ECE 5390 Practicum Assignment 6

Modeling a Potentiometer Employed in Angular Position Sensing

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I. Introduction

Practicum 5 introduces electromechanical systems, while Practicum 6 further explores the idea through the angular position sensing of a potentiometer. The goal of this practicum is to derive the voltage output of the potentiometer with respect to its angular position.

The rest of this report is organized as follows. Section 2 identifies the constants characterizing a potentiometer, while Section 3 relates its mechanical and electrical components before exploring alterations to the system. Section 4 concludes with a discussion on the practicum and the resulting transfer functions.

II. Understanding the Potentiometer and its Characteristics

To understand encoding the angular position of the potentiometer the useful degrees of rotation must first be decided. The resistive strip within the potentiometer that allows it to serve as a voltage divider is not present for all 360°, and so there are points where the wiper terminals lose contact, and no useful output is generated. This relationship can be seen in **Figure 1**, and the useful degrees, or radians, of rotation can be seen as $\pi - 2\pi$ in terms of a unit circle.

With the useful degrees of rotation found, several potentiometer constants can be defined. The ohmic transfer coefficient is defined as

$$K_R = \frac{R}{\pi}$$

where R is the maximum resistance of the potentiometer and π is derived from the degrees of rotation $|\Theta_1 - \Theta_2| = |\pi - 2\pi|$. The angular transfer coefficient can be similarly derived as

$$K_P = \frac{V_2 - V_1}{\pi}$$

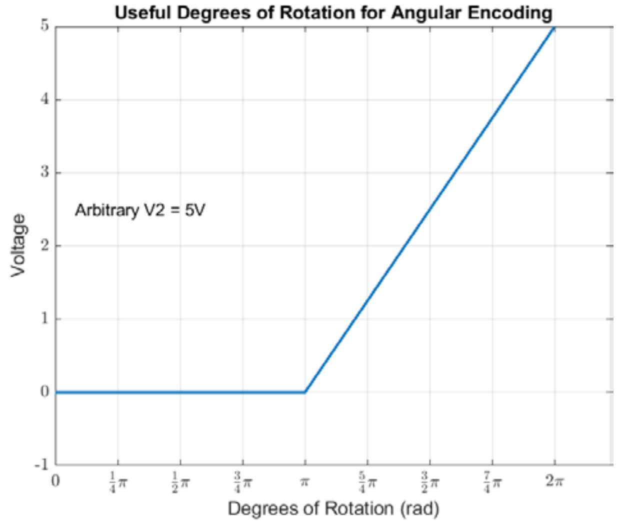

Fig. 1: Plot of V_o with respect to potentiometer rotation

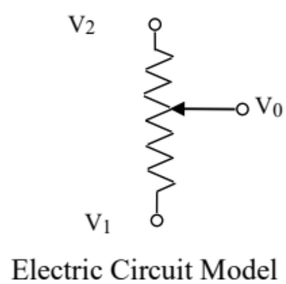

Fig. 2: Electric circuit model of a potentiometer and its I/O

V_2 and V_1 are defined in **Figure 2**, with respect to the potentiometer. If $V_1 = 0\,V$ and $V_2 = 10\,V$, then $K_P = \frac{10}{\pi}$.

With an understanding of the potentiometer and its characteristics, the output voltage V_o can be expressed as a linear function of θ normalized to pass through the origin:

$$V_o = K_P\left(\theta + \frac{\pi}{2}\right) - \frac{V_2 - V_1}{2}$$

Plotting this expression for degrees of rotation normalized to $-\frac{\pi}{2} - \frac{\pi}{2}$ can be seen as the linear function in **Figure 3**.

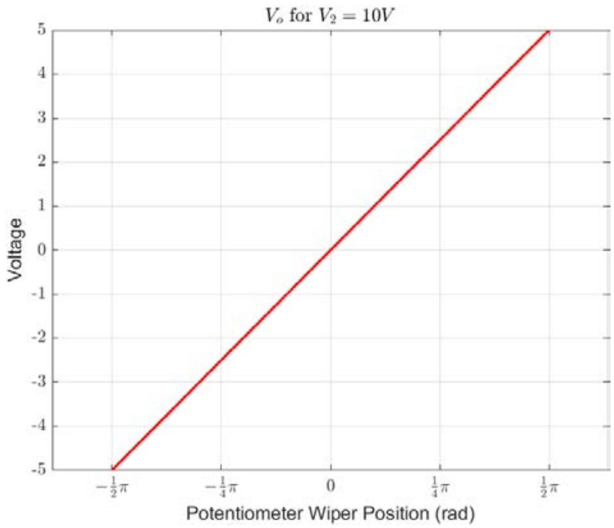

Fig. 3: Normalized output voltage for a linear system

III. Modeling the Electromechanical System

To model the output voltage of the potentiometer with respect to the force applied to its dial—the torque—the representation of **Figure 4** can be used. It shows the inertia and dampening of the wheel that turns the potentiometer's wiper.

The transfer function of such a system $\frac{\theta_P(s)}{\tau(s)}$ is expressed as

$$\frac{\theta_P(s)}{\tau(s)} = \frac{1}{J_P s^2 + D_P s}$$

As before, an equation for V_o can be derived as a function of θ_P using the angular transfer coefficient K_P. The transfer function:

$$\frac{V_o(s)}{\tau(s)} = \frac{K_P}{J_P s^2 + D_P s}$$

Normalized to pass through the origin:

$$\frac{V_o(s)}{\tau(s)} = \frac{K_P}{J_P s^2 + D_P s} - \frac{V_2 - V_1}{2} + K_P \frac{\pi}{2}$$

Should inertia and dampening be neglected, the resulting transfer function is simply

$$\frac{\theta_P(s)}{\tau(s)} = 1$$

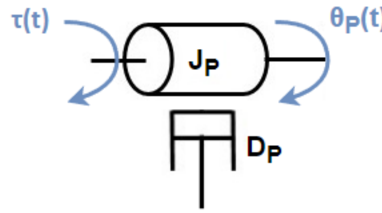

Fig. 4: Inertia of potentiometer wheel

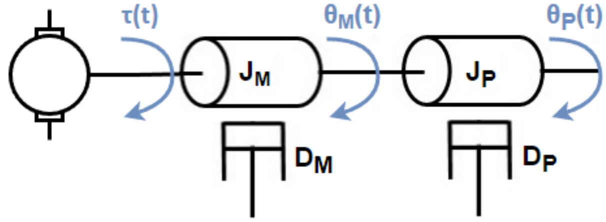

Fig. 5: Potentiometer attached to motor

The angle of the potentiometer follows the torque exactly and is not a physically realizable system. Should the inertia and dampening be neglected in the output voltage response as a result, the voltage remains constant:

$$\frac{V_o(s)}{\tau(s)} = \frac{V_2 - V_1}{2} + K_P \frac{\pi}{2} = V_2 - V_1$$

There is the case of assuming negligible inertia and dampening and so the system in **Figure 5** is used to compare when connected to a motor with inertia and dampening. The transfer function for the response of the potentiometer angle to the motor's torque is

$$\frac{\theta_P(s)}{\tau(s)} = \frac{J_M}{(J_M D_P + J_P D_M)s + D_M D_P}$$

Assuming negligible potentiometer inertia and dampening—i.e., $J_P \ll J_M$ and $D_P \ll D_M$—component $J_{Total} = J_M + J_P$ and component $D_{Total} = D_M + D_P$ are considered to generate:

$$\frac{\theta_P(s)}{\tau(s)} = \frac{1}{(J_M + J_P)s^2 + (D_M + D_P)s}$$

The motor components will overpower the potentiometer components and the resulting response will be an endlessly increasing potentiometer angle as seen in **Figure 6**. Some spring friction must be incorporated for a settling system as seen in **Figure 7**.

$$\frac{\theta_P(s)}{\tau(s)} = \frac{K}{(J_M + J_P)s^2 + (D_M + D_P)s + K}$$

IV. Discussion and Conclusion

Practicum 6 is the first real-world application of the course's teachings. It leaves many aspects open-ended for student discovery, like the angular encoding portion of the lab where the measured voltage of a potentiometer could be used to encode the angular position of a motor fixed to it.

The resulting transfer function considering spring friction within the potentiometer allows for the angular response to the torque to settle and derive an encodable position.

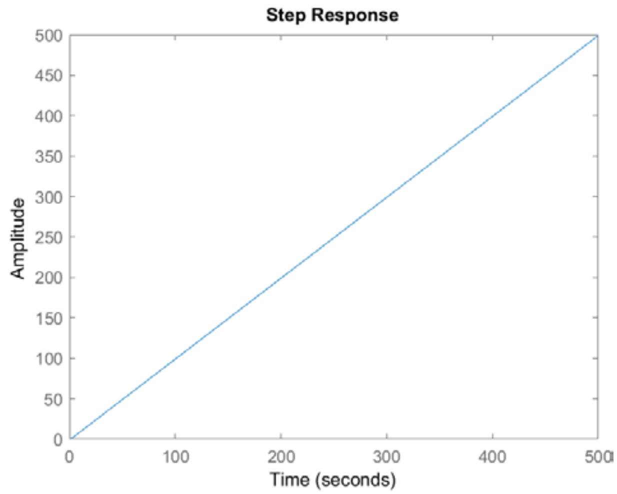

Fig. 6: Frictionless system with negligible potentiometer

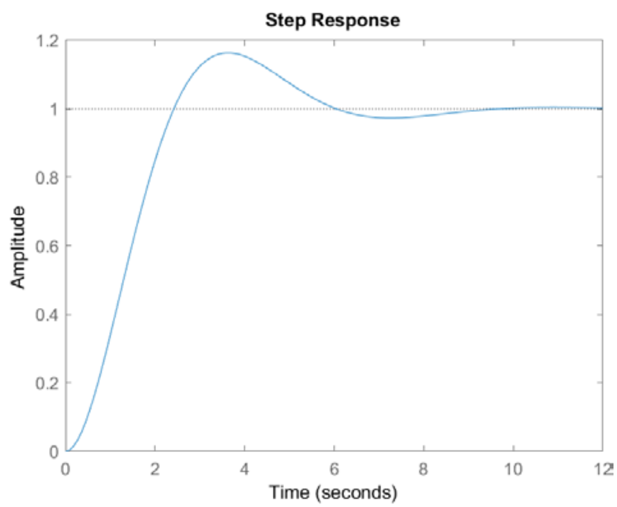

Fig. 7: Simulated system with spring friction