

hw08

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0.1 This Jupyter notebook answers HW08 questions for PHY 981 Nuclear Structure.

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0.1.1 HW08 question 3

Calculate the magnetic moments for ^{17}O and ^{17}F . Compare to experiment.

Eq. 18.24

$$\frac{\mu}{\mu_N} = j \left[g_q^\ell \pm \frac{g_q^s - g_q^\ell}{2\ell + 1} \right]$$

In ^{17}O we look at the neutron, since the protons are in a closed shell. Looking at the $0d_{5/2}$ neutron orbit, we calculate μ/μ_N as

$$\frac{\mu}{\mu_N} = \frac{5}{2} \left[0 + \frac{-3.826 - 0}{2(2) + 1} \right] = -1.913$$

The experimental ^{17}O $\mu = -1.89379$.

In ^{17}F we look at the proton, since the neutrons are in a closed shell. Again, looking at the $0d_{5/2}$ proton orbit, the magnetic moment is

$$\frac{\mu}{\mu_N} = \frac{5}{2} \left[1 + \frac{5.586 - 1}{2(2) + 1} \right] = 4.793$$

The experimental ^{17}F $\mu = 4.7223$

0.1.2 HW08 question 4

Calculate the quadrupole moments for ^{17}O and ^{17}F . Use harmonic oscillator- radial wave functions with $\hbar\omega = 14$ MeV. Compare to experiment.

Eq. 18.31 (single-particle quadrupole moment)

$$\frac{Q(i)}{e} = - \left(\frac{2\ell}{2\ell + 3} \right) \langle n\ell j | r^2 | n\ell j \rangle e_q$$

The quadrupole moment for ^{17}O is zero, because the protons are in a closed shell and the neutron charge e_n is zero.

The quadrupole moment for ^{17}F (neutrons are in a closed shell; single proton particle state above the closed shell) is

$$\frac{Q(i)}{e} = - \left(\frac{2(2)}{2(2) + 3} \right) \langle 0d5/2 | r^2 | 0d5/2 \rangle e_p$$

The matrix element $\langle 0d5/2 | r^2 | 0d5/2 \rangle$ we compute using the nushell app **rme**, multiplied by $b^2 = 41.1/14 \text{ fm}^2$ with $\hbar\omega=14 \text{ MeV}$.

$$\frac{Q(i)}{e} = - \left(\frac{4}{7} \right) 3.5 * \frac{41.1}{14} (1) = -5.91 \text{ fm}^2$$

0.1.3 HW08 question 5

Calculate the quadrupole moments for ^{17}O and ^{17}F . Use Woods-Saxon radial matrix elements given in the output of **wspot**.

The quadrupole moment for ^{17}O is zero because the protons are in a closed shell and the neutron charge e_n is zero.

For ^{17}F , we have the same equation as the previous question, except the r^2 matrix element will be computed in a Woods-Saxon potential. From **wspot**, the RMS radius is 3.719, so the r^2 matrix element is 13.83.

$$\frac{Q(i)}{e} = - \left(\frac{4}{7} \right) 13.83(1) = -7.90 \text{ fm}^2$$

0.1.4 HW08 question 6

Calculate the quadrupole moments for ^{17}O and ^{17}F . Use radial matrix elements given in the output of **dens** with the Skyrme **skx** EDF potential.

Again, the ^{17}O quadrupole moment is zero.

For ^{17}F , we get the r^2 matrix element from the output of **dens**. For **ia,iz=17,9**, **dens** gives the RMS radius of 2.853 which means the r^2 matrix element computed in a Skyrme EDF potential is 8.140

$$\frac{Q(i)}{e} = - \left(\frac{4}{7} \right) 8.140(1) = -4.65 \text{ fm}^2$$

0.1.5 HW08 question 7

Obtain the values of c in Eq. 19.46 from the Fermi gas model, for symmetric nuclear matter, for neutron matter and for the symmetry energy.

The value of c in Eq 19.46 depends on asymmetry factor $F_{5/3}(I) = \frac{1}{2} [(1 + I)^{5/3} + (1 - I)^{5/3}]$ where $I = (N - Z)/A$.

For symmetric nuclear matter, $N=Z$ and $I=0$. Therefore,

$$F_{5/3}(I = 0) = \frac{1}{2} \left[(1)^{5/3} + (1)^{5/3} \right] = 0.5$$

And

$$c = \frac{3}{5} \frac{\hbar^2}{2m} \left(\frac{3\pi^2}{2} \right)^{2/3} \times \frac{1}{2}$$

For neutron matter, $Z=0$ and $I=1$. Therefore,

$$F_{5/3}(I = 1) = \frac{1}{2} \left[(2)^{5/3} + (0)^{5/3} \right] = 1.587$$

And

$$c = \frac{3}{5} \frac{\hbar^2}{2m} \left(\frac{3\pi^2}{2} \right)^{2/3} \times 1.587$$

The symmetry energy comes from the difference between the symmetric nuclear matter and neutron matter $[E/A]$, which is dependent on the factor $(F_{5/3}(I = 1) - F_{5/3}(I = 0)) = (1.587 - 0.5) = 1.087$. In this case,

$$c = \frac{3}{5} \frac{\hbar^2}{2m} \left(\frac{3\pi^2}{2} \right)^{2/3} \times 1.087$$

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