hw05

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0.1 This Jupyter notebook answers HW05 questions for PHY 981 Nuclear Structure.

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0.1.1 HW05 question 3

Use the liquid drop model to calculate the energy of the T=2 state in 20Ne that is the IAS of the ground state of 20O. Compare to experiment.

The shift does not depend on the Coulomb term. The shift *does* come from the symmetry term multiplied by α_4. The difference in the symmetry term between the T=2 states of 20Ne and 20O is

$$-\alpha_4 \frac{0^2 - 4^2}{20} = 18.08 \text{ MeV}$$

The ground state (0^+) of the T=2 20Ne is 16.732 MeV according to NNDC. The isospin shifts the energy by about 1.3 MeV.

0.1.2 HW05 question 4

Derive Eq. 13.8 from 13.7

Eq. 13.7

$$|\Psi >= \sum_a c_a |\Phi_a >$$

Eq. 13.8

$$\sum_a < \Phi_b | H | \Phi_a > c_a = E c_b$$

We start we Eq 13.7 and multiply $< \Phi_b | H$ from the left.

$$< \Phi_b | H | \Psi >= \sum_a < \Phi_b | H | \Phi_a > c_a$$

Note that $H|\Psi >= E|\Psi >$ (the time-independent Schrodinger equation).

$$E < \Phi_b|\Psi >= \sum_a < \Phi_b|H|\Phi_a > c_a$$

Note that $< \Phi_b|\Psi >= c_a\delta_{ab} = c_b$ given that $\{|\Phi_a >\}$ span a complete orthonormal basis. Finally,

$$Ec_b = \sum_a < \Phi_b|H|\Phi_a > c_a. \quad \blacksquare$$

0.1.3 HW05 question 5

Derive Eq. 13.19 for two-particles (n = 2) wavefunctions.

For two particles, $|\Phi_a >= \frac{1}{\sqrt{2}}(\phi_\alpha(1)\phi_\beta(2) - \phi_\beta(1)\phi_\alpha(2))$. The one-body operator is $T + U = T_1 + U_1 + T_2 + U_2$. The diagonal matrix element is

$$\frac{1}{\sqrt{2}}(\phi_\alpha(1)\phi_\beta(2)-\phi_\beta(1)\phi_\alpha(2))^*(T_1+U_1+T_2+U_2)\frac{1}{\sqrt{2}}(\phi_\alpha(1)\phi_\beta(2)-\phi_\beta(1)\phi_\alpha(2)) = \frac{1}{2}(\phi_\alpha(1)\phi_\beta(2)-\phi_\beta(1)\phi_\alpha(2))^*((T_1+$$

We pause for a moment to realize that $T_k + U_k$ will not connect two different single particle states. We have that $(T + U)|\alpha >= \epsilon_\alpha|\alpha >$ and $< \alpha_m|\alpha_n >= \delta_{mn}$. So we can eliminate cross terms in the above equation, and only deal with terms $\phi_S(k)^*(T_k + U_k)\phi_S(k)$ where S can be α or β. Then,

$$= \frac{1}{2}(\phi_\alpha(1)^*(T_1+U_1)\phi_\alpha(1)\phi_\beta(2)^*\phi_\beta(2)+\phi_\beta(1)^*(T_1+U_1)\phi_\beta(1)\phi_\alpha(2)^*\phi_\alpha(2)+\phi_\alpha(1)^*\phi_\alpha(1)\phi_\beta(2)^*(T_2+U_2)\phi_\beta(2)+\phi_\beta(1)$$

Finally, we consolidate terms.

$$= \frac{1}{2}\left(\sum_S \phi_S(1)^*(T_1 + U_1)\phi_S(1) + \sum_S \phi_S(2)^*(T_2 + U_2)\phi_S(2)\right)$$

And then we sum over *all states occupied by 1 and 2* in each term. The unrestricted sum brings an extra factor of two that cancels the 1/2. Finally, we can combine the terms to get Eq. 13.19

$$= \sum_{S=\alpha,\beta} \phi_S^*(T + U)\phi_S. \quad \blacksquare$$

0.1.4 HW05 question 6

Derive Eq. 13.21 for two-particles (n = 2) wavefunctions.

For two particles, the interaction $\sum_{k<l} V_{kl} = V_{12}$. This time we will write $|\Phi_a >= \frac{1}{\sqrt{2}}(|\alpha\beta > -|\beta\alpha >)$ where

$$< \Phi_a|V_{12}|\Phi_a >= \frac{1}{2}(< \alpha\beta|- < \beta\alpha|)V_{12}(|\alpha\beta > -|\beta\alpha >) = \frac{1}{2}(< \alpha\beta|V_{12}|\alpha\beta > - < \alpha\beta|V_{12}|\beta\alpha > - < \beta\alpha|V_{12}|\alpha\beta > +$$

At this point, we recall that the Slater determinant ensures $|\alpha\beta>= -|\beta\alpha>$. Then, we can combine terms to get

$$=<\alpha\beta|V_{12}|\alpha\beta> - <\alpha\beta|V_{12}|\beta\alpha> \equiv <\alpha\beta|V|\alpha\beta>_A .$$

If we unrestrict the sum $\alpha < \beta$, and instead sum over all $\alpha\beta$, then we just have to multiply by 1/2 to remove the double counting.

0.1.5 HW05 question 7

Derive Eq. 13.29.

Eq. 13.29 is

$$E(Ch^{-1}h'^{-1}) = E(C) - e_h - e_{h'} - <hh'|V|hh'>$$

The energy of a configuration C is

$$E(C) = <C|H|C> = \sum_\alpha <\alpha|T|\alpha> + \frac{1}{2}\sum_{\alpha\beta} <\alpha\beta|V|\alpha\beta>$$

The energy of a configuration with two states h and h' *removed* from C is simply the negative of the energy of a configuration with two states h and h' *added* to C. We can understand the energy of state Chh' using Eq. 13.24, 13.25, and 13.26.

$$E(Ch) = E(C) + <h|T|h> + \sum_\alpha <h\alpha|V|h\alpha> = E(Ch'; h \to h')$$

$$E(Chh') = E(C) + <h|T|h> + \sum_\alpha <h\alpha|V|h\alpha> + <h'|T|h'> + \sum_\alpha <h'\alpha|V|h'\alpha> + <hh'|V|hh'>$$

The extra term comes from the additional energy contributed by the interaction between states h and h'.

If we let

$$e_h = E(Ch) - E(C) = <h|T|h> + \sum_\alpha <h\alpha|V|h\alpha>,$$

then

$$E(Chh') = E(C) + e_h + e_{h'} + <hh'|V|hh'>$$

We realize that the configuration $Ch^{-1}h'^{-1}$ simply removes the single particle energys $e_h, e_{h'}$ and the interaction energy $<hh'|V|hh'>$ from the configuration C, so that we arrive at

$$E(Ch^{-1}h'^{-1}) = E(C) - e_h - e_{h'} - < hh'|V|hh' > \quad \blacksquare$$

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