This Jupyter notebook answers HW11 questions for PHY 981 Nuclear Structure.

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HW11 question 3

For the neutron configuration $(d_{1/2})^n$, what are the allowed (J, T) values for n = 1; 2; 3; 4; 5; 6?

In [1]:

```
import itertools
import numpy as np
import pandas as pd

def m_scheme(j, n):
    m2_vals = np.array([j-i for i in range(int(2*j+1))])

    nucleons = np.ones(n,dtype=int)
    occupations = np.append(nucleons, np.zeros(len(m2_vals)-len(nucleons),dtyp
    occupations_str = ''.join([str(x) for x in occupations])
    configs = np.sort(list(set(map("".join, itertools.permutations(occupation

    df_list = []
    for config in configs:
        config_arr = np.array([int(x) for x in list(config)])
        row = pd.DataFrame([np.append(config_arr, [config_arr.dot(m2_vals)])],
                               columns=np.append([str(x) for x in m2_vals],['M'])
        df_list.append(row)
    df = pd.concat(df_list).reset_index(drop=True)
    return df

scheme_df = m_scheme(7/2,2)
print('M scheme table')
print(scheme_df)
print('\nNum unique M values')
print(scheme_df['M'].value_counts(sort=False))
```

```
M scheme table
    3.5  2.5  1.5  0.5  -0.5  -1.5  -2.5  -3.5    M
0   1.0  1.0  0.0  0.0   0.0   0.0   0.0   0.0  6.0
1   1.0  0.0  1.0  0.0   0.0   0.0   0.0   0.0  5.0
2   1.0  0.0  0.0  1.0   0.0   0.0   0.0   0.0  4.0
3   1.0  0.0  0.0  0.0   1.0   0.0   0.0   0.0  3.0
4   1.0  0.0  0.0  0.0   0.0   1.0   0.0   0.0  2.0
5   1.0  0.0  0.0  0.0   0.0   0.0   1.0   0.0  1.0
6   1.0  0.0  0.0  0.0   0.0   0.0   0.0   1.0  0.0
7   0.0  1.0  1.0  0.0   0.0   0.0   0.0   0.0  4.0
8   0.0  1.0  0.0  1.0   0.0   0.0   0.0   0.0  3.0
9   0.0  1.0  0.0  0.0   1.0   0.0   0.0   0.0  2.0
10  0.0  1.0  0.0  0.0   0.0   1.0   0.0   0.0  1.0
11  0.0  1.0  0.0  0.0   0.0   0.0   1.0   0.0  0.0
12  0.0  1.0  0.0  0.0   0.0   0.0   0.0   1.0 -1.0
13  0.0  0.0  1.0  1.0   0.0   0.0   0.0   0.0  2.0
14  0.0  0.0  1.0  0.0   1.0   0.0   0.0   0.0  1.0
15  0.0  0.0  1.0  0.0   0.0   1.0   0.0   0.0  0.0
16  0.0  0.0  1.0  0.0   0.0   0.0   1.0   0.0 -1.0
17  0.0  0.0  1.0  0.0   0.0   0.0   0.0   1.0 -2.0
18  0.0  0.0  0.0  1.0   1.0   0.0   0.0   0.0  0.0
```

```
 19  0.0  0.0  0.0  1.0   0.0   1.0   0.0   0.0 -1.0
 20  0.0  0.0  0.0  1.0   0.0   0.0   1.0   0.0 -2.0
 21  0.0  0.0  0.0  1.0   0.0   0.0   0.0   1.0 -3.0
 22  0.0  0.0  0.0  0.0   1.0   1.0   0.0   0.0 -2.0
 23  0.0  0.0  0.0  0.0   1.0   0.0   1.0   0.0 -3.0
 24  0.0  0.0  0.0  0.0   1.0   0.0   0.0   1.0 -4.0
 25  0.0  0.0  0.0  0.0   0.0   1.0   1.0   0.0 -4.0
 26  0.0  0.0  0.0  0.0   0.0   1.0   0.0   1.0 -5.0
 27  0.0  0.0  0.0  0.0   0.0   0.0   1.0   1.0 -6.0

Num unique M values
  6.0    1
  5.0    1
  4.0    2
  3.0    2
  2.0    3
  1.0    3
  0.0    4
 -1.0    3
 -2.0    3
 -3.0    2
 -4.0    2
 -5.0    1
 -6.0    1
Name: M, dtype: int64
```

n = 1

j = J = 5/2 and T = 1/2

n = 2

T = 1

J = 4,2,0

n = 3

T = 3/2

J = 9/2, 5/2, 3/2

n = 4

T = 2

J = 4,2,0

n = 5

T = 5/2

J = 5/2

n = 6

T = 3

J = 0

HW11 question 4

For the configuration $(\pi d_{5/2})(\nu d^2_{5/2})$, what the number of states for each of the allowed (J,T) values? Remember that the states with T = 3/2 are theisobaric analogue states of the $(\nu d_{5/2})^3$ configuation.

In this configuration, we must have have T$_z$ = 1/2. The j = 5/2 proton is coupled to the configuration $(5/2)^2$ with J$_n$ = 0,2,4. The possible J values according to the triangle rule (5/2 coupled to 0, 2, and 4) are now
$J = (5/2),(9/2,7/2,5/2,3/2,1/2),(13/2,11/2,9/2,7/2,5/2,3/2)$

(J, T)	Num states in T=1/2	Num states in T=3/2
(13/2, 1/2)	1	0
(11/2, 1/2)	1	0
(9/2, 1/2)	1	1
(7/2, 1/2)	2	0
(5/2, 1/2)	2	1
(3/2, 1/2)	1	1
(1/2, 1/2)	1	0

HW11 question 5

For the configuration $(\pi d_{5/2})(\nu d_{5/2})^5$, what the number of states for each of the allowed (J, T) values?

Here the T$_z$ = 2. We couple the j = 5/2 with J$_n$ = 5/2 neutrons (this is the only possible J value in $(\nu d_{5/2})^5$. Triangle rule gives us possible J values $J = (0,1,2,3,4,5)$ the highest allowed isospin for 6 neutrons T = T$_z$ = 3. In this case, J = 0 is the only allowed J. (T,T$_z$) = (3,2) is an isobaric analogue of (T,T$_z$) = (3,3). Therefore, the only J = 0 in the configuraiton $(\pi d_{5/2})(\nu d_{5/2})^5$ comes from this isobaric analogue. The rest are counted once.

HW11 question 6

Calculate the magnetic moment for the 5+ state in 18F assuming a $(0d_{5/2})^2$ configuration. Use free-nucleon g-factors. Compare to experiment. (I attach a compilation of experimental moments.)

Use one-body operator magnetic moment Eq. 18.20

$$\frac{\hat{\mu}}{\mu_N} = \hat{\ell}_z g_q^\ell + \hat{s}_z g_q^s$$

The matrix element $\langle C(0d_{5/2})^2|\frac{\mu_z}{\mu_N}|C(0d_{5/2})^2\rangle$ is a sum over occupied single-particle states with respect to the closed shell 16O. In other words, we can compute the magnetic moment according to the sum,

$$\langle C(0d_{5/2})^2|\frac{\hat{\mu}_z}{\mu_N}|C(0d_{5/2})^2\rangle = \langle(\pi 0d_{5/2})|\frac{\hat{\mu}_z}{\mu_N}|(\pi 0d_{5/2})\rangle + \langle(\nu 0d_{5/2})|\frac{\hat{\mu}_z}{\mu_N}|(\nu 0d_{5/2})\rangle$$

.

The matrix element (in units of μ_N) is

$$\langle\mu_z\rangle = jg_q^\ell + \frac{[j(j+1)+s(s+1)-\ell(\ell+1)]\left(g_q^s - g_q^\ell\right)}{2(j+1)}$$

HW11 question 7

Calculate the magnetic moment for the 4+ state in 18O assuming a $(0f_{7/2})^2$ configuration. Use free-nucleon g-factors. Compare to experiment.

Similar to question 6 except the two single-particle states with respect to the closed shell 16O are both neutron states. The 4+ state requires that the wavefunction be a linear combination of the configurations j_1, j_2 = 7/2,1/2 and j_1, j_2 = 5/2,3/2.

use eq. 18.9. The matrix element of a general M is related to the matrix element of a state where $M = J_{max}$ by a ratio of Clebsch Gordon coefficients.

$$< J, M|O^\lambda|J, M >= \frac{< J, M, \lambda, 0|J, M >}{< J, J_{max}, \lambda, 0|J, J_{max} >} < J, J_{max}|O^\lambda|J, J_{max} >$$

For a single neutron in this shell, the J_{max} = 7/2. When we compute another M, we multiply by the appropriate Clebsch Gordon coefficient ratio.

The answer I get is -2.186. The literature gives a magnitude of μ = 2.5(4) with no sign determined by the experiment. Calculated results are consistent within error of the experiment, but sign cannot be compared.

In [8]:
```
def mu(j, l, q):
    if q == 0:
        gl = 1
        gs = 5.586
    elif q == 1:
        gl = 0
        gs = -3.826
    else:
        return "q must be 0 (proton) or 1 (neutron)"

    dot_me = (j*(j+1) + 1/2*(1/2+1) - l*(l+1))/2

    return j*gl + (dot_me*(gs-gl))/(j+1)

tj_max = 0.2415321
tj_half = 0.0345032
tj_thalf = 0.1035098
tj_fhalf = 0.1725164


print(mu(7/2,3,1)+ tj_half/tj_max*mu(7/2,3,1))
print(tj_fhalf/tj_max*mu(7/2,3,1)+ tj_thalf/tj_max*mu(7/2,3,1))
```

```
-2.186274739051248
-2.1862026645733628
```

HW11 question 8

What are the partitions allowed for the $(0d_{5/2}, 0d_{3/2}, 1s_{1/2})$ set of orbitals for 19O. What is the maximum J value allowed?

Allowed partitions:

$$(0d_{5/2})^0(0d_{3/2})^1(1s_{1/2})^2$$

$$(0d_{5/2})^1(0d_{3/2})^0(1s_{1/2})^2$$

$$(0d_{5/2})^0(0d_{3/2})^2(1s_{1/2})^1$$

$$(0d_{5/2})^2(0d_{3/2})^0(1s_{1/2})^1$$

$$(0d_{5/2})^1(0d_{3/2})^1(1s_{1/2})^1$$

$$(0d_{5/2})^1(0d_{3/2})^2(1s_{1/2})^0$$

$$(0d_{5/2})^2(0d_{3/2})^1(1s_{1/2})^0$$

$$(0d_{5/2})^0(0d_{3/2})^3(1s_{1/2})^0$$

$$(0d_{5/2})^3(0d_{3/2})^0(1s_{1/2})^0$$

Maximum allowed J is 5/2+3/2+1/2 = 9/2

HW11 question 9

Evaluate the spin matrix elements $\langle S|X^q|S'\rangle$ for Eqs. 25.25-27.

$$\langle S|X^{(0)}|S'\rangle = \delta_{SS'}$$

$$\langle S|X^{(1)}|S'\rangle = \langle S|\vec{S}|S'\rangle = \delta_{SS'}\sqrt{S(S+1)(2S+1)}$$

$$\langle S|X^{(2)}|S'\rangle = \langle S|\sqrt{6}[\vec{\sigma}_1 \otimes \vec{\sigma}_2]^{(2)}|S'\rangle = (-1)^S\sqrt{2S+1(5)(2S'+1)}\begin{pmatrix} S & 2 & S' \\ 0 & 0 & 0 \end{pmatrix}$$

In [ ]: