25/08/20

Nome: Davi Augusto Neves Leite RA: 191027383

5ª Lista de Exercícios
(Integração e Sistemas Não Lineares)

① a) $\int_{-1}^{2} e^x \, dx$ $\quad * h = \frac{(2+1)}{6} = 0,5$

X	-1	-0,5	0	0,5	1	1,5	2
e^x	0,3679	0,6065	1	1,6487	2,7183	4,4817	7,3891

• Regra dos Trapézios

$$\int_{-1}^{2} e^x \, dx \approx \frac{h}{2} [F(X_0) + F(X_6) + 2.(F(X_1) + F(X_2) + F(X_3) + F(X_4) + F(X_5))]$$

$$\Rightarrow \int_{-1}^{2} e^x \, dx \approx \frac{0,5}{2} [0,3679 + 7,3891 + 2.(0,6065 + 1 + 1,6487 + 2,7183 + 4,4817)]$$

$$\boxed{\therefore \int_{-1}^{2} e^x \, dx \approx 7,1669}$$

• Regra 1/3 de Simpson

$$\int_{-1}^{2} e^x \, dx \approx \frac{h}{3} [F(X_0) + F(X_6) + 4.(F(X_1) + F(X_3) + F(X_5)) + 2.(F(X_2) + F(X_4))]$$

$$\boxed{\therefore \int_{-1}^{2} e^x \, dx \approx 7,0235}$$

spiral

• Regra 3/8 de Simpson

$$\int_{-1}^{2} e^x\,dx \approx \frac{3h}{8}\left[F(x_0)+F(x_6)+3.(F(x_1)+F(x_2)+F(x_4)+F(x_5)) + 2.(F(x_3))\right]$$

$$\boxed{\therefore \int_{-1}^{2} e^x\,dx \approx 7{,}0264}$$

b) $\int_{1}^{7} \sqrt{x}\,dx$ $\quad * h = \frac{(7-1)}{6} = 1$

x	1	2	3	4	5	6	7
$\sqrt{x}$	1	1,4142	1,7321	2	2,2361	2,4495	2,6458

• Regra dos Trapézios

$$\int_{1}^{7} \sqrt{x}\,dx \approx \frac{1}{2}\left[1+2{,}6458+2.(1{,}4142+1{,}7321+2+2{,}2361+2{,}4495)\right]$$

$$\boxed{\therefore \int_{1}^{7} \sqrt{x}\,dx \approx 11{,}6547}$$

• Regra 1/3 de Simpson

$$\int_{1}^{7} \sqrt{x}\,dx \approx (1/3)\left[1+2{,}6458+4.(1{,}4142+2+2{,}4495)+2.(1{,}7321+2{,}2361)\right]$$

spiral

$$\therefore \int_1^7 \sqrt{x}\,dx \approx 11,6789$$

- Regra 3/8 de Simpson

$$\int_1^7 \sqrt{x}\,dx \approx \left(\frac{3.1}{8}\right)[1 + 2,6458 + 3.(1,4142 + 1,7321 + 2,2361 + 2,4495) + 2.(2)]$$

$$\therefore \int_1^7 \sqrt{x}\,dx \approx 11,6780$$

c) $\int_0^9 (x^3 - 3x)\,dx$ $\quad * h = \frac{9-0}{6} = 1,5$

x	0	1,5	3	4,5	6	7,5	9
x^3-3x	0	-1,125	18	77,625	198	399,375	702

- Regra dos trapézios

$$\int_0^9 (x^3 - 3x)\,dx \approx \left(\frac{1,5}{2}\right)[0 + 702 + 2.((-1,125) + 18 + 77,625 + 198 + 399,375)]$$

$$\therefore \int_0^9 (x^3 - 3x)\,dx \approx 1564,3125$$

spiral

• Regra 1/3 de Simpson

$$\int_0^9 (x^3-3x)\,dx \approx \left(\frac{1,5}{3}\right)[0+702+4.(-1,125+77,625+399,375)+2.(18+198)]$$

$$\boxed{\therefore \int_0^9 (x^3-3x)\,dx \approx 1518,75}$$

• Regra 3/8 de Simpson

$$\int_0^9 (x^3-3x)\,dx \approx \left(\frac{3.1,5}{8}\right)[0+702+3.(-1,125+18+198+399,375)+2.(77,625)]$$

$$\boxed{\therefore \int_0^9 (x^3-3x)\,dx \approx 1518,75}$$

2) $* \; h = \frac{10 \text{ min}}{60 \text{ min}} \approx 0,1667$

• Regra dos Trapézios

$$\int_8^{8,6667} v(t)\,dt \approx \left(\frac{0,1667}{2}\right)[24,2+34,2+2.(35+41,3+42,8)]$$

$$\boxed{\therefore \int_8^{8,6667} v(t)\,dt = \text{distância percorrida entre 8h e 8h40} \approx 24,8833 \text{ Km}}$$

/ /

③ a) $\int_0^2 f(x)\,dx$ pela regra 1/3 de Simpson

$* h = 0,5$

$$\int_0^2 f(x)\,dx \approx \left(\frac{0,5}{3}\right)[1+8,695+4.(2,119+3,945)$$
$$+2.(2,910+5,720)]$$

$$\therefore \boxed{\int_0^2 f(x)\,dx \approx 6,1327}$$

b) • Aproximar f por um polinômio de 2º grau e obter F(3)

• Método Newton e pontos: (1,5 ; 3,945) ; (2 ; 5,720) e (2,5 ; 8,695)

X	DF	1ª ordem	2ª ordem
1,5	3,945		
		3,55	
2	5,720		2,4
		5,95	
2,5	8,695		

$$\Rightarrow P_2(x) = F(x_0) + (x-x_0)\cdot F[x_0,x_1] + (x-x_0)(x-x_1)F[x_0,x_1,x_2]$$
$$P_2(x) = 3,945 + (x-1,5)(3,55) + (x-1,5)(x-2)(2,4)$$
$$\therefore \boxed{F(x) \approx P_2(x) = 2,4x^2 - 4,85x + 5,82}$$

spiral

/ /

$\therefore \boxed{F(3) \approx 2{,}4 \cdot 3^2 - 4{,}85 \cdot 3 + 5{,}82 = 12{,}87}$

- Estimar $\int_0^3 F(x)\,dx$ pela regra 3/8 de Simpson

$$\int_0^3 F(x)\,dx \approx \left(\frac{3 \cdot 0{,}5}{8}\right)[1 + 12{,}87 + 3 \cdot (2{,}119 + 2{,}910 + 5{,}720 + 8{,}695) + 2 \cdot (3{,}945)]$$

$$\boxed{\therefore \int_0^3 F(x)\,dx \approx 15{,}0173}$$

4) Método de Newton e $\varepsilon = 0{,}01$.

$$\begin{cases} x_1^2 + x_2 = 3 \\ x_1 + x_2^2 = 5 \end{cases} \quad \text{com } X^0 = [2;2]^t$$

1) $J = \begin{bmatrix} 2x_1 & 1 \\ 1 & 2x_2 \end{bmatrix}$ e $F(x_1,x_2) = \begin{bmatrix} x_1^2 + x_2 - 3 \\ x_1 + x_2^2 - 5 \end{bmatrix}$

2) Iterações

- $K = 0$

$$J \cdot D = -F(x_1^0, x_2^0)$$

$$\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} dx_1^0 \\ dx_2^0 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

$\Rightarrow dX_1^0 = -0{,}7333$
$\Rightarrow dX_2^0 = -0{,}0667$

$\hookrightarrow X_1^1 = X_1^0 + dX_1^0 = 2 - 0{,}7333 = 1{,}2667$
$\hookrightarrow X_2^1 = X_2^0 + dX_2^0 = 2 - 0{,}0667 = 1{,}9333$

Erro: $\dfrac{MAX\{|1{,}2667-2|;|1{,}9333-2|\}}{MAX\{|1{,}2667|;|1{,}9333|\}} = 0{,}3793 > \varepsilon$

• $K = 1$

$$\begin{bmatrix} 2{,}5334 & 1 \\ 1 & 3{,}8666 \end{bmatrix} \begin{bmatrix} dX_1^1 \\ dX_2^1 \end{bmatrix} = \begin{bmatrix} -0{,}5378 \\ -0{,}0043 \end{bmatrix}$$

$\Rightarrow dX_1^1 = -0{,}2359$
$\Rightarrow dX_2^1 = 0{,}0599$

$\hookrightarrow X_1^2 = 1{,}2667 - 0{,}2359 = 1{,}0308$
$\hookrightarrow X_2^2 = 1{,}9333 + 0{,}0599 = 1{,}9932$

Erro: $\dfrac{MAX\{|-0{,}2359|;|0{,}0599|\}}{MAX\{|1{,}0308|;|1{,}9932|\}} = 0{,}1184 > \varepsilon$

• $K = 2$

$$\begin{bmatrix} 2{,}0616 & 1 \\ 1 & 3{,}9864 \end{bmatrix} \begin{bmatrix} dX_1^2 \\ dX_2^2 \end{bmatrix} = \begin{bmatrix} -0{,}0557 \\ -0{,}0036 \end{bmatrix}$$

$\Rightarrow dx_1^2 = -0,0303$
$\Rightarrow dx_2^2 = 0,0067$

$\hookrightarrow x_1^3 = 1,0308 - 0,0303 = 1,0005$
$\hookrightarrow x_2^3 = 1,9932 + 0,0067 = 1,9999$

Erro: $\dfrac{MAX\{|-0,0303|;|0,0067|\}}{MAX\{|1,0005|;|1,9999|\}} = 0,0152 > \varepsilon$

• $K=3$

$$\begin{bmatrix} 2,001 & 1 \\ 1 & 3,9998 \end{bmatrix} \begin{bmatrix} dx_1^3 \\ dx_2^3 \end{bmatrix} = \begin{bmatrix} -0,0009 \\ -0,0001 \end{bmatrix}$$

$\Rightarrow dx_1^3 = -0,0005$
$\Rightarrow dx_2^3 = 0,0001$

$\hookrightarrow x_1^4 = 1,0005 - 0,0005 = 1$
$\hookrightarrow x_2^4 = 1,9999 + 0,0001 = 2$

Erro: $\dfrac{MAX\{|-0,0005|;|0,0001|\}}{MAX\{|1|;|2|\}} = 0,0003 < \varepsilon$

$$\boxed{\therefore \bar{X} \cong \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

spiral

⑤ Método de Newton e $\varepsilon = 0{,}01$

$$\begin{cases} x_1 + x_2 = 5 \\ x_1^2 - x_2^2 = 5 \end{cases} \quad \text{com } X^0 = [1;5]^t$$

1) $J = \begin{bmatrix} 1 & 1 \\ 2x_1 & -2x_2 \end{bmatrix}$ e $F(x_1, x_2) = \begin{bmatrix} x_1 + x_2 - 5 \\ x_1^2 - x_2^2 - 5 \end{bmatrix}$

2) Iterações

• $k = 0$

$$\begin{bmatrix} 1 & 1 \\ 2 & -10 \end{bmatrix} \begin{bmatrix} dx_1^0 \\ dx_2^0 \end{bmatrix} = \begin{bmatrix} -1 \\ 29 \end{bmatrix}$$

$\Rightarrow dx_1^0 = 1{,}5833$

$\Rightarrow dx_2^0 = -2{,}5833$

$\hookrightarrow x_1^1 = 1 + 1{,}5833 = 2{,}5833$

$\hookrightarrow x_2^1 = 5 - 2{,}5833 = 2{,}4167$

Erro: $\dfrac{\max\{|1{,}5833| ; |-2{,}5833|\}}{\max\{|2{,}5833| ; |2{,}4167|\}} = 1 > \varepsilon$

• $k = 1$

$$\begin{bmatrix} 1 & 1 \\ 5{,}1666 & -4{,}8334 \end{bmatrix} \begin{bmatrix} dx_1^1 \\ dx_2^1 \end{bmatrix} = \begin{bmatrix} 0 \\ -4{,}167 \end{bmatrix}$$

spiral

$\Rightarrow dx_1^1 = 0{,}4167$
$\Rightarrow dx_2^1 = -0{,}4167$

$\hookrightarrow x_1^2 = 2{,}5833 + 0{,}4167 = 3$
$\hookrightarrow x_2^2 = 2{,}4167 - 0{,}4167 = 2$

Erro: $\dfrac{MAX\{|0{,}4167|;|0{,}4167|\}}{MAX\{|3|;|2|\}} = 0{,}1389 > \varepsilon$

• $K = 2$

$$\begin{bmatrix} 1 & 1 \\ 6 & -4 \end{bmatrix} \begin{bmatrix} dx_1^2 \\ dx_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow dx_1^2 = 0$
$\Rightarrow dx_2^2 = 0$

$\hookrightarrow x_1^3 = 3$
$\hookrightarrow x_2^3 = 2$

Erro: $\dfrac{MAX\{|0|;|0|\}}{MAX\{|3|;|2|\}} = 0 < \varepsilon$

$$\boxed{\therefore \bar{x} \cong \begin{bmatrix} 3 \\ 2 \end{bmatrix}}$$