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• Método de Cholesky

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 8 & 10 \\ 4 & 10 & 26 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 10 \end{bmatrix}$$

(A)



I) \* A matriz (A) é simétrica. ( $a_{12} = a_{21}$ ;  $a_{13} = a_{31}$ ;  
 $a_{23} = a_{32}$ )

$$* \det(A_1) = 1$$

$$\det(A_2) = 8 - 4 = 4$$

$$\det(A_3) = 36$$

$\Rightarrow$  A matriz (A) é definida positiva.



$$\text{II) } A = G \cdot G^+$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 8 & 10 \\ 4 & 10 & 26 \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \cdot \begin{bmatrix} g_{11} & g_{21} & g_{31} \\ 0 & g_{22} & g_{32} \\ 0 & 0 & g_{33} \end{bmatrix}$$

$$\begin{cases} (g_{11})^2 = a_{11} \Rightarrow g_{11} = \sqrt{1} = 1 \end{cases}$$

$$\begin{cases} (g_{21})(g_{11}) = a_{21} \Rightarrow g_{21} = 2/1 = 2 \end{cases}$$

$$\begin{cases} (g_{31})(g_{11}) = a_{31} \Rightarrow g_{31} = 4/1 = 4 \end{cases}$$

$$\begin{cases} (g_{22})^2 = a_{22} - (g_{21})^2 \Rightarrow g_{22} = \sqrt{8 - 2^2} = \sqrt{4} = 2 \end{cases}$$

$$\begin{cases} (g_{31})(g_{21}) + (g_{32})(g_{22}) = a_{32} \Rightarrow g_{32} = (10 - 4 \cdot 2)/2 = 1 \end{cases}$$

$$\begin{cases} (g_{33})^2 = a_{33} - (g_{31})^2 - (g_{32})^2 \Rightarrow g_{33} = \sqrt{26 - 4^2 - 1^2} = 3 \end{cases}$$

$$\Rightarrow G = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 1 & 3 \end{bmatrix}$$

$$G^+ = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$



$$\text{III) } \det(A) = (g_{11} \cdot g_{22} \cdot g_{33})^2$$

$$\det(A) = (1 \cdot 2 \cdot 3)^2$$

$$\det(A) = 36$$

$$\text{IV) } * GY = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 10 \end{bmatrix}$$

$$\rightarrow \begin{cases} y_1 = 1 \\ y_2 = (-4 - 2)/2 = -3 \\ y_3 = (10 + 3 - 4)/3 = 3 \end{cases} \Rightarrow Y^* = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$$

$$* G^T X = Y$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$$



$$IV) * GY = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 10 \end{bmatrix}$$

$$\rightarrow \begin{cases} y_1 = 1 \\ y_2 = (-4 - 2)/2 = -3 \\ y_3 = (10 + 3 - 4)/3 = 3 \end{cases} \Rightarrow Y^* = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$$

$$* G^T X = Y$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$$

$$\rightarrow \begin{cases} x_3 = 1 \\ x_2 = (-3 - 1)/2 = -2 \\ x_1 = (1 - 4 + 4)/1 = 1 \end{cases} \Rightarrow X^* = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$