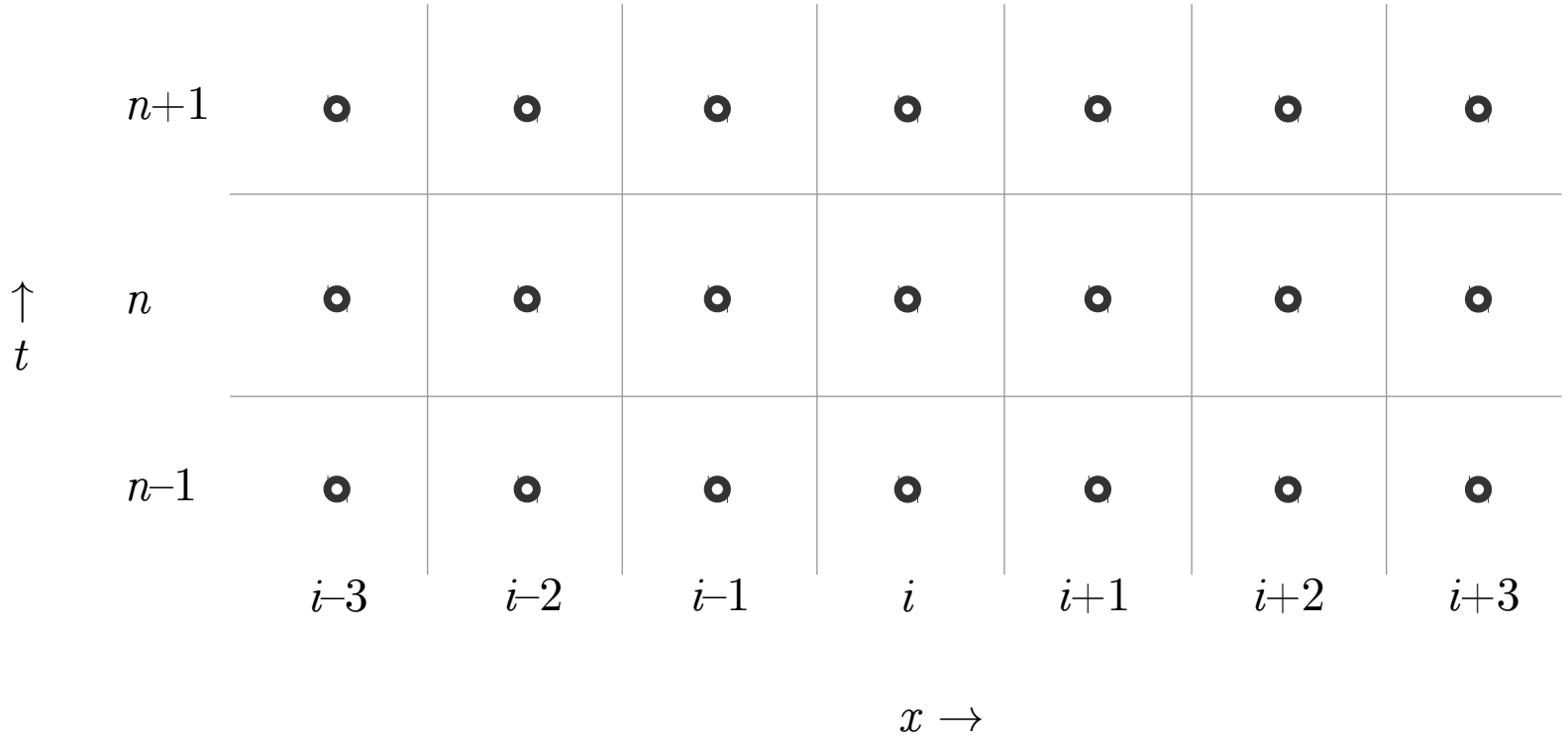
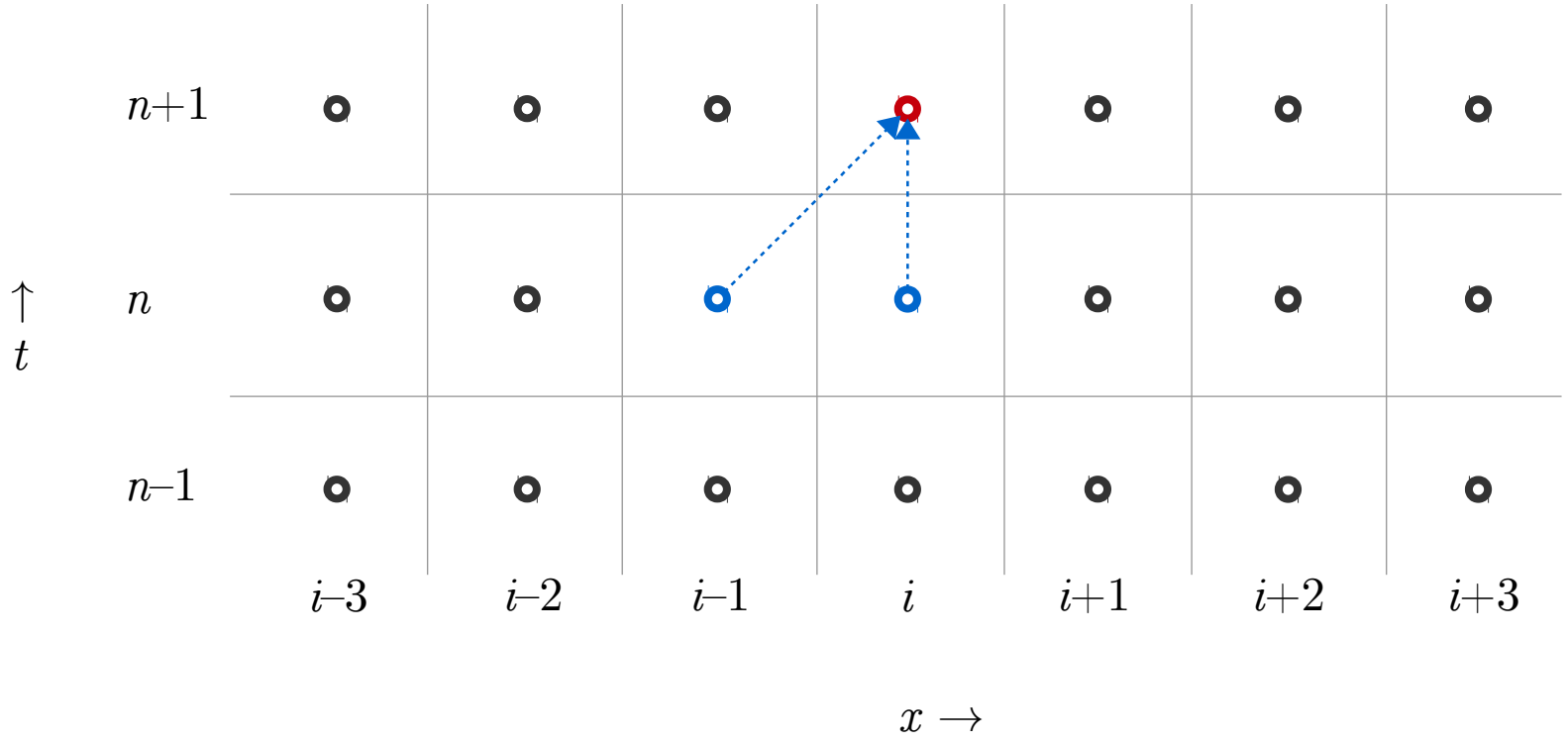
Solvers and Discretization Schemes



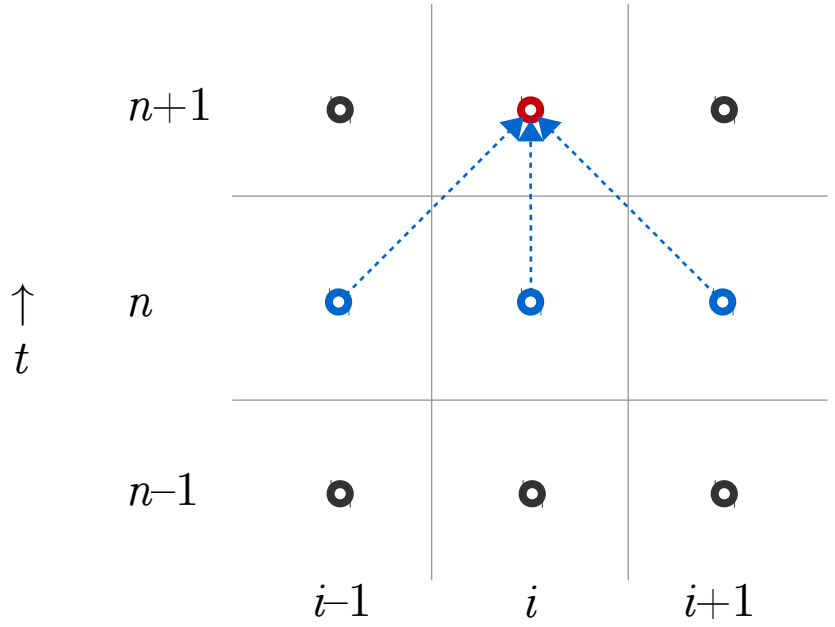
$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x}$$

linear advection



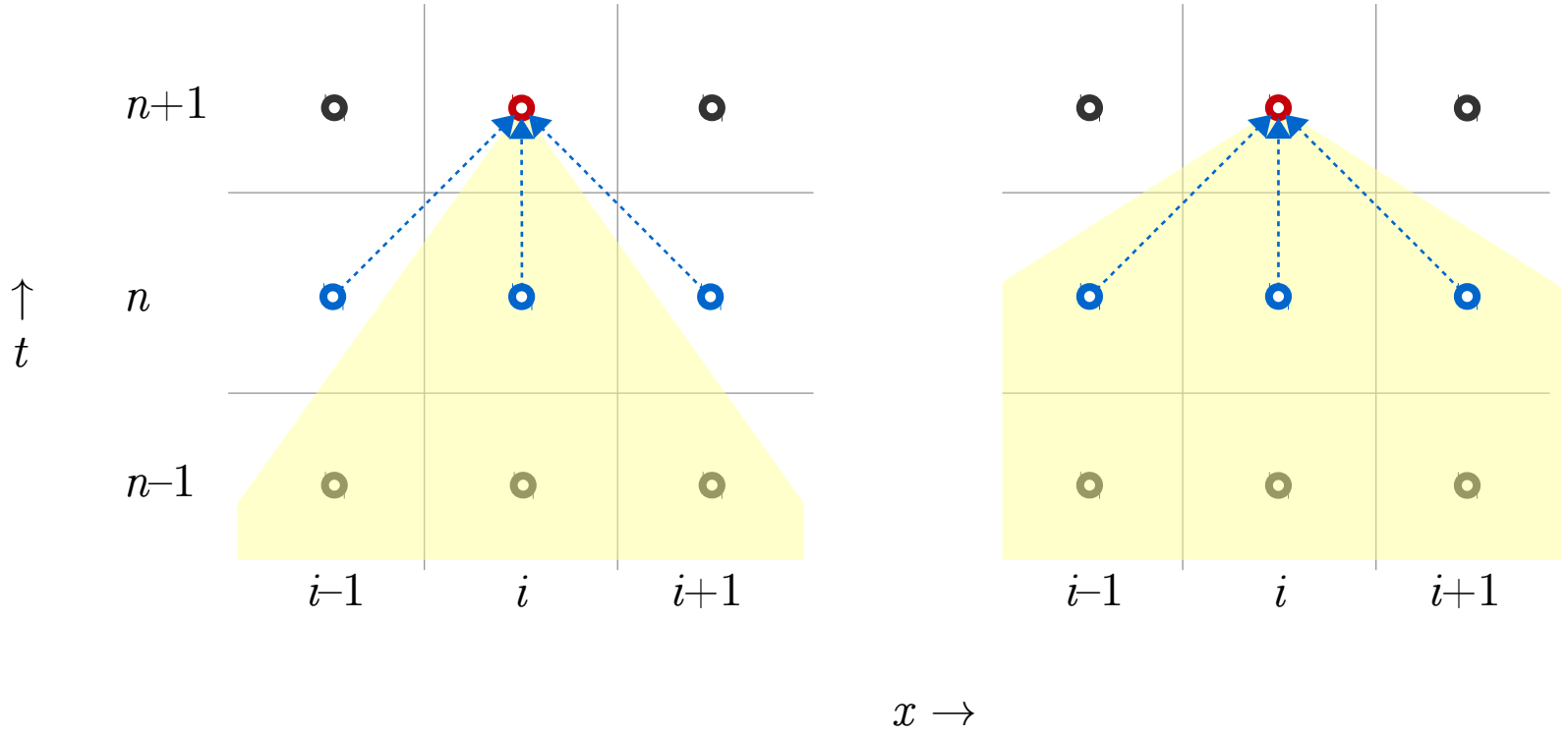
$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -a \frac{u_i^n - u_{i-1}^n}{\Delta x}$$

first-order upwind



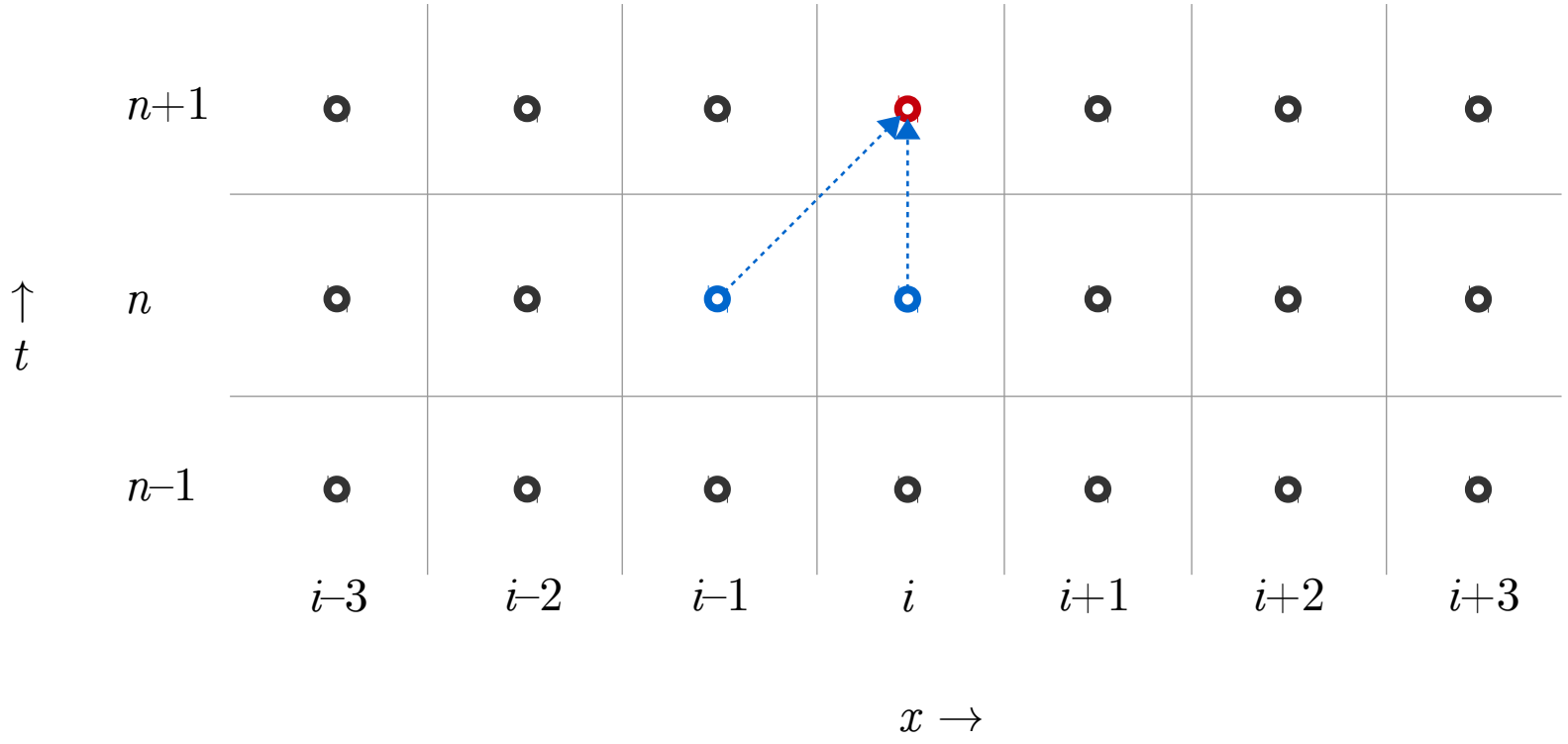
$$c = \left| \frac{a\Delta t}{\Delta x} \right| \leq 1$$

CFL stability criterion



$$c = \left| \frac{a\Delta t}{\Delta x} \right| \leq 1$$

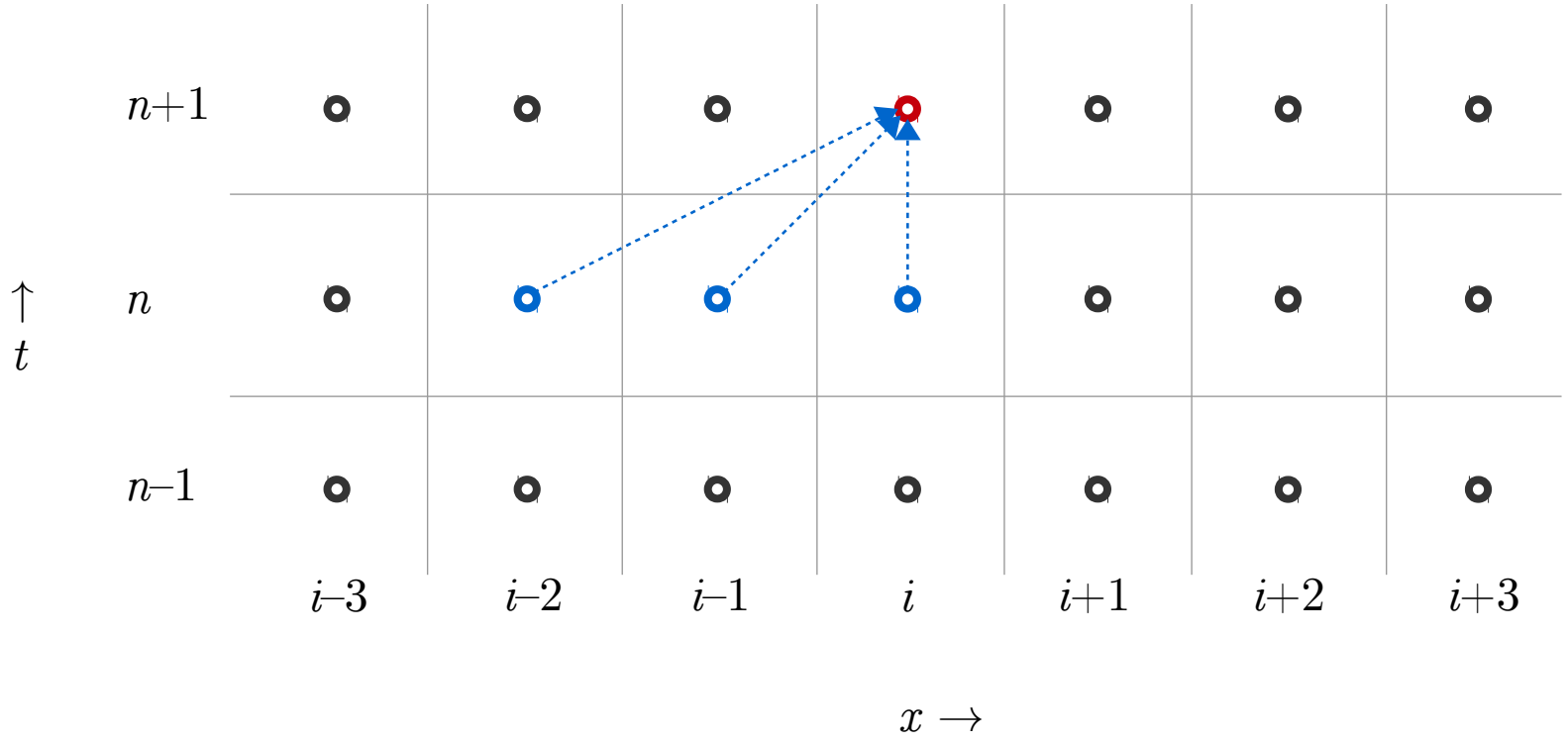
CFL stability criterion



$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x}$$

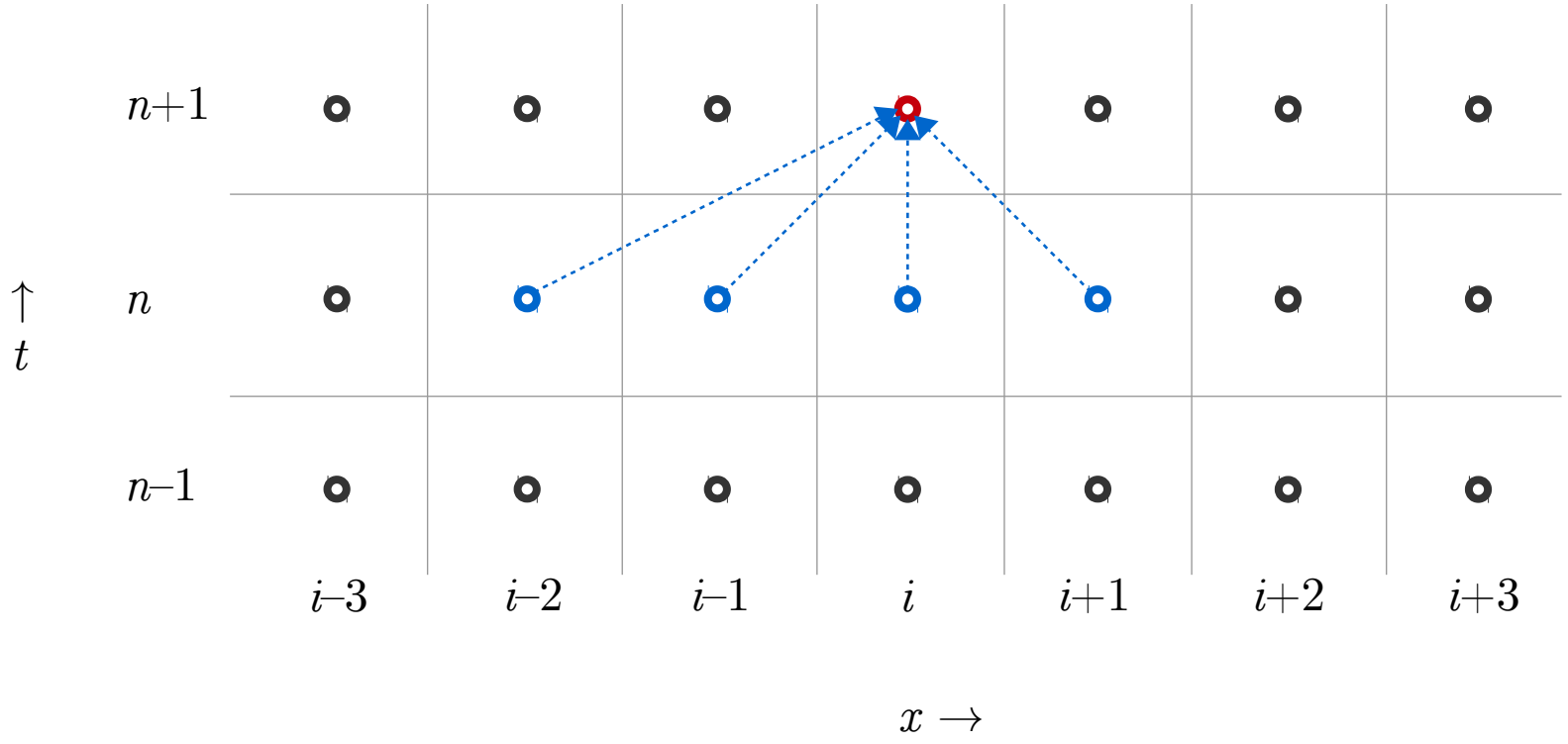
$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x} + \left(\frac{\partial^2 u}{\partial t^2}\frac{\Delta t}{2} - a\frac{\partial^2 u}{\partial x^2}\frac{\Delta x}{2}\right) + \mathrm{HOT}$$

first-order upwind modified equation



$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -a \frac{3u_i^n - 4u_{i-1}^n + u_{i-2}^n}{2\Delta x}$$

second-order upwind



$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -a \frac{2u_{i+1}^n + 3u_i^n - 6u_{i-1}^n + u_{i-2}^n}{6\Delta x}$$ third-order upwind

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = F(u^n)$$

explicit
(cond. stable)
density-based

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = F(u^{n+1})$$

implicit
(stable)
pressure-,
density-based

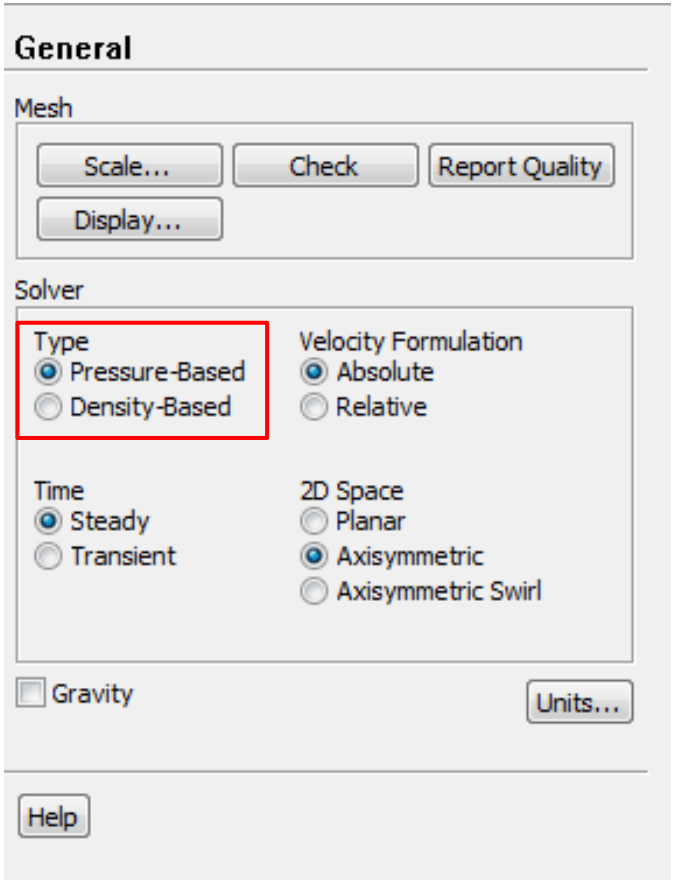
$$u^{n+1/2} = u_n + \frac{1}{2}\beta_{n+1/2}\left(u^n - u^{n-1}\right)$$

$$u^{n-1/2} = u_{n-1} + \frac{1}{2}\beta_{n-1/2}\left(u^{n-1} - u^{n-2}\right)$$

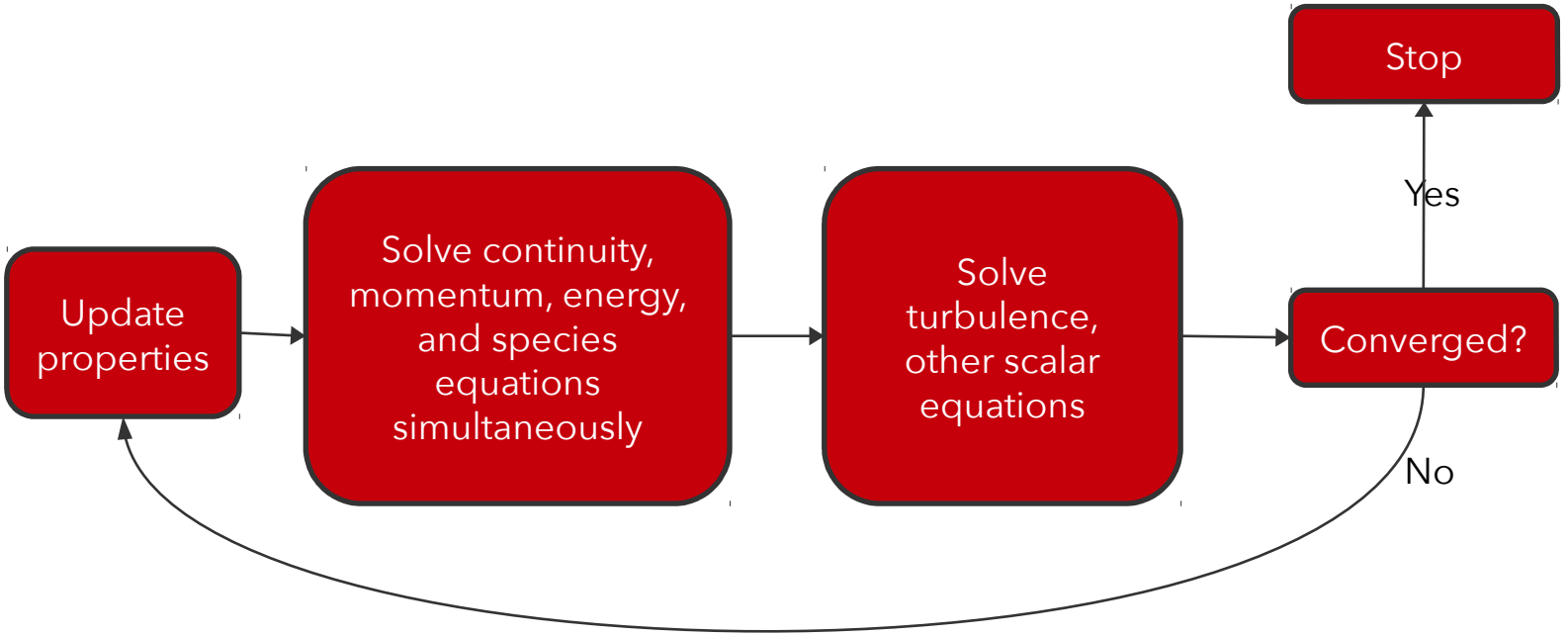
second-order
implicit
(stable)
pressure-based

Solvers

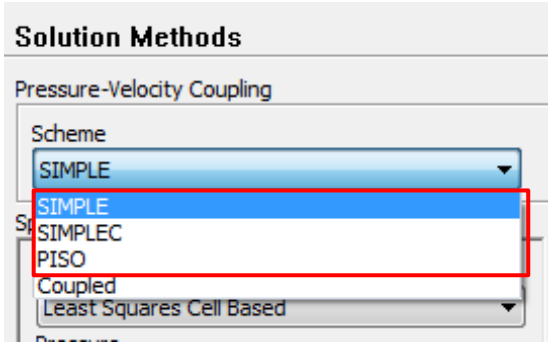
- Pressure-based
  - Segregated–converges slowly, saves memory
    - SIMPLE, SIMPLEC, PISO
  - Coupled – converges fast but costs memory
    - Coupled
- Density-based

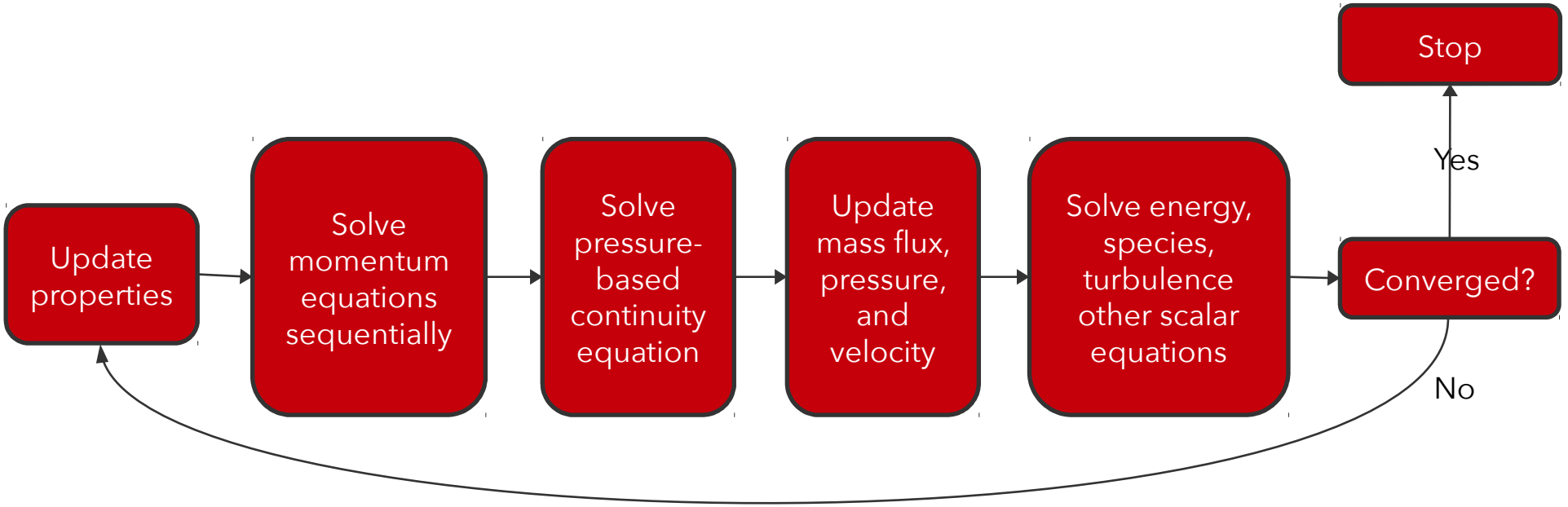


Density-based solver

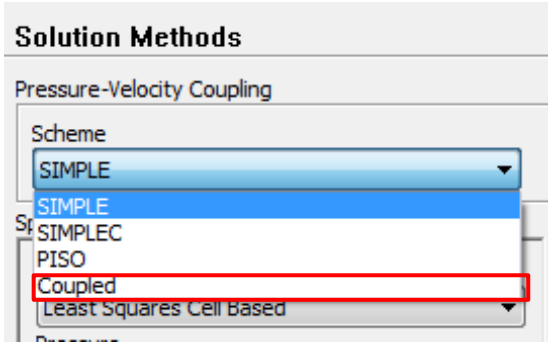


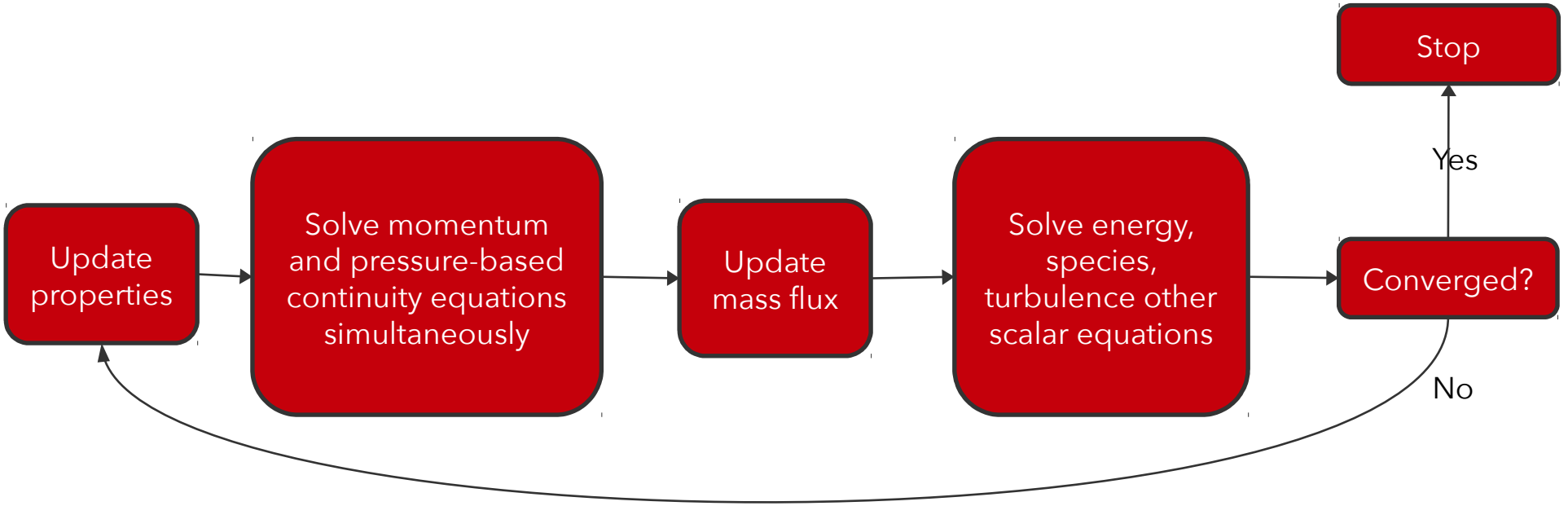
Segregated pressure-based solver



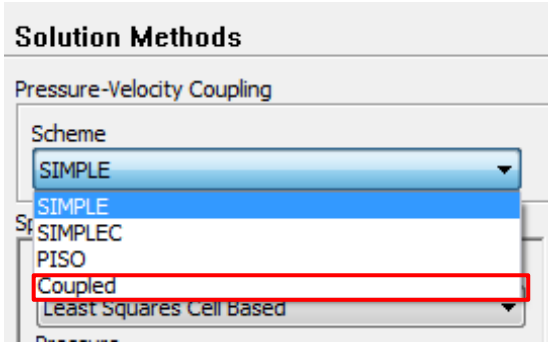


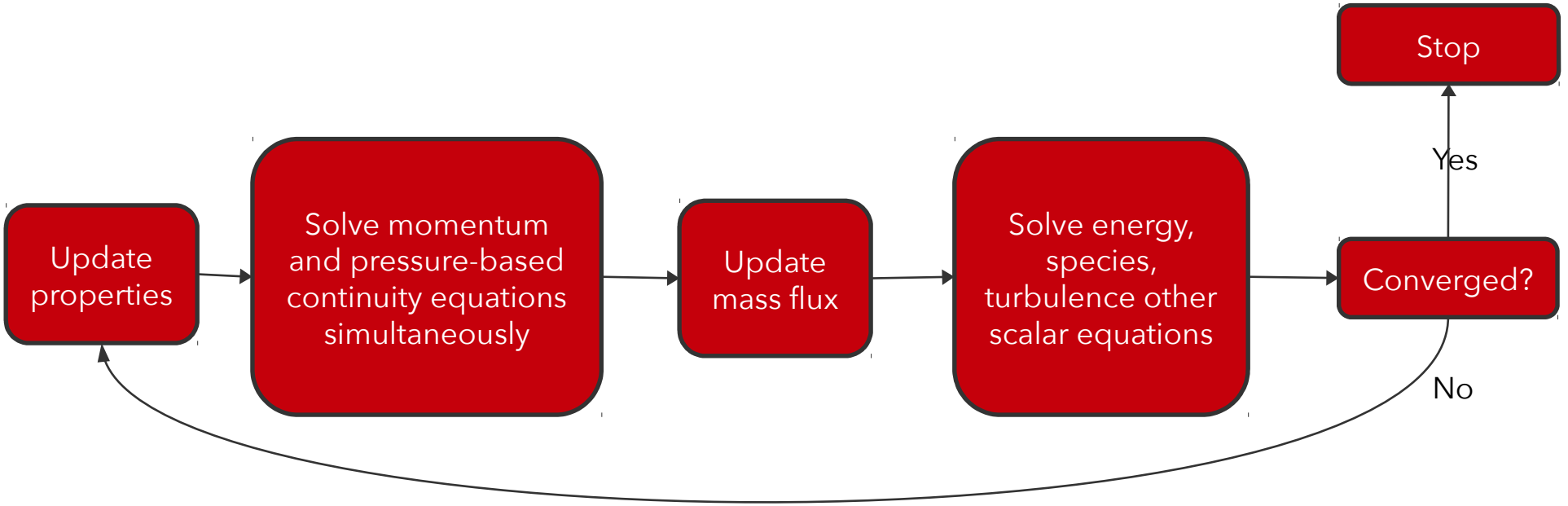
Coupled pressure-based solver





Coupled pressure-based solver





Density-based solver