Question 1

Write about how the hardware and software of computers has advanced since the time of Charles Babbage. Give specific details about the places, people and times when advances in computing hardware and software were made. (400 words approx.)

The invention of the first electronic digital computer, the ENIAC (Electronic Numerical Integrator and Computer), completed in 1945 at the University of Pennsylvania by John Mauchly and J. Presper Eckert is probably the first milestone since the time of Charlse Babbage. ENIAC was a massive machine that could perform complex calculations much faster than previous mechanical devices, marking the dawn of electronic computing.

Following the ENIAC, the development of transistors in the late 1940s / early 1950s by William Shockley, John Bardeen, and Walter Brattain at Bell Labs revolutionized computer hardware. Transistors replaced vacuum tubes, making computers smaller, faster, and more reliable. This advancement led to the creation of the first commercially successful transistorized computer, the IBM 1401, in 1959.

The concept of integrated circuits emerged in the 1960s, enabling multiple transistors to be fabricated onto a single silicon chip. Jack Kilby at Texas Instruments and Robert Noyce at Fairchild Semiconductor independently invented the integrated circuit, which in turn made developing microprocessors possible. The Intel 4004, released in 1971 by Intel Corporation, was the world's first microprocessor, consolidating the central processing unit (CPU) onto a single chip and sparking the personal computing revolution.

Advancements in hardware continued with Moore's Law, which predicted that the number of transistors on a microchip would double approximately every two years. This exponential growth led to increasingly powerful and compact devices, such as smartphones and tablets, which have become a stable of modern life.

Simultaneously, in the 1950s, Grace Hopper developed the first compiler, a program that translates high-level programming languages into machine code. This innovation simplified programming and laid the groundwork for modern software development. The 1970s witnessed the development of UNIX, an operating system created by Ken Thompson, Dennis Ritchie, and others at Bell Labs. UNIX introduced concepts like multi-tasking and file system organization, shaping the foundation of subsequent operating systems. Graphical user interfaces (GUIs) were being developed in the 1980s with the introduction of the Xerox Star, Apple Lisa, and the Macintosh. These systems, along with Microsoft's Windows operating environment, made computers more accessible to non-technical users, fueling widespread adoption.

The internet became publicly available in the 1990s, revolutionizing communication and information exchange. Tim Berners-Lee's invention of the World Wide Web in 1989 laid the groundwork for the internet, which became publicly available in the 1990s, enabling easy access to vast amounts of information and facilitating global connectivity.

a) Convert to 100111_2 to decimal. Show your workings.

binary position →	32	16	8	4	2	1		
binary number →	1	0	0	1	1	1		
add them up →	32	0	0	4	2	1	=	**39**

b) Convert 100.0011_2 to decimal. Show your workings.

binary position →	4	2	1	0.5	0.25	0.125	0.0625		
binary number →	1	0	0	0	0	1	1		
add them up →	4	0	0	0	0	0.125	0.0625	=	**4.1875**

c) Convert 186_{10} to binary. Show your workings.

decimal number →	186	93	46	23	11	5	2	1	0
quotient →	93	46	23	11	5	2	1	0	0
binary number →	0	**1**	**0**	**1**	**1**	**1**	**0**	**1**	**0**

d) Convert the hexadecimal number $41B2_{16}$ to decimal. Show your workings.

Original = 41B2
B = 11 in hex

hexadecimal position →	4096	256	16	1		
hexadecimal number →	4	1	11	2		
add them up →	16384	256	176	2	=	**16818**

Question 2

a) Calculate the binary sum 110011 + 11101. Show your workings.

carries →	1	1	1	1	1	1	
		1	1	0	0	1	1
		+	1	1	1	0	1
	1	**0**	**1**	**0**	**0**	**0**	**0**

b) Write the number 435000 in normalized scientific notation.

4.35 E 5 or 4.35×10^5

c) What is -102_{10} as a two's complement 32-bit binary number? Explain your answer.

Convert absoulte value to binary

decimal number →	102	51	25	12	6	3	1	0
quotient →	51	25	12	6	3	1	0	0
binary number →	0	1	1	0	0	1	1	0

Place that into 32-bit notation

$+102_{10}$	=	0000	0000	0000	0000	0000	0000	0110	0110_2

Invert the bits and add 1

-102_{10}	=	**1111**	**1111**	**1111**	**1111**	**1111**	**1111**	**1001**	**1010_2**

d) Calculate the difference 1100011 – 110111 by adding the two's complement of the number to be subtracted. Show your workings.

Get the 2's compliment of 110111

1's = 001000
2's = 001001

Add 2's compliment to 1100011

carries →					1	1	
	1	1	0	0	0	1	1
	+	0	0	1	0	0	1
overflow bit →	~~1~~	**1**	**0**	**1**	**1**	**0**	**0**

Question 3

a) Which of these sentences are propositions? What (if any) are the truth values of those that are propositions?

i) London is the capital of France.

This is a proposition with a truth value of FALSE.

ii) Open the window.

This is not a proposition. It is a command.

iii) X + 5 = 10

This is not a proposition. It can be either TRUE or FALSE.

b) Let p be "The table is green" and q be "the ball is yellow".
Write each of the following statements in symbolic form using p and q:

i) If the table is green, then the ball is yellow.

$p \rightarrow q$

ii) The table is not green or the ball is yellow.

$\neg p \vee q$

iii) It is not true that the table is green and the ball is yellow.

$\neg(p \wedge q)$

c) Consider the statements below:

S1: Charlie is not a cook.
S2: Alice is an architect or Bob is a builder.
S3: If Bob is a builder, then Charlie is a cook.

Which of the following arguments are logically correct (show your workings as part of the answer):

S1 puts the X for Charlie as Cook
S3 puts the X for Bob as Builder, following S1
S2 puts the ✓ for Alice as Architect, following S3
The remaining two ✓ are for Bob as Cook and Charlie as Builder

	Alice	Bob	Charlie
Architect	✓	x	x
Builder	x	x	✓
Cook	x	✓	x

i) Suppose S1, S2 and S3 are all true. Then Alice is an architect.

This argument is correct

ii) Suppose S1, S2 and S3 are all true. Then Bob is a builder.

This argument is incorrect

iii) Suppose S1, S2 and S3 are all true. Then Charlie is a builder.

This argument is correct

iv) Suppose S1, S2 and S3 are all true. Then Charlie is not a builder.

This argument is incorrect

Question 4

a) Describe each of the following sets in terms of a property of its elements (that is, using the 'description by common property' notation).

i) The set of dates in the month of March

{x | x is a date in the month of March}

ii) {8,10,12,14,16}

{x | x is an even number between 8 and 16}

iii) The set of positive rational numbers

Q + or {x/y | x, y ∈ Z +, y ≠ 0} where Z + = positive integer numbers

b) Consider the following sets:

Universal set U = {1, 2, 4, 5, 6, 7, 12, blue, red, green, pink, water}

A = {blue, red, green, pink}
B = {12, 5, 6, 7}
C = {green, 4, 12, 7, water}
D = {1, 2, 5}

Find the following:

i) (A ∪ D) ∩ C

{green}

ii) (B ∩ D) ∩ A

Ø or { } or empty set

iii) D – B

{1, 2}

iv) A' ∩ (B ∪ D)'
(i.e., the intersection of the complement of A and the complement of B union D)

{4, water}

v) A X D
(The Cartesian product of A and D)

{(blue, 1), (blue, 2), (blue, 5), (red, 1), (red, 2), (red, 5), (green, 1), (green, 2), (green, 5), (pink, 1), (pink, 2), (pink, 5)}

c) Consider the sets P = {21, 12, 5, 22} and Q = {4, 3}. Suppose that the relation R expresses "is divisible by (with no remainder)". Define R from P to Q as a set of ordered pairs.

R = {(21, 3), (12, 4), (12, 3)}

Question 5

a) Let A = {1,2} and B={a,b,c}

What is:

i) A X A

{(1, 1), (1, 2), (2, 1), (2, 2)}

ii) B X A

{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)}

iii) A X B

{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}

(Where X denotes the Cartesian product)

b) For each of the following relations on the set {1, 2, 3, 4}, decide whether it is reflexive, and/or irreflexive, and/or symmetric, and/or antisymmetric, and/or transitive:

i) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

reflective:	x
irreflective:	x
symmetric:	x
antisymmetric:	x
transitive:	x

ii) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}

reflective:	x
irreflective:	x
symmetric:	✓
antisymmetric:	x
transitive:	x

iii) {(2, 4), (4, 2)}

reflective:	x
irreflective:	✓
symmetric:	✓

antisymmetric: x
transitive: x

c) Consider the following sets:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}
A = {1, 2, 3, 4, 5}
B = {4, 5, 6, 7}
C = {5, 6, 7, 8, 9}
D = {1, 3, 5, 7, 9}
E = {2, 4, 6, 8}
F = {1, 5, 9}

What is:

i) A ∩ (B ∪ E)

{2, 4, 5}

ii) (B ∩ F) ∪ (C ∩ E)

{5, 6, 8}

iii) B' (that is: "the complement of B")

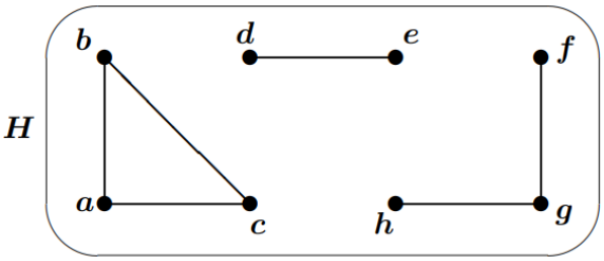
{1, 2, 3, 8, 9}

iv) (A ∩ D) – B

{1, 3}

Question 6

a) Consider the graph H, below:



i) What is the degree of the node a?

2

ii) How many vertices are in the graph H?

8

iii) How many connected components are in H?

3

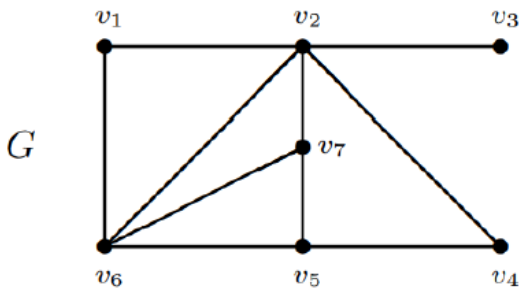
iv) Identify a simple cycle of length 3 in H.

(a, b, c, a)

v) Is the cycle you have identified Hamiltonian? Why or why not?

No. Not all vertices are attached so Hamiltonian is impossible on this graph.

b) Consider the graph G, below:



i) Which vertex is pendant?

v3

ii) Is the sequence (v1, v6, v2, v5, v4) a path? Explain your answer.

No. v2 to v5 is not an edge.

iii) Is the sequence (v6, v7, v5, v6, v2, v1, v6) a simple cycle? Explain your answer.

Yes. No edges were used twice.

iv) Is G a connected graph?

Yes

v) Which vertices are adjacent to v6?

v1, v2, v5, v7

c) Represent G and H as adjacency matrices.

G	v1	v2	v3	v4	v5	v6	v7
v1	0	1	0	0	0	1	0
v2	1	0	1	1	0	1	1
v3	0	1	0	0	0	0	0
v4	0	1	0	0	1	0	0
v5	0	0	0	1	0	1	1
v6	1	1	0	0	1	0	1
v7	0	1	0	0	1	1	0

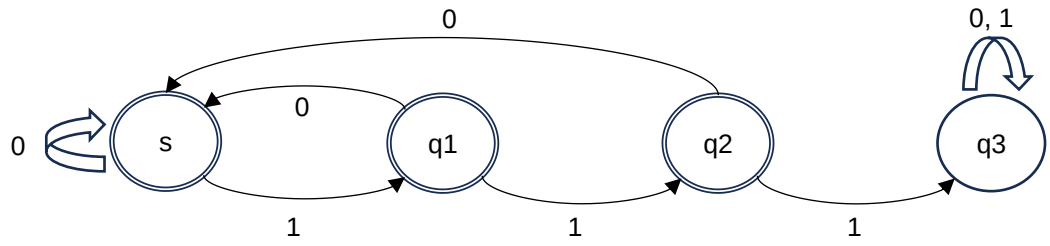
H	a	b	c	d	e	f	g	h
a	0	1	1	0	0	0	0	0
b	1	0	1	0	0	0	0	0
c	1	1	0	0	0	0	0	0
d	0	0	0	0	1	0	0	0
e	0	0	0	1	0	0	0	0
f	0	0	0	0	0	0	1	0
g	0	0	0	0	0	1	0	1
h	0	0	0	0	0	0	1	0

Question 7

a) Consider the finite automaton A = (Q, Σ, δ, s, F), where Q = {s, q1, q2, q3}, Σ = {0, 1}, F = {s, q1, q2}, and the transition function δ is given by this transition table.

δ	0	1
s	s	q_1
q_1	s	q_2
q_2	s	q_3
q_3	q_3	q_3

i) Draw a graphical representation of A.



ii) Does A accept the words 000110100? Show the computation of A on these input words.

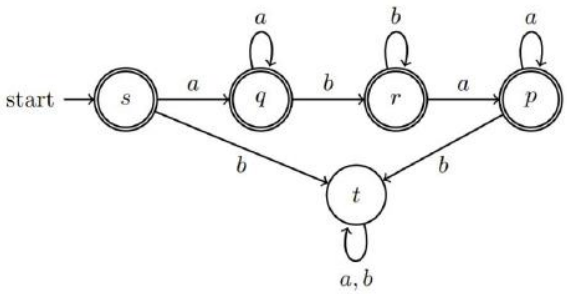
Yes

(s, 000110100), (s, 00110100), (s, 0110100), (s, 110100), (q1, 10100), (q2, 0100), (s, 100), (q1, 00), (s, 0), (s, ε)

iii) Describe (in ordinary words) the language accepted by A.

L(A) = all words that do not contain 1 three times in a row

b) Consider the following Deterministic Finite State automaton:



i) Give the formal description of the automaton, using a transition table.

A = (Q, Σ, δ, s, F), where Q = {s, q, r, p, t}, Σ = {a, b}, F = {s, q, r, p}, and the transition function δ is given by this transition table.

δ	a	b
s	q	t
q	q	r
r	p	r
p	p	t
t	t	t

ii) Find the computation of the automaton on the input string aabbaab and determine if the string is accepted.

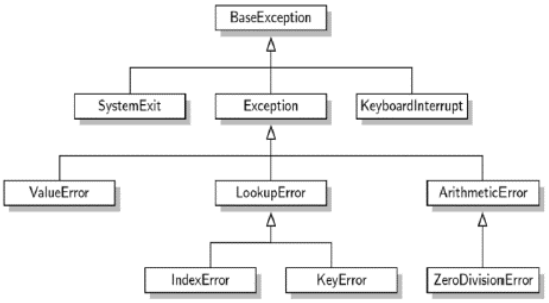
(s, aabbaab), (q, abbaab), (q, bbaab), (r, baab), (r, aab), (p, ab), (p, b), (t, ε)

This string is not accepted

iii) Describe in ordinary words the language accepted by the automaton.

L(A) = all the words that do not contain the pattern bab or start with b

Question 8



a) Answer the following questions with respect to the tree above:

i) What are the leaf nodes of the tree?

ValueError, IndexError, KeyError, ZeroDivisionError, SystemExit, KeyboardInterrupt

ii) What is the height of the tree?

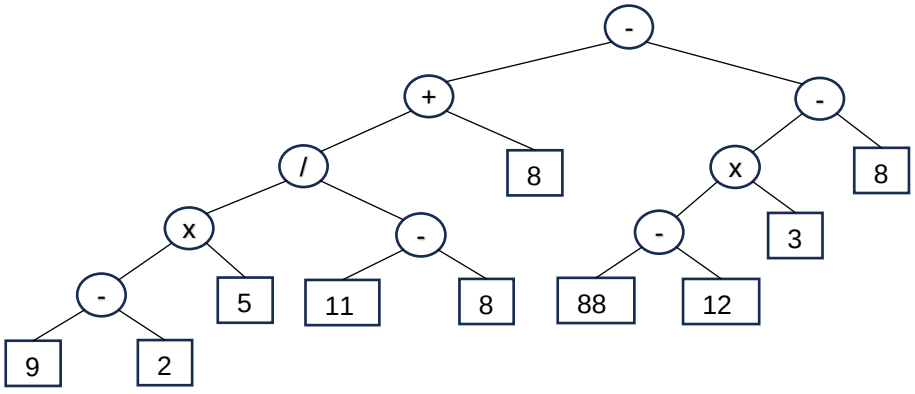
4

iii) What are the siblings of the ArithmeticError node?

ValueError, LookupError

b)

i) Represent the following arithmetic expression as a binary tree:
(((9-2) x 5)/(11-8) +8) – ((3 x (88-12) – 8))



ii) How many levels does the resulting tree have?

6

iii) What is the maximum number of nodes that can be at level 5 of a binary tree?

$2^{l-1} = 2^{5-1} =$ **16**

iv) Which additional accessor methods should a binary tree Abstract Data Type implement in addition to those of an ordinary tree?

T.left(p): Return the position that represents the left child of p, or None if p has no left child

T.right(p): Return the position that represents the right child of p, or None if p has no right child

T.sibling(p): Return the position that represents the sibling of p, or None if p has no sibling

Question 9

a)

i) Is it easier to insert and delete elements into a linear data structure if the data structure is based on an array or a linked list? Explain why.

An array. When dealing with linked lists, you will need to reorganize the connecting nodes and/or the head and tail nodes

ii) Explain the differences between the str, list, and tuple data structures in python, with reference to their memory allocation and how they store elements.

str stores characters directly, whereas lists and tuples store references, resulting in a str creating more memory.
lists are dynamic arrays, meaning they can change size, wheres strs and tuples are immutable or fixed-sized.

b)

i) If S is a stack, what is the output of the following pseudocode program? Draw the state of the stack at each operation.

	Stack	Output
S.push(5)	[5]	
S.push(3)	[5, 3]	
S.push(22)	[5, 3, 22]	
print(S.top())	[5, 3, 22]	22
print(S.pop())	[5, 3]	22
S.push(7)	[5, 3, 7]	
print(S.pop())	[5, 3]	7
print(S.pop())	[5]	3

ii) If Q is a queue, what is the output of the following pseudocode program? Draw the state of the queue at each operation.

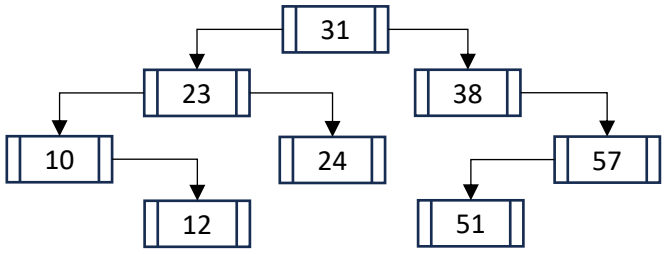
	Stack	Output
Q.enqueue(15)	[15]	
Q.enqueue(31)	[15, 31]	
Q.enqueue(4)	[15, 31, 4]	
print(Q.dequeue())	[31, 4]	15
Q.enqueue(11)	[31, 4, 11]	
print(Q.dequeue())	[4, 11]	31
print(Q.dequeue())	[11]	4
print(Q.dequeue())	[ ]	11
print(Q.dequeue())	error - empty queue	

Question 10

a)

i) Draw the binary search tree obtained by inserting items with the following keys into an initially empty unbalanced binary search tree:

31, 38, 23, 57, 51, 24, 10, 12



ii) Write the order that the nodes would be visited in a pre-order traversal of the tree.

31, 23, 10, 12, 24, 38, 57, 51

iii) Write the order that the nodes would be visited in a post-order traversal of the tree.

12, 10, 24, 23, 51, 57, 38, 31

iv) Write the order that the nodes would be visited in an in-order traversal of the tree.

10, 12, 23, 24, 31, 38, 51, 57

b) Show how binary trees might be used in a real-world context. In your answer, give an example showing how building and operating using a binary tree helps solve a problem.

I can think of plenty examples for a normal tree, but a binary tree? That is tricky.
Upon doing some research, I found a very interesting use case for a binary tree.
Morse Code can actually be laid out in a binary tree where the left is the dot, and right is dash