A Tailor for Giants
VAR Models and Macroeconomic Indicators

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April 23, 2017

Abstract

How do the giants of macroeconomic indicators interact with each other? How can they be predicted? This paper applies the methods used in Stock and Watson (2001) and extends the method by using updated data, which includes interest rates, inflation, and the unemployment rate in the US economy from 1950s until now, to determine how these macroeconomic indicators interact with each other, if this concurrent with the surrounding literature, and how to predict these economic indicators in the future. First, we determine that a VAR(2) model best describes the data. We then perform Granger causality tests to determine if one time series is useful in predicting another. We plug in historical values to do one-step forecast and perform impulse response. After running these tests, we conclude that: (1) the federal funds rate can be used to predict both inflation and unemployment rate, which is consistent with the current literature; (2) inflation can be used to predict federal funds rate, but not the unemployment rate; (3) unemployment cannot be used to predict neither federal funds rate nor inflation, which is consistent with the LR Model, but inconsistent with Stock and Watson's conclusion; (4) VAR (2) Model can successfully forecast economic indicators for at least 15 months; (5) after conducting impulse response on the VAR (2) Model, it is found to portray a more complicated, dynamic economic system.

1 Introduction

Macroeconomics deals with the structure, behavior, and decision-making of the economy. In order to study the interrelations among economic indicators, such as GDP, unemployment rate, inflation rate, etc., economists develop models that explain the relationship between these indicators, such as Fisher effect which describes the relationship between inflation and both real and nominal interest rates, and Phillips Curve which indicates the inverse relationship between unemployment rates and inflation.

The federal funds rate is the interest rate at which depository institutions, such as banks and credit unions, lend reserve balance to other depository institutions overnight. Raising the federal funds rate will keep depository institutions from borrowing money, which makes cash much harder to procure. Conversely, decreasing the federal funds rate will encourage banks to borrow money and therefore it will be easier to make investments. According to the relationship, federal funds rate is used as a regulatory tool to control how freely the national economy operates. The unemployment rate is calculated by diving the number of job seeking individuals by the total number of employed individuals and job seeking individuals. During periods of recession, an economy usually experiences a higher unemployment rate. The inflation is the increase in the price level of goods and services in an economy over a period of time. When the price level rises, each unit of currency buys fewer goods and services; therefore, inflation reflects a reduction in the purchasing power per unit of money. Although a high inflation rate will reduce the public's ability to make long-term decisions, a proper inflation rate will lead to the central banks adjusting interest rates in order to stabilize the economy and reduce the unemployment rate. Therefore, many central banks have an inflation target. The Federal Reserve aims for two percent inflation over time. Deflation is harmful, because it may increase the real value of debt, and may also aggravate recessions and lead to a deflationary spiral, which is happening in Japan's economy.

Because of the significance of economic indicators, it is vital to find out the interactions among them and furthermore forecast these indicators. Four decades ago, Laffer and Ranson (1971) created a model which is used to develop statistical relationships among different economic variables. By using the formal model they have created, Laffer and Ranson concluded that there is not a significant partial relationship between inflation and unemployment rate and therefore the existence of a Phillips Curve cannot be confirmed. Expanding on this idea, MacRae (1972) claimed that there is an implicit relationship between these two parameters, because the change in money supply influences both variables. After examining the relation in both the short run and the long run, MacRae concluded that there is a tradeoff between unemployment rate and inflation associated with monetary policy only exists in the short run and not in the long run, which agrees with the LR model.

In investigating the relationship between unemployment rate and inflation, Stock and Watson (2001) built a VAR model using data from 1960 to 2000, and concluded that unemployment rate causes inflation. The authors used F-statistics for Granger causality testing and got a p-value of 0.02, which shows that it is statistically significant to use unemployment to predict inflation. This is consistent with economic theory, as a higher unemployment rate will urge the Federal Reserve to boost the economy with a lower interest rate; however, it is contradictory to the Laffer-Ranson model. We will perform the Granger causality test using more recent data to see whether the conclusion is still true. The paper of Stock and Watson (2001) also states that inflation forecasts through the Phillips Curve are more accurate than interest rates. They also cite a Ball (1999) paper that countries that did not undergo expansionary monetary policy in the early 1980s resulted in interest rates that raised the natural rate of unemployment. Therefore, we would anticipate interest rates to cause unemployment. We will test this causal relationship by testing for Granger Causality for interest rates and unemployment rates in our VAR model.

Instead of the relationship between unemployment rate and inflation, many economists have studied the relationship between interest rate and inflation. Lee (2009) employed Johansen's method to investigate for the existence of a long-run Fisher effect over the period of 1976 to 2006. He found interest rate and inflation do have a positive relationship, while he rejected the notion of a full one-to-one Fisher Effect. Alveraz et al. (2001) and Mankiw (2001) also discussed the short-run tradeoff between inflation and unemployment, and they concurred with that a positive relationship exists. In general, as the Federal Reserve employs contractionary monetary policies, such as raising interest rate, it reduces the supply of money, and thus make money scarcer, which will cause deflation.

Given the economic literature surrounding these indicators, the paper determines how interest rates, inflation, and the unemployment rate interact with each other, if this concurrent with the surrounding literature, and how to predict these economic indicators in the future. This paper applies the statistical analysis methods used in Stock and Watson (2001), but it brings recent data to the forefront. This paper can thus be viewed as an update of Stock and Watson (2001) in the era of low federal funds rates.

2 Data

All of our data for this paper was sourced from the Federal Reserve Economic Database (FRED) of St. Louis. From this database we retrieved the Consumer Price Index (CPI), the Federal Funds Reserve Rate (FFR), and the Unemployment Rate (UNEM) for the United States. We were able to retrieve monthly data for these three macroeconomic indicators from July, 1954 until March, 2017. For the purposes of this paper, we removed the last 15 months of data in order to have a substantial out of sample data set to test our forecasts. 15 was chosen as we removed all the months available for 2016 and 2017 and two months due to lagging our data by two months in order to test breakpoints.

Due to the theoretical lack of stationarity of CPI, we will be converting CPI into inflation rate. Also, we are using inflation rate because Stock and Watson (2001) also used it in their paper. It would thus be beneficial to use the same metric in order to increase the comparability of our papers. Inflation rate is a measure of the annual increase of CPI at a moment in time. It is calculated by 12*100*ln(CPI_t/CPI_{t-1}). We scaled the natural log of change of CPI by 12 to make the metric annual and by 100 to make it a percentage.

3 Methods

The first step in our methodology is to determine a reasonable breakpoint for beginning our statistical analysis. Our full data set has over 60 years of monthly data for these macroeconomic indicators. It would be irresponsible to assume that the relationships between these macroeconomic indicators has not changed in over 60 years. Stock and Watson (2001) used 40 years of quarterly data (1960-2000), but he did not give reasoning as to why he chose that range of years. We will use the Chow Test, the dynamic programming algorithm for Simultaneous Estimation of Breakpoints developed by Bai and Perron (2003), and Recursive Cumulative Sum to determine a reasonable breakpoint to begin our data analysis. We will then use these three methods of determining breakpoints for each of our indicators at first assuming a

VAR(1) structure and then assuming a VAR(2) structure.

After determining a reasonable starting point to subset our data, we will then determine the optimal order for a VAR model depending on which order minimizes AIC and BIC. After determining the order for our model, we will then perform Granger causality tests with the order specified by our model on all the permutations of our variables (i.e., FFR on UNEM, FFR on INFL, etc.). We will then compare the results of these Granger causality tests with our aforementioned hypotheses regarding how these macroeconomic indicators should predict each other. Our Granger causal relations will be tested with these hypotheses.

Hypothesis 1: Because we expect inflation rate to predict federal funds rate, a Granger test of inflation rate on federal funds rate should be statistically significant.

Hypothesis 2: Because we do not expect inflation rate to predict unemployment rate, a Granger test of inflation rate on unemployment should be statistically insignificant.

Hypothesis 3: Because we do expect federal funds rate to predict the unemployment rate, a Granger test of federal funds rate on unemployment rate should be statistically significant.

Hypothesis 4: Because we do expect federal funds rate to predict the inflation rate, a Granger test of federal funds rate on inflation rate should be statistically significant.

Hypothesis 5a: Because we do expect unemployment rate to predict the inflation rate, a Granger test of unemployment rate on inflation rate should be statistically significant. This hypothesis would support Stock and Watson (2001).

Hypothesis 5b: Because we do not expect unemployment rate to predict the inflation rate, a Granger test of unemployment rate on inflation rate should be statistically insignificant. This hypothesis would support Laffer and Ranson (1971).

Hypothesis 6: Because we do expect unemployment rate to predict the federal funds rate, a Granger test of unemployment rate on federal funds rate should be statistically significant.

We will then generate forecasted out of sample predictions and test to see whether or not they were accurate.

Hypothesis 7: The VAR model's 95% prediction confidence will be able to accurately contain 95% of our observed out of sample data.

Finally, to see the interconnectedness of our indicators, we will be performing impulse response tests on all our variables and evaluating if a shock in one variable causes other variables to change.

Hypothesis 8: Our impulse response tests will show evidence of relationship dynamics, similar to those expected in our Granger test hypotheses.

4 Results

4.1 Finding a Breakpoint

All the tests for finding a breakpoint require a linear model to test for a structural break. The problem in this paper is that in order to develop our model we must determine where to subset our data via a breakpoint. In order to circumvent this problem, for all the structural break tests that we perform we will test with both a VAR(1) and a VAR(2) model. Fortunately, upon writing this paper we have the foresight that the resulting model will be a VAR(2), but in the original process of the paper we did not know this. During the actual process of the paper, we only completed structural break tests for the VAR(1) model and then checked to see if the structural break identified was consistent for VAR(2) (it was).

Chow Test:

A Chow Test was performed on every month in the data set for all the underlying univariate linear models for the VAR(1) model and then the VAR(2) model. A Chow Test is an F-Test to see if the estimates for the models before an after a specified breakpoint are the statistically different.

These underlying models are as follows:

VAR(1): $FFR_t = \theta_{1,1}{*}FFR_{t-1} + \theta_{1,2}{*}INFL_{t-1} + \theta_{1,3}{*}UNEM_{t-1}$

$IR_t = \theta_{2,1}{*}FFR_{t-1} + \theta_{2,2}{*}INFL_{t-1} + \theta_{2,3}{*}UNEM_{t-1}$

$UNEM_t = \theta_{3,1}{*}FFR_{t-1} + \theta_{3,2}{*}INFL_{t-1} + \theta_{3,3}{*}UNEM_{t-1}$

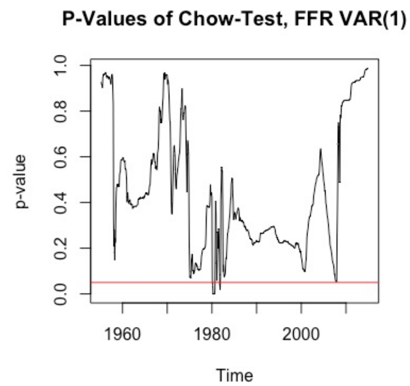
VAR(2): $FFR_t = \theta_{1,1,1}{*}FFR_{t-1} + \theta_{1,2,1}{*}INFL_{t-1} + \theta_{1,3,1}{*}UNEM_{t-1} +$ $\theta_{1,1,2}{*}FFR_{t-2} + \theta_{1,2,2}{*}INFL_{t-2} + \theta_{1,3,2}{*}UNEM_{t-2}$

$IR_t = \theta_{1,1,1}{*}FFR_{t-1} + \theta_{1,2,1}{*}INFL_{t-1} + \theta_{1,3,1}{*}UNEM_{t-1} + \theta_{1,1,2}{*}FFR_{t-2}$ $+ \theta_{1,2,2}{*}INFL_{t-2} + \theta_{1,3,2}{*}UNEM_{t-2}$

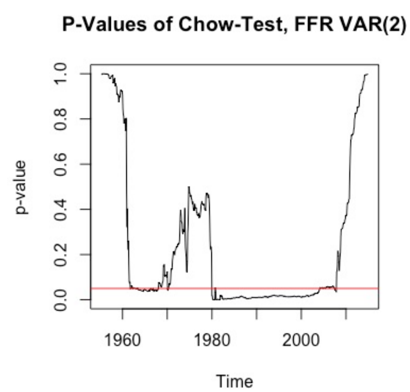
$UNEM_t = \theta_{1,1,1}{*}FFR_{t-1} + \theta_{1,2,1}{*}INFL_{t-1} + \theta_{1,3,1}{*}UNEM_{t-1} + \theta_{1,1,2}{*}FFR_{t-2}$ $+ \theta_{1,2,2}{*}INFL_{t-2} + \theta_{1,3,2}{*}UNEM_{t-2}$

As seen in Figure 1, the Chow Test for federal funds rate for the VAR(1) model suggests that there are structural breaks between August, 1979 to February, 1980 and March, 1981 to April, 1981. The Chow Test for federal funds rate for the VAR(2) model suggests that there are two large steaks of possible structural breaks between March, 1961 to September, 1961 and June, 1979 to July, 2003. Both of these results suggest that there should be some type of structural break for the Federal Reserve during the 80's. The likely cause of this structural break is the change in chairman of the federal reserve to either Paul Volcker (1979-1987) or the illustrious Alan Greenspan (1987-2006).

Figure 1: P-values for Chow Test for Federal Funds Rate



(a) VAR(1)



(b) VAR(2)

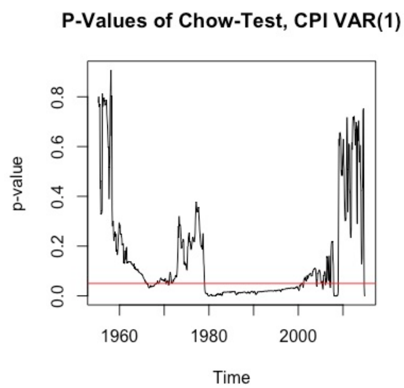
Similarly, the Chow Test for inflation rate for the VAR(1) model suggests that there are structural breaks between July, 1978 to August, 2003 in Figure 2. The VAR(2) model for inflation rate suggests a much larger range of possible structural breaks between January, 1958 to March, 2013. The VAR(1) result seems consistent with what was thought previously, that there should be a structural break somewhere during the 1980's. Although the VAR(2) result for inflation rate supports this, it doesn't necessarily specifically support it with a large date range.

Finally, the Chow Test for unemployment rate for the VAR(1) model as seen in Figure 3 also gave a large range of possible breakpoints including the range between March, 1974 to March, 2008. Similar to the VAR(1) results, the VAR(2) results for the Chow Test gives a large range of possible breakpoints between March, 1980 to March, 2008. Again, the 1980's as suggested by the Chow Test results for the federal funds rate are included in these range of data, but then again there isn't much more specification due to the large range of dates.

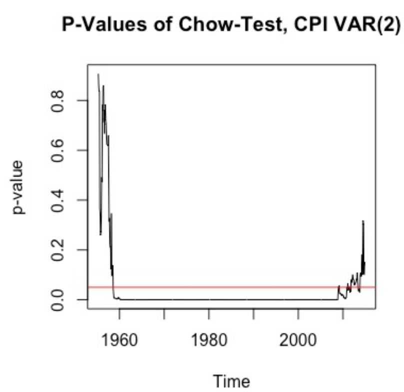
Overall, the Chow Test for this analysis is helpful in determining a large range of plausible breakpoints; however, because the range is so large there is not a definitive choice as to when specifically we should to subset our data. The results of both the VAR(1) and VAR(2) Chow Test for federal funds rate were more specific and suggest a breakpoint during the 1980's. This will be noted when comparing other methods of determining a breakpoint.

Simultaneous Estimation of Multiple Breakpoints:

Figure 2: P-values for Chow Test for Inflation Rate

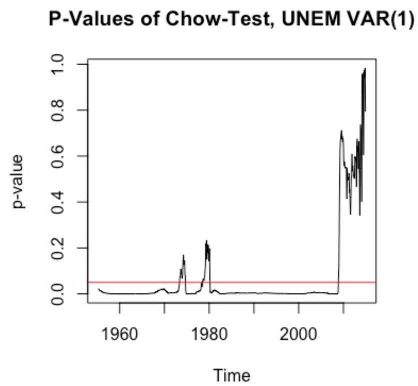


(a) VAR(1)

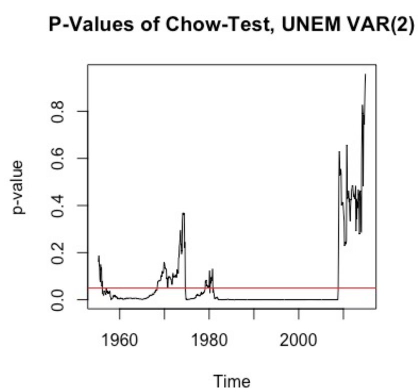


(b) VAR(2)

Figure 3: P-values for Chow Test for Unemployment Rate



(a) VAR(1)



(b) VAR(2)

In order to get a definitive answer with regards for a breakpoint in our data set, we applied the algorithm proposed by Bai and Perron (2003), Simultaneous estimation of multiple breakpoints. Essentially, the algorithm computes the residual sum of squares for all the data given a certain amount of breakpoint divisions and uses the Bellman Principle to eliminate breakpoints that could not result in the minimal residual sum of squares (Bai and Perron 2003). Eventually, through the process of elimination an optimal breakpoint can be found. Simultaneous estimation of multiple breakpoints solves the aforementioned problem of the Chow Test as it gives a direct answer as to what the breakpoint should be. Again, similar to the Chow Test, we will be conducting Simultaneous estimation of multiple breakpoints for both the VAR(1) and the VAR(2) model for each of our three variables.

Using Simultaneous estimation of multiple breakpoints, breakpoints were identified at February, 1971 and June, 1980 for the VAR(1) federal funds rate model and no breakpoints were identified for the VAR(2) federal funds rate model. As for inflation rate, no breakpoints were found for the VAR(1) model, but the VAR(2) model had a breakpoint at May, 1981. Finally, for unemployment rate both the VAR(1) model and the VAR(2) had no breakpoints according to simultaneous estimation of multiple breakpoints.

Our analysis from Simultaneous estimate of multiple breakpoints supports our analysis from our Chow tests results that there's a point in 1980 in which to subset our data. The latest point possible according to our Simultaneous estimation of breakpoints is May, 1981. This is appealing as it is right after the first 100 days of Ronald Reagan as president and 2 years into Paul Volcker's term as chairman of the federal reserve.

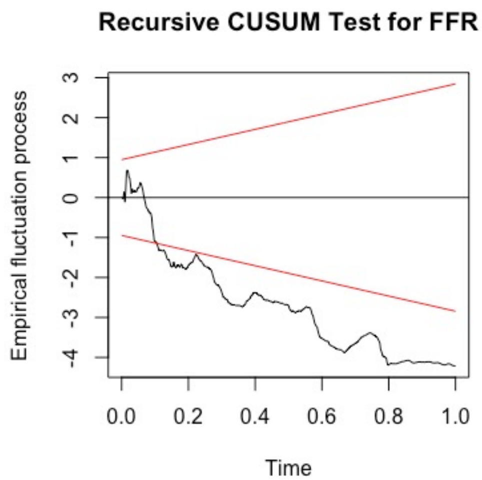
Recursive Cumulative Sum:

The last breakpoint analysis we performed involved Recursive Cumulative Sum. Recursive Cumulative Sum involves summing all of the one-step prediction errors for a model. If the cumulative sum of the predictive residuals passes a certain 95% confident threshold, then there is evidence of a structural break.

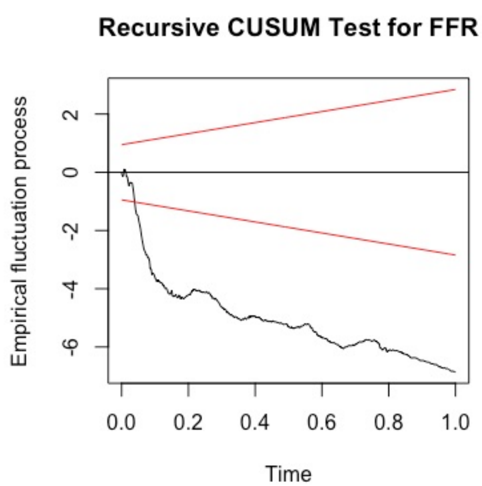
Performing a recursive cumulative sum on the data subset at May, 1981 has a statistically significant result for the VAR(1) and VAR(2) models for federal funds rate (Figure 4) and the VAR(1) and VAR(2) models for unemployment rate (Figure 5). The black line is a progression of the cumulative sum of prediction residuals, while the red line represents the threshold on which to claim that there is a structural break. This suggests that May, 1981 would have a structural break for both unemployment and federal funds rate and would not be a good choice for a breakpoint for our data.

January, 1984 on the other hand is fairly close to May, 1981 (two and a half years) and has the best evidence for being an appropriate break point. First, January, 1984 satisfies all of the Chow Test requirements for a structural break for a VAR(2) model. Secondly, this breakpoint is fairly close to the breakpoint indicated by the simultaneous estimation of multiple breakpoints for inflation rate and as stated previously federal funds rate and unemployment rate had no breakpoints. Finally, January, 1984 passes the recursive cu-

Figure 4: Recursive Cumulative Sum Test for Federal Funds Rate at May, 1981

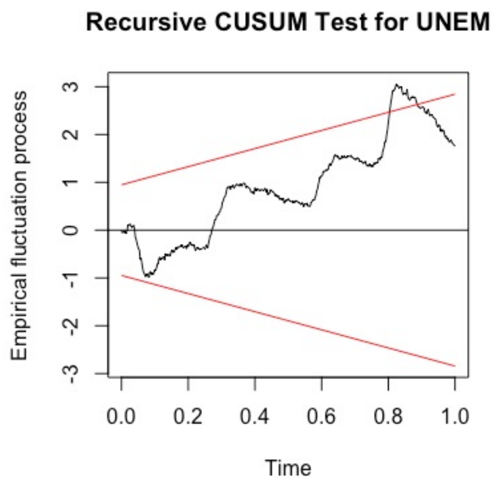


(a) VAR(1)

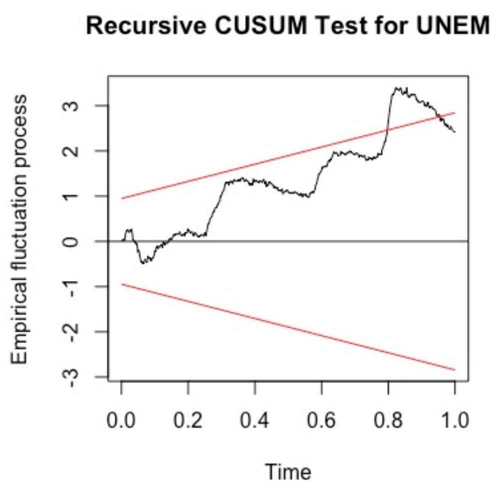


(b) VAR(2)

Figure 5: Recursive Cumulative Sum Test for Unemployment Rate at May, 1981



(a) VAR(1)



(b) VAR(2)

mulative sum test for inflation rate and unemployment rate for both VAR(1) and VAR(2) (Figures 6 and 7) and is the closest to passing federal funds rate for both VAR(1) and VAR(2) (Figure 8). This date is appealing as it is around the middle of the Reagan administration and three years before Alan Greenspan's term as chairman of the federal reserve.

4.2 Model Selection

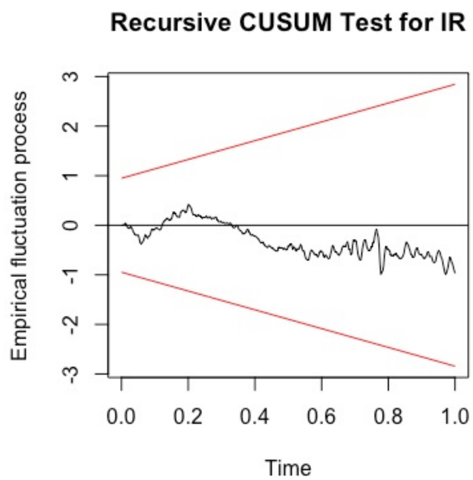
We selected our model by observing the AIC and BIC values. The AIC was lowest at VAR(7) with a value of 0.0000, whereas the BIC value for VAR(2) produces the lowest value of 522.9426. The BIC produces a better model than AIC because it is more aggressive at penalizing unnecessary lags. In addition, the AIC values decrease at a smaller rate when we add more lags after VAR(2). In the end, we selected the VAR(2) model for the best fit. These results contrast with the VAR(1) model used by Stock and Watson (2001) who did not take into consideration AIC and BIC. We thus view our paper as an improvement on the Stock and Watson (2001) due to our analysis of model selection.

Here are the formulas of the result of our VAR(2) model.

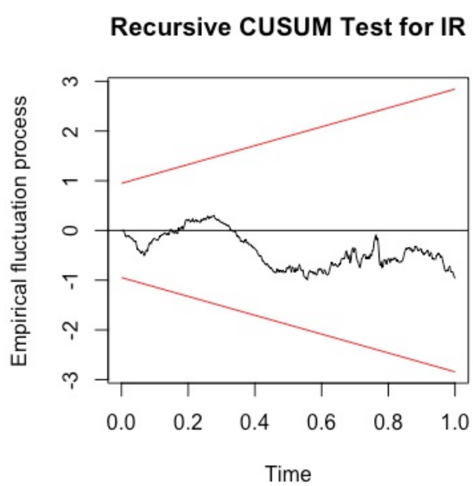
4.3 Granger Causality Test Results

In order to interpret our forecast, we ran the Granger causality test for the six possible combinations of two indicators using a VAR(2) order. The p-value

Figure 6: Recursive Cumulative Sum Test for Inflation Rate at January, 1984

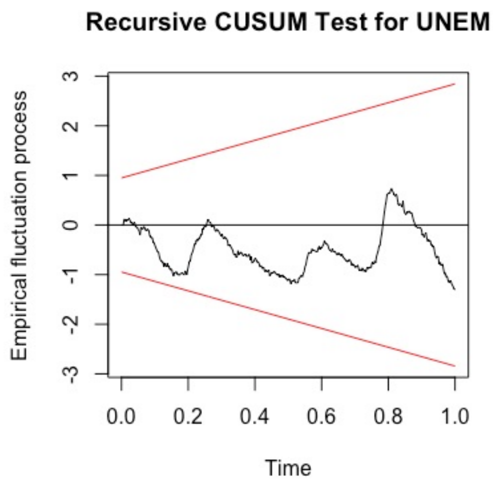


(a) VAR(1)

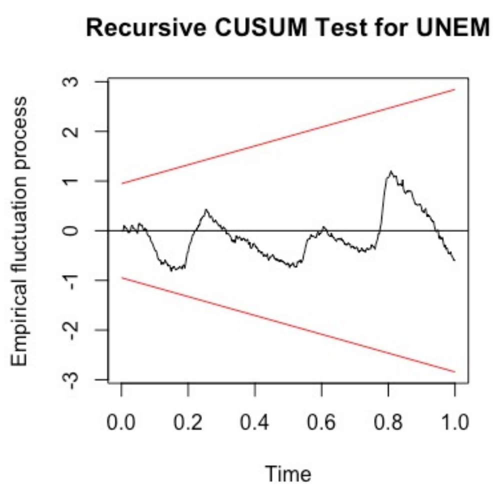


(b) VAR(2)

Figure 7: Recursive Cumulative Sum Test for Unemployment Rate at January, 1984

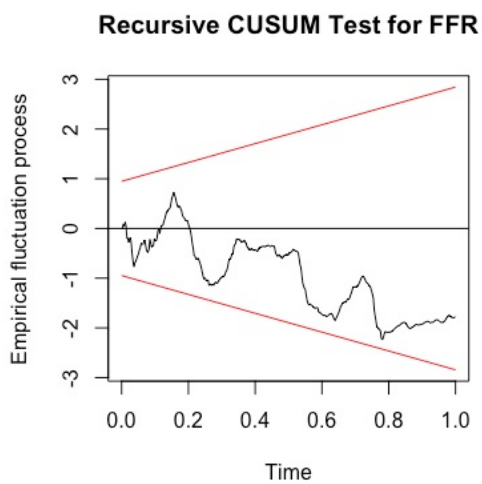


(a) VAR(1)

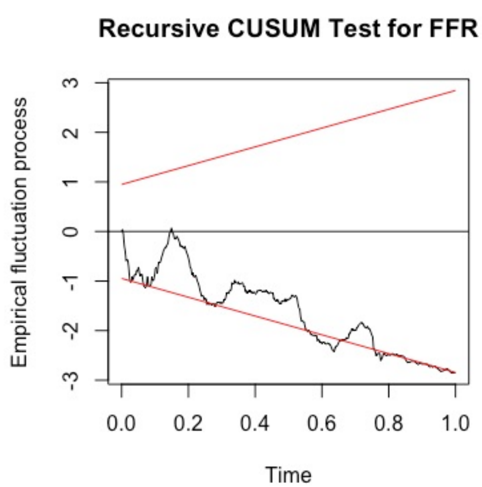


(b) VAR(2)

Figure 8: Recursive Cumulative Sum Test for Federal Funds Rate at January, 1984



(a) VAR(1)



(b) VAR(2)

Table 1: Model Selection

VAR order	AIC (Δ AIC(7))	BIC
1	297.691454	570.8351
2	82.49051	522.9426
3	71.325834	552.468
4	45.63097	560.5105
5	59.777101	607.5544
6	6.926987	563.3937
7	0	570.8351

Table 2: VAR(2) Model, First Lag Matrix

	FFR	UNEM	INFL
FFR	1.458	-0.126	0.0292
UNEM	-.1429	1.031	.00004
INFL	-0.0883	-0.4321	1.4166

Table 3: VAR(2) Model, Second Lag Matrix

	FFR	UNEM	INFL
FFR	-0.4665	0.1206	-0.0219
UNEM	.1429	-.0396	-.00386
INFL	0.1509	0.4438	-0.651

results of the Granger causality test tend to confirm the hypotheses we made before running our data analysis. We hypothesized that the federal funds rate was shown to predict both unemployment and inflation, which we found to be statistically significant. Unemployment was found to not be a useful predictor for the federal funds rate or inflation. However, it was not surprising that unemployment was not a good predictor of inflation since the McRae 1972 paper found no relationship between unemployment and inflation (we based this hypothesis on testing the Stock 2001 paper which argued in favor for a relationship of unemployment and inflation, despite being contrary to McRae). Finally, inflation was found to be a good predictor for the federal funds rate while not having a relationship with unemployment, which is consistent with our hypotheses.

Table 4: Granger Causality Results

Granger	p-value
FFR -> UNEM	.000245
FFR -> INF	.000687
UNEM -> FFR	.139
UNEM -> INF	.2318
INF -> FFR	.004828
INF -> UNEM	.2579

4.4 Forecasting

Based on our VAR model, we go one step further and forecast future values of these macroeconomics indicators. A good forecast is not only useful in application, but also shows our model has successfully captured the dynamic of the economic system and is not susceptible to overfitting.

Our forecast is divided into two parts. In the first part, we utilize data up to October, 2015, forecast the next 15 months, and compare the forecast to newly observed data values. This kind of forecast is important in real world application as it gives the forecast of macroeconomics indicator values in future periods, and helps to predict economic trends in the future. However, at the same time, we notice that the forecast in the long term is usually inaccurate and has a large confidence interval, and short-term forecast is more accurate and has more potential decision making application. Therefore, it's particularly necessary to test the quality of short-term forecasts. Therefore in the second part, we focus more on examining and testing the one-step forecast based on our model. To be specific, we recursively add new data to our data set, conduct one-step forecasts and specify its 95% prediction limit, and in this way generate a series of prediction and prediction limits. Then we compare them to the observed data. We will perform a z-test on our prediction success rate to test our forecasting quality.

Long Term Forecasting

In this part, we fit a VAR(2) model to data from January, 1984 to October, 2015, and generate the forecasted value and the 95% prediction limits from November, 2015 to January, 2017 based on our model. The forecasted value and 95% prediction limits are generated from the R function by a similar methodology as we've discussed for ARMA model, and the results are shown in the following graphs. In the graphs, FFR refers to federal fund rates, UNEM refers to unemployment rate, and INFL refers to inflation rate. The black line shows the observed data value, the solid red line shows the predicted value, and the two dotted red lines show our 95% prediction limit.

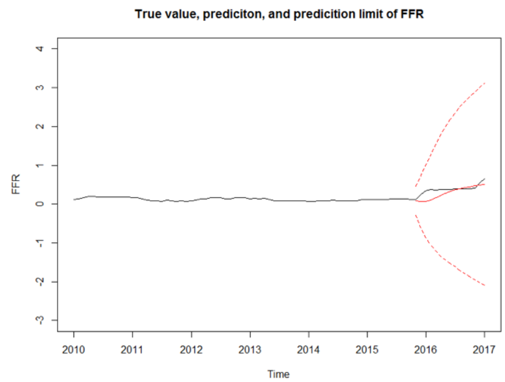
From Figure 9, we can see that our forecast is consistent with the trend of observed data, and all the observed data falls into our 95% prediction limits. However, as mentioned above, the 95% confidence interval gets really large when we try to do long-term forecasts due to the nature of the VAR model. So we will focus more on the quality of short-term forecasts in the next section.

Recursive One Step Forecasting

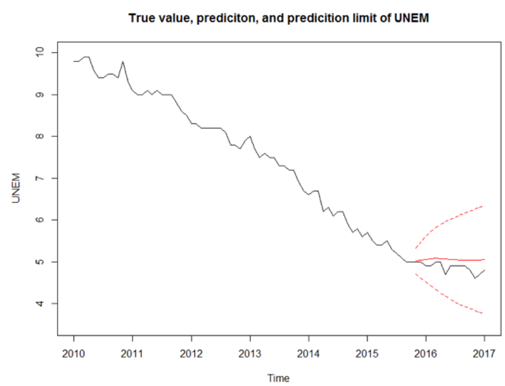
This section aims at examining and testing the quality of one-step forecast based on our model. In this part, we initially fit a VAR(2) model to data from January, 1984 to December 2010, and get the predicted values and 95% prediction limits of January, 2011. Then we add observed (true) values from January, 2011 to our data set, reevaluate a VAR(2) model and get the prediction for the next month. In this way, we recursively add new data to our data set, get our new forecasts, until we get the forecast for January, 2017. We've finally got a series of one-step predicted values and 95% prediction limits of the 3 variables, which can be utilized to examine the one-step forecast quality. First, the graphs of these predictions are shown in Figure 10.

Because the graph above is not easy to read, we also separately plot the

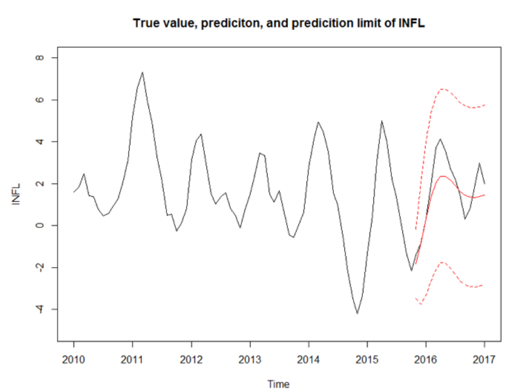
Figure 9: Long Term Forecasting



(a) Federal Funds Rate

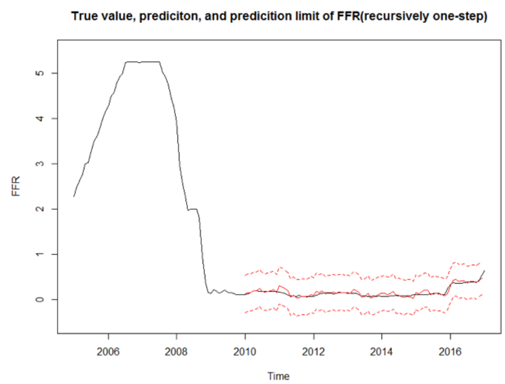


(b) Unemployment Rate

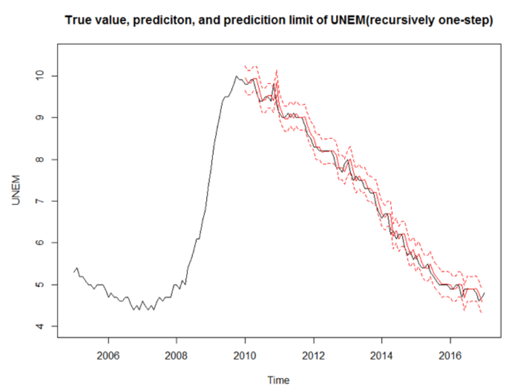


(c) Inflation Rate

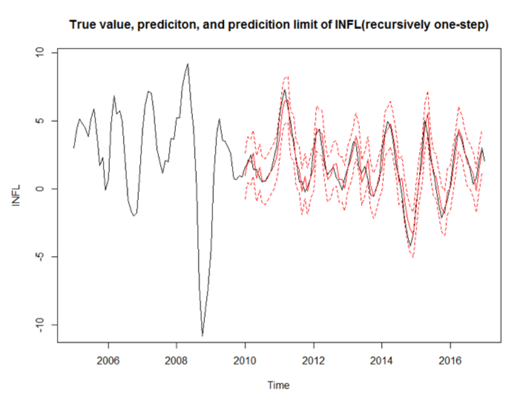
Figure 10: Recursive One Step Forecasts



(a) Federal Funds Rate

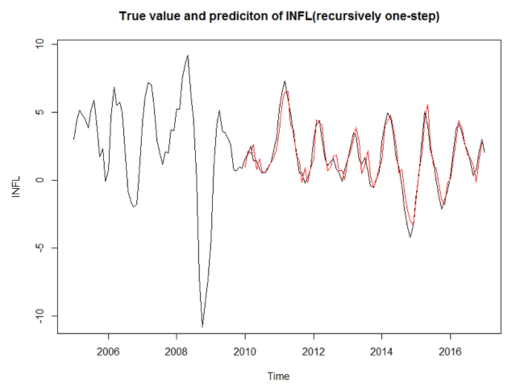


(b) Unemployment Rate

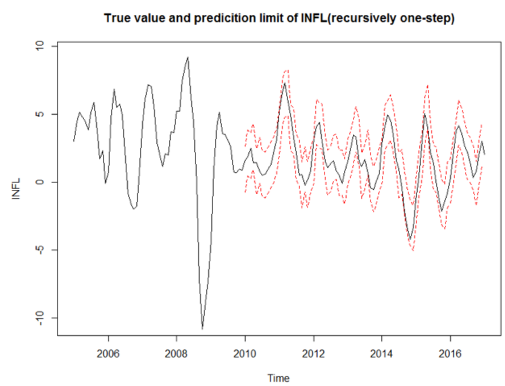


(c) Inflation Rate

Figure 11: Recursive One Step Forecast for Inflation Rates



(a) Predicted Value



(b) 95% Confidence Interval

predicted value and the 95% prediction limits in Figure 11.

As shown in the graphs, our forecasts correctly predicts the trend of the 3 variables. They are accurate as well, and nearly all the observed data points fall in to the 95% prediction limits.

To further quantify our forecasting quality, we calculate the forecasting errors as observed value minus forecasting value. Also, we plot our forecasting errors in Figure 12 and calculate the mean and standard deviation in Table 5.

Table 5: One Step Forecasting Errors

	Federal Funds Rate	Unemployment	Inflation
Forecasting Errors Mean	-0.0168	-0.0668	-0.0461
Forecasting Errors Standard Deviation	0.0473	0.1527	0.7399

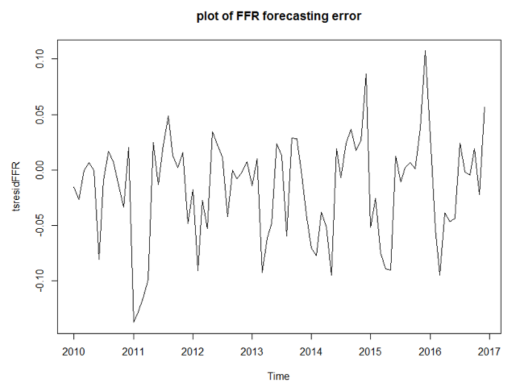
As for forecasting errors, the graph shows no general pattern, the nearly zero error means indicate unbiasedness, and the forecasting standard deviations are truly low compared to how these data range over time (especially for unemployment rate). These all show good forecasting quality.

Furthermore, we do a z-test to test whether our 95% prediction limits are truly accurate. To be specific, we calculate the proportion of observations that fall in our 95% prediction limits for federal fund rates, unemployment and inflation rate, and do a z-test to test whether this proportion equals to 95% (Table 6).

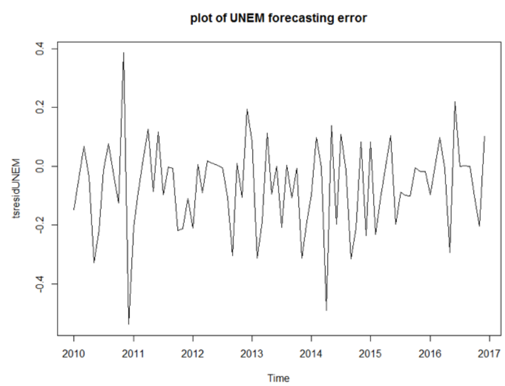
Table 6: Z-Test on Success Rate

	Federal Funds Rate	Unemployment	Inflation
Sample Size	84	84	84
Theoretical proportion	0.95	0.95	0.95
Standard error under H0	0.02378	0.02378	0.02378
Observations in 95% prediction limit	84	76	84
Observed proportion	1	0.9047	1
Z-score	2.10	-1.90	2.10
p-value	0.0352	0.0574	0.0352

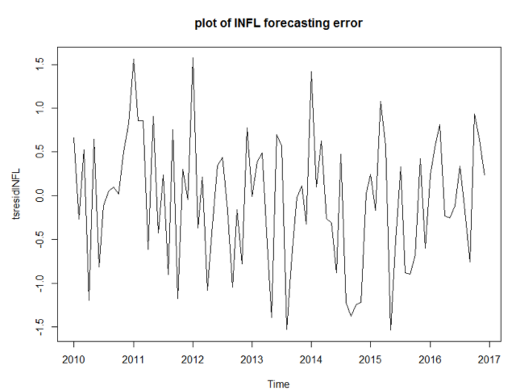
Figure 12: One Step Forecasting Errors



(a) Federal Funds Rate



(b) Unemployment Rate



(c) Inflation Rate

The result shows that though the forecast of unemployment rate is comparatively inaccurate, it's still consistent with the theoretical result of 95% as p-value equals 0.0574. For federal fund rate and inflation rate, the p-values are small due to all the observed data points falling into the 95% prediction limits, implying that our forecasting is even better than the theoretical forecasting quality.

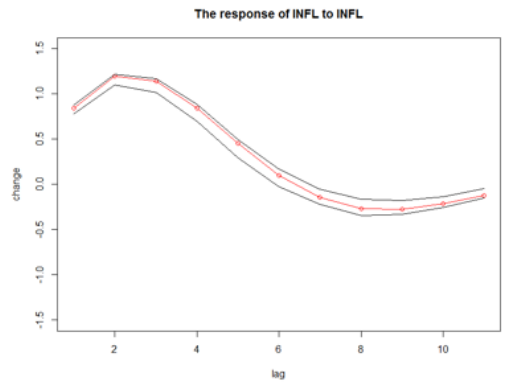
4.5 Impulse Response

Impulse responses trace out the response of current and future values of each of the variables to a one-unit increase in the current value of one of the VAR errors, called a shock. This process assumes that this error returns to zero in subsequent periods and that all other errors are equal to zero. This can be regarded as a thought experiment showing how the whole economic system will react to a potential shock in one macroeconomic indicator. The method of impulse response is validly used in macro-econometrics when a VAR model is built. We also use this method to examine how the variables interact and how the system will react to a potential shock.

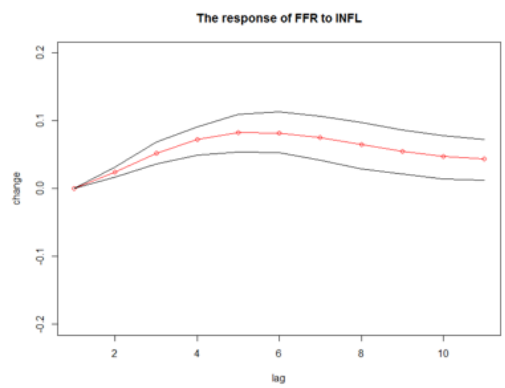
The black lines in Figures 13, 14, and 15 show the effect of an expected 1 unit increase on all three variables in the following 12 months. Also plotted are ± 1 standard error bands, which are shown in red lines and yield an approximate 66 percent confidence interval for each of the impulse responses.

As is shown in the graphs, all the 3 variables will respond to a shock of itself. The response of inflation on itself (Figure 13a) is significant but fades

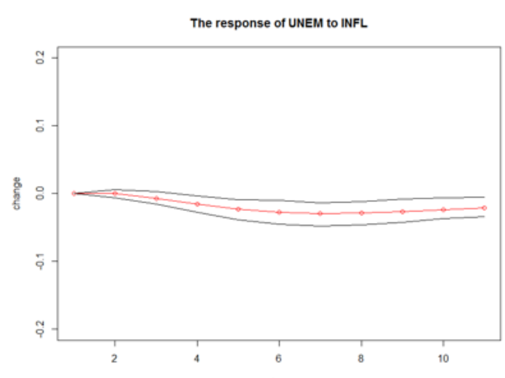
Figure 13: Impulse Response from Shock in Inflation Rate



(a) Inflation Rate

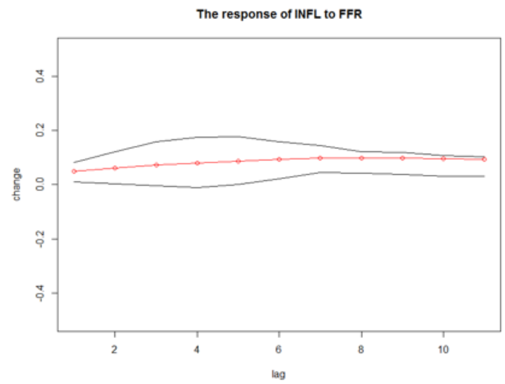


(b) Federal Funds Rate

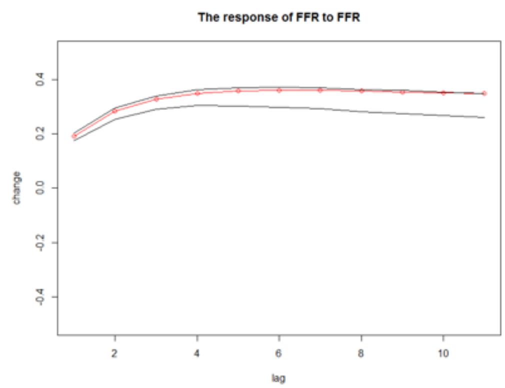


(c) Unemployment

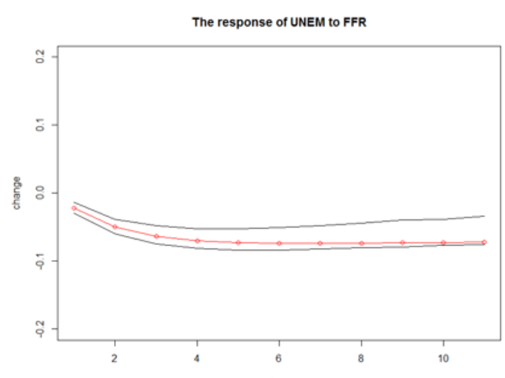
Figure 14: Impulse Response from Shock in Federal Funds Rate



(a) Inflation Rate

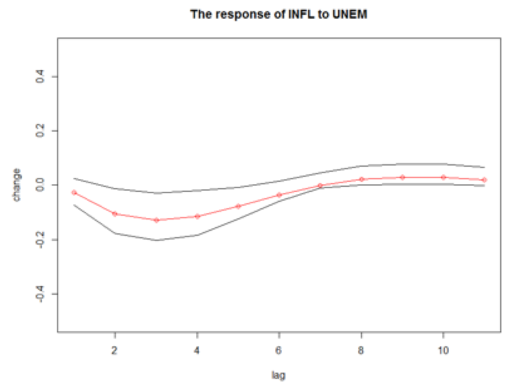


(b) Federal Funds Rate

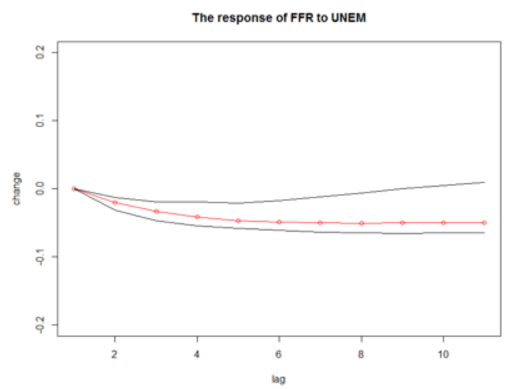


(c) Unemployment

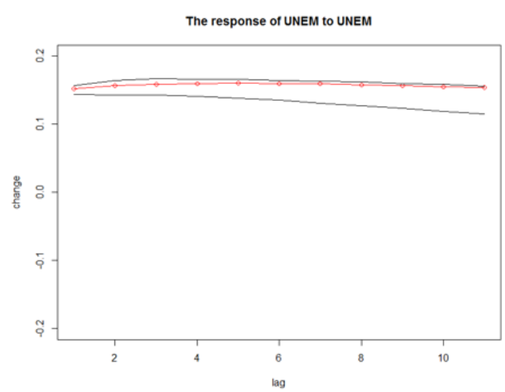
Figure 15: Impulse Response from Shock in Unemployment Rate



(a) Inflation Rate



(b) Federal Funds Rate



(c) Unemployment

away after several months, while the effect of a shock in federal fund rates on itself and a shock in unemployment on itself (Figure 14b, 15c) are relatively small but long-lasting. Also, the graphs show that federal funds rate will react positively to an increase in inflation rate (Figure 13b), but negatively to a rise in unemployment rate (Figure 15b). Also a shock in federal funds rate will lead to a decrease of unemployment rate (Figure 14c). Other effects are vague in the impulse responses analysis.

5 Discussion

Overall, we view our paper as an improvement and an update of Stock and Watson (2001). Our paper is an improvement as we determined an appropriate time to begin the subset of our data and selected the order of our VAR model based off of analysis of AIC and BIC, rather than arbitrarily selecting an order of 1. Of course, our paper is an update as our data spans until 2015. Similar to Stock and Watson (2001), we conclude also that the VAR model is an immensely powerful and predictive tool to both predict and analyze macroeconomic indicators.

5.1 Granger Causality

The Granger causality test confirmed most of our hypotheses. The relationship between inflation and federal funds rate was found to be significant by both p-values and thus Hypothesis 1 was confirmed. Hypothesis 2 was confirmed by an insignificant p-value for the non-existent relationship inflation on unemployment. Hypothesis 3 and 4 were found to be significant which proves that the federal funds rate is a good predictor of the other indicators. Hypotheses 5a and 5b are testing the results of two conflicting papers of whether unemployment is a good predictor for inflation. The Granger causality test showed that the relationship is insignificant, which proclaims Hypothesis 5b to be correct. Finally, Hypothesis 6 was proven to be insignificant which disproved our hypothesis that unemployment is able to predict the federal funds rate.

5.2 Forecasting

Moreover, our forecast based on the VAR model is of good quality. All of our forecasts are out-of-sample so the forecasting quality is not susceptible to overfitting due to our overall low residuals. Our long-term forecast helps to predict the future trend of the whole economy, while the high-quality short-term forecast justified by small forecasting errors and z-tests have potential application in short-term policy making.

5.3 Impulse Response

As for impulse response, we've got slightly different result from the Granger test, but the results are not contradictory. Granger test mainly focuses on pair-wise between two variables, while impulse response shows more about the dynamics of the whole system. Also, the Granger test is more rigorous for causality testing, while impulse response is mainly regarded as a thought experiment and does not rigorously test on causal relationship. These all lead to potentially different results from Granger test and impulse result. From impulse response, the positive response of federal fund rates to a shock in inflation rate, and the negative response of unemployment rate to a shock in interest rate are consistent with the Granger test. However, we fail to observe significant response of inflation to federal funds rate, but get the negative response of federal funds rate to unemployment. The reason for this difference may be due to aforementioned difference between granger test and impulse response, potential existence of reversed causality, and correlation between errors across VAR equations which will affect accuracy of impulse response as theories suggest.

5.4 Next Steps

Future steps for this project would include using a recursive VAR model in conducting impulse response, as Stock and Watson (2001) claim that this results in clearer impulse response results. Also, other macroeconomic concepts such as stock market aggregates could be integrated and tested. Finally, as stated in Choi (1999) the relationships between these macroeconomic indicators change due to monetary regime type. In future work, we could take into consideration the regime type at certain periods in U.S. economic history into our analysis.

A Data Appendix

The monthly modified data employed in the paper are:

Effective Federal Funds Rate, Percent, Monthly, Seasonally Adjusted (source: FRED)

Consumer Price Index for All Urban Consumers: All Items, Index 1982-1984=100, Monthly, Seasonally Adjusted (source: FRED)

Civilian Unemployment Rate, Percent, Monthly, Seasonally Adjusted (source: FRED)

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