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Nome: Davi Augusto Neves Leite RA: 191027383

P1 - M.N.C. (tipo B)

① $F(x) = e^{x-2} + x$ e $\varepsilon = 10^{-3}$

• $F'(-1)$

K	h	$x+h$	$F(x+h)$	$x-h$	$F(x-h)$	$F'(x)$	Erro
1	1,0000	0	0,1353	-2,0000	-1,9817	1,0585	—
2	0,5000	-0,5000	-0,4179	-1,5000	-1,4698	1,0519	0,0063
3	0,2500	-0,7500	-0,6861	-1,2500	-1,2112	1,0503	0,0015
4	0,1250	-0,8750	-0,8186	-1,1250	-1,0811	1,0499	0,0004

$\therefore F'(-1) \approx 1,0499$

③ Cholesky

$$\underbrace{\begin{bmatrix} 1 & \alpha & -1 \\ 1 & 5 & \beta \\ \gamma & -1 & 3 \end{bmatrix}}_{(A)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}$$

I) Para utilizar o método de Cholesky, a matriz A deve ser simétrica ($a_{ij} = a_{ji}$). Desta forma:

$* \alpha = 1 \quad * \beta = -1 \quad * \gamma = -1$

Além disso, é necessário verificar se essa matriz é definida positiva. Para isso:

FORONI

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 5 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

I) $\det(A_1) = 1$

II) $\det(A_2) = 5 - 1 = 4$

III) $\det(A) = 8$

↳ Como (I), (II) e (III) não são nulos, a matriz A é definida positiva.

II) $A = G \cdot G^T$

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 5 & -1 \\ -1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \cdot \begin{bmatrix} g_{11} & g_{21} & g_{31} \\ 0 & g_{22} & g_{32} \\ 0 & 0 & g_{33} \end{bmatrix}$$

$(g_{11})^2 = a_{11} \Rightarrow g_{11} = \sqrt{1} = 1$

$(g_{21})(g_{11}) = a_{21} \Rightarrow g_{21} = 1/1 = 1$

$(g_{31})(g_{11}) = a_{31} \Rightarrow g_{31} = -1/1 = -1$

$(g_{22})^2 = a_{22} - (g_{21})^2 \Rightarrow g_{22} = \sqrt{5 - 1^2} = 2$

$(g_{31})(g_{21}) + (g_{32})(g_{22}) = a_{32} \Rightarrow g_{32} = (-1+1)/2 = 0$

$(g_{33})^2 = a_{33} - (g_{31})^2 = (g_{32})^2 \Rightarrow g_{33} = \sqrt{3 - 1 - 0} = \sqrt{2}$

$$\Rightarrow G = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & \sqrt{2} \end{bmatrix} \quad e \quad G^T = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

III) $\det(A) = (g_{11} \cdot g_{22} \cdot g_{33})^2$
$\det(A) = (1 \cdot 2 \cdot \sqrt{2})^2$
$\det(A) = 4 \cdot 2 = 8$

IV) * $Gy = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & \sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}$$

$$\begin{cases} y_1 = 2 \\ y_2 = (6-2)/2 = 2 \\ y_3 = (0+2)/\sqrt{2} = \sqrt{2} \end{cases} \Rightarrow y^* = \begin{bmatrix} 2 \\ 2 \\ \sqrt{2} \end{bmatrix}$$

* $G^T x = y$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ \sqrt{2} \end{bmatrix}$$

$$\begin{cases} x_3 = 1 \\ x_2 = 1 \\ x_1 = 2 \end{cases}$$

$\therefore x^* = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

a) LU

$A = LU$

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

• 1ª linha (U): $\begin{cases} u_{11} = 1 \\ u_{12} = -1 \\ u_{13} = 0 \end{cases}$ | 1ª coluna (L): $\begin{cases} l_{21} = 2/1 = 2 \\ l_{31} = -1 \end{cases}$

• 2ª linha (U): $\begin{cases} u_{22} = 2 - (2 \cdot (-1)) = 4 \\ u_{23} = 3 - (2 \cdot 0) = 3 \end{cases}$

2ª coluna (L): $\{ l_{32} = (3 - (-1) \cdot (-1)) / 4 = 1/2$

• 3ª linha (U): $\{ u_{33} = 2 - (-1) \cdot 0 - \left(\frac{1}{2}\right) \cdot 3 = \frac{1}{2}$

$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1/2 & 1 \end{bmatrix}$ e $U = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & 1/2 \end{bmatrix}$

c) $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\begin{cases} y_1 = 2 \\ y_2 = -1 - 4 = -5 \\ y_3 = 4 + 2 + 5/2 = 17/2 \end{cases} \Rightarrow y = \begin{bmatrix} 2 \\ -5 \\ 17/2 \end{bmatrix}$$

$\Rightarrow Ux = y$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 17/2 \end{bmatrix}$$

$$\begin{cases} x_3 = 17 \\ x_2 = -14 \\ x_1 = -12 \end{cases}$$

$$\therefore X^* = \begin{bmatrix} -12 \\ -14 \\ 17 \end{bmatrix}$$

b) $\det(A) = 4 + 3 - (9 - 4) = 7 - 5 = 2$

5) $$\begin{bmatrix} 10 & -2 & -2 \\ 4 & 6 & 0 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 0 \end{bmatrix}$$

a) Critério de Sassenfeld

$$* \ B = \begin{bmatrix} 0 & 2/10 & 2/10 \\ -2/3 & 0 & 0 \\ 1/3 & -1/3 & 0 \end{bmatrix} \quad e \quad g = \begin{bmatrix} 8/10 \\ 7/6 \\ 0 \end{bmatrix}$$

$$\Rightarrow \beta_1 = \left|\frac{2}{10}\right| + \left|\frac{2}{10}\right| = \frac{2}{5} = 0,4$$

$$\Rightarrow \beta_2 = \left|-\frac{2}{3}\right| \cdot \frac{2}{5} + |0| = \frac{4}{15} \cong 0,2667$$

$$\Rightarrow \beta_3 = \left|\frac{1}{3}\right| \cdot \frac{2}{5} + \left|-\frac{1}{3}\right| \cdot \frac{4}{15} = \frac{2}{15} + \frac{4}{45} = \frac{10}{45} \cong 0,2222$$

$$\beta = \text{MAX}\left\{\frac{2}{5}; \frac{4}{15}; \frac{10}{45}\right\} = \boxed{\frac{2}{5} < 1}$$

$\therefore$ Converge.

5) Método de Gauss - Seidel

* $X^{(0)} = (0, 0, 0)^t$ e $\varepsilon = 0{,}001$

* $\begin{cases} X_1^{(k)} = (8 + 2X_2 + 2X_3)/10 \\ X_2^{(k)} = (7 - 4X_1)/6 \\ X_3^{(k)} = (X_1 - X_2)/3 \end{cases}$

$\Rightarrow \begin{cases} X_1^{(1)} = 0{,}8 \\ X_2^{(1)} = (7 - 4 . 0{,}8)/6 = 0{,}6333 \\ X_3^{(1)} = (0{,}8 - 0{,}6333)/3 = 0{,}0556 \end{cases}$

$$Erro = \frac{\|X^{(1)} - X^{(0)}\|_\infty}{\|X^{(1)}\|_\infty} = \frac{MAX\{|0-0{,}8|; |0-0{,}6333|; |0-0{,}0556|\}}{MAX\{|0{,}8|; |0{,}6333|; |0{,}0556|\}}$$

$\Rightarrow Erro = \frac{0{,}8}{0{,}8} = 1 > \varepsilon$

$\Rightarrow \begin{cases} X_1^{(2)} = 0{,}9378 \\ X_2^{(2)} = 0{,}5415 \\ X_3^{(2)} = 0{,}1321 \end{cases}$

$$Erro = \frac{0{,}1378}{0{,}9378} = 0{,}1469 > \varepsilon$$

$\Rightarrow \begin{cases} X_1^{(3)} = 0{,}9347 \\ X_2^{(3)} = 0{,}5435 \\ X_3^{(3)} = 0{,}1304 \end{cases}$

$$Erro = \frac{0{,}0031}{0{,}9347} = 0{,}0033 > \varepsilon$$

$$\Rightarrow \begin{cases} X_1^{(4)} = 0,9348 \\ X_2^{(4)} = 0,5435 \\ X_3^{(4)} = 0,1304 \end{cases}$$

$$\text{Erro} = \frac{0,0001}{0,9348} = 0,0001 < \varepsilon$$

$$\therefore X^* = \begin{bmatrix} 0,9348 \\ 0,5435 \\ 0,1304 \end{bmatrix}$$

② $X = \sqrt[5]{15}$ com $X_0 = 2$ e $\varepsilon = 0,001$

$\forall$ $F(X) = X - \sqrt[5]{15}$ e $F'(X) = 1$

- Newton

$$X_1 = X_0 - \frac{F(X_0)}{F'(X_0)} = 2 - \frac{0,2812}{1} = 1,7188$$

$$\text{Erro} = \frac{|2 - 1,7188|}{|1,7188|} = 0,1636 > \varepsilon$$

$$X_2 = 1,7188 - \frac{0}{1} = 1,7188$$

$$\text{Erro} = \frac{|1,7188 - 1,7188|}{|1,7188|} = 0 < \varepsilon$$

$\therefore X = 1,7188$