This Jupyter notebook answers HW06 questions for PHY 981 Nuclear Structure.

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HW06 question 3

132Sn is a good doubly-magic nucleus. What are the (n, ℓ, j) values associated with the ground state J^π of 131Sn, 133Sn, 131In and 133Sb. Use binding energies to find the single-particle (hole) energies associated with these states. What is the single-particle energy of the proton $0h_{11/2}$ state?

Single-particle energies can be computed empirically, according ot HF theory. HF theory stipulates the single-particle energy:

$$\epsilon(\alpha) = E(C,\alpha) - E(C) = E(C) - E(C,\alpha^{-1})$$
$$E = -BE + E_x$$

where E_x is the excitation energy associated with the single particle state α.

We will need the ground state binding energies of the isotopes of interest.

Isotope	N	Z	BE (MeV)	J^π	(n, ℓ, j)
132Sn	82	50	1102.843	0^+	(0,0,0)
131Sn	81	50	1095.490	$3/2^+$	(1,2,3/2)
133Sn	83	50	1105.242	$7/2^-$	(1,3,7/2)
131In	82	49	1087.033	$9/2^+$	(0,4,9/2)
133Sb	82	51	1112.509	$7/2^+$	(0,4,7/2)

Relative to 132Sn, the remaining isotopes can be used to find the single-particle energies of the HF potential, according to the equation above.

$$\begin{aligned}
E(132Sn) &= E(C) = 1102.843\,\text{MeV} \\
\epsilon(0p_{3/2};\text{neutron}) &= E(132Sn) - E(132Sn,(0p_{3/2})^{-1};neutron) \\
&= E(132Sn) - E(131Sn) = -17.353\,\text{MeV} \\
\epsilon(0f_{7/2};\text{neutron}) &= E(132Sn,(0f_{7/2});neutron) - E(132Sn) \\
&= E(133Sn) - E(132Sn) = -2.399\,\text{MeV} \\
\epsilon(0g_{9/2};\text{proton}) &= E(132Sn) - E(132Sn,(0g_{9/2})^{-1};proton) \\
&= E(132Sn) - E(131In) = -15.810\,\text{MeV} \\
\epsilon(0f_{7/2};\text{proton}) &= E(132Sn,(0f_{7/2});proton) - E(132Sn) \\
&= E(133Sb) - E(132Sn) = -9.666\,\text{MeV}
\end{aligned}$$

To find the single-particle energy of the proton in the $0h_{11/2}$ state, we will need the E_x

associated with the $11/2^-$ state in 133Sb. This is $E_{11/2^-}$ = 2.792 MeV. Then, we find that

$$E(132Sn, 0h_{11/2^-}; proton) = -BE(133Sb) + E_{11/2^-} = -1112.509 + 2.972 \text{ (MeV)}.$$

Finally,

$$\begin{aligned}\epsilon(0h_{11/2^-}; proton) &= E(132Sn, 0h_{11/2^-}; proton) - E(132Sn)\\ &= -BE(133Sb) + E_{11/2^-} - E(132Sn) = -6.694 \text{ MeV}\end{aligned}$$

HW06 question 4

Use the harmonic-oscillator model with $\hbar\omega$=14 MeV to find the rms proton radius for 20Ne. What is the total kinetic energy for 20Ne?

Ne $\implies$ Z=10. Which means that N_{max} = 2. However, only the N=0,1 levels are completely filled; N=2 is partially filled. There are only two states that will be filled in the N=2 major shell. Then, to find the R_p^2, we use the Eq. 16.11

$$\begin{aligned}R_p^2 &= \frac{D_0 < 0|r^2|0> + D_1 < 1|r^2|1> + 2 < 2|r^2|2>}{2+6+2} && (1)\\ &= \frac{2 \times 3/2b^2 + 6 \times 5/2b^2 + 2 \times 7/2b^2}{10} && (2)\end{aligned}$$

where b^2 is given by

$$b^2 = \frac{41.4 \text{ MeV fm}^2}{\hbar\omega}$$

and in our problem, $\hbar\omega = 14$ MeV.

Finally, our equation becomes

$$\begin{aligned}R_p^2 &= \frac{25}{10}\frac{41.4\text{MeV fm}^2}{14 \text{ MeV}} = 7.39 \text{ fm}^2 && (3)\\ R_p &= 2.72 \text{ fm} && (4)\end{aligned}$$

Lastly, according to virial theorem, the total kinetic energy should be

$$T = \frac{\hbar\omega}{2}(2 \times \frac{3}{2} + 6 \times \frac{5}{2} + 2\frac{7}{2}) = \frac{25}{2}\hbar\omega = 175 \text{ MeV}$$

HW06 question 5

Derive Eq. 16.45.

Eq. 16.45

$$\sum_i^A \vec{\rho}_i = 0$$

where $\vec{\rho}_i = \vec{r}_i - \vec{R}$ and $\vec{R} = 1/A\sum_i^A \vec{r}_i$

We begin by writing out the LHS of the sum in terms of $\vec{r}_i$ and $\vec{R}$.

$$\sum_i^A(\vec{r}_i - \vec{R}) = \sum_i^A\left(\vec{r}_i - \frac{1}{A}\sum_i^A \vec{r}_i\right) \tag{5}$$

$$= \sum_i^A \vec{r}_i - \frac{1}{A}\sum_i^A\sum_i^A \vec{r}_i = \sum_i^A \vec{r}_i - \frac{A}{A}\sum_i^A \vec{r}_i \tag{6}$$

$$= 0 \tag{7}$$

HW06 question 6

Derive Eq. 16.59.

Eq. 16.59

$$\sum_i^A U^{HO}(r_i) =$$

$$\frac{1}{2}m\omega^2\sum_i^A r_i^2 = \frac{1}{2}m\omega^2\sum_i^A \rho_i^2 + \frac{1}{2}\omega^2 MR^2$$

We want to show, in the bottom equation, that the LHS comes to the RHS when we switch to the relative coordinate system. We begin with the LHS writing r_i^2 in terms of the relative coordinate.

$$\frac{1}{2}m\omega^2\sum_i^A r_i^2 = \frac{1}{2}m\omega^2\sum_i^A(\vec{\rho}_i + \vec{R})^2 \tag{8}$$

$$= \frac{1}{2}m\omega^2\sum_i^A(\rho_i^2 + R^2 + 2\vec{\rho}_i\cdot\vec{R}) \tag{9}$$

$$= \frac{1}{2}m\omega^2\sum_i^A \rho_i^2 + \frac{1}{2}(mA)\omega^2R^2 + 2\frac{1}{2}m\omega^2\left(\sum_i^A \vec{\rho}_i\right)\cdot\vec{R} \tag{10}$$

The result of question 5, combined with the fact that $(mA) = M$ where M is the total nuclear mass (assuming proton and neutron mass is approximately the same), gives us the result we want

$$\frac{1}{2}m\omega^2\sum_i^A r_i^2 = \frac{1}{2}m\omega^2\sum_i^A \rho_i^2 + \frac{1}{2}M\omega^2R^2$$

HW06 question 7

Use the harmonic-oscillator model with $\hbar\omega$=14 MeV to find the center-of- mass correction to the energy of 20Ne.

We correct for the kinetic energy of the center-of-mass using Eq. 16.66

$$H + H'_{cm} - \frac{3}{2}\hbar\omega.$$

The expression for H'_{cm} comes from Eq. 16.65

$$H'_{cm} = \beta\left(\frac{Q^2}{2Am} + \frac{1}{2}Am\omega^2R^2 - \frac{3}{2}\hbar\omega\right)$$

HW06 question 8

Use wspot with the default Bohr-Mottelson set of Woods-Saxon param- eters. Assume that 16O is a doubly closed shell nucleus. What are the single-particle energies for the bound states? How do they compare with experiment? What is the rms charge radius? How does it compare to experiment?

Proton single-particle energies in 16O

(n,ℓ,2j)	s-p energy (MeV)	Experiment $\epsilon(\alpha)$ (MeV)	RMS
(0,0,1)	-25.586	-53	2.173
(0,1,3)	-13.612	-18.45	2.847
(0,1,1)	-7.437	-12.13	2.953
(0,2,5)	-1.995	-0.60	3.651

The experimental rms charge radius for 16O is 2.710(15) according to Kim et al(1978) (tabulated in Brown et al 1984 https://people.nscl.msu.edu/~brown/brown-all-papers/065-1984-jpg.10.1683-rms-charge-radii.pdf)

Neutron single-particle energies in 16O

(n,ℓ,2j)	s-p energy (MeV)	Experiment $\epsilon(\alpha)$ (MeV)
(0,0,1)	-30.381	-57
(0,1,3)	-17.833	-21.84
(0,1,1)	-11.611	-15.66
(1,0,1)	-2.950	-3.27
(0,2,5)	-5.567	-4.14

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