Homework 5 - Reaction-Diffusion Equations

Submission open until 11:59:59pm Thursday December 9, 2021

A reaction-diffusion system can be modeled with the following system:

$$\begin{aligned} U_t &= \lambda(A)U - \omega(A)V + D_1\nabla^2 U \\ V_t &= \omega(A)U + \lambda(A)V + D_2\nabla^2 V \end{aligned} \tag{1}$$

where $A^2 = U^2 + V^2$ and $\nabla^2 = \partial_x^2 + \partial_y^2$. We will consider a particular system

$$\begin{aligned} \lambda(A) &= 1 - A^2 \\ \omega(A) &= -\beta A^2 \end{aligned}$$

although you can try other choices of λ and ω in your investigation of the system. Initiate the system with spiral initial conditions:

```
[X,Y]=meshgrid(x,y);
m=1; % number of spirals
u=tanh(sqrt(X.^2+Y.^2)).*cos(m*angle(X+i*Y)-(sqrt(X.^2+Y.^2)));
v=tanh(sqrt(X.^2+Y.^2)).*sin(m*angle(X+i*Y)-(sqrt(X.^2+Y.^2)));
```

and investigate its solutions with the following boundary conditions/ numerical methods:

- Periodic boundary conditions - use the spectral FFT method
- **Dirichlet** boundaries – use the Chebyshev polynomials

and advance the solution in time with `ode45`.

ANSWERS:
(a) With $x, y \in [-10, 10], n = 64, \beta = 1, D_1 = D_2 = 0.1, m = 1, \text{tspan} = 0 : 0.5 : 4$ and u_f stacked on top of v_f, save the solution of your numerical evolution from ode45 with periodic boundary conditions as A1 (9×8192 for the real part) and A2 (9×8192 for the imaginary part). (NOTE: your solution will be in the Fourier domain when you save it.)

(b) With $x, y \in [-10, 10], n = 30$ **(i.e. use cheb(n))**, $\beta = 1, D_1 = D_2 = 0.1, m = 1, \text{tspan} = 0 : 0.5 : 4$ and u stacked on top of v, write out the solution of your numerical evolution from ode45 with Dirichlet boundaries (**first and last row of the Laplacian matrix should be zero**) as A3 (9×1922 matrix). (NOTE: be sure to remember that you have to rescale the problem to -1 to 1 for cheb.m.)

Besides these answers, investigate and construct various one- and two-armed ($m = 1, 2$) spirals for this system. Also investigate when the solutions become unstable and "chaotic" in nature. Investigate the system for all three (no-flux, pinned and periodic) boundary conditions. Note that for $\beta > 0$ and further consider the diffusion to be not too large, but big enough to kill the Gibbs phenomena at the boundary, i.e., $D_1 = D_2 = 0.1$.