PROB 4.15, PART 3

pg#1

$$\dot{\psi} = -k\psi + k_1\phi \qquad k>0,\ k_1 \neq 0$$
$$\ddot{\phi} = k_2 u \qquad k_2 \neq 0$$

$$C = [0 \quad 1 \quad 0]$$

$$A = \begin{bmatrix} -k & k_1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\dot{\tilde{X}} = [A + GC]\tilde{X} \qquad A + GC = A_c = \begin{bmatrix} -k & k_1+g_1 & 0 \\ 0 & g_2 & 1 \\ 0 & g_3 & 0 \end{bmatrix}$$

$$\det(\lambda I - A_c) = \begin{vmatrix} \lambda+k & k_1+g_1 & 0 \\ 0 & \lambda-g_2 & 1 \\ 0 & g_3 & \lambda \end{vmatrix}$$

$$= (\lambda+k)\left[(\lambda-g_2)\lambda - g_3\right] = (\lambda+k)\left[\lambda^2 - g_2\lambda - g_3\right] = 0$$

$\lambda_1 = -k$ ✓

↳ NEGATIVE

$$\lambda_{2,3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad +b = -g_2 \quad c = -g_3$$

$$= \frac{g_2 \pm \sqrt{g_2^2 + 4g_3}}{2}$$

* CANNOT PLACE THIS E-VALUE, BUT IT IN THE LEFT HALF PLANE!
→ ERROR GOES TO ZERO ASYMPTOTICALLY ANYWAYS

$$\lambda_2 = \frac{g_2 + \sqrt{g_2^2 + 4g_3}}{2} \qquad \lambda_3 = \frac{g_2 - \sqrt{g_2^2 + 4g_3}}{2}$$

↳ CAN DESIGN g SO λ_2 & λ_3 ARE NEGATIVE

// FOR OBSERVABILITY

$$\mathcal{O} = \begin{bmatrix} -C- \\ -CA- \\ -CA^2- \end{bmatrix} \qquad A^2 = \begin{bmatrix} -k & k_1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -k & k_1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} k^2 & -kk_1 & k_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$CA^2 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \qquad \mathcal{O} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

↳ EVEN THOUGH THE $\text{Rank}(\mathcal{O}) = 2$

SYSTEM IS NOT OBSERVABLE IT IS <u>DETECTABLE</u>, SINCE WE CAN PLACE λ_2 & λ_3 & λ_1 IS ALWAYS NEGATIVE.

SO, THE SYSTEM CAN BE RECONSTRUCTED

TO ENSURE STABILITY ON ESTIMATION ERROR:

$\lambda_2 < 0$ & $\lambda_3 < 0$

$g_2 < -\sqrt{g_2^2 + 4g_3}$ $\qquad g_2 < \sqrt{g_2^2 + 4g_3}$

$g_2^2 > g_2^2 + 4g_3$

$\boxed{0 > g_3}$ $\qquad$ ALSO $\boxed{g_2 < 0}$ & $\boxed{g_2^2 + 4g_3 > 0}$