29/07/20

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3ª Lista de Exercícios
(Sistemas Lineares)

① $\begin{cases} x_1 + 2x_2 + 4x_3 = 30 & (A) \\ 2x_1 + 0x_2 + 2x_3 = 20 & (B) \\ 4x_1 + x_2 + 3x_3 = 40 & (C) \end{cases}$

$$\Rightarrow \underbrace{\begin{bmatrix} 1 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 1 & 3 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 30 \\ 20 \\ 40 \end{bmatrix}}_{b}$$

• Método de Eliminação de Gauss

$$\Rightarrow A^{(1)} = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 30 \\ 2 & 0 & 2 & 20 \\ 4 & 1 & 3 & 40 \end{array}\right]$$

I) Pivô $= a_{11}^{(1)} = 1$

• 2ª linha

$$m_{21} = \frac{2}{1} = 2$$

$a_{21} = 2 - (2 \cdot 1) = 0$
$a_{22} = 0 - (2 \cdot 2) = -4$
$a_{23} = 2 - (2 \cdot 4) = -6$

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$b_{21} = 20 - (2 \cdot 30) = -40$

• 3ª linha

$m_{31} = \frac{4}{1} = 4$

$a_{31} = 4 - (4 \cdot 1) = 0$
$a_{32} = 1 - (4 \cdot 2) = -7$
$a_{33} = 3 - (4 \cdot 4) = -13$

$b_3 = 40 - (4 \cdot 30) = -80$

$$\Rightarrow A^{(2)} = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 30 \\ 0 & -4 & -6 & -40 \\ 0 & -7 & -13 & -80 \end{array}\right]$$

II) $P_{ivô} = a_{22}^{(2)} = -4$

• 3ª linha

$m_{32} = \frac{-7}{-4} = \frac{7}{4}$

$a_{32} = (-7) - (7/4 \cdot (-4)) = 0$
$a_{33} = (-13) - (7/4 \cdot (-6)) = -5/2$

$b_3 = (-80) - (7/4 \cdot (-40)) = -10$

$$\Rightarrow A^{(3)} = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 30 \\ 0 & -4 & -6 & -40 \\ 0 & 0 & -5/2 & -10 \end{array}\right]$$

$$\rightarrow X_3 = (-10) \cdot \left(\frac{2}{-5}\right) = 4$$

$$\rightarrow X_2 = (-40 + 4 \cdot 4) / (-6) = 4$$

$$\rightarrow X_1 = (30 - 2 \cdot 4 - 4 \cdot 4) / 1 = 6$$

$$\Rightarrow X^* = \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$$

∴ Pode-se produzir: 6 unidades do produto 1, 4 unidades do produto 2 e 4 unidades do produto 3.

3 a)
$$\begin{cases} 2X_1 + 3X_2 - 1X_3 = 1 \\ 4X_1 + 4X_2 - 3X_3 = -8 \\ 2X_1 - 3X_2 + 1X_3 = -9 \end{cases}$$

$$\rightarrow \underbrace{\begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ 2 & -3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ -8 \\ -9 \end{bmatrix}}_{b}$$

$$\Rightarrow A^{(1)} = \left[\begin{array}{ccc|c} 2 & 3 & -1 & 1 \\ 4 & 4 & -3 & -8 \\ 2 & -3 & 1 & -9 \end{array}\right] \text{trocar}$$

spiral

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$$\Rightarrow A^{(1)} = \left[\begin{array}{ccc|c} 4 & 4 & -3 & -8 \\ 2 & 3 & -1 & 1 \\ 2 & -3 & 1 & -9 \end{array}\right]$$

I) $Pivô = a_{11}^{(1)} = 4$

• 2ª linha

$$m_{21} = \frac{2}{4} = \frac{1}{2}$$

$$a_{21} = 2 - ((1/2) \cdot 4) = 0$$
$$a_{22} = 3 - ((1/2) \cdot 4) = 1$$
$$a_{23} = -1 - ((1/2) \cdot (-3)) = 1/2$$

$$b_2 = 1 - ((1/2) \cdot (-8)) = 5$$

• 3ª linha

$$m_{31} = \frac{2}{4} = \frac{1}{2}$$

$$a_{31} = 2 - ((1/2) \cdot 4) = 0$$
$$a_{32} = -3 - ((1/2) \cdot 4) = -5$$
$$a_{33} = 1 - ((1/2) \cdot (-3)) = 5/2$$

$$b_3 = (-9) - ((1/2) \cdot (-8)) = -5$$

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$\Rightarrow A^{(2)} = \left[\begin{array}{ccc|c} 4 & 4 & -3 & -8 \\ 0 & 1 & 1/2 & 5 \\ 0 & -5 & 5/2 & -5 \end{array}\right]$ trocar

$\Rightarrow A^{(2)} = \left[\begin{array}{ccc|c} 4 & 4 & -3 & -8 \\ 0 & -5 & 5/2 & -5 \\ 0 & 1 & 1/2 & 5 \end{array}\right]$

II) Pivô $= a_{22}^{(2)} = -5$

• 3º linha

$m_{32} = \dfrac{1}{-5}$

$a_{32} = 1 - ((-1/5) \cdot (-5)) = 0$
$a_{33} = (1/2) - ((-1/5) \cdot (5/2)) = 1$

$b_3 = 5 - ((-1/5) \cdot (-5)) = 4$

$\Rightarrow A^{(3)} = \left[\begin{array}{ccc|c} 4 & 4 & -3 & -8 \\ 0 & -5 & 5/2 & -5 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$\rightarrow X_3 = 4$
$\rightarrow X_2 = (-5 - (5 \cdot 4)/2)/-5 = 3$
$\rightarrow X_1 = (-8 - 4 \cdot 3 + 3 \cdot 4)/4 = -2$

$\therefore X^* = [-2; 3; 4]^t$

spiral

3) b) $\begin{cases} 4x_1 + 3x_2 = -2 \\ -x_1 + 3x_2 - 3x_3 = 14 \\ -3x_2 + x_3 - 2x_4 = -6 \\ +3x_3 - 2x_4 = -4 \end{cases}$

$$\rightarrow \underbrace{\begin{bmatrix} 4 & 3 & 0 & 0 \\ -1 & 3 & -3 & 0 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 3 & -2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -2 \\ 14 \\ -6 \\ -4 \end{bmatrix}}_{b}$$

$$\Rightarrow A^{(1)} = \left[\begin{array}{cccc|c} 4 & 3 & 0 & 0 & -2 \\ -1 & 3 & -3 & 0 & 14 \\ 0 & -3 & 1 & -2 & -6 \\ 0 & 0 & 3 & -2 & -4 \end{array}\right]$$

I) $P_{w2} = a_{11}^{(1)} = 4$

• 2ª linha

$m_{21} = -\frac{1}{4}$

$a_{21} = -1 - ((-1/4) \cdot 4) = 0$
$a_{22} = 3 - ((-1/4) \cdot 3) = 15/4$
$a_{23} = -3 - ((-1/4) \cdot 0) = -3$
$a_{24} = 0 - ((-1/4) \cdot 0) = 0$

$b_2 = 14 - ((-1/4) \cdot (-2)) = 27/2$

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$$\Rightarrow A^{(2)} = \left[\begin{array}{cccc|c} 4 & 3 & 0 & 0 & -2 \\ 0 & 15/4 & -3 & 0 & 27/2 \\ 0 & -3 & 1 & -2 & -6 \\ 0 & 0 & 3 & -2 & -4 \end{array}\right]$$

II) $Pivô = a_{22}^{(2)} = 15/4$

• 3ª linha

$$m_{32} = (-3) \cdot \left(\frac{4}{15}\right) = -\frac{4}{5}$$

$$a_{32} = -3 - ((-4/5) \cdot (15/4)) = 0$$
$$a_{33} = 1 - ((-4/5) \cdot (-3)) = -7/5$$
$$a_{34} = -2 - ((-4/5) \cdot 0) = -2$$

$$b_3 = -6 - ((-4/5) \cdot 27/2) = 24/5$$

$$\Rightarrow A^{(3)} = \left[\begin{array}{cccc|c} 4 & 3 & 0 & 0 & -2 \\ 0 & 15/4 & -3 & 0 & 27/2 \\ 0 & 0 & -7/5 & -2 & 24/5 \\ 0 & 0 & 3 & -2 & -4 \end{array}\right] \text{trocar}$$

$$\Rightarrow A^{(3)} = \left[\begin{array}{cccc|c} 4 & 3 & 0 & 0 & -2 \\ 0 & 15/4 & -3 & 0 & 27/2 \\ 0 & 0 & 3 & -2 & -4 \\ 0 & 0 & -7/5 & -2 & 24/5 \end{array}\right]$$

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III) Pivô $= a_{33}^{(3)} = 3$

• 4ª linha

$$m_{43} = \left(-\frac{7}{5}\right) \cdot \frac{1}{3} = -\frac{7}{15}$$

$$a_{43} = (-7/5) - ((-7/15) \cdot 3) = 0$$
$$a_{44} = (-2) - ((-7/15) \cdot (-2)) = -44/15$$

$$b_4 = 24/5 - ((-7/15) \cdot (-4)) = 44/15$$

$$\Rightarrow A^{(4)} = \left[\begin{array}{cccc|c} 4 & 3 & 0 & 0 & -2 \\ 0 & 15/4 & -3 & 0 & 27/2 \\ 0 & 0 & 3 & -2 & -4 \\ 0 & 0 & 0 & -44/15 & 44/15 \end{array}\right]$$

$\rightarrow x_4 = -1$
$\rightarrow x_3 = (-4 + 2 \cdot (-1)) / 3 = -2$
$\rightarrow x_2 = (27/2 + 3 \cdot (-2)) / (15/4) = 2$
$\rightarrow x_1 = (-2 - 3 \cdot 2)/4 = -2$

$$\therefore X^* = \begin{bmatrix} -2 \\ 2 \\ -2 \\ -1 \end{bmatrix}$$

④ a) $\begin{cases} 10x_1 + x_2 - x_3 = 10 \\ x_1 + 10x_2 + x_3 = 12 \\ 2x_1 - x_2 + 10x_3 = 11 \end{cases}$ * $\det(A_1) = 10$
* $\det(A_2) = 99$

* $A = LU$ e $\det(A) = 1023$

$$\begin{bmatrix} 10 & 1 & -1 \\ 1 & 10 & 1 \\ 2 & -1 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

• 1ª linha (U): $\begin{cases} u_{11} = 10 \\ u_{12} = 1 \\ u_{13} = -1 \end{cases}$ | 1ª coluna (L): $\begin{cases} l_{21} = 1/10 \\ l_{31} = 2/10 \end{cases}$

• 2ª linha (U): $\begin{cases} u_{22} = 10 - (1/10) \cdot 1 = 99/10 \\ u_{23} = 1 - (1/10) \cdot (-1) = 11/10 \end{cases}$

2ª coluna (L): $\{ l_{32} = (-1 - (2/10) \cdot 1) / (99/10) = -4/33$

• 3ª linha (U): $\{ u_{33} = 10 - \left(\frac{2}{10}\right)(-1) - \left(\frac{-4}{33}\right) \cdot \left(\frac{11}{10}\right) = \frac{31}{3}$

→ $L = \begin{bmatrix} 1 & 0 & 0 \\ 1/10 & 1 & 0 \\ 2/10 & -4/33 & 0 \end{bmatrix}$ e $U = \begin{bmatrix} 10 & 1 & -1 \\ 0 & 99/10 & 11/10 \\ 0 & 0 & 31/3 \end{bmatrix}$

spiral

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* $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/10 & 1 & 0 \\ 2/10 & -4/33 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 11 \end{bmatrix}$$

$$\begin{cases} y_1 = 10 \\ y_2 = 11 \\ y_3 = 31/3 \end{cases}$$

* $Ux = y$

$$\begin{bmatrix} 10 & 1 & -1 \\ 0 & 99/10 & 11/10 \\ 0 & 0 & 31/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 31/3 \end{bmatrix}$$

$$\begin{cases} x_3 = 1 \\ x_2 = (11 - 11/10) / (99/10) = 1 \\ x_1 = 1 \end{cases}$$

$\therefore x^* = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

4 b) $\begin{cases} 5x_1 + 2x_2 + x_3 = -12 \\ -x_1 + 4x_2 + 2x_3 = 20 \\ 2x_1 - 3x_2 + 10x_3 = 3 \end{cases}$ * $\det(A_1) = 5$
* $\det(A_2) = 22$

* $A = LU$ e $\det(A) = 253$

$$\begin{bmatrix} 5 & 2 & 1 \\ -1 & 4 & 2 \\ 2 & -3 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

• 1ª linha (U): $\begin{cases} u_{11} = 5 \\ u_{12} = 2 \\ u_{13} = 1 \end{cases}$ | 1ª coluna (L): $\begin{cases} l_{21} = -1/5 \\ l_{31} = 2/5 \end{cases}$

• 2ª linha (U): $\begin{cases} u_{22} = 4 - (-1/5) \cdot 2 = 22/5 \\ u_{23} = 2 - (-1/5) \cdot 1 = 11/5 \end{cases}$

2ª coluna (L): $\{ l_{32} = (-3 - (2/5) \cdot 2) / (22/5) = -19/22$

• 3ª linha (U): $\{ u_{33} = 10 - \left(\frac{2}{5}\right)(1) - \left(-\frac{19}{22}\right) \cdot \left(\frac{11}{5}\right) = \frac{23}{2}$

→ $L = \begin{bmatrix} 1 & 0 & 0 \\ -1/5 & 1 & 0 \\ 2/5 & -19/22 & 1 \end{bmatrix}$ e $U = \begin{bmatrix} 5 & 2 & 1 \\ 0 & 22/5 & 11/5 \\ 0 & 0 & 23/2 \end{bmatrix}$

spiral

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* $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/5 & 1 & 0 \\ 2/5 & -19/22 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 20 \\ 3 \end{bmatrix}$$

$$\begin{cases} y_1 = -12 \\ y_2 = 88/5 \\ y_3 = 23 \end{cases}$$

* $Ux = y$

$$\begin{bmatrix} 5 & 2 & 1 \\ 0 & 22/5 & 11/5 \\ 0 & 0 & 23/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 88/5 \\ 23 \end{bmatrix}$$

$$\begin{cases} x_3 = 2 \\ x_2 = 3 \\ x_1 = -4 \end{cases}$$

$\therefore x^* = \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}$

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5) $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 2 \\ 0 & 2 & 2 \end{bmatrix}$ e $b = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

a) Para utilizar o método de Cholesky, a matriz A deve ser simétrica ($a_{ij} = a_{ji}$). Desta forma:

$$a_{12} = 1 = a_{21}$$
$$a_{13} = 0 = a_{31}$$
$$a_{23} = 2 = a_{32}$$

Logo, a matriz A é simétrica. Além disso, é necessário verificar se essa matriz é definida positiva. Para isso:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

I) $\det(A_1) = 1$
II) $\det(A_2) = 5 - 1 = 4$
III) $\det(A) = 4$

↳ Como (I), (II) e (III) não são nulos, a matriz A é definida positiva.

∴ É possível utilizar Cholesky.

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b) $A = G \cdot G^T$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 2 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \cdot \begin{bmatrix} g_{11} & g_{21} & g_{31} \\ 0 & g_{22} & g_{32} \\ 0 & 0 & g_{33} \end{bmatrix}$$

$(g_{11})^2 = a_{11} \Rightarrow g_{11} = 1$
$(g_{21})(g_{11}) = a_{21} \Rightarrow g_{21} = 1$
$(g_{31})(g_{11}) = a_{31} \Rightarrow g_{31} = 0$

$(g_{22})^2 = a_{22} - (g_{21})^2 \Rightarrow g_{22} = 2$
$(g_{31})(g_{21}) + (g_{32})(g_{22}) = a_{32} \Rightarrow g_{32} = 1$
$(g_{33})^2 = a_{33} - (g_{31})^2 - (g_{32})^2 \Rightarrow g_{33} = 1$

$$\Rightarrow G = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad e \quad G^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

c) $\det(A) = (g_{11} \cdot g_{22} \cdot g_{33})^2$
$\det(A) = (1 \cdot 2 \cdot 1)^2$
$\det(A) = 4$

d) $Gy = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$$

spiral

$$\begin{cases} Y_1 = 3 \\ Y_2 = -1/2 \\ Y_3 = 3/2 \end{cases}$$

$* \; G^T X = Y$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1/2 \\ 3/2 \end{bmatrix}$$

$$\begin{cases} X_3 = 3/2 \\ X_2 = -1 \\ X_1 = 4 \end{cases}$$

$$\therefore X^* = \begin{bmatrix} 4 \\ -1 \\ 3/2 \end{bmatrix}$$

6) $A = \begin{bmatrix} 10 & 7 & 8 \\ 7 & 5 & 6 \\ 8 & 6 & 10 \end{bmatrix}$ e $b = \begin{bmatrix} -3 \\ -1 \\ 7 \end{bmatrix}$

a) Para utilizar o método de Cholesky, a matriz A deve ser simétrica ($a_{ij} = a_{ji}$). Desta forma:

$$a_{12} = 7 = a_{21}$$
$$a_{13} = 8 = a_{31}$$
$$a_{23} = 6 = a_{32}$$

spiral

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Logo, a matriz A é simétrica. Além disso, é necessário verificar se essa matriz é definida positiva. Para isso:

$$\Rightarrow A = \begin{bmatrix} 10 & 7 & 8 \\ 7 & 5 & 6 \\ 8 & 6 & 10 \end{bmatrix}$$

(I) (II) (III)

I) $\det(A_1) = 10$

II) $\det(A_2) = 1$

III) $\det(A) = 2$

↳ Como (I), (II) e (III) não são nulos, a matriz A é definida positiva.

∴ É possível utilizar Cholesky.

b) $A = G \cdot G^T$

$$\Rightarrow \begin{bmatrix} 10 & 7 & 8 \\ 7 & 5 & 6 \\ 8 & 6 & 10 \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \cdot \begin{bmatrix} g_{11} & g_{21} & g_{31} \\ 0 & g_{22} & g_{32} \\ 0 & 0 & g_{33} \end{bmatrix}$$

$(g_{11})^2 = a_{11} \Rightarrow g_{11} = \sqrt{10}$

$(g_{21})(g_{11}) = a_{21} \Rightarrow g_{21} = 7/\sqrt{10}$

$(g_{31})(g_{11}) = a_{31} \Rightarrow g_{31} = 8/\sqrt{10}$

$(g_{22})^2 = a_{22} - (g_{21})^2 \Rightarrow g_{22} = 1/\sqrt{10}$

$(g_{31})(g_{21}) + (g_{32})(g_{22}) = a_{32} \Rightarrow g_{32} = 2\sqrt{10}/5$

$(g_{33})^2 = a_{33} - (g_{31})^2 - (g_{32})^2 \Rightarrow g_{33} = 2\sqrt{5}/\sqrt{10}$

spiral

$$* \; G = \begin{bmatrix} \sqrt{10} & 0 & 0 \\ 7/\sqrt{10} & 1/\sqrt{10} & 0 \\ 8/\sqrt{10} & 2\sqrt{10}/5 & 2\sqrt{5}/\sqrt{10} \end{bmatrix}$$

$$G^t = \begin{bmatrix} \sqrt{10} & 7/\sqrt{10} & 8/\sqrt{10} \\ 0 & 1/\sqrt{10} & 2\sqrt{10}/5 \\ 0 & 0 & 2\sqrt{5}/\sqrt{10} \end{bmatrix}$$

c) $\det(A) = (g_{11} \cdot g_{22} \cdot g_{33})^2$

$\det(A) = (\sqrt{10} \cdot (1/\sqrt{10}) \cdot (2\sqrt{5}/\sqrt{10}))^2$

$\det(A) = 2$

d) $* \; Gy = b$

$$\begin{bmatrix} \sqrt{10} & 0 & 0 \\ 7/\sqrt{10} & 1/\sqrt{10} & 0 \\ 8/\sqrt{10} & 2\sqrt{10}/5 & 2\sqrt{5}/\sqrt{10} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 7 \end{bmatrix}$$

$$\begin{cases} y_1 = -3/\sqrt{10} \\ y_2 = 11/\sqrt{10} \\ y_3 = 5/\sqrt{2} \end{cases}$$

$* \; G^t x = y$

$$\begin{bmatrix} \sqrt{10} & 7/\sqrt{10} & 8/\sqrt{10} \\ 0 & 1/\sqrt{10} & 2\sqrt{10}/5 \\ 0 & 0 & 2\sqrt{5}/\sqrt{10} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3/\sqrt{10} \\ 11/\sqrt{10} \\ 5/\sqrt{2} \end{bmatrix}$$

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$$\begin{cases} x_3 = 5/2 \\ x_2 = 1 \\ x_1 = -3 \end{cases}$$

$$\therefore X^* = \begin{bmatrix} -3 \\ 1 \\ 5/2 \end{bmatrix}$$

7) $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ 4 & -1 & 3 \end{bmatrix}$

• A^{-1} (deixar A em I)

$$\left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 4 & -1 & 3 & 0 & 0 & 1 \end{array}\right] \text{ trocar}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 1 & 0 & 0 \\ 4 & -1 & 3 & 0 & 0 & 1 \end{array}\right] \cdot(-2) + \quad \cdot(-4) +$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -5 & 1 & -2 & 0 \\ 0 & -1 & -5 & 0 & -4 & 1 \end{array}\right] \cdot(1) +$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -5 & 1 & -2 & 0 \\ 0 & 0 & -10 & 1 & -6 & 1 \end{array}\right] \div (-10)$$

spiral

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -5 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1/10 & 3/5 & -1/10 \end{array}\right] \begin{array}{l} \\ \\ .(5)\ + \quad .(-2)\ + \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & -1/5 & 1/5 \\ 0 & 1 & 0 & 1/2 & 1 & -1/2 \\ 0 & 0 & 1 & -1/10 & 3/5 & -1/10 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1/5 & -1/5 & 1/5 \\ 1/2 & 1 & -1/2 \\ -1/10 & 3/5 & -1/10 \end{bmatrix}$$

8)
$$\begin{cases} 10x_1 + x_2 + x_3 = 10 \\ 2x_1 + 10x_2 + 8x_3 = 20 \\ 7x_1 + x_2 + 10x_3 = 20 \end{cases}$$

a) Para utilizar o método iterativo de Jacobi, é necessário verificar a convergência da matriz B abaixo. Desta forma:

$$B = \begin{bmatrix} 0 & -1/10 & -1/10 \\ -2/10 & 0 & -8/10 \\ -7/10 & -1/10 & 0 \end{bmatrix} \quad e \quad g = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

* Critério das colunas

$\rightarrow |-2/10| + |-7/10| = 9/10$
$\rightarrow |-1/10| + |-1/10| = 1/5$
$\rightarrow |-1/10| + |-8/10| = 9/10$

spiral

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$\rightarrow MAX = \left\{ \frac{9}{10} ; \frac{1}{5} ; \frac{9}{10} \right\} = \frac{9}{10} < 1$

$\therefore$ Como a matriz B converge, é possível utilizar o método.

b) $* X^{(0)} = (-1; 2; -1)^t$ e $\varepsilon = 0{,}01$

$$\begin{cases} X_1^{(k)} = (10 - \cancel{x_2} - X_3)/10 \\ X_2^{(k)} = (20 - 2X_1 - 8X_3)/10 \\ X_3^{(k)} = (20 - 7X_1 - X_2)/10 \end{cases}$$

$\rightarrow X^{(1)} = (0{,}9 ; 3 ; 2{,}5)$

$\text{Erro} = \dfrac{\| X^{(1)} - X^{(0)} \|_\infty}{\| X^{(1)} \|_\infty} = \dfrac{3{,}5}{3} = 1{,}1667 > \varepsilon$

$\rightarrow X^{(2)} = (0{,}45 ; -0{,}18 ; 1{,}07)$

$\text{Erro} = \dfrac{3{,}18}{1{,}07} = 2{,}9720 > \varepsilon$

$\rightarrow X^{(3)} = (0{,}911 ; 1{,}054 ; 1{,}703)$

$\text{Erro} = \dfrac{1{,}234}{1{,}703} = 0{,}7246 > \varepsilon$

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$\rightarrow X^{(4)} = (0{,}7243 ; 0{,}4554 ; 1{,}2569)$

$$\text{Erro} = \frac{0{,}5986}{1{,}2569} = 0{,}4763 > \varepsilon$$

$\rightarrow X^{(5)} = (0{,}8288 ; 0{,}8496 ; 1{,}4475)$

$$\text{Erro} = \frac{0{,}3942}{1{,}4475} = 0{,}2724 > \varepsilon$$

$\rightarrow X^{(6)} = (0{,}7703 ; 0{,}6763 ; 1{,}3349)$

$$\text{Erro} = \frac{0{,}1733}{1{,}3349} = 0{,}1299 > \varepsilon$$

$\rightarrow X^{(7)} = (0{,}7989 ; 0{,}7780 ; 1{,}3932)$

$$\text{Erro} = \frac{0{,}1017}{1{,}3932} = 0{,}073 > \varepsilon$$

$\rightarrow X^{(8)} = (0{,}7829 ; 0{,}7257 ; 1{,}3630)$

$$\text{Erro} = \frac{0{,}0523}{1{,}3630} = 0{,}0384 > \varepsilon$$

$\rightarrow X^{(9)} = (0{,}7911 ; 0{,}7530 ; 1{,}3794)$

$$\text{Erro} = \frac{0{,}0274}{1{,}3794} = 0{,}0198 > \varepsilon$$

spiral

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$\rightarrow x^{(10)} = (0{,}7868 ; 0{,}7382 ; 1{,}3709)$

$\text{Erro} = \dfrac{0{,}0148}{1{,}3709} = 0{,}0108 > \varepsilon$

$\rightarrow x^{(11)} = (0{,}7891 ; 0{,}7459 ; 1{,}3755)$

$\text{Erro} = \dfrac{0{,}0077}{1{,}3755} = 0{,}0056 < \varepsilon$

$\therefore \bar{x} = [0{,}7891 ; 0{,}7459 ; 1{,}3755]^t$

9) $A = \begin{bmatrix} 5 & 2 & 2 \\ 1 & 6 & 3 \\ 2 & 2 & 7 \end{bmatrix}$ e $b = \begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix}$

a) Pelo critério de Sassenfeld:

$B = \begin{bmatrix} 0 & -2/5 & -2/5 \\ -1/6 & 0 & -1/2 \\ -2/7 & -2/7 & 0 \end{bmatrix}$ e $g = \begin{bmatrix} 9/5 \\ 10/6 \\ 11/7 \end{bmatrix}$

$\Rightarrow \beta_1 = \left|-\dfrac{2}{5}\right| + \left|-\dfrac{2}{5}\right| = \dfrac{4}{5} = 0{,}8$

$\Rightarrow \beta_2 = \left|-\dfrac{1}{6}\right| \cdot \dfrac{4}{5} + \left|-\dfrac{1}{2}\right| = \dfrac{17}{30} = 0{,}5667$

$\Rightarrow \beta_3 = \left|-\dfrac{2}{7}\right| \cdot \dfrac{4}{5} + \left|-\dfrac{2}{7}\right| \cdot \dfrac{17}{30} = \dfrac{41}{105} = 0{,}3905$

spiral

$\Rightarrow \beta = \max\{0,8;\ 0,5667;\ 0,3905\} = 0,8 < 1$

∴ Há convergência.

b) * $X^{(0)} = (0,0,0)$ e $\varepsilon = 0,01$.

$$\begin{cases} x_1^{(k)} = (9 - 2x_2 - 2x_3)/5 \\ x_2^{(k)} = (10 - x_1 - 3x_3)/6 \\ x_3^{(k)} = (11 - 2x_1 - 2x_2)/7 \end{cases}$$

$$\Rightarrow \begin{cases} x_1^{(1)} = 1,8 \\ x_2^{(1)} = (10 - 1,8 - 3.0)/6 = 1,3667 \\ x_3^{(1)} = (11 - 2.1,8 - 2.1,3667)/7 = 0,6667 \end{cases}$$

$$\text{Erro} = \frac{\|X^{(1)} - X^{(0)}\|_\infty}{\|X^{(1)}\|_\infty} = \frac{1,8}{1,8} = 1 > \varepsilon$$

$\Rightarrow X^{(2)} = (0,9867;\ 1,1689;\ 0,9556)$

$$\text{Erro} = \frac{0,8133}{1,1689} = 0,6958 > \varepsilon$$

$\Rightarrow X^{(3)} = (0,9502;\ 1,0305;\ 1,0055)$

$$\text{Erro} = \frac{0,1384}{1,0305} = 0,1343 > \varepsilon$$

spiral

$\Rightarrow X^{(4)} = (0,9856 ; 0,9997 ; 1,0042)$

$$Erro = \frac{0,0354}{1,0042} = 0,0352 > \varepsilon$$

$\Rightarrow X^{(5)} = (0,9985 ; 0,9982 ; 1,0010)$

$$Erro = \frac{0,0129}{1,0010} = 0,0129 > \varepsilon$$

$\Rightarrow X^{(6)} = (1,0004 ; 0,9995 ; 1,0001)$

$$Erro = \frac{0,0019}{1,0004} = 0,0019 < \varepsilon$$

$\therefore \bar{X} = [1,0004 ; 0,9995 ; 1,0001]^t$

10)
$$\begin{cases} 4x_1 + x_2 + x_3 + x_4 = 11 \\ 2x_1 - 8x_2 + x_3 - x_4 = -4 \\ x_1 + 2x_2 - 5x_3 + x_4 = 0 \\ x_1 + x_2 + x_3 - 4x_4 = 0 \end{cases}$$

e $X^{(0)} = (1, 1, 1, 1)^t$ e $\varepsilon = 0,0001$

I) $B = \begin{bmatrix} 0 & -1/4 & -1/4 & -1/4 \\ 1/4 & 0 & 1/8 & -1/8 \\ 1/5 & 2/5 & 0 & 1/5 \\ 1/4 & 1/4 & 1/4 & 0 \end{bmatrix}$ e $g = \begin{bmatrix} 11/4 \\ 1/2 \\ 0 \\ 0 \end{bmatrix}$

Verificando convergência (critério das linhas):

* $\left|-\frac{1}{4}\right| \cdot 3 = \frac{3}{4} = 0{,}75$

* $\left|\frac{1}{4}\right| + \left|\frac{1}{8}\right| + \left|-\frac{1}{8}\right| = \frac{1}{2} = 0{,}5$

* $\left|\frac{1}{5}\right| \cdot 2 + \left|\frac{2}{5}\right| = \frac{4}{5} = 0{,}8$

* $\left|\frac{1}{4}\right| \cdot 3 = \frac{3}{4} = 0{,}75$

$\Rightarrow MAX = \{0{,}75; 0{,}5; 0{,}8; 0{,}75\} = \boxed{0{,}8 < 1}$

Logo, pode utilizar o método.

II) $*X^{(0)} = (1, 1, 1, 1)^t$ e $\varepsilon = 0{,}0001$

$$\begin{cases} x_1^{(k)} = (11 - x_2 - x_3 - x_4)/4 \\ x_2^{(k)} = (-4 - 2x_1 - x_3 + x_4)/(-8) \\ x_3^{(k)} = (-x_1 - 2x_2 - x_4)/(-5) \\ x_4^{(k)} = (-x_1 - x_2 - x_3)/(-4) \end{cases}$$

$$\Rightarrow \begin{cases} x_1^{(1)} = 2 \\ x_2^{(1)} = (-4 - 2.2 - 1 + 1)/(-8) = 1 \\ x_3^{(1)} = (-2 - 2.1 - 1)/(-5) = 1 \\ x_4^{(1)} = (-2 - 1 - 1)/(-4) = 1 \end{cases}$$

$$\text{Erro} = \frac{\| X^{(1)} - X^{(0)} \|_\infty}{\| X^{(1)} \|_\infty} = \frac{1}{2} = 0{,}5 > \varepsilon$$

spiral

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$\Rightarrow X^{(2)} = (2;1;1;1)$

$\text{Erro} = \frac{0}{2} = \boxed{0 < \varepsilon}$

$\therefore \bar{x} = [2;1;1;1]^{t}$

2 $* \; b = \frac{a}{3} - 1$ e $b = \frac{i}{2} - 3$

$* \; t + a = (g + i) + 3$

$* \; g + a = \frac{(a+b+g+i+t)}{2} - 1$

$* \; i + g = \frac{7(a+b+g+i+t)}{16}$

$$\Rightarrow \begin{cases} (-a/3) + b + 0g + 0i + 0t = -1 \\ 0a + b + 0g - i/2 + 0t = -3 \\ a + 0b - g - i + t = 3 \\ a - b + g - i - t = -2 \\ -7a - 7b + 9g + 9i - 7t = 0 \end{cases}$$

$$\Rightarrow \begin{bmatrix} -1/3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 1 & 0 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & -1 \\ -7 & -7 & 9 & 9 & -7 \end{bmatrix} \begin{bmatrix} a \\ b \\ g \\ i \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

Legenda: a - alemão; b - brasileiro; g - grego; i - italiano; t - turco

spiral

Utilizando o Método de Eliminação de Gauss:

$$\Rightarrow A^{(1)} = \left[\begin{array}{ccccc|c} -1/3 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 & 0 & -3 \\ 1 & 0 & -1 & -1 & 1 & 3 \\ 1 & -1 & 1 & -1 & -1 & -2 \\ -7 & -7 & 9 & 9 & -7 & 0 \end{array}\right]$$

I) Pivô $= a_{11}^{(1)} = -\frac{1}{3}$

• 3ª linha

$$m_{31} = \frac{1}{(-1/3)} = -3$$

$a_{31} = 1 - ((-3) \cdot (-1/3)) = 0$
$a_{32} = 0 - ((-3) \cdot 1) = 3$
$a_{33} = -1 - 0 = -1$
$a_{34} = -1 - 0 = -1$
$a_{35} = 1 - 0 = 1$

$b_3 = -3 - ((-3) \cdot (-1)) = 0$

• 4ª linha

$m_{41} = -3$

spiral

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$a_{41} = 0$
$a_{42} = 2$
$a_{43} = 1$
$a_{44} = -1$
$a_{45} = -1$

$b_4 = -5$

• 5º linha

$m_{51} = 21$

$a_{51} = 0$
$a_{52} = -28$
$a_{53} = 9$
$a_{54} = 9$
$a_{55} = -7$

$b_5 = 21$

$$\Rightarrow A^{(2)} = \left[\begin{array}{ccccc|c} -1/3 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 & 0 & -3 \\ 0 & 3 & -1 & -1 & 1 & 0 \\ 0 & 2 & 1 & -1 & -1 & -5 \\ 0 & -28 & 9 & 9 & -7 & 21 \end{array}\right]$$

II) $\text{Pivô} = a_{22}^{(2)} = 1$

spiral

- 3ª linha

$m_{32} = 3$

$a_{32} = 0$
$a_{33} = -1$
$a_{34} = 1/2$
$a_{35} = 1$

$b_3 = 9$

- 4ª linha

$m_{42} = 2$

$a_{42} = 0$
$a_{43} = 1$
$a_{44} = 0$
$a_{45} = -1$

$b_4 = 1$

- 5ª linha

$m_{52} = -28$

$a_{52} = 0$
$a_{53} = 9$
$a_{54} = -5$

spiral

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$a_{55} = -7$

$b_5 = -63$

$$\Rightarrow A^{(3)} = \left[\begin{array}{ccccc|c} -1/3 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 & 0 & -3 \\ 0 & 0 & -1 & 1/2 & 1 & 9 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 9 & -5 & -7 & -63 \end{array}\right]$$

III) $P_{\text{ivô}} = a_{33}^{(3)} = -1$

- 4ª linha

$m_{43} = -1$

$a_{43} = 0$
$a_{44} = 1/2$
$a_{45} = 0$

$b_4 = 10$

- 5ª linha

$m_{53} = -9$

$a_{53} = 0$
$a_{54} = -1/2$
$a_{55} = 2$

$b_5 = 18$

spiral

$$\Rightarrow A^{(4)} = \left[\begin{array}{ccccc|c} -1/3 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 & 0 & -3 \\ 0 & 0 & -1 & 1/2 & 1 & 9 \\ 0 & 0 & 0 & 1/2 & 0 & 10 \\ 0 & 0 & 0 & -1/2 & 2 & 18 \end{array}\right]$$

IV) $P_{\text{ivô}} = a_{44}^{(4)} = 1/2$

• 5ª linha

$m_{54} = -1$

$a_{54} = 0$
$a_{55} = 2$

$b_5 = 28$

$$\Rightarrow A^{(5)} = \left[\begin{array}{ccccc|c} -1/3 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 & 0 & -3 \\ 0 & 0 & -1 & 1/2 & 1 & 9 \\ 0 & 0 & 0 & 1/2 & 0 & 10 \\ 0 & 0 & 0 & 0 & 2 & 28 \end{array}\right]$$

$\rightarrow t = 14$
$\rightarrow i = 20$
$\rightarrow g = 15$
$\rightarrow b = 7$
$\rightarrow a = 24$

∴ A quantidade de membros de cada nacionalidade da companhia é: 24 alemães, 7 brasileiros, 15 gregos, 20 italianos e 14 turcos.