PROB 4.11
pg#1

$\dot{r} = v$
$\dot{v} = a$

$Y_a = a + g\psi + w_a$

$\dot{\psi} = \rho$
$\dot{\rho} = w_p$ → WHITE NOISE

(A) $[\hat{r}, \hat{v}, \hat{\psi}, \hat{\rho}]^T = \hat{x}(t)^T$, FROM Y_a ONLY

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + \cancel{B(t)u(t)} + G(t)\left(Y_A(t) - C\hat{x}(t)\right)$$
NEGLECT

$\hat{x}(t_0) = \hat{x}_0$, $t > t_0$

LET, $G = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$$\underline{X(t)} = \begin{bmatrix} r \\ v \\ \psi \\ \rho \end{bmatrix} \qquad \underline{\dot{X}(t)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{pmatrix} r \\ v \\ \psi \\ \rho \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} a + \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_\rho \end{bmatrix}$$

$\dot{\hat{v}}(t) =$ ACCELERATION

$$Y_a(t) = \begin{bmatrix} 0 & 0 & g & 0 \end{bmatrix}\begin{bmatrix} r \\ v \\ \psi \\ \rho \end{bmatrix} + a + w_a$$
→ ACCELEROMETER READING

$$\dot{\hat{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\hat{x}(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\left(Y_A(t) - g\hat{\psi}(t)\right)$$

THUS,

$$\dot{\hat{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\hat{x}(t) + \begin{bmatrix} 0 \\ Y_a(t) \\ 0 \\ 0 \end{bmatrix}$$

THE ESTIMATION ERROR IS: $\tilde{x}(t) = x(t) - \hat{x}(t)$ → NOT STABLE SINCE ONE OF THE EIGENVALUES WILL BE ZERO!

$$\dot{\tilde{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\tilde{x}(t) + \begin{bmatrix} 0 \\ -w_a \\ 0 \\ w_\rho \end{bmatrix}$$

(B) $Y_r = r + w_r$

$$Y(t) = \begin{bmatrix} Y_a(t) \\ Y_r(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & g & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} X(t) + \begin{bmatrix} W_a \\ W_r \end{bmatrix}$$

$C_2 = [1\ 0\ 0\ 0]$

ADDING THIS TO THE PREVIOUS ESTIMATION; AGAIN NEGLECTING INPUT:

$$\dot{\hat{X}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \hat{X}(t) + \begin{bmatrix} 0 \\ Y_A(t) \\ 0 \\ 0 \end{bmatrix} + G(t)\left[C_2 \hat{X}(t) - Y_r(t)\right]$$

$$\dot{\hat{X}}(t) = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \hat{X}(t) + \begin{bmatrix} 0 \\ Y_A(t) \\ 0 \\ 0 \end{bmatrix} + G\left(\hat{r}(t) - Y_r(t)\right)$$

THEN THE ERROR ESTIMATE EQUATIONS: $\dot{\tilde{X}}(t) = \dot{X}(t) - \dot{\hat{X}}(t)$

$$\dot{\tilde{X}}(t) = \underbrace{(A + G_1 C_2)}_{\text{DETERMINISTIC PART}} \tilde{X}(t) + \underbrace{\begin{bmatrix} 0 \\ -W_a \\ 0 \\ W_p \end{bmatrix} + G_2 W_r}_{\text{STOCASTIC PART}}$$

SIMILAR TO EXAMPLE 4.10 ON PG # 103, THE GAINS CAN BE SELECTED USING THE <u>ARE</u>:

$$\dot{P} = 0 = AP + PA^T - PC^T R_v^{-1} CP + R_w \Rightarrow P$$

TO FIND THE OPTIMAL GAIN:

$$G = -PC^T R_v^{-1}$$

PROBLEM 4.12
Pg #1

GIVEN:

$$\dot{X}(t) = f(x(t), u(t), t)$$
$$Y(t) = g(x(t), t)$$

SHOW THAT IF :

$$\frac{\partial f}{\partial t} = 0 \quad \& \quad \frac{\partial g}{\partial t} = 0$$

IT IS NOT POSSIBLE TO CORRECT THE ONBOARD CLOCK USING RECURSIVE NAVIGATION.

SOLUTION:

$$\dot{x}(t) = f(x(t), u(t), \tau(t))$$
$$\dot{\tau}(t) = 1 + w_\tau(t)$$
$$Y(t) = g(x(t), \tau(t))$$

SO, THE STATE VECTOR IS $\begin{bmatrix} x \\ \tau \end{bmatrix}$

DEFINE A NOMINAL TRAJECTORY $x^\circ(t)$, $u^\circ(t)$, & $y^\circ(t)$

THEN PERTURBATION VARIABLES:

$$\delta x(t) = x(t) - x^\circ(t)$$
$$\delta u(t) = u(t) - u^\circ(t)$$
$$\delta y(t) = y(t) - y^\circ(t)$$

THE JACOBIANS ARE:

$$A(t) = \frac{\partial f}{\partial x}\bigg|_\circ, \quad B(t) = \frac{\partial f}{\partial u}\bigg|_\circ, \quad C(t) = \frac{\partial g}{\partial x}\bigg|_\circ$$

PROB 4.17
Pg #2

LOOKING AT $A(t)$,

THE ORIGINAL STATE EQUATIONS DID NOT HAVE τ!

$$A(t) = \frac{\partial f}{\partial x}\Big|_0 = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & 0 \\ 0 & 0 & \cdots & \frac{\partial f_\tau}{\partial \tau} \end{bmatrix}$$

$\frac{\partial f_\tau}{\partial \tau} \rightarrow 1 + W_\tau(t)$

LOOKING AT $C(t) \rightarrow$

OUTPUT EQS DID NOT HAVE τ!

$$C(t) = \frac{\partial g}{\partial x}\Big|_0 = \underbrace{\begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_N} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial g_M}{\partial x_1} & \frac{\partial g_M}{\partial x_2} & \cdots & \frac{\partial g_M}{\partial x_N} & 0 \end{bmatrix}}$$

$\begin{bmatrix} x \\ \tau \end{bmatrix} \rightarrow \in \mathbb{R}^N$

$\rightarrow$ # OF STATES $= N+1$

THEN, $\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-1} \end{bmatrix}$ WILL HAVE A MAXIMUM RANK OF N!