final_exam

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0.1 This notebook answers questions for the PHY 981 final exam.

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0.1.1 Question 1

The 0+ state at 3.067 MeV in 20Na has a proton decay width of 36 keV. It decays to the 1/2+ ground state of 19Ne. Obtain the experimental spectroscopic factor for this decay by using the wspot app to calculate the single-particle proton decay width.

For this problem, we take the excitation energy of this particular 0+ state in 20Na, and then add the S+p energy in 19Ne to find the one-proton resonance energy that we will use as input to `wspot`:

3.067 MeV + (-2.190 MeV) = 0.877 MeV

Input to `wspot`:

```
18 10 1 1
0.877 0.877 0.5 1.5
1 0 1
```

Output from `wspot`:

```
ws paramters, v0,v1,r0,a0,vs,rc:
 -51.000 -33.000   1.270   0.670  22.000   1.200
 input: iat,izt,iap,izp =  18 10  1  1

 input: emin,emax,vnmin,vnmax =      0.877     0.877     0.500     1.500
 defaults:  vn,adif,rr0,vnls,r0c =   1.0000  0.6700  1.2700  1.0000  1.2000
 input:     vn,adif,rr0,vnls,r0c =   0.0000  0.0000  0.0000  0.0000  0.0000

 resonance at vn =   0.995
 resonance at vn =   0.995
 resonance at vn =   0.995
 resonance at ei =   0.877 with  G =    0.06577 MeV    half-life (ps) =   0.6933E-08
 resonance at ei =   0.877 with  G =    0.05788 MeV    half-life (ps) =   0.7878E-08
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```

Which means we find the decay width of 57.88 keV. The spectroscopic factor is the ratio of experiment over theory, S = 36/57.88 = 0.62.

0.1.2 Question 2

This 0+ state in 20Na is calculated to decay the 1+ state at 0.984 MeV with a B(M1) = 1.85 μ_N^2. If the proton decay and this gamma decay are the only modes of decay, what is the branching ratio for the gamma decay?

The decay width and the transition rate are related by:

$$\Gamma = \hbar W,$$

where W is the transition rate and Γ is the decay width. The width for proton decay in 20Na is 36 keV. We must calculate the width of M1 gamma decay from the B(M1) value. We have the B(M1) in Weiskoff units. We use this value as input to `bem`, including the gamma energy (3.068-0.984 = 2.102 MeV), and mass of 20.

Input to `bem`:

```
1.85,2.102,20
```

In this way, the output gives us the half life of each transition associated with that gamma energy:

```
B(E) in units of e^2 fm^2l   T =    1.850 ps     E =    2.102 MeV
B(M) in units of (u_N)^2 fm^[2(l-1)]
l=lamda  B(E)          B(M)         B(E)/WUE     B(M)/WUM        WUE        WUM
  1    0.2532E-04   0.2297E-02    0.5332E-04   0.1283E-02          0.47          1.79
  2    0.7438E+01   0.6747E+03    0.2306E+01   0.5549E+02          3.22         12.16
  3    0.3613E+07   0.3278E+09    0.1521E+06   0.3659E+07         23.76         89.58
  4    0.2751E+13   0.2496E+15    0.1485E+11   0.3574E+12        185.23        698.30
  5    0.3056E+19   0.2772E+21    0.2031E+16   0.4887E+17       1504.64       5672.53
  6    0.4682E+25   0.4246E+27    0.3712E+21   0.8930E+22      12613.72      47554.04
```

The M1 transition has half-life $T_{1/2} = 0.002297$ ps. Therefore, the decay width Γ is

$$\Gamma_{M1} = \frac{\hbar \ln 2}{T_{1/2}} = \frac{0.456 \text{ eV fs}}{2.297 \text{ fs}} = 0.190 \text{ ev}.$$

The branching fraction from state i to f is the ratio of the particular transition rate $W_{i,f}$ over the sum over all transition rates in each decay channel. In this case, the only decay channels are proton and gamma decay. Therefore,

$$b_{M1} = \frac{W_{M1}}{W_p + W_{M1}}$$

Note that the $\hbar$ factors in each W on the LHS will cancel if we express in terms of Γ so that

$$b_{M1} = \frac{\Gamma_{M1}}{\Gamma_p + \Gamma_{M1}}.$$

Finally the M1 branching fraction is

$$b_{M1} = \frac{0.190 \text{ eV}}{(36000 + 0.190) \text{ eV}} = 5.278 \times\ 10^{-6}$$

0.1.3 Question 3

In the 0f7/2 model space for the calcium isotopes, (a) what is the spectro- scopic factor for 45Ca 7/2- to 44Ca 0+? (b) What is the sum over all states in 44Ca?

(a) Using the fp model space restricted to the 0f7/2 subshell, we get a spectroscopic factor of 0.5.

```
! model space = fp
! interaction = usdb

( Ai   Tzi)  ( Af   Tzf) (type n,l,2j)    Ji    Jf     ni    nf     C^2S        Ei         Ef
( 44   2.0)  ( 45   2.5)  ( n   1 3  7)  0.0+  3.5-     1     1   0.5000     0.000     0.000   72.0
                                                             sum   0.5000

                                                       total sum   0.5000
```

(b) We run the same code, but computing the transition into all 44Ca states. The sum over all states is 1.2993.

```
! model space = fp
! interaction = usdb

( Ai   Tzi)  ( Af   Tzf) (type n,l,2j)    Ji    Jf     ni    nf     C^2S        Ei         Ef
( 44   2.0)  ( 45   2.5)  ( n   1 3  7)  0.0+  3.5-     1     1   0.5000     0.000     0.000   72.0
( 44   2.0)  ( 45   2.5)  ( n   1 3  7)  2.0+  3.5-     1     1   0.7852     0.000     0.000   72.0
( 44   2.0)  ( 45   2.5)  ( n   1 3  7)  4.0+  3.5-     1     1   0.0141     0.000     0.000   72.0
                                                             sum   1.2993

                                                       total sum   1.2993
```

0.1.4 Question 4

What are the maximum J and T values allowed for 46V in the (0f1p) model space?

The fp model space contains orbitals $(0f_{7/2}, 1p_{3/2}, 0f_{5/2}, 1p_{1/2})$. 46V contains 23 protons and 23 neutrons, which means 3 protons and 3 neutrons are treated as valence nucleons with a 40Ca core.

The isospin projection for this nucleus is $T_z = 0$, which means the minimum T = 0 and the maximum T = 3/2+3/2 = 3 (adding proton and neutron). The maximum J value comes from the partition $(0f_{7/2})^2(1p_{3/2})^0(0f_{5/2})^1(1p_{1/2})^0 \to J = 17/2$ for both proton and neutron, which means the max J = 34/2 = 17.

0.1.5 Question 5

What are the spatial tensor ranks of the following operators (put "none" if it is not a tensor)?

a) The creation operator $a_k^\dagger$ for the proton orbital $\alpha = (n, \ell, j) = (0, 2, 5/2)$.

b) The destruction operator a_k for proton orbital $\alpha = (n, \ell, j) = (0, 2, 5/2)$.

c) The two-body isospin operator $\tau_{zi}\tau_{zj}$.

d) The δ function interaction between two nucleons.

e) The two-body Coulomb interaction.

f) The magnetic moment operator.

g) rank 1

h) none (can written as tensor by multiplying a phase proportional to $(-1)^{j_\alpha}$)

i) none (can be written as a sum of rank 0 and rank 2 tensors)

j) rank 0

k) rank 2

l) rank 1 (unless you include rank 2 correction that accounts wavefunction contributions outside sd shell)

0.1.6 Question 6

What are the isospin tensor ranks for the above (put "none" if it is not a tensor)?

a) rank 0, 1

b) none

c) none

d) rank 0

e) rank 0, 1, 2

f) rank 0, 1

[ ]: