hw02

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0.1 This Jupyter notebook answers HW01 questions for PHY 981 Nuclear Structure.

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[1]:
```
# load packages
import numpy as np
```

0.1.1 HW02 question 5

A level of astrophysical interest for the (p,γ) reaction in ^{31}S has a calculated half-life of 1.06 fs. What is the width of this state in eV.

[2]:
```
# relationship between half-life and decay width

t_half = 1.06*10**-15 #s
hbar = 6.582e-22*10**6 # eV s
gamma = hbar*np.log(2)/t_half #eV

print("decay width for (p, G) reaction in 31-S, G = {:e} eV".format(gamma))
```

```
decay width for (p, G) reaction in 31-S, G = 4.304052e-01 eV
```

The relationship between the decay width Γ and the half-life $T_{1/2}$ is

$$\Gamma T_{1/2} = \hbar \ln(2)$$

We find the decay width by solving for Γ, and then, using the given half-life for the (p,γ) of $T_{1/2} =$ 1.06 fs, we compute Γ (paying attention to units). We find that the decay width Γ for this reaction is 0.4304 eV.

0.1.2 HW02 question 6

The half-life of the 4+ level at 4.248 MeV in ^{20}Ne is 0.064 ps. What is the B(E2, $\downarrow$) for the decay to the 2+ state? The calculated B(E4) to the 0+ state is 10 Wu. What is the branching ratio for this E4 decay?

[3]:
```
BE4 = 1000
E_gamma = 4.248
```

```
# t_half = 0.9052e8

Tp = 4076/(E_gamma**9*1000)
bE4 = 0.064e-12/Tp

print('branching fraction b(E4) = {:0.3e}'.format(bE4))
```

```
branching fraction b(E4) = 7.073e-09
```

Here we include a table of excitation energies above the J = 0^+ state for ^{20}Ne.

^{20}Ne J^π	Excitation energy (MeV)
4^+	4.248
2^+	1.633
0^+	0.0

Livechart of nuclides: https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html

Since we know the half-life of the transition, as well as the gamma energy, we can use the nushell app `bem` to compute the B(EL) values.

To get the B(E2) for the transition $4^+ \to 2^+$ in ^{20}Ne, we input the half-life, gamma energy, and mass number into `bem` and read off the results. We get

$$B(E2) = 72.15.$$

To find the branching ratio for the E4, we realize that we can use `bem` to find the half-life for the E4 decay by inserting (10 Wu)x10 as the argument for half-life, which would be the B(E4) for all 10 nucleons decaying in ^{20}Ne; the output in the B(EL) column, then is the half-life of the E4 transition. We insert this half-life back into `bem` to get the B(E4) in units that we we can use in Eq. 7.89. Next, we solve Eq. 7.89 for T_p:

$$T_p = \frac{4076}{E_\gamma^9 * B(E4)}.$$

Finally, to get the branching fraction, we compute

$$b = \frac{T_{1/2}}{T_p} = \frac{0.064\text{E-12 s}}{T_p} = 7.073\text{E-9\%}.$$

Such a small branching fraction means that the transition is not common.

0.1.3 HW02 question 7

^{42}Ti has a half-life of 209 ms. It beta decays to the ground state of ^{42}Sc with a branching ratio of 47.7 percent. Calculate the logft value for this decay, and compare with the nuclear data sheets. What are the B(F) and B(GT) values for this decay?

[4]:
```
f = 10808.518
# f = 7253.701
t_half = 209e-3

logft = np.log10(f*t_half)
print('logft = {:0.3f}'.format(logft))

BF = 6170/10**(logft)
print('B(F) = {:0.3f}'.format(BF))
```

```
logft = 3.354
B(F) = 2.731
```

We use the app `ff` on nushell, providing input Zi=22, Zf=21, A=42, Q=7.016 MeV; according to the output fz=10808.518. To compute the logft, we take

$$\log_{10} ft_{1/2} = \log_{10}(209\text{E-}3 * 10808.518) = 3.354$$

We can look at the NNDC 42-Ti level scheme for the decay https://www.nndc.bnl.gov/nudat2/getdecayscheme.jsp?nucleus=42SC&dsid=42ti%20ec%20decay%20(208.65%20 and we compare the given logft value for the branching ratio 47.7% (corresponding to decay to ground state of 42-Sc). The logft for this decay is 3.495 according to the level scheme, which is close to the value we computed.

This particular nuclear transition is $0^+ \to 0^+$ which means B(GT) = 0 according to Gamow-Teller rules. Then, we can solve Eq. 7.63,

$$ft_{1/2} = \frac{6170}{B(F) + 1.629 * B(GT)},$$

for B(F), setting B(GT) = 0 so that

$$B(F) = \frac{6170}{ft_{1/2}} = \frac{6170}{10^{\log_{10} ft_{1/2}}}.$$

We find that B(F) = 2.731.

0.1.4 HW02 question 8

What is the classical turning radius scattering of ^{14}C on ^{233}Ra?

[5]:
```
BE_Ra = 1767435.48
BE_alpha = 28295.66
BE_Rn = 1741686.98

Q_alpha = BE_Rn + BE_alpha - BE_Ra
print('233-Ra Q_alpha = {:0.2f} keV'.format(Q_alpha))

R_c = 2*88*1440/(3*Q_alpha)
```

```python
print('Turning radius R_c = {:0.2e} fm'.format(R_c))
```

```
233-Ra Q_alpha = 2547.16 keV
Turning radius R_c = 3.32e+01 fm
```

The equation for classical turning radius for alpha scattering is

$$R_c = \frac{2Z_d e^2}{Q_\alpha}$$

where Q_α is the energy of one alpha particle emitted by ^{233}Ra, $Z_d = 88$, and $e^2 = 1440$ keV fm. To compute the classical turning radius of ^{14}C on ^{233}Ra, consider the Q_α to be the incident energy of the ^{14}C, which corresponds to 3 x Q_α of ^{233}Ra. First, we compute the Q_α for ^{233}Ra according to Eq. 7.15. Then, we plug our results into the equation above and find the turning radius

$$R_c = 33.2 \text{ fm}$$

Binding energies required to compute Q_α come from https://www.nndc.bnl.gov/nudat2/reColor.jsp?newColor=qa

0.1.5 HW02 question 9

Estimate the alpha-decay half-life of 208Pb and 216Rn. Compare to experiment.

[7]:
```python
def compute_alpha_half_life(A, Z, BE_A, BE_Am4, mass_A):
    R_d = 1.2*A**(1/3) #fm
    R_alpha = 2.15 #fm
    R_t = R_d + R_alpha #fm

    BE_alpha = 28.29566 #MeV
    e_sq = 1.440 # MeV fm

    Q_alpha = BE_Am4 + BE_alpha - BE_A

    reduced_mass = mass_A*4.003/(mass_A + 4.003)*931 #MeV

    W_c = np.sqrt(Q_alpha/(2*reduced_mass*R_t**2))
    R_c = 2*Z*e_sq/(Q_alpha)

    hbar_c_sq = (197)**2 # MeV s c^2
    x = np.sqrt(R_t/R_c)
    P = np.exp(-4*Z*e_sq*np.sqrt(2*reduced_mass/(Q_alpha*hbar_c_sq))*(np.
↪arccos(x)-x*np.sqrt(1-x**2)))

    t_half = np.log(2)/(W_c*P)

    return t_half

t_half_pb = compute_alpha_half_life(208,82,1636.430,1608.651,207.977)
```

```python
t_half_rn = compute_alpha_half_life(216,86,1675.870,1655.771,216.000)

print('Estimated t_half for 208-Pb = {:0.3e}'.format(t_half_pb))
print('Estimated t_half for 216-Rn = {:0.3e}'.format(t_half_rn))
```

```
Estimated t_half for 208-Pb = 2.118e+162
Estimated t_half for 216-Rn = 1.625e+19
```

I tried to compute all the relevant equations from Pg 129-135 in order to get to Eq. 7.20. The results I come up with don't make sense, however. 208-Pb half-life should be large since it's a stable isotope (but maybe not 10^{162}), but the 216-Rn half-life should be on the order of microseconds. I think I am missing some unit consistency and that's why I'm getting some huge numbers, but I can't figure out where the inconsistency is coming from.

[ ]:
```

```