

Relatório Trabalho 2:

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Parte 1:

Dados do trabalho 1:

Espessura h (mm): 4 Largura b (mm): 103 Comprimento total L_{tot} (cm): 61,2

Massa total (g): 160,6

Momento de inércia I calculado (mm^4): 2,67E-09

Densidade ρ (kg/m^3): 636,938

Comprimento da viga livre do experimento L (cm): 50

E (GPa): 0,2751 (calculado no trabalho 1)

Frequências obtidas do experimento com o Tracker:

ω_{n1} (rad/s): 23,52 f_{n1} (Hz): 3,74

Cálculo das 4 primeiras frequências para essa placa engastada do trabalho 1 utilizando a teoria do contínuo: (ver anexo 1 os valores de βL):

$$\omega_n = (\beta L)^2 \sqrt{\frac{EI}{\rho AL^4}}$$

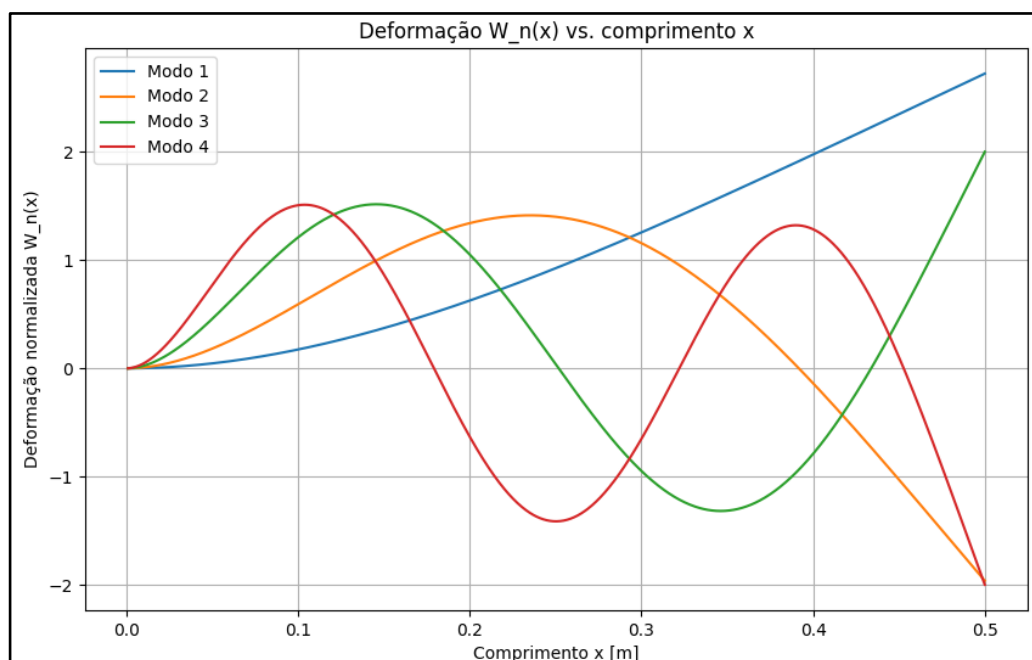
$$\omega_1 = 23,52959488 \quad f_1 = 3,744851334 \text{ Hz}$$

$$\omega_2 = 147,4574565 \quad f_2 = 23,4685831 \text{ Hz}$$

$$\omega_3 = 412,8851253 \quad f_3 = 65,71270862 \text{ Hz}$$

$$\omega_4 = 809,0901869 \quad f_4 = 128,7707026 \text{ Hz}$$

Gráfico desses **4 modos de vibrações verticais**, $W(x)$, correspondendo às frequências acima, com valor de $C_n = 1$:



Print da tela de resultados dos primeiros 10 modos da placa deformada nesses modos:

Step/Frame

Step Name		Description	
Step-1			

Frame

Index	Description		
0	Increment	0: Base State	
1	Mode	1: Value = 116.82 Freq = 1.7202 (cycles/time)	
2	Mode	2: Value = 4579.8 Freq = 10.771 (cycles/time)	
3	Mode	3: Value = 10792. Freq = 16.534 (cycles/time)	
4	Mode	4: Value = 36103. Freq = 30.240 (cycles/time)	
5	Mode	5: Value = 70781. Freq = 42.343 (cycles/time)	
6	Mode	6: Value = 1.01233E+05 Freq = 50.638 (cycles/time)	
7	Mode	7: Value = 1.39869E+05 Freq = 59.522 (cycles/time)	
8	Mode	8: Value = 3.03889E+05 Freq = 87.736 (cycles/time)	
9	Mode	9: Value = 3.85848E+05 Freq = 98.862 (cycles/time)	
10	Mode	10: Value = 6.62301E+05 Freq = 129.52 (cycles/time)	

OK

Apply

Field Output...

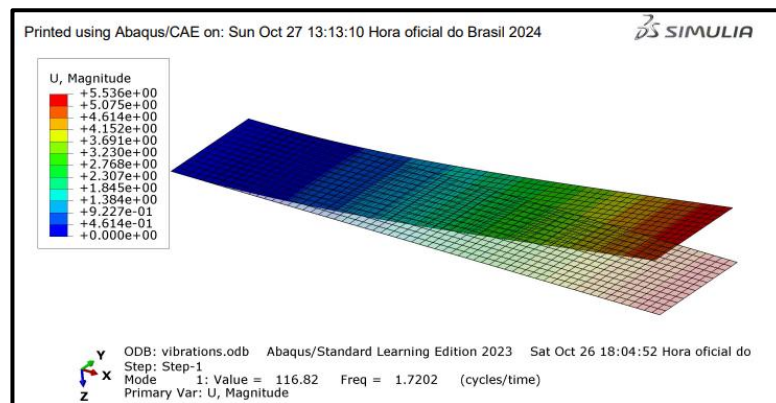
Cancel

O resultado dos modos de vibração gerados em CAE revelam bastante divergência quando comparados ao resultado dos modos de vibração calculados. Isso provavelmente se deve à quantidade limitada de elementos do *Abaqus Student Edition*.

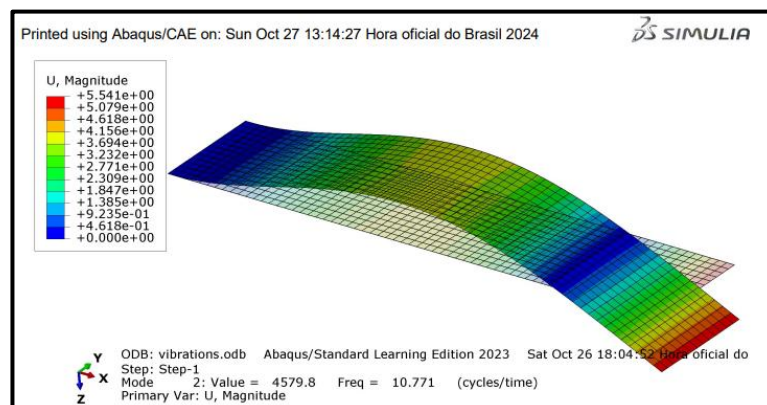
Mesh: 819 elementos criados

Modos para placa livre de 50 cm. Há 5 modos de vibração vertical que estão hachurados.

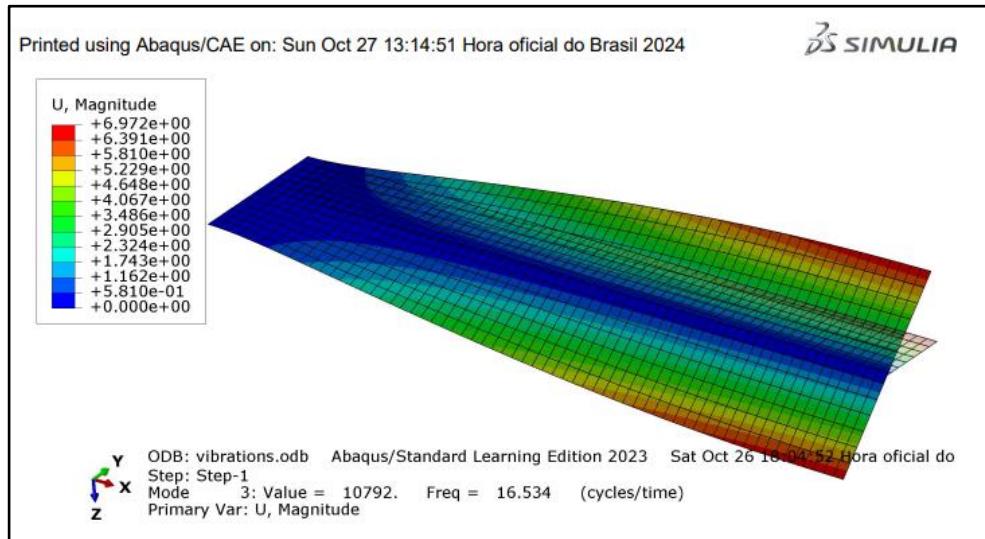
Modo 1 (1,7202 Hz) correspondente ao modo 1 de vibração vertical



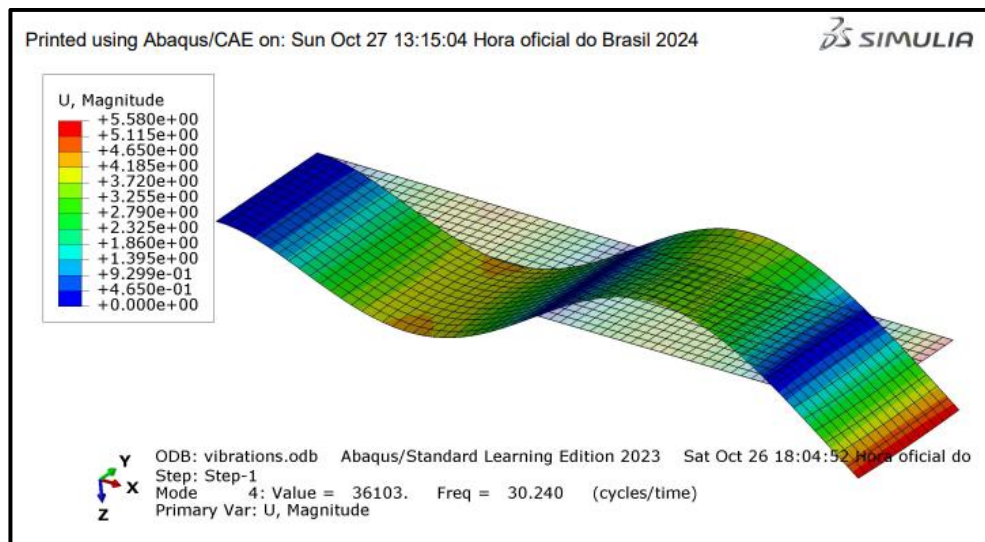
Modo 2 (10,771 Hz) correspondente ao modo 2 de vibração vertical



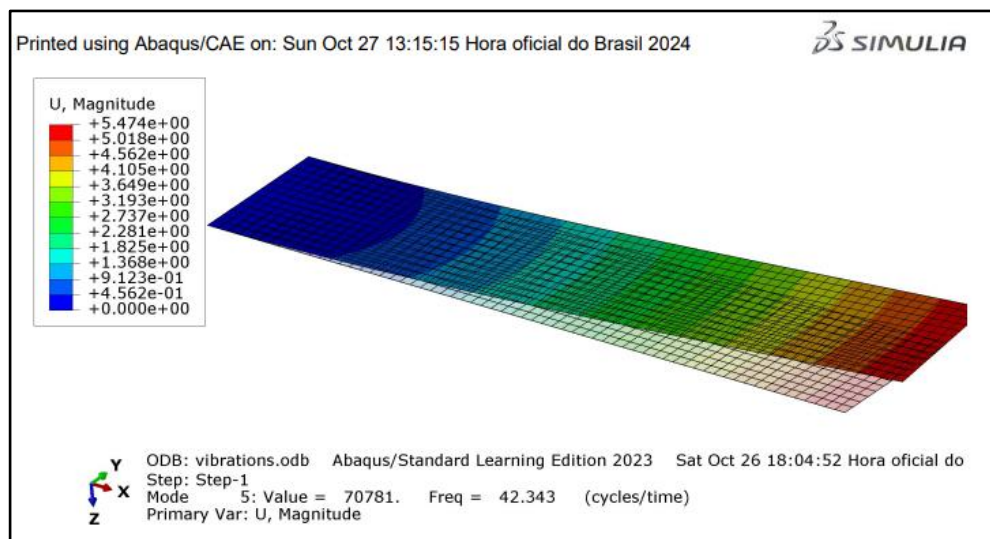
Modo 3 (16,534 Hz)



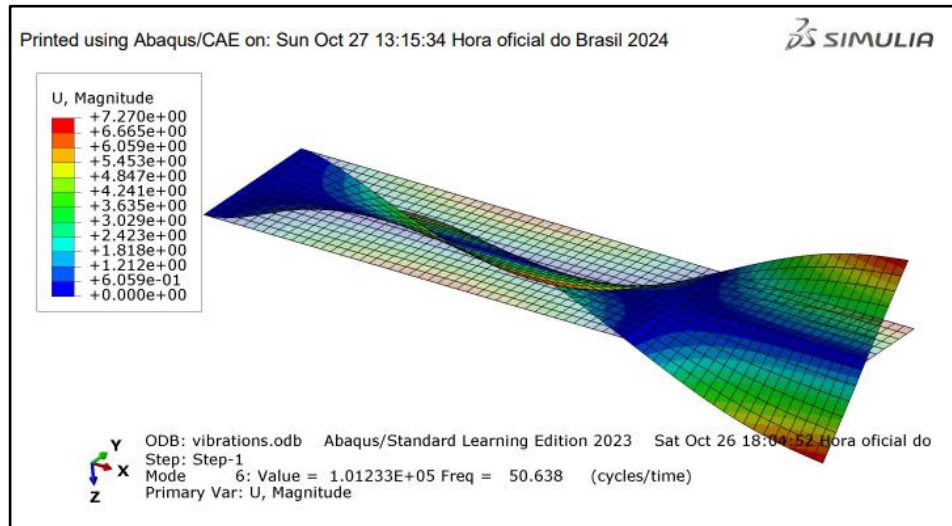
Modo 4 (30,240 Hz) correspondente ao modo 3 de vibração vertical



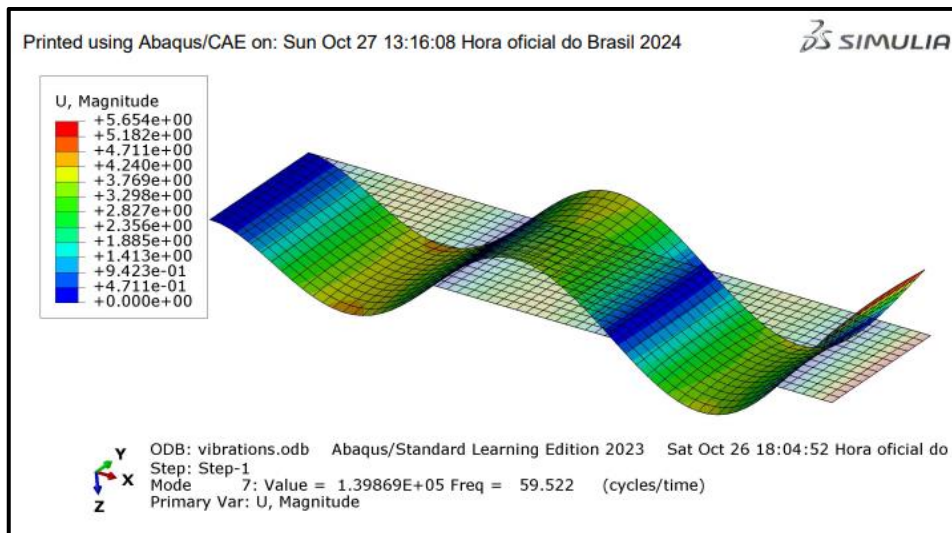
Modo 5 (42,343 Hz)



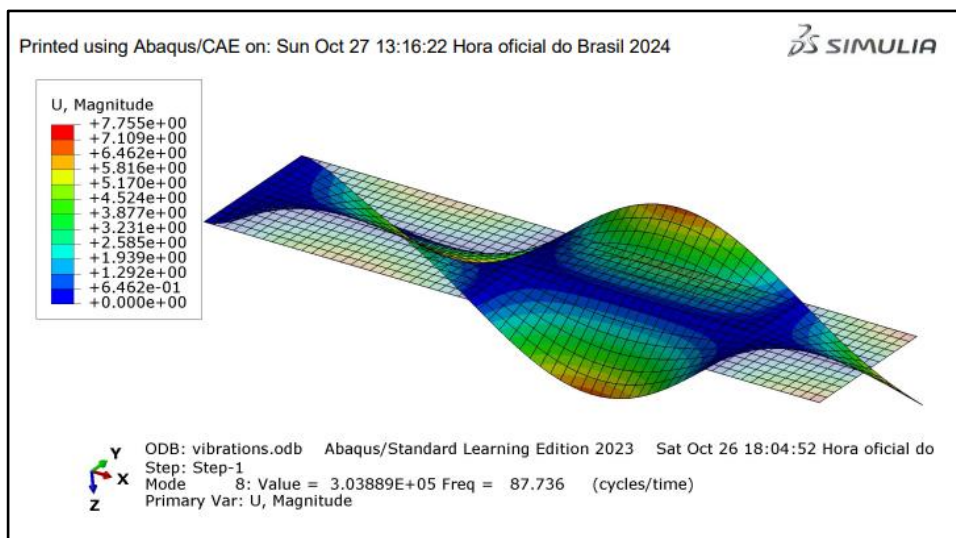
Modo 6 (50,638 Hz)



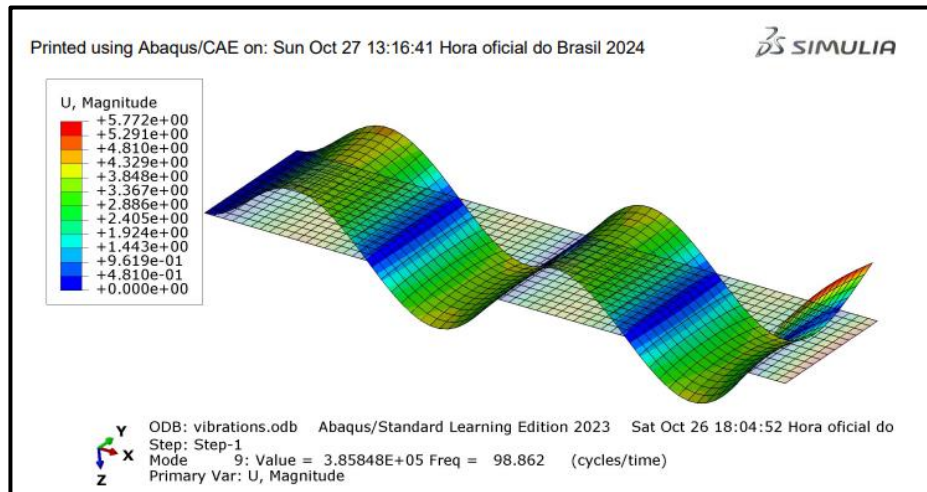
Modo 7 (59,522 Hz) correspondente ao modo 4 de vibração vertical



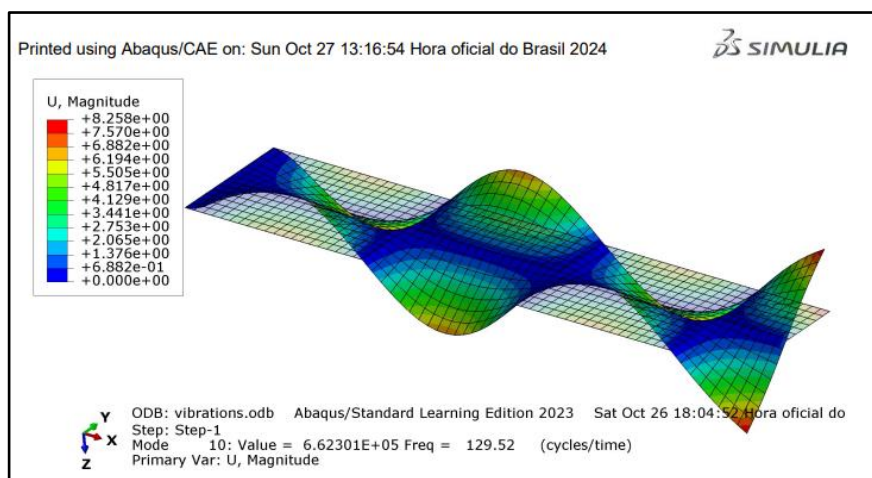
Modo 8 (87,736 Hz)



Modo 9 (98,862 Hz) correspondente ao modo 5 de vibração vertical



Modo 10 (129,52 Hz)



Parte 2:

Realizar o mesmo que a parte 1, mas com outra condição de contorno escolhida da tabela em anexo 1 (não escolher engastada-livre, nem engastada-engastada). Se quiser pode escolher outras dimensões e material ou usar os mesmos da etapa anterior.

Espessura h (mm): 4 Largura b (mm): 103 Comprimento L (cm): 50

Momento de inércia I calculado (mm⁴): 2,67E-09

Densidade ρ (kg/m³): 636,938

E (GPa): 0,2751

Condição de contorno escolhida: *pinned-free* (rotação ao longo do eixo Y, somente)

Mesh: 819 elementos criados

Cálculo das 4 primeiras frequências utilizando a teoria do contínuo:

$$\omega_n = (\beta L)^2 \sqrt{\frac{EI}{\rho AL^4}}$$

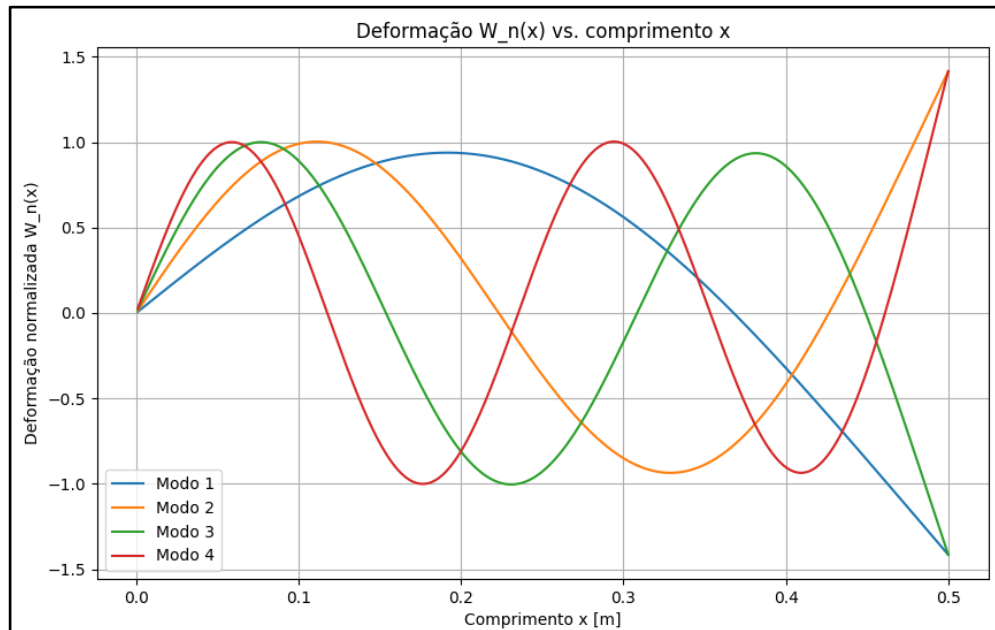
$$\omega_1 = 103,1804687 \quad f_1 = 16,42168162$$

$$\omega_2 = 334,3708838 \quad f_2 = 53,21677897$$

$$\omega_3 = 697,6380926 \quad f_3 = 111,0325509$$

$$\omega_4 = 1193,002309 \quad f_4 = 189,8722146$$

Gráfico desses 4 modos de vibrações, $W(x)$, correspondendo às frequências acima:



Com software de FEM mostrar print da tela de resultados dos primeiros 10 modos e da placa deformada nesses modos (aqui, se desejar, pode tentar usar outros elementos (viga, sólido, etc), desde que chegue nos resultados próximos ao teórico):

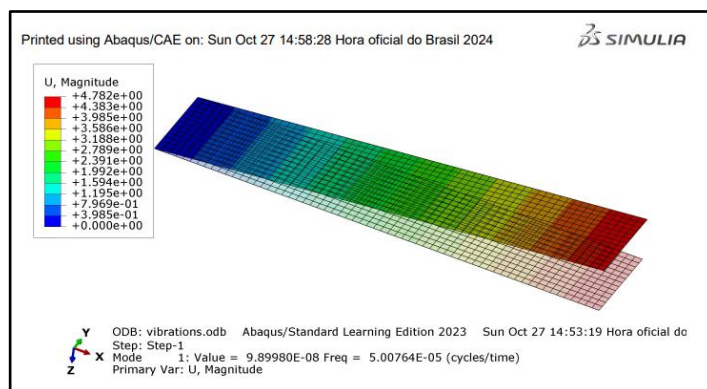
Simulação usando “shell”

Step/Frame	
Step Name	Description
Step-1	
Frame	
Index	Description
0	Increment 0: Base State
1	Mode 1: Value = 9.89980E-08 Freq = 5.00764E-05 (cycles/time)
2	Mode 2: Value = 2201.5 Freq = 7.4676 (cycles/time)
3	Mode 3: Value = 9748.1 Freq = 15.714 (cycles/time)
4	Mode 4: Value = 23376. Freq = 24.333 (cycles/time)
5	Mode 5: Value = 70781. Freq = 42.343 (cycles/time)
6	Mode 6: Value = 91472. Freq = 48.135 (cycles/time)
7	Mode 7: Value = 1.03055E+05 Freq = 51.092 (cycles/time)
8	Mode 8: Value = 2.74848E+05 Freq = 83.438 (cycles/time)
9	Mode 9: Value = 3.05012E+05 Freq = 87.898 (cycles/time)
10	Mode 10: Value = 5.99952E+05 Freq = 123.28 (cycles/time)

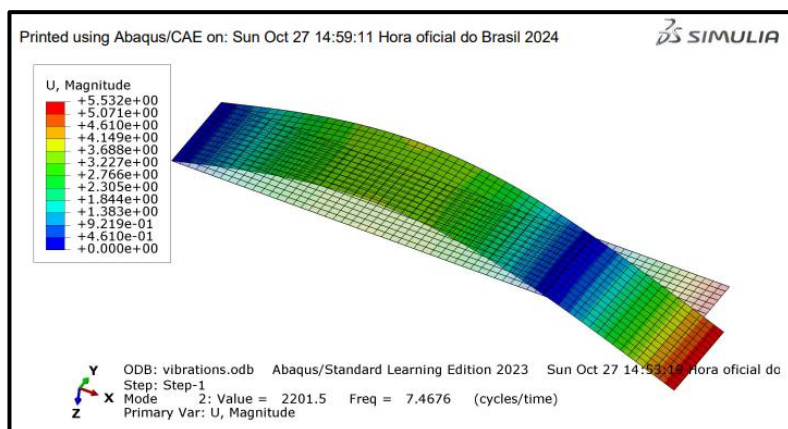
O resultado dos modos de vibração gerados em CAE revelam bastante divergência quando comparados ao resultado dos modos de vibração calculados. Isso provavelmente se deve à quantidade limitada de elementos do *Abaqus Student Edition*.

Mesh: 819 elementos criados

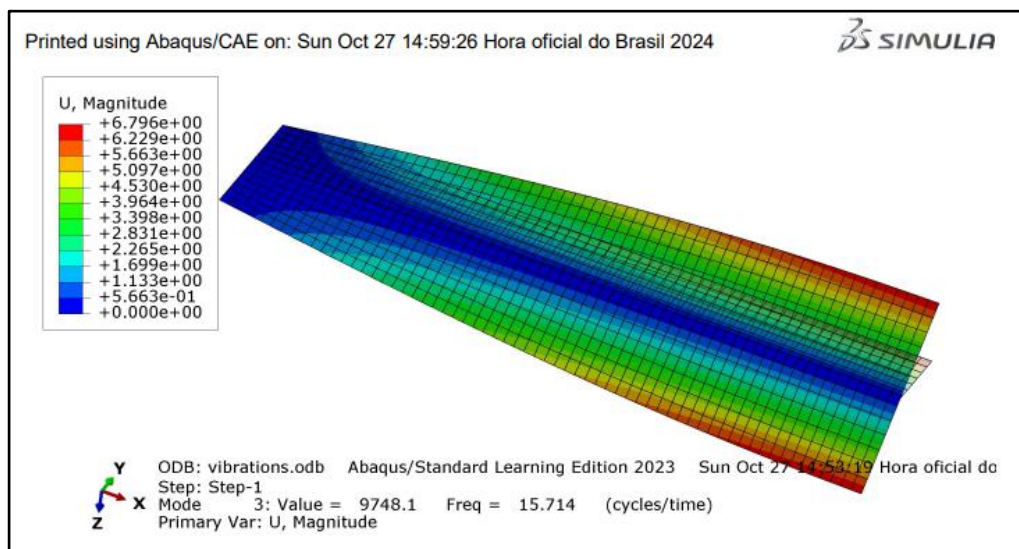
Modo 1 ($5,007\text{e-}5$ Hz)



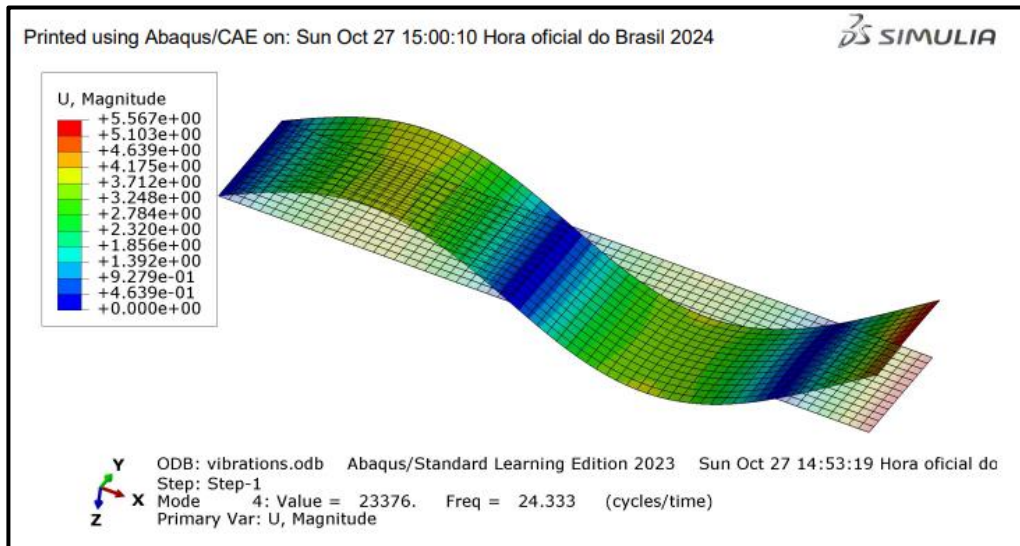
Modo 2 (7,4676 Hz) correspondente ao modo 1 de vibração vertical



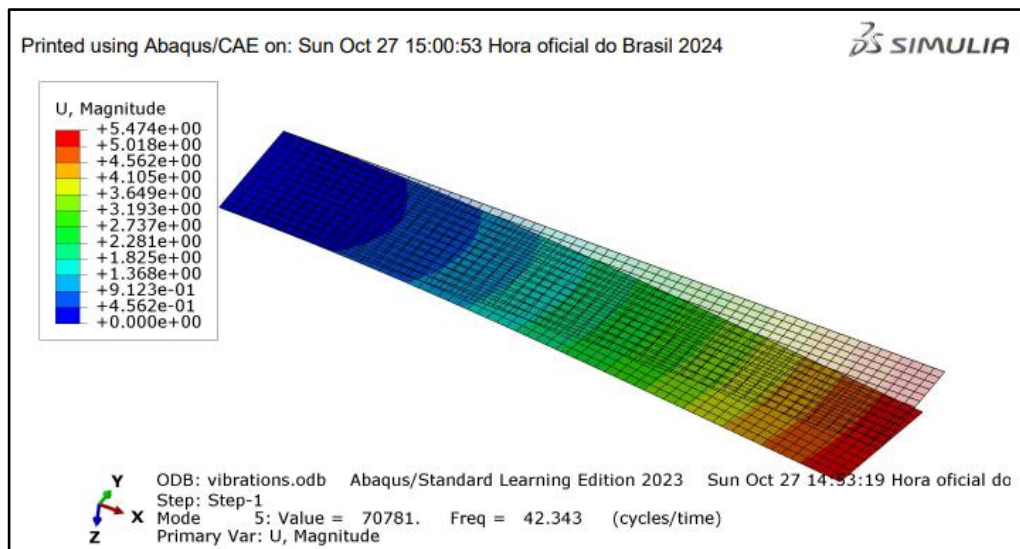
Modo 3 (15,714 Hz)



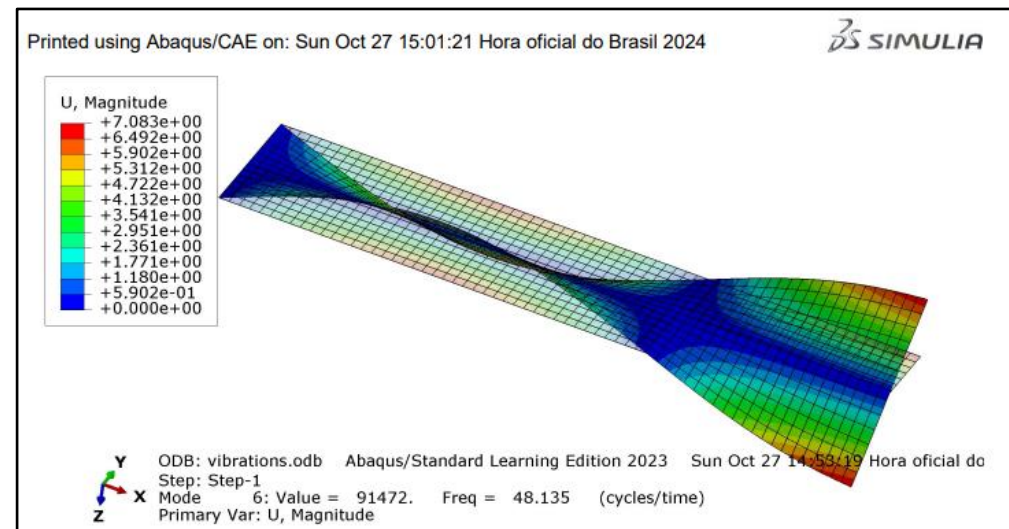
Modo 4 (24,333 Hz) correspondente ao modo 2 de vibração vertical



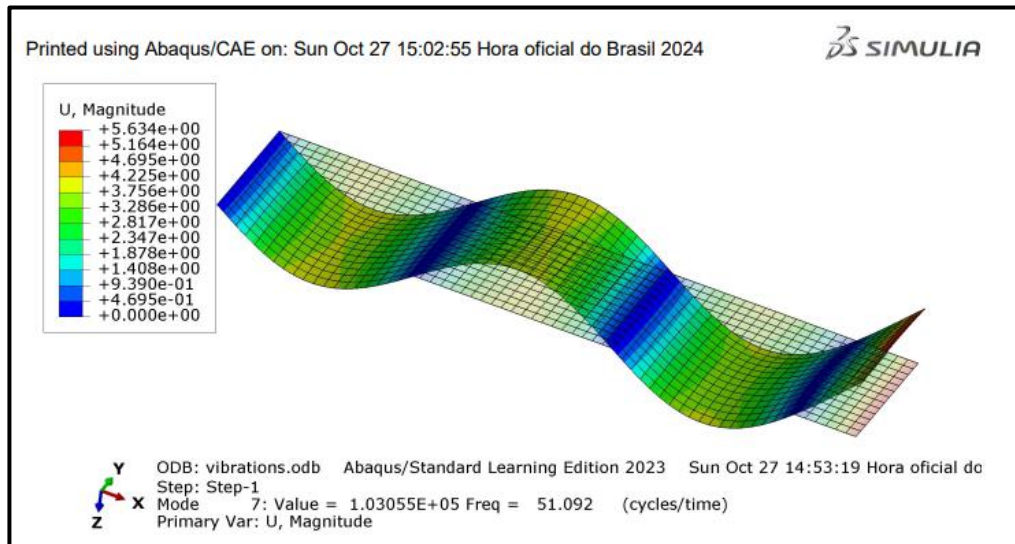
Modo 5 (42,343 Hz)



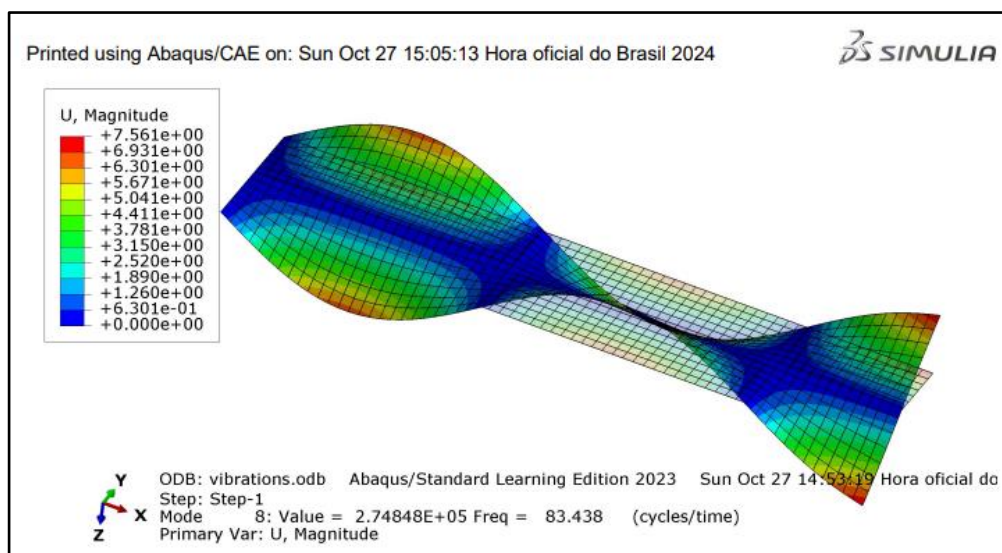
Modo 6 (48,135 Hz)



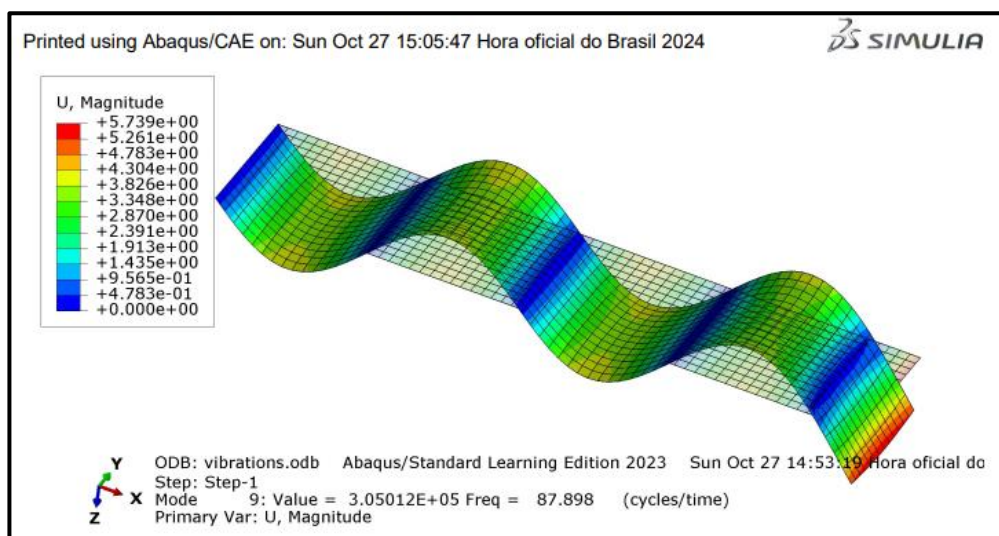
Modo 7 (51,092 Hz) correspondente ao modo 3 de vibração vertical



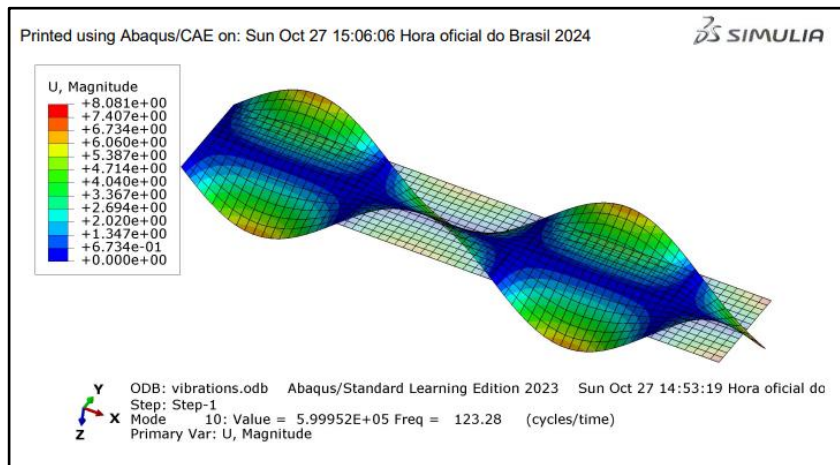
Modo 8 (83,438 Hz)



Modo 9 (87,898 Hz) correspondente ao modo 4 de vibração vertical

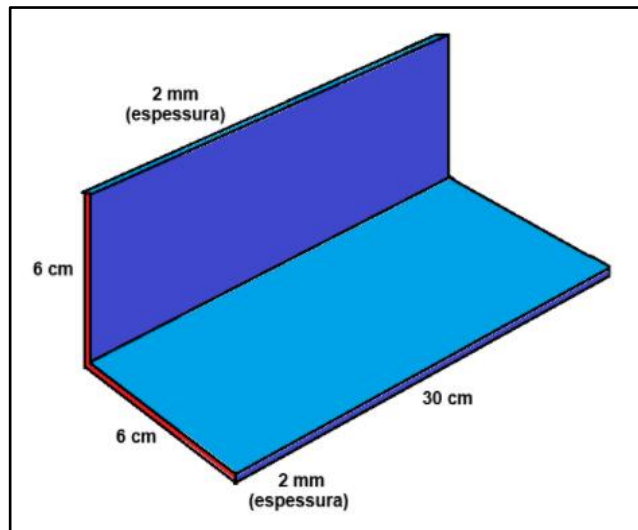


Modo 10 (123,28 Hz)

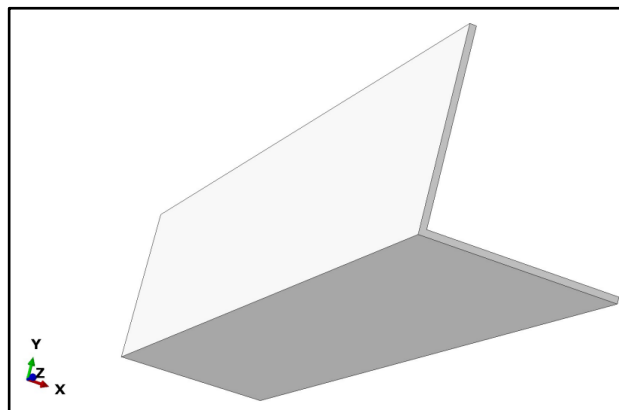


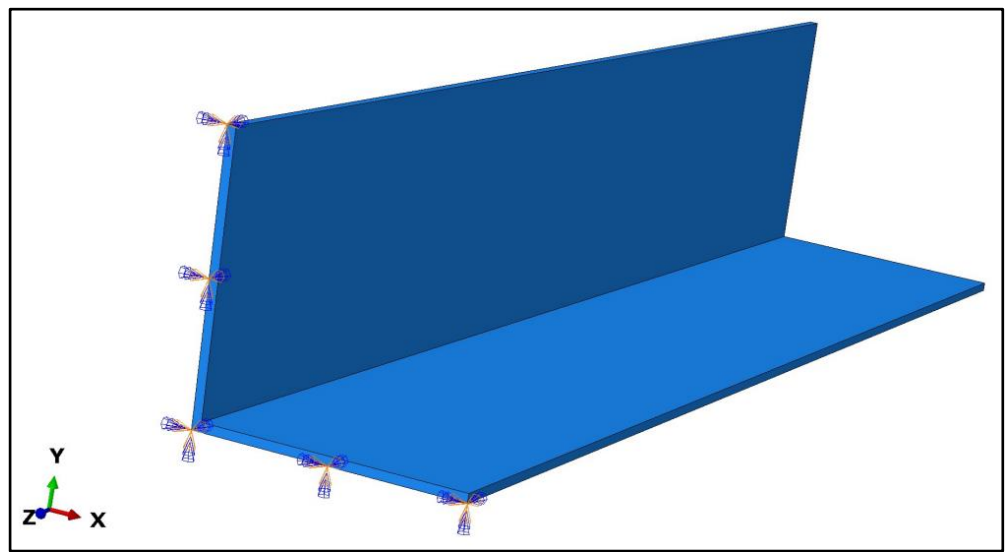
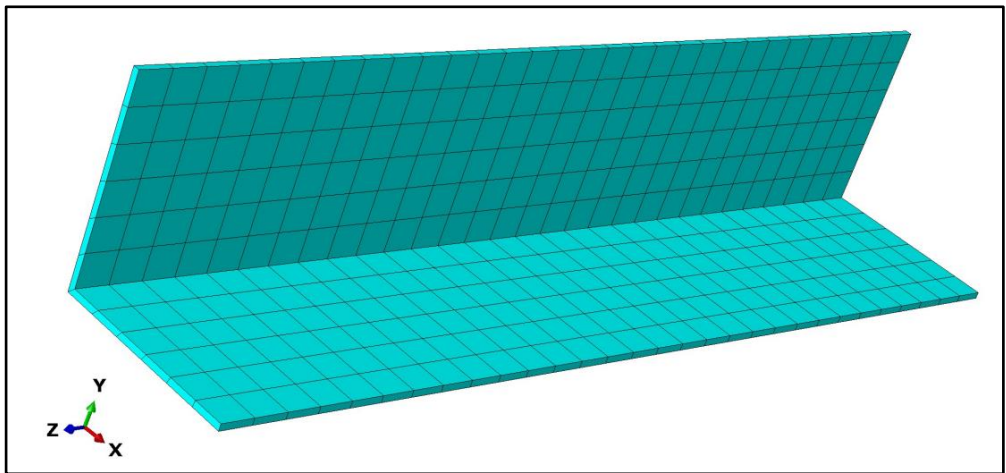
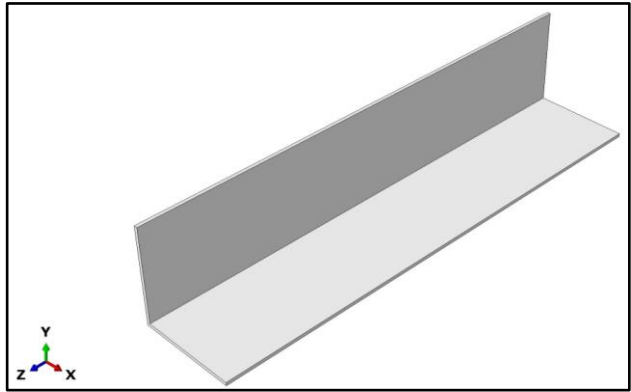
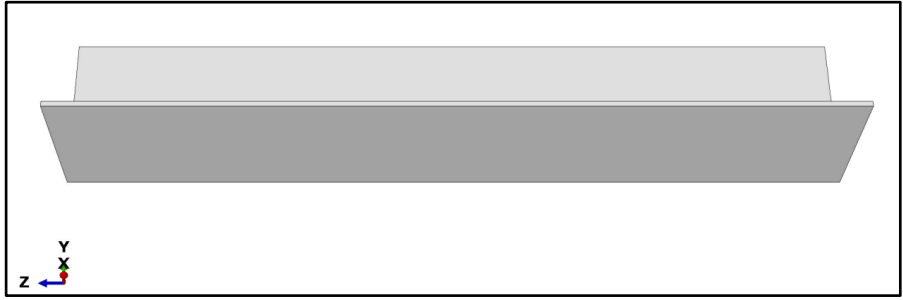
Parte 3:

Criar alguma outra geometria e mostrar os resultados dos 5 primeiros modos.



A análise vibracional do sólido maciço foi feita usando “solid/homogeneous” para a descrição da peça, que tem engaste na face lateral destacada em vermelha. Foi considerado alumínio de $E = 69\text{GPa}$, $\nu = 0,33$ e $\rho = 2700\text{ kg/m}^3$. A malha tem *approximate global size* de 0,01 e isso gerou 360 elementos.

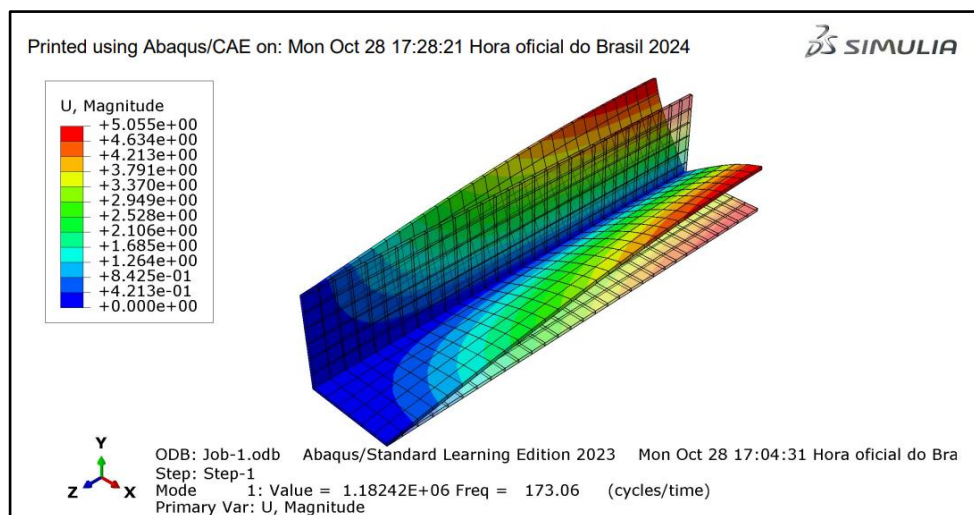




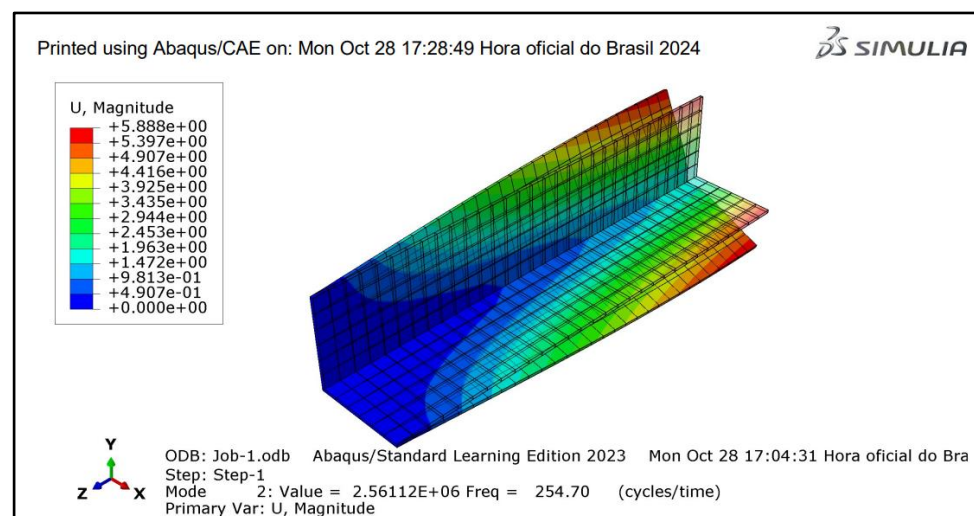
Frame			
Index	Description		
0	Increment	0: Base State	
1	Mode	1: Value = 1.18242E+06 Freq = 173.06 (cycles/time)	
2	Mode	2: Value = 2.56112E+06 Freq = 254.70 (cycles/time)	
3	Mode	3: Value = 8.73382E+06 Freq = 470.35 (cycles/time)	
4	Mode	4: Value = 9.92898E+06 Freq = 501.50 (cycles/time)	
5	Mode	5: Value = 1.50414E+07 Freq = 617.25 (cycles/time)	
6	Mode	6: Value = 2.18051E+07 Freq = 743.19 (cycles/time)	
7	Mode	7: Value = 2.77242E+07 Freq = 838.01 (cycles/time)	
8	Mode	8: Value = 3.29727E+07 Freq = 913.90 (cycles/time)	
9	Mode	9: Value = 3.69442E+07 Freq = 967.37 (cycles/time)	
10	Mode	10: Value = 4.35833E+07 Freq = 1050.7 (cycles/time)	

OK Apply Field Output... Cancel

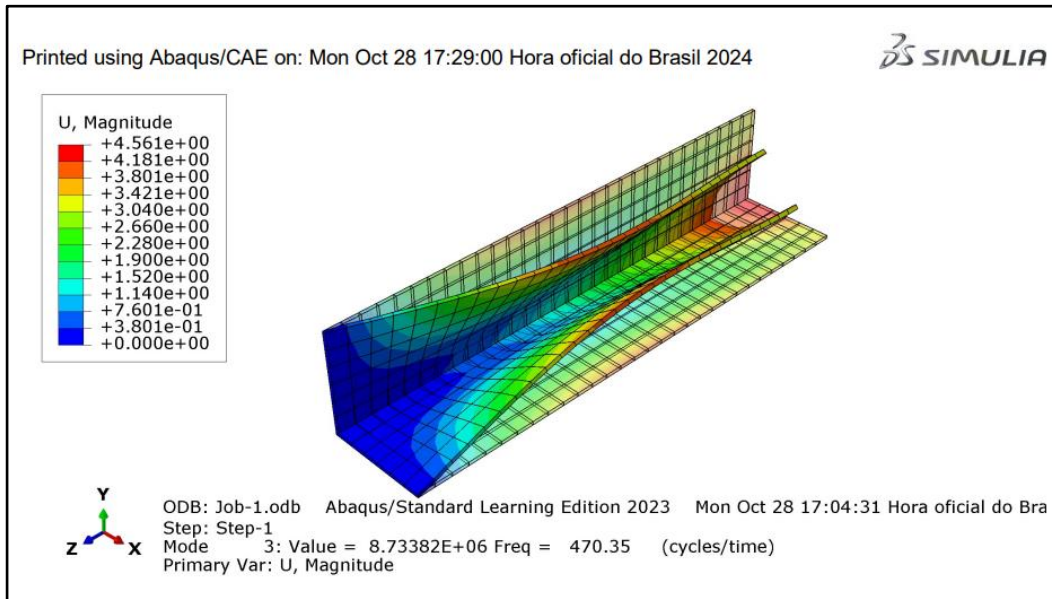
Modo 1 (173,07 Hz)



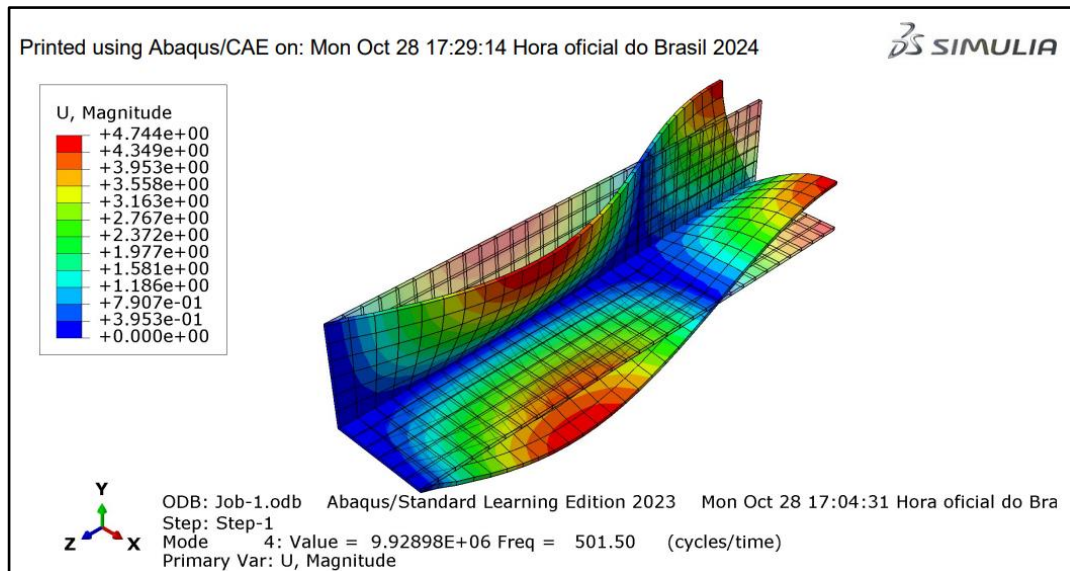
Modo 2 (254,70 Hz)



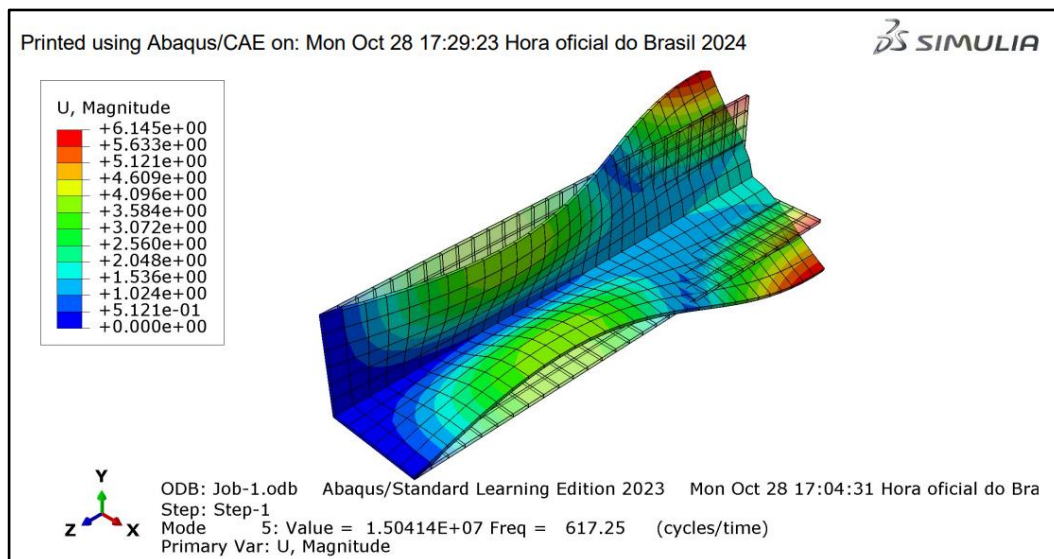
Modo 3 (470,35 Hz)



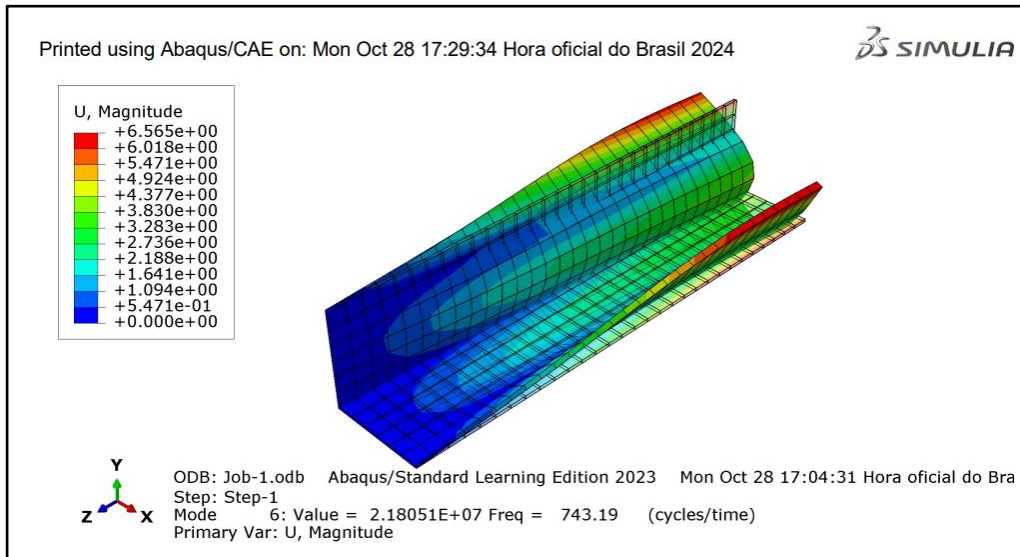
Modo 4 (501,50 Hz)



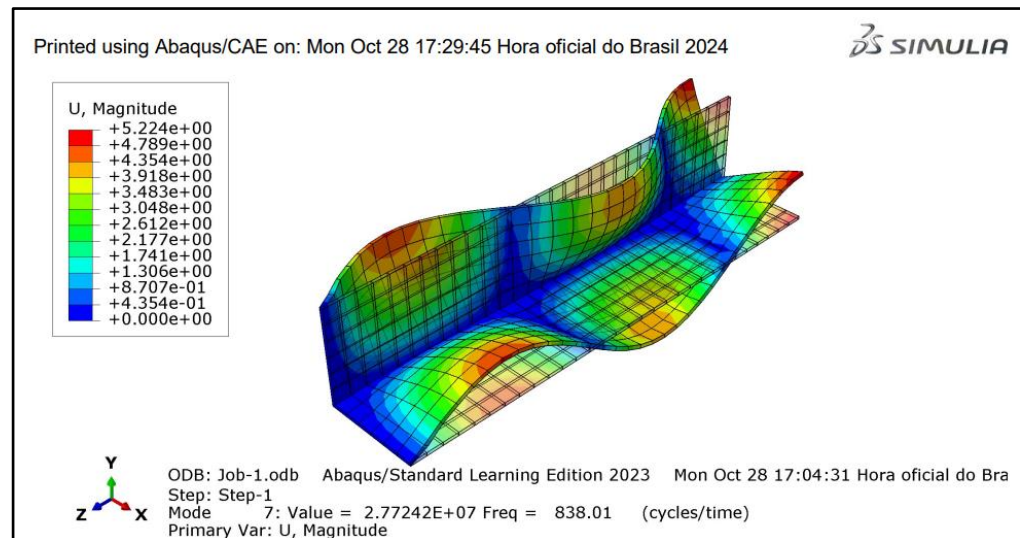
Modo 5 (617,25 Hz)



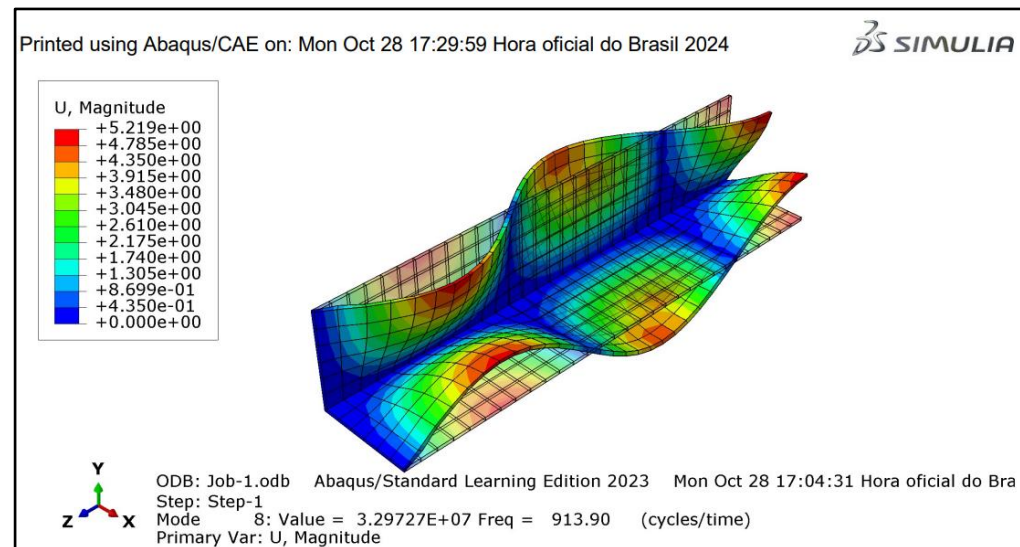
Modo 6 (743,19 Hz)



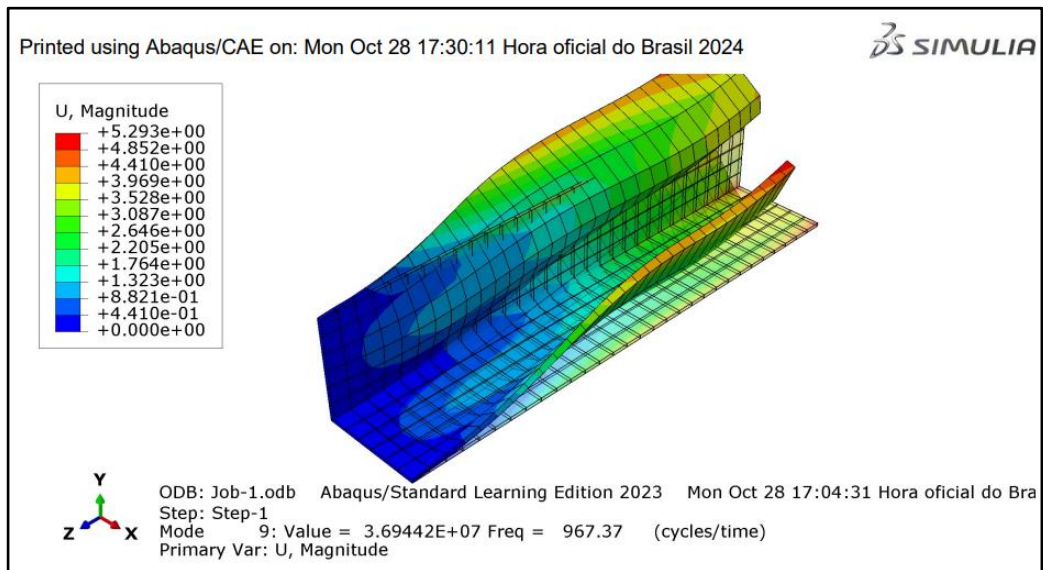
Modo 7 (838,01 Hz)



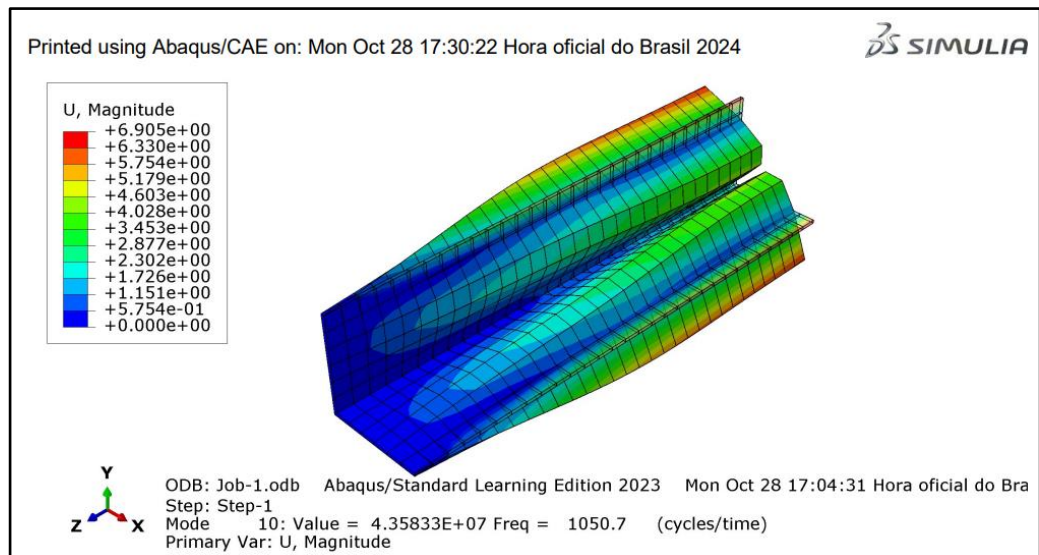
Modo 8 (913,90 Hz)






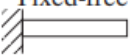

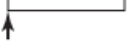
Modo 9 (967,37 Hz)



Modo 10 (1050,7 Hz)



Anexo 1:

End Conditions of Beam	Frequency Equation	Mode Shape (Normal Function)	Value of $\beta_n l$
Pinned-pinned 	$\sin \beta_n l = 0$	$W_n(x) = C_n [\sin \beta_n x]$	$\beta_1 l = \pi$ $\beta_2 l = 2\pi$ $\beta_3 l = 3\pi$ $\beta_4 l = 4\pi$
Free-free 	$\cos \beta_n l \cdot \cosh \beta_n l = 1$	$W_n(x) = C_n [\sin \beta_n x + \sinh \beta_n x$ $+ \alpha_n (\cos \beta_n x + \cosh \beta_n x)]$ where $\alpha_n = \left(\frac{\sin \beta_n l - \sinh \beta_n l}{\cosh \beta_n l - \cos \beta_n l} \right)$	$\beta_1 l = 4.730041$ $\beta_2 l = 7.853205$ $\beta_3 l = 10.995608$ $\beta_4 l = 14.137165$ ($\beta l = 0$ for rigid-body mode)
Fixed-fixed 	$\cos \beta_n l \cdot \cosh \beta_n l = 1$	$W_n(x) = C_n [\sinh \beta_n x - \sin \beta_n x$ $+ \alpha_n (\cosh \beta_n x - \cos \beta_n x)]$ where $\alpha_n = \left(\frac{\sinh \beta_n l - \sin \beta_n l}{\cos \beta_n l - \cosh \beta_n l} \right)$	$\beta_1 l = 4.730041$ $\beta_2 l = 7.853205$ $\beta_3 l = 10.995608$ $\beta_4 l = 14.137165$
Fixed-free 	$\cos \beta_n l \cdot \cosh \beta_n l = -1$	$W_n(x) = C_n [\sin \beta_n x - \sinh \beta_n x$ $- \alpha_n (\cos \beta_n x - \cosh \beta_n x)]$ where $\alpha_n = \left(\frac{\sin \beta_n l + \sinh \beta_n l}{\cos \beta_n l + \cosh \beta_n l} \right)$	$\beta_1 l = 1.875104$ $\beta_2 l = 4.694091$ $\beta_3 l = 7.854757$ $\beta_4 l = 10.995541$
Fixed-pinned 	$\tan \beta_n l - \tanh \beta_n l = 0$	$W_n(x) = C_n [\sin \beta_n x - \sinh \beta_n x$ $+ \alpha_n (\cosh \beta_n x - \cos \beta_n x)]$ where $\alpha_n = \left(\frac{\sin \beta_n l - \sinh \beta_n l}{\cos \beta_n l - \cosh \beta_n l} \right)$	$\beta_1 l = 3.926602$ $\beta_2 l = 7.068583$ $\beta_3 l = 10.210176$ $\beta_4 l = 13.351768$
Pinned-free 	$\tan \beta_n l - \tanh \beta_n l = 0$	$W_n(x) = C_n [\sin \beta_n x + \alpha_n \sinh \beta_n x]$ where $\alpha_n = \left(\frac{\sin \beta_n l}{\sinh \beta_n l} \right)$	$\beta_1 l = 3.926602$ $\beta_2 l = 7.068583$ $\beta_3 l = 10.210176$ $\beta_4 l = 13.351768$ ($\beta l = 0$ for rigid-body mode)