CS341 Assignment 1 Fall 2009

Due date: Thurs. Oct. 1, at 5:00 pm (3rd floor MC "pink boxes")

OR hand in early at start of class on Tues. Sept. 29 to get bonus 5 marks.

Please read http://www.student.cs.uwaterloo.ca/~cs341/Assignment_Guidelines.pdf for guidelines on submission. Note that in this assignment, and in all future assignments, any reference to the text will use page numbers that are consistent with the second edition first published in 2001. The assignment will be marked out of 80.

Problem 1 [2+2+3+3+3+3=16 marks]

Consider each of the following statements:

a) $f(n) \in O(g(n))$ and $g(n) \in O(h(n)) \Rightarrow f(n) \in O(h(n))$.

b) $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n)) \Rightarrow f_1(n) \cdot f_2(n) \in O(g_1(n) \cdot g_2(n))$.

c) $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n)) \Rightarrow f_1(n) + f_2(n) \in O(\max(g_1(n), g_2(n)))$.

d) $f_1(n) \in \Theta(g(n))$ and $f_2(n) \in \Theta(g(n)) \Rightarrow f_1(n) - f_2(n) \in \Theta(1)$.

e) $f_1(n) \in \Theta(g(n))$ and $f_2(n) \in \Theta(g(n)) \Rightarrow f_1(n) / f_2(n) \in \Theta(1)$.

f) $\log(f(n)) \in \Omega(\log(g(n))) \Rightarrow f(n) \in \Omega(g(n))$.

For each statement: If the statement is true then provide a proof that starts with the formal definition of the order notation utilized in the statement. If the statement is false then you may provide a proof that also relies on formal definitions OR you may provide a counter example and then demonstrate why the statement is false. All functions are nonnegative functions.

Problem 2 [3+3+3+4+5=18 marks]

Consider the relative asymptotic behaviour of the following functions:

a) $f(n) = 16^{\log\sqrt{n}}$ versus $g(n) = n^2$.

b) $f(n) = n^2 + 5^n$ versus $g(n) = n^{5^2} + 2^n$.

c) $f(n) = n^4 / \log^2 n$ versus $g(n) = n^3$.

d) $f(n) = n^5 / (n^3 + 8)$ versus $g(n) = (n^4 - 2n^3) / (3n^2)$.

e) $f(n) = \log(n^2 + 8)$ versus $g(n) = \sqrt{n}$.

For each pair of functions fill in the correct asymptotic notation using either Θ, o, or ω in the statement $f(n) \in \_\_(g(n))$. Provide a brief justification of your answers. In your justification you may quote any relationship that is described on page 49 or in section 3.2 of your text.

Problem 3 [4+5+2=11 marks]

a) Let $f(n)$, $g(n)$, and $h(n)$ denote positive-valued functions defined on integers $n \geq 1$. **Prove** that if $f(n) = \mathbf{O}(\log n \cdot g(n))$ and $g(n) = \mathbf{O}(1)$, then $f(n) = \boldsymbol{o}(n)$.

b) Consider the functions $f(n) = 2^{\sqrt{n}}$ and $g(n) = n^{(\log n)^2}$.

Prove or disprove the following:

i) $f(n) \in \omega(g(n))$

ii) $f(n) \in O(g(n))$

Problem 4 [12 marks]

Analyze the following pseudo-code and give a $\Theta(.)$ bound on the running time as a function on n.

Show your work.

sum = 0

for i = 1 to n **do**

  for j = 1 to ni **do**

    for k = 1 to $\lceil \log j \rceil$ **do**

      for m = 1 to 10 **do**

        sum = sum + i + j + k + m

Problem 5 [5+6 =11 marks]

For both (a) and (b) analyze the following pseudo-code and give a tight **Θ** bound on the running time as a function of n. Show your work. In the pseudo-code "..." refers to some constant-time operations that do not change the values of i, j, k, or n.

(a)

```
k = 1
while (k < n) do
    for (i= 1 to k) do
        for (j = n down to n – k) do
            …
    for (i = k to 3*k) do
        …
    k = 3*k
```

Problem 6 [6 marks]

Consider the statement:

$$\sum_{i=1}^{n}\left\lfloor \log\left(\frac{n}{i}\right)\right\rfloor \in o(n)$$

Clearly state whether or not this statement is true and then provide an argument that supports your claim. You may assume that n is even.

Problem 7 [6 marks]

"I'm a Mac" and "I'm a PC"

Consider the following recurrence problem: There is a row of n chairs and two types of people labelled with single letters: M for Mac people and P for PC people. You want to assign one person to each seat but you can never seat two PC people together or they will start talking about virus protection and everyone else in the room will get bored. For example, if $n = 3$, the following are some valid seating arrangements: MMM, MPM, PMP, and MMP. However, the following is an invalid seating: MPP. In this problem, your goal is as follows. Let $f(n)$ be the number of valid seating arrangements when there are n chairs in a row. Give a recurrence relation for $f(n)$. Give a well reasoned argument that supports your claim. Does $f(n)$ have exponential growth in n?