITS_cutoff <- 2 * sqrt(p/n)
pts_DFFITS_cutoff <- which(abs(influence_stats$DFFITS) > DFFITS_cutoff)

# DFBETAS cutoff
DFBETAS_cutoff <- 2/(sqrt(n))

# Loop through subset of matrix containing DFBETAS values
count <- 0
pts_DFBETAS_cutoff <- vector() # initialize an empty vector
for (row in 1:n) {
  for (col in 6:10) {

    if (abs(out[row,col]) > DFBETAS_cutoff) {
        count <- count + 1
        pts_DFBETAS_cutoff[count] <- row    # get obs from out matrix
      }
  }
}

if (count > 0) {
  pts_DFBETAS <- unique(pts_DFBETAS_cutoff)
  }
```

Flagged Observations

1. Point(s) $6, 8, 9, 10, 11, 15, 16, 20, 21$ exceed the cutoff $COVRATIO_i$ limits of $1 - \frac{3p}{n} = 0.625$ and $1 + \frac{3p}{n} = 1.375$.
2. Point(s) $10, 11$ exceed h_{ii} cutoff of $\frac{2p}{n} = 0.25$.
3. Point(s) $$ exceed D_i cutoff of 1.
4. Point(s) $20, 21, 28$ where $|DFFITS_i|$ exceed cutoff of $2\sqrt{\frac{p}{n}} = 0.7071068$.
5. Point(s) $11, 20, 21, 25, 28, 29, 37$ where $|DFBETAS_{j,i}|$ exceed cutoff of $\frac{2}{\sqrt{n}} = 0.3162278$.

Summary couple of leverage points Point 20 drops DFFITS Point 20 is a challenge to the model Model A mitigates issues with points 20 and 28.

Model B: per video

```
library(car) # Needed for vif() function

# linear regression model - Model A
model.B <- lm(thrust~primary+fuel+exhaust+ambient+press)
vif(model.B)
```

```
##   primary      fuel   exhaust   ambient     press
## 286.35594 126.48916  32.34355   7.59230 219.44392
```

```
summary(model.B)
```

```
##
## Call:
## lm(formula = thrust ~ primary + fuel + exhaust + ambient + press)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -52.113 -18.321  -0.732  18.010  47.433
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3982.1058  2236.9356  -1.780 0.083989 .
## primary         1.0964     0.2835   3.867 0.000473 ***
## fuel            0.1843     0.1018   1.810 0.079193 .
## exhaust         0.8343     0.3484   2.394 0.022308 *
## ambient       -16.2781     2.4658  -6.602 1.44e-07 ***
## press           3.7456     2.6681   1.404 0.169432
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.36 on 34 degrees of freedom
## Multiple R-squared:  0.9976, Adjusted R-squared:  0.9973
## F-statistic:  2852 on 5 and 34 DF,  p-value: < 2.2e-16
```

```
anova(model.B)
```

	Df <int>	Sum Sq <dbl>	Mean Sq <dbl>	F value <dbl>	Pr(>F) <dbl>
primary	1	9833159.525	9833159.5248	14149.865292	3.873765e-46
fuel	1	29183.216	29183.2157	41.994495	2.062293e-07
exhaust	1	4567.425	4567.4254	6.572501	1.494946e-02
ambient	1	40128.663	40128.6628	57.744937	7.905673e-09
press	1	1369.567	1369.5666	1.970799	1.694316e-01
Residuals	34	23627.605	694.9296	NA	NA

6 rows

```
pr <- resid(model.B)/(1 - lm.influence(model.B)$hat)
press_stat <- sum(pr^2)

# Press Statistic
press_stat
```

```
## [1] 34081.58
```

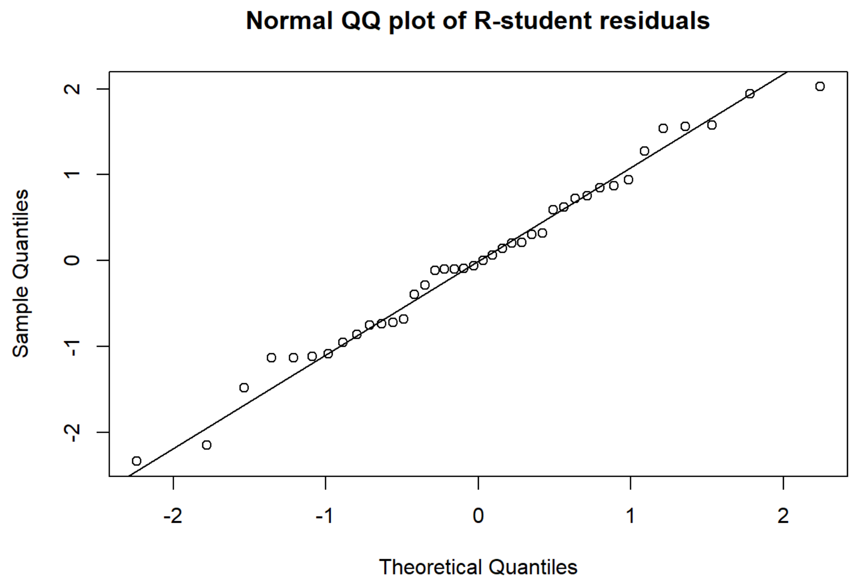
Model B initial review 5 term model – overall significance R^2 a little higher … but, we are using 5 terms Press: 34081

VIF worse than model but better than full model might not need pressure

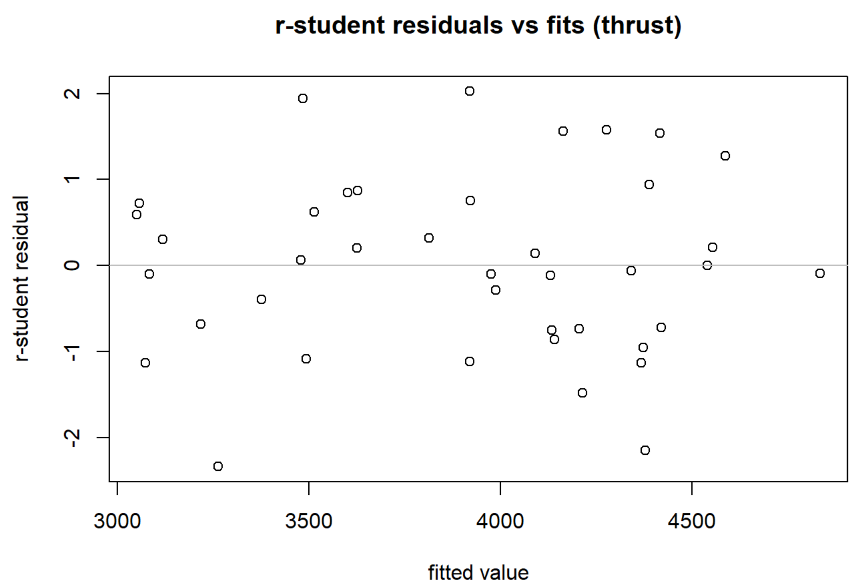
trade off between rule and pressure.

Perform full analysis on Model B

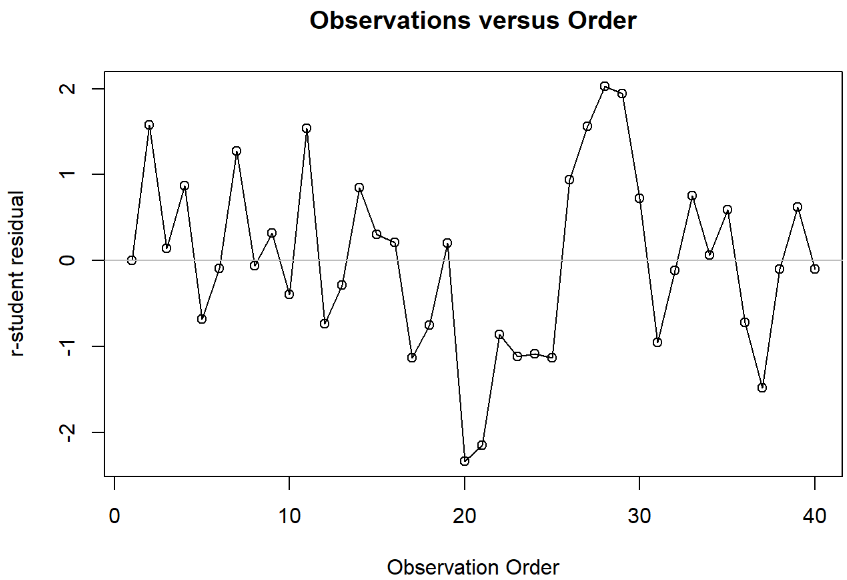
```
qqnorm(rstudent(model.B),main="Normal QQ plot of R-student residuals")
qqline(rstudent(model.B))
```



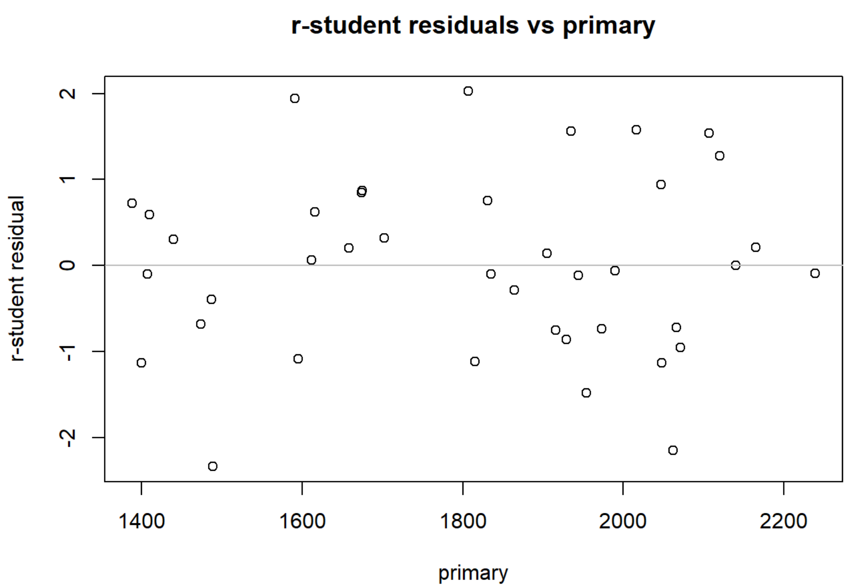

```
plot(model.B$fitted.values,rstudent(model.B), main="r-student residuals vs fits (thrust)",
     ylab="r-student residual",
     xlab="fitted value")
abline(0, 0, col="gray")
```



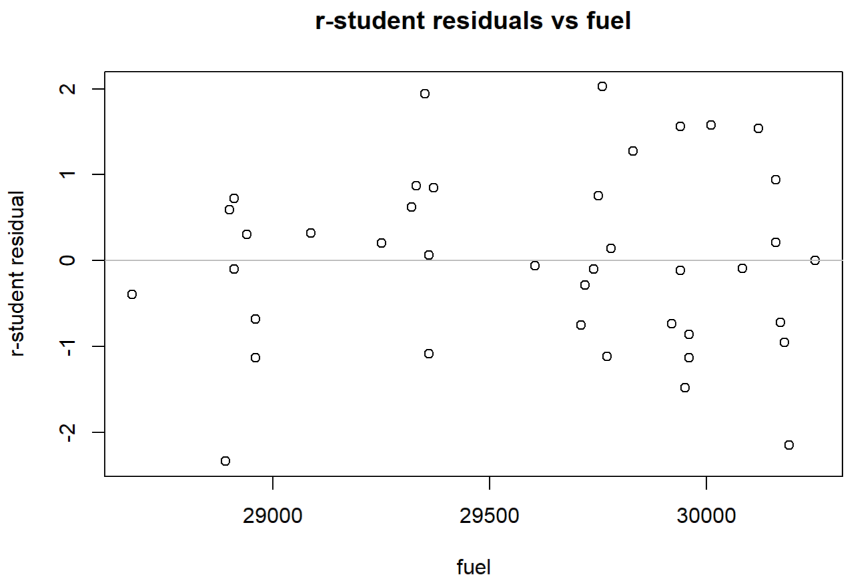

```
Obs <- seq(1, length(thrust))
plot(Obs, rstudent(model.B),  main = "Observations versus Order", xlab = "Observation Order", ylab = "r-student residual")
lines(Obs, rstudent(model.B))
abline(0, 0, col="gray")
```



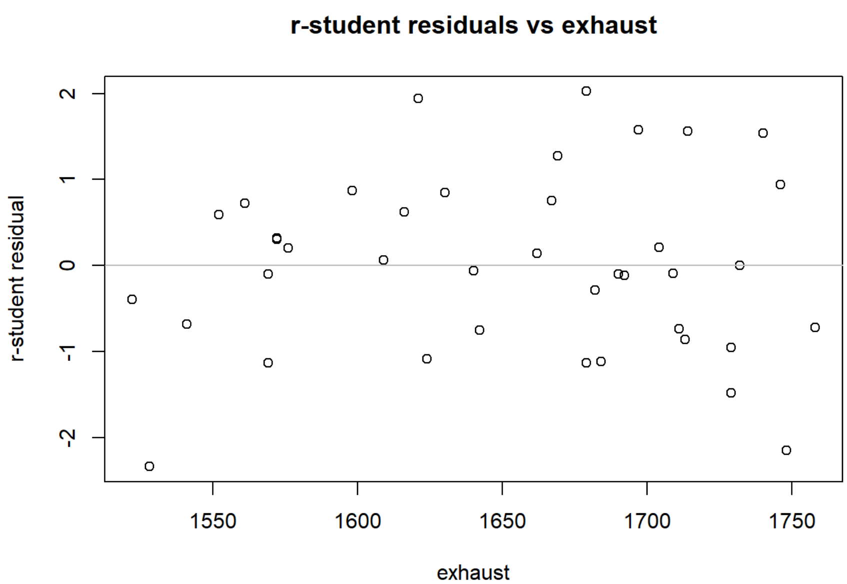

```
plot(primary,rstudent(model.B), main="r-student residuals vs primary",
     ylab="r-student residual",
     xlab="primary")
abline(0, 0, col="gray")
```



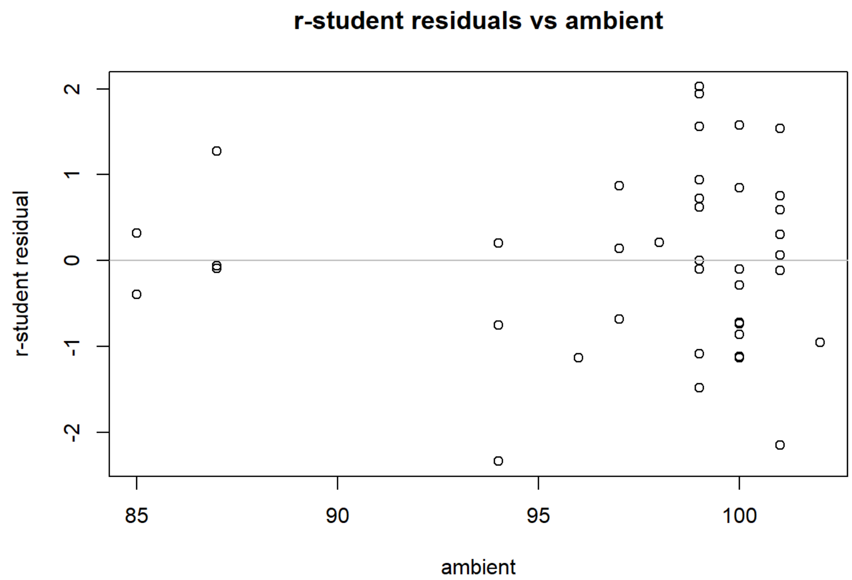

```
plot(fuel,rstudent(model.B), main="r-student residuals vs fuel",
     ylab="r-student residual",
     xlab="fuel")
abline(0, 0, col="gray")
```



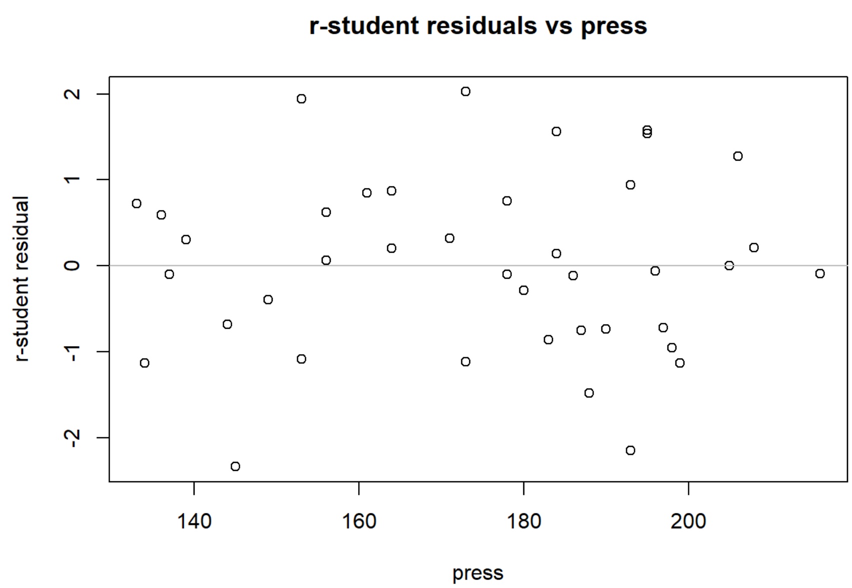

```
plot(exhaust,rstudent(model.B), main="r-student residuals vs exhaust",
     ylab="r-student residual",
     xlab="exhaust")
abline(0, 0, col="gray")
```




```
plot(ambient,rstudent(model.B), main="r-student residuals vs ambient",
     ylab="r-student residual",
     xlab="ambient")
abline(0, 0, col="gray")
```




```
plot(press,rstudent(model.B), main="r-student residuals vs press",
     ylab="r-student residual",
     xlab="press")
abline(0, 0, col="gray")
```



Analysis summary: minor improvements in normality and residual vs fits, regressors

Influence analysis of model B

```
# sequence of observations
Obs <- seq(1, length(thrust))

influence_stats <- data.frame(cbind(Obs))

# R-student
r_student <- rstudent(model.B)
influence_stats$r_student <- data.frame(cbind(r_student))

# calculate hat matrix automatically
hat_diags <- lm.influence(model.B)$hat
influence_stats$hat_ii <- data.frame(cbind(hat_diags))

#### this section as a check on observation 14
X <- cbind(matrix(1,length(thrust),1),
           as.matrix(primary),
           as.matrix(fuel),
           as.matrix(exhaust),
           as.matrix(ambient),
           as.matrix(press))
y <- as.matrix(thrust)

xTx <- t(X) %*% X
H_matrix <- X %*% ginv(xTx, tol=.Machine$double.eps) %*% t(X)

# get the diagonal
diag(H_matrix)
```

```
##  [1] 0.09885473 0.12174357 0.06319091 0.07248289 0.13234159 0.25247271
##  [7] 0.20652084 0.17675472 0.26643787 0.30653319 0.49030000 0.14575268
## [13] 0.13436464 0.09508737 0.21105667 0.21523997 0.11824033 0.10646424
## [19] 0.13533578 0.19192605 0.16081874 0.08720993 0.09067565 0.12753370
## [25] 0.12683926 0.15634179 0.08443508 0.13800374 0.12089448 0.13759532
## [31] 0.10307717 0.06789838 0.12103101 0.08583125 0.16670955 0.15121335
## [37] 0.18667110 0.14804969 0.04490281 0.15316787
```

```
#### this section as a check on observation 14

# Cooks D
D_i_auto <- cooks.distance(model.B)
influence_stats$Cooks_D <- c(D_i_auto)

#### as a check on observation 14
# Calculate studentized residuals, r_i (eqn 4.8)
e_i <- model.B$residuals
MS_Res <- anova(model.B)$'Mean Sq'[8]
r_i <- e_i/sqrt(MS_Res * (1-hat_diags))

p <- sum(hat_diags)

D_i <- ((r_i)^2/p) * (hat_diags/(1-hat_diags))

# Calculate DFFITS and DFBETAS
influence_stats$DFFITS <- c(dffits(model.B))
dfbetas.col <- dfbetas(model.B)
influence_stats$DFBETAS_0 <- c(dfbetas.col[,1])
influence_stats$DFBETAS_1 <- c(dfbetas.col[,2])
influence_stats$DFBETAS_2 <- c(dfbetas.col[,3])
influence_stats$DFBETAS_3 <- c(dfbetas.col[,4])
influence_stats$DFBETAS_4 <- c(dfbetas.col[,5])
influence_stats$DFBETAS_5 <- c(dfbetas.col[,6])


# Calculate Covariance Ratio
influence_stats$covratio <- c(covratio(model.B))

# Identify observations that exceed limits of 1 +/- 3p/n for COVRATIO
n <- length(thrust)
limit_plus <- (1 + 3*p/n)
limit_minus <- (1 - 3*p/n)
points <- which(influence_stats$covratio > limit_plus | influence_stats$covratio < limit_minus)

# hat matrix cutoff
hm_cutoff <- 2*p/n
pts_hm_cutoff <- which(influence_stats$hat_ii > hm_cutoff)

# Cook's cutoff
Di_cutoff <- 1.0
pts_Di_cutoff <- which(influence_stats$Cooks_D > Di_cutoff)

# DFFITS cutoff
DFFITS_cutoff <- 2 * sqrt(p/n)
pts_DFFITS_cutoff <- which(abs(influence_stats$DFFITS) > DFFITS_cutoff)

# DFBETAS cutoff
DFBETAS_cutoff <- 2/(sqrt(n))

# Loop through subset of matrix containing DFBETAS values
count <- 0
pts_DFBETAS_cutoff <- vector() # initialize an empty vector
for (row in 1:n) {
  for (col in 6:11) {

    if (abs(out[row,col]) > DFBETAS_cutoff) {
        count <- count + 1
        pts_DFBETAS_cutoff[count] <- row    # get obs from out matrix
      }
  }
}

if (count > 0) {
  pts_DFBETAS <- unique(pts_DFBETAS_cutoff)
  }
```

Flagged Observations

1. Point(s) $6, 8, 9, 10, 11, 15, 16$ exceed the cutoff $COVRATIO_i$ limits of $1 - \frac{3p}{n} = 0.55$ and $1 + \frac{3p}{n} = 1.45$.
2. Point(s) $10, 11$ exceed h_{ii} cutoff of $\frac{2p}{n} = 0.3$.
3. Point(s) $$ exceed D_i cutoff of 1.
4. Point(s) $11, 20, 21, 28$ where $|DFFITS_i|$ exceed cutoff of $2\sqrt{\frac{p}{n}} = 0.7745967$.
5. Point(s) $11, 20, 21, 25, 28, 29, 37$ where $|DFBETAS_{j,i}|$ exceed cutoff of $\frac{2}{\sqrt{n}} = 0.3162278$.

Summary Not as well as model A. 4 obs flagged in DFFITS. point 11 has popped up. Model A has done a better job on influence points, however, model B has done a better job in the residual analysis

Model C: per video

```
# linear regression model - Model C
model.C <- lm(thrust~primary+press+exhaust+ambient)
vif(model.C)
```

```
##    primary      press    exhaust    ambient
## 263.589214 212.440398  17.585611   3.477889
```

```
summary(model.C)
```

```
##
## Call:
## lm(formula = thrust ~ primary + press + exhaust + ambient)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.116 -16.513  -0.828  14.978  60.854
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.6198   273.0182   0.138   0.8912
## primary       1.2411     0.2807   4.421 9.08e-05 ***
## press         4.6082     2.7092   1.701   0.0978 .
## exhaust       1.2603     0.2652   4.753 3.38e-05 ***
## ambient     -12.9932     1.7223  -7.544 7.69e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 27.2 on 35 degrees of freedom
## Multiple R-squared:  0.9974, Adjusted R-squared:  0.9971
## F-statistic:  3346 on 4 and 35 DF,  p-value: < 2.2e-16
```

```
anova(model.C)
```

	Df <int>	Sum Sq <dbl>	Mean Sq <dbl>	F value <dbl>	Pr(>F) <dbl>
primary	1	9833159.5248	9833159.5248	1.328628e+04	9.310675e-47
press	1	30783.6204	30783.6204	4.159394e+01	1.985085e-07
exhaust	1	65.8449	65.8449	8.896772e-02	7.672572e-01
ambient	1	42123.5532	42123.5532	5.691612e+01	7.687113e-09
Residuals	35	25903.4567	740.0988	NA	NA

5 rows

```
pr <- resid(model.C)/(1 - lm.influence(model.C)$hat)
press_stat <- sum(pr^2)

# Press Statistic
press_stat
```

```
## [1] 33334.3
```

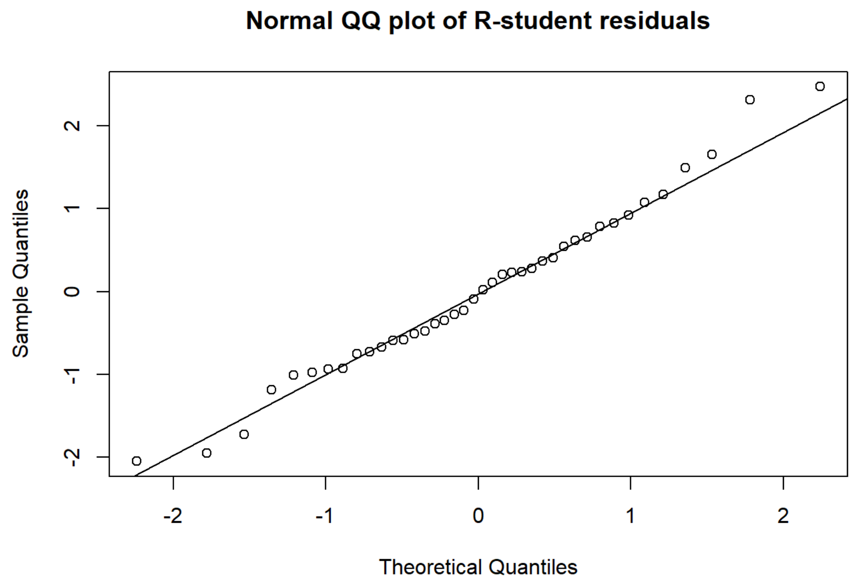
Model B initial review 5 term model – overall significance R^2 a little higher … but, we are using 5 terms Press: 34081

VIF worse than model but better than full model might not need pressure

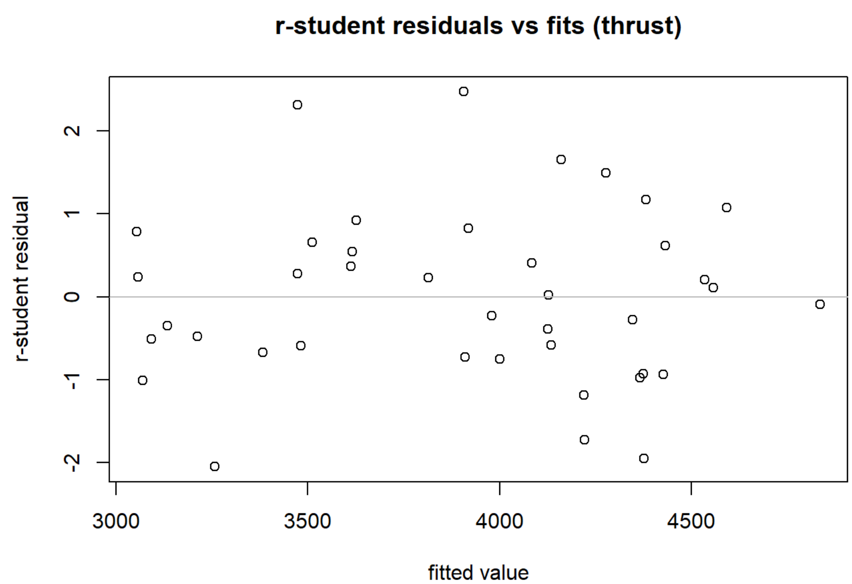
trade off between rule and pressure.

Perform full analysis on Model C

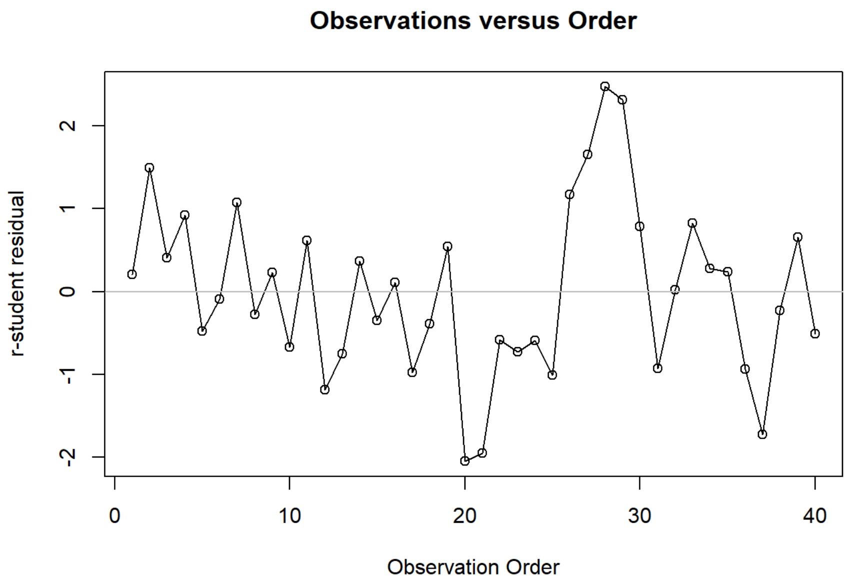
```
qqnorm(rstudent(model.C),main="Normal QQ plot of R-student residuals")
qqline(rstudent(model.C))
```



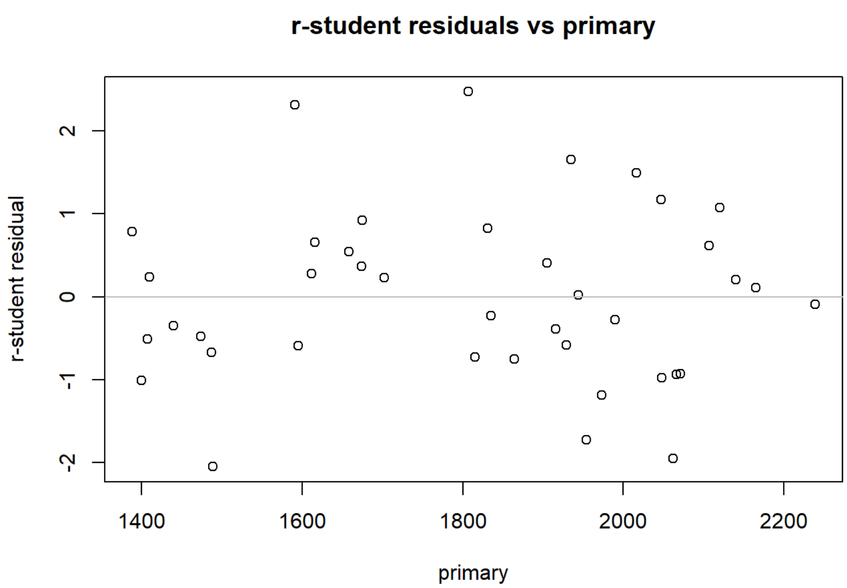

```
plot(model.C$fitted.values,rstudent(model.C), main="r-student residuals vs fits (thrust)",
     ylab="r-student residual",
     xlab="fitted value")
abline(0, 0, col="gray")
```



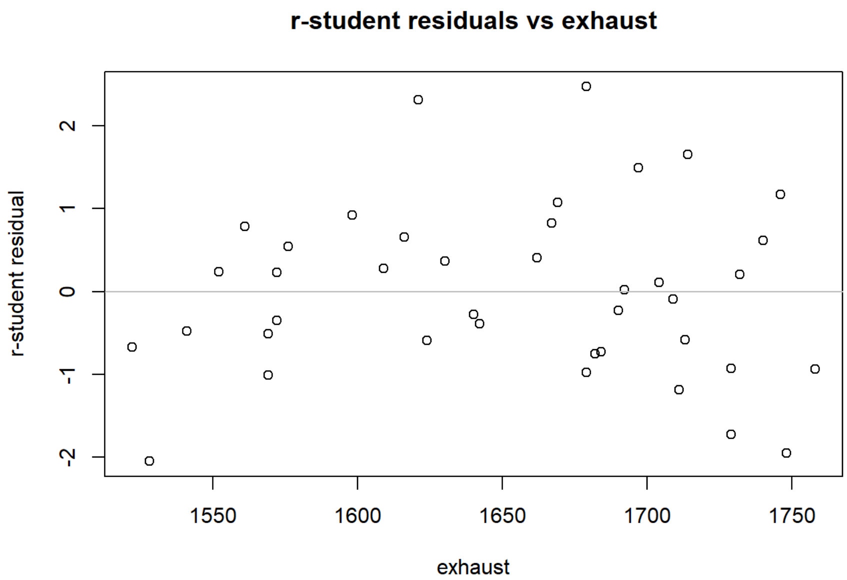

```
Obs <- seq(1, length(thrust))
plot(Obs, rstudent(model.C),  main = "Observations versus Order", xlab = "Observation Order", ylab = "r-student residual")
lines(Obs, rstudent(model.C))
abline(0, 0, col="gray")
```



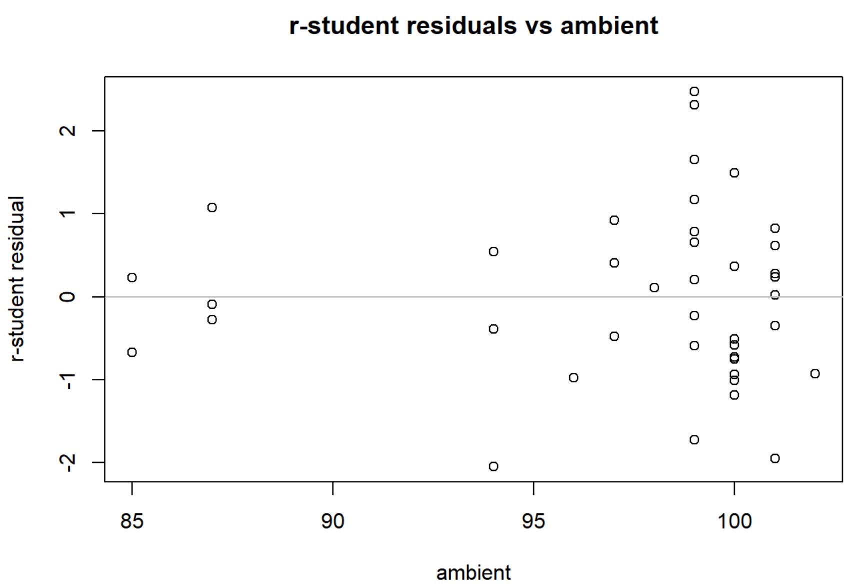

```
plot(primary,rstudent(model.C), main="r-student residuals vs primary",
     ylab="r-student residual",
     xlab="primary")
abline(0, 0, col="gray")
```



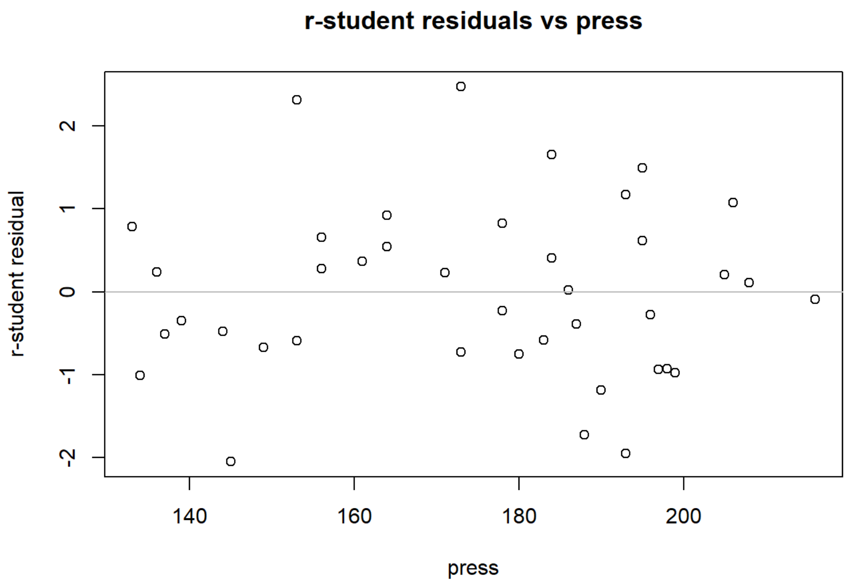

```
plot(exhaust,rstudent(model.C), main="r-student residuals vs exhaust",
     ylab="r-student residual",
     xlab="exhaust")
abline(0, 0, col="gray")
```




```
plot(ambient,rstudent(model.C), main="r-student residuals vs ambient",
     ylab="r-student residual",
     xlab="ambient")
abline(0, 0, col="gray")
```




```
plot(press,rstudent(model.C), main="r-student residuals vs press",
     ylab="r-student residual",
     xlab="press")
abline(0, 0, col="gray")
```



Analysis summary: minor improvements in normality and residual vs fits, regressors

Influence analysis of model C

```
# sequence of observations
Obs <- seq(1, length(thrust))

influence_stats <- data.frame(cbind(Obs))

# R-student
r_student <- rstudent(model.C)
influence_stats$r_student <- data.frame(cbind(r_student))

# calculate hat matrix automatically
hat_diags <- lm.influence(model.C)$hat
influence_stats$hat_ii <- data.frame(cbind(hat_diags))

#### this section as a check on observation 14
X <- cbind(matrix(1,length(thrust),1),
           as.matrix(primary),
           as.matrix(exhaust),
           as.matrix(ambient),
           as.matrix(press))
y <- as.matrix(thrust)

xTx <- t(X) %*% X
H_matrix <- X %*% ginv(xTx, tol=.Machine$double.eps) %*% t(X)

# get the diagonal
diag(H_matrix)
```

```
##  [1] 0.08620745 0.12154395 0.04008740 0.07052370 0.12329572 0.25247271
##  [7] 0.20094042 0.16292634 0.26479315 0.28701912 0.38919734 0.07500691
## [13] 0.06240178 0.03927925 0.09487565 0.21308785 0.11469893 0.07437154
## [19] 0.09931486 0.18490570 0.15794279 0.07013410 0.05748348 0.07381070
## [25] 0.12492490 0.13707559 0.07849526 0.05886297 0.06703799 0.13553688
## [31] 0.10307351 0.06266659 0.11863646 0.07089815 0.13727191 0.13469772
## [37] 0.16471548 0.14322376 0.04395591 0.10260608
```

```
#### this section as a check on observation 14

# Cooks D
D_i_auto <- cooks.distance(model.C)
influence_stats$Cooks_D <- c(D_i_auto)

#### as a check on observation 14
# Calculate studentized residuals, r_i (eqn 4.8)
e_i <- model.C$residuals
MS_Res <- anova(model.C)$'Mean Sq'[8]
r_i <- e_i/sqrt(MS_Res * (1-hat_diags))

p <- sum(hat_diags)

D_i <- ((r_i)^2/p) * (hat_diags/(1-hat_diags))

# Calculate DFFITS and DFBETAS
influence_stats$DFFITS <- c(dffits(model.C))
dfbetas.col <- dfbetas(model.C)
influence_stats$DFBETAS_0 <- c(dfbetas.col[,1])
influence_stats$DFBETAS_1 <- c(dfbetas.col[,2])
influence_stats$DFBETAS_2 <- c(dfbetas.col[,3])
influence_stats$DFBETAS_3 <- c(dfbetas.col[,4])
influence_stats$DFBETAS_4 <- c(dfbetas.col[,5])

# Calculate Covariance Ratio
influence_stats$covratio <- c(covratio(model.C))

# Identify observations that exceed limits of 1 +/- 3p/n for COVRATIO
n <- length(thrust)
limit_plus <- (1 + 3*p/n)
limit_minus <- (1 - 3*p/n)
points <- which(influence_stats$covratio > limit_plus | influence_stats$covratio < limit_minus)

# hat matrix cutoff
hm_cutoff <- 2*p/n
pts_hm_cutoff <- which(influence_stats$hat_ii > hm_cutoff)

# Cook's cutoff
Di_cutoff <- 1.0
pts_Di_cutoff <- which(influence_stats$Cooks_D > Di_cutoff)

# DFFITS cutoff
DFFITS_cutoff <- 2 * sqrt(p/n)
pts_DFFITS_cutoff <- which(abs(influence_stats$DFFITS) > DFFITS_cutoff)

# DFBETAS cutoff
DFBETAS_cutoff <- 2/(sqrt(n))

# Loop through subset of matrix containing DFBETAS values
count <- 0
pts_DFBETAS_cutoff <- vector() # initialize an empty vector
for (row in 1:n) {
  for (col in 6:10) {

    if (abs(out[row,col]) > DFBETAS_cutoff) {
        count <- count + 1
        pts_DFBETAS_cutoff[count] <- row    # get obs from out matrix
      }
  }
}

if (count > 0) {
  pts_DFBETAS <- unique(pts_DFBETAS_cutoff)
  }
```

Flagged Observations

1. Point(s) $6, 9, 10, 11, 16, 28, 29$ exceed the cutoff $COVRATIO_i$ limits of $1 - \frac{3p}{n} = 0.625$ and $1 + \frac{3p}{n} = 1.375$.
2. Point(s) $6, 9, 10, 11$ exceed h_{ii} cutoff of $\frac{2p}{n} = 0.25$.
3. Point(s) $$ exceed D_i cutoff of 1.
4. Point(s) $20, 21, 37$ where $|DFFITS_i|$ exceed cutoff of $2\sqrt{\frac{p}{n}} = 0.7071068$.
5. Point(s) $11, 20, 21, 25, 28, 29, 37$ where $|DFBETAS_{j,i}|$ exceed cutoff of $\frac{2}{\sqrt{n}} = 0.3162278$.

Summary Multicollinearity exist … as it does in all models A and B. Normal probability plot looks O.K. Residuals vs fits looks good. Model C. shows most leverage points.

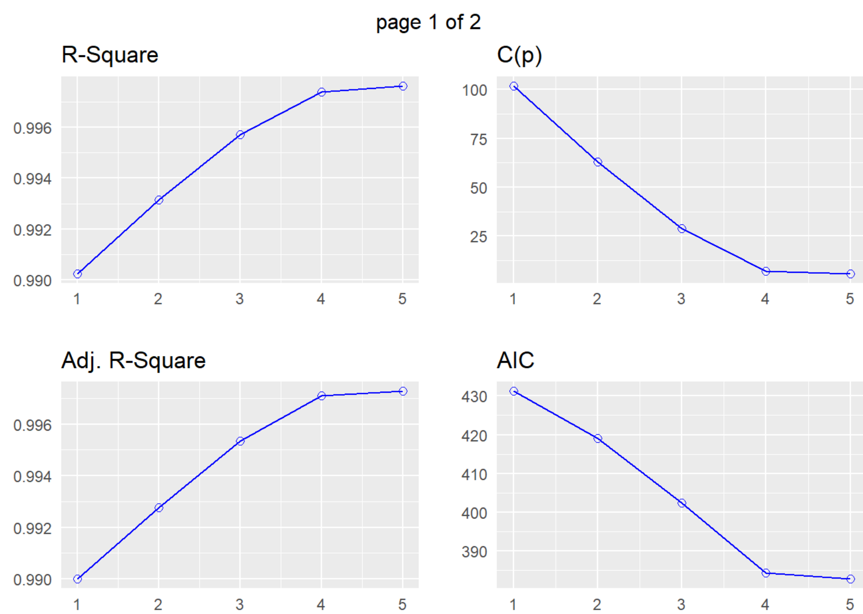
```r
library(olsrr)
```

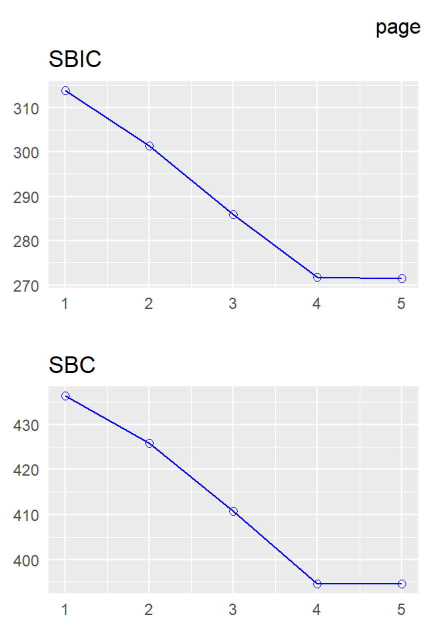
```
##
## Attaching package: 'olsrr'
```

```
## The following object is masked from 'package:MASS':
##
##     cement
```

```
## The following object is masked from 'package:datasets':
##
##     rivers
```

```r
k <- ols_step_forward_p(model, penter=0.1)
plot(k)
```

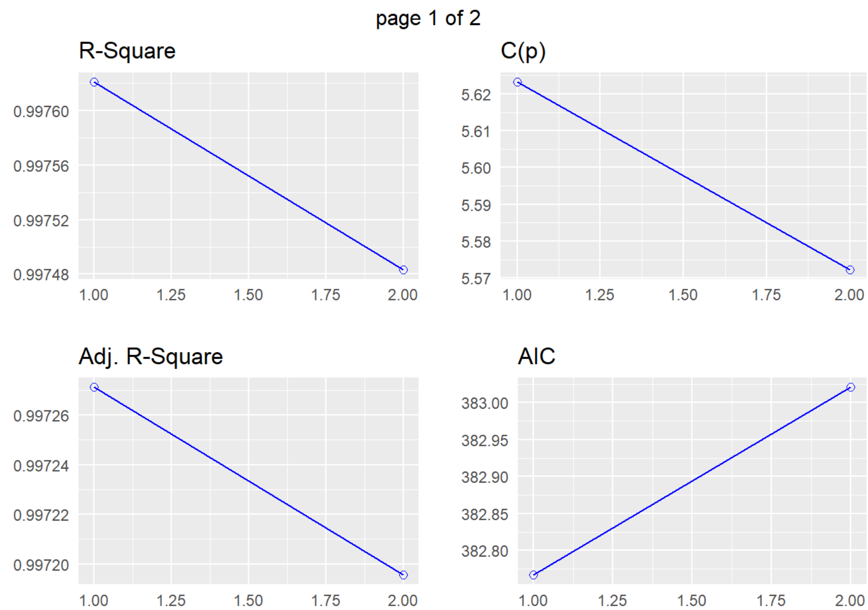


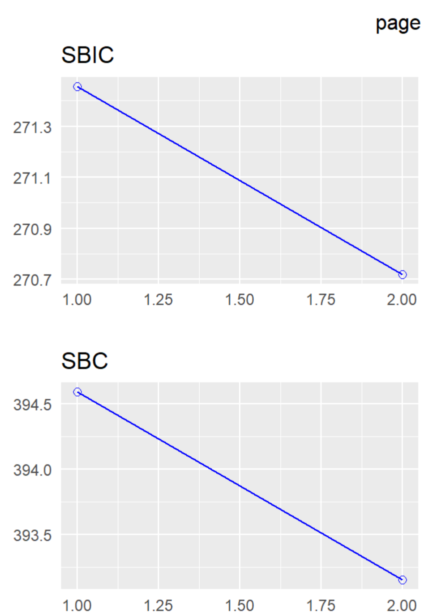



```r
# final model
k$model
```

```
##
## Call:
## lm(formula = paste(response, "~", paste(preds, collapse = " + ")),
##     data = l)
##
## Coefficients:
## (Intercept)        press      primary      ambient      exhaust         fuel
##  -3982.1058       3.7456       1.0964     -16.2781       0.8343       0.1843
```

```
k <- ols_step_backward_p(model, prem =0.1)
plot(k)
```

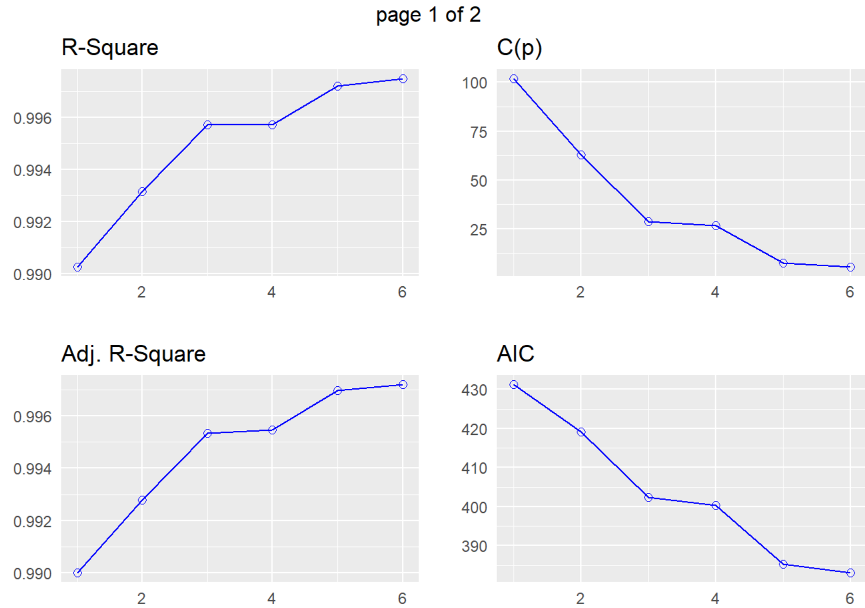


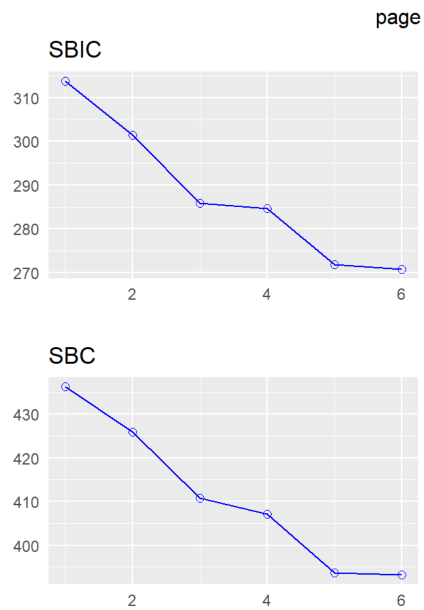



```
# final model
k$model
```

```
##
## Call:
## lm(formula = paste(response, "~", paste(preds, collapse = " + ")),
##     data = l)
##
## Coefficients:
## (Intercept)      primary         fuel      exhaust      ambient
##  -4280.1645       1.4420       0.2098       0.6467     -17.5103
```

```
k <- ols_step_both_p(model, prem =0.1)
plot(k)
```






```
# final model
k$model
```

```
##
## Call:
## lm(formula = paste(response, "~", paste(preds, collapse = " + ")),
##     data = l)
##
## Coefficients:
## (Intercept)      primary      ambient         fuel      exhaust
##  -4280.1645       1.4420     -17.5103       0.2098       0.6467
```