UNIVERSITY OF MICHIGAN

# AERO 584, Homework 6

Huckleberry Febbo

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PROB 4.11
pg#1

$\dot{r} = v$
$\dot{v} = a$

$Y_a = a + g\psi + w_a$ $\qquad \dot{\psi} = \rho$
$\dot{\rho} = w_\rho$ → WHITE NOISE

(A) $[\hat{r}, \hat{v}, \hat{\psi}, \hat{\rho}]^T = \hat{x}(t)^T$, FROM $Y_a$ ONLY

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + \underbrace{B(t)u(t)}_{\text{NEGLECT}} + G(t)\left(Y_A(t) - C\hat{x}(t)\right)$$

LET, $G = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$\hat{x}(t_0) = \hat{x}_0$, $t > t_0$

$$\underline{X}(t) = \begin{bmatrix} r \\ v \\ \psi \\ \rho \end{bmatrix} \qquad \dot{\underline{X}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ v \\ \psi \\ \rho \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} a + \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_\rho \end{bmatrix}$$

$\dot{\hat{v}}(t) = \text{ACCELERATION}$

$$Y_a(t) = \begin{bmatrix} 0 & 0 & g & 0 \end{bmatrix}\begin{bmatrix} r \\ v \\ \psi \\ \rho \end{bmatrix} + a + w_a$$

$Y_a(t)$ → ACCELEROMETER READING

$$\dot{\hat{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\hat{x}(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\left(Y_A(t) - g\hat{\psi}(t)\right)$$

THUS,

$$\dot{\hat{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\hat{x}(t) + \begin{bmatrix} 0 \\ Y_a(t) \\ 0 \\ 0 \end{bmatrix}$$

THE ESTIMATION ERROR IS:

$\tilde{x}(t) = x(t) - \hat{x}(t)$ → NOT STABLE SINCE ONE OF THE EIGENVALUES WILL BE ZERO!

$$\dot{\tilde{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\tilde{x}(t) + \begin{bmatrix} 0 \\ -w_a \\ 0 \\ w_\rho \end{bmatrix}$$

PROB 4.1
PG #2

(B) $Y_r = r + w_r$

$$Y(t) = \begin{bmatrix} Y_a(t) \\ Y_r(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} X(t) + \begin{bmatrix} W_a \\ W_r \end{bmatrix}$$

$C_2 = [1\ 0\ 0\ 0]$

ADDING THIS TO THE PREVIOUS ESTIMATION, AGAIN NEGLECTING INPUT:

$$\dot{\hat{X}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \hat{X}(t) + \begin{bmatrix} 0 \\ Y_A(t) \\ 0 \\ 0 \end{bmatrix} + G(t) \left[ C_2 \hat{X}(t) - Y_r(t) \right]$$

$$\dot{\hat{X}}(t) = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \hat{X}(t) + \begin{bmatrix} 0 \\ Y_A(t) \\ 0 \\ 0 \end{bmatrix} + G\left(\hat{r}(t) - Y_r(t)\right)$$

THEN THE ERROR ESTIMATE EQUATIONS: $\dot{\tilde{X}}(t) = \dot{X}(t) - \dot{\hat{X}}(t)$

$$\dot{\tilde{X}}(t) = \underbrace{(A + G_1 C_2)}_{\text{DETERMINISTIC PART}} \tilde{X}(t) + \underbrace{\begin{bmatrix} 0 \\ -W_a \\ 0 \\ W_p \end{bmatrix} + G_2 W_r}_{\text{STOCASTIC PART}}$$

SIMILAR TO EXAMPLE 4.10 ON PG #103, THE GAINS CAN BE SELECTED USING THE ARE:

$$\dot{P} = 0 = AP + PA^T - PC^T R_v^{-1} CP + R_w \Rightarrow P$$

TO FIND THE OPTIMAL GAIN:

$$G = -PC^T R_v^{-1}$$

PROBLEM 4.12
Pg #1

GIVEN:

$$\dot{X}(t) = f(x(t), u(t), t)$$
$$Y(t) = g(x(t), t)$$

SHOW THAT IF:

$$\frac{\partial f}{\partial t} = 0 \quad \& \quad \frac{\partial g}{\partial t} = 0$$

IT IS NOT POSSIBLE TO CORRECT THE ONBOARD CLOCK USING RECURSIVE NAVIGATION.

SOLUTION:

$$\dot{X}(t) = f(x(t), u(t), \tau(t))$$
$$\dot{\tau}(t) = 1 + w_\tau(t)$$
$$Y(t) = g(x(t), \tau(t))$$

SO, THE STATE VECTOR IS $\begin{bmatrix} x \\ \tau \end{bmatrix}$

DEFINE A NOMINAL TRAJECTORY $x^\circ(t)$, $u^\circ(t)$, & $y^\circ(t)$

THEN PERTURBATION VARIABLES:

$$\delta x(t) = x(t) - x^\circ(t)$$
$$\delta u(t) = u(t) - u^\circ(t)$$
$$\delta y(t) = y(t) - y^\circ(t)$$

THE JACOBIANS ARE:

$$A(t) = \left.\frac{\partial f}{\partial x}\right|_\circ, \quad B(t) = \left.\frac{\partial f}{\partial u}\right|_\circ, \quad C(t) = \left.\frac{\partial g}{\partial x}\right|_\circ$$

PROB 4.12
Pg #2

LOOKING AT $A(t)$,

$$A(t) = \frac{\partial f}{\partial x}\Big|_0 = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & 0 \\ 0 & 0 & \cdots & \frac{\partial f_\tau}{\partial \tau} \end{bmatrix}$$

THE ORIGINAL STATE EQUATIONS DID NOT HAVE $\tau$!

$\frac{\partial f_\tau}{\partial \tau} \rightarrow 1 + W_\tau(t)$

LOOKING AT $C(t)$

$$C(t) = \frac{\partial g}{\partial x}\Big|_0 = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_N} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial g_M}{\partial x_1} & \frac{\partial g_M}{\partial x_2} & \cdots & \frac{\partial g_M}{\partial x_N} & 0 \end{bmatrix}$$

OUTPUT EQS DID NOT HAVE $\tau$!

$\begin{bmatrix} x \\ \tau \end{bmatrix} \in \mathbb{R}^N$

# OF STATES $= N+1$

THEN,

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-1} \end{bmatrix}$$ WILL HAVE A MAXIMUM RANK OF $N$!

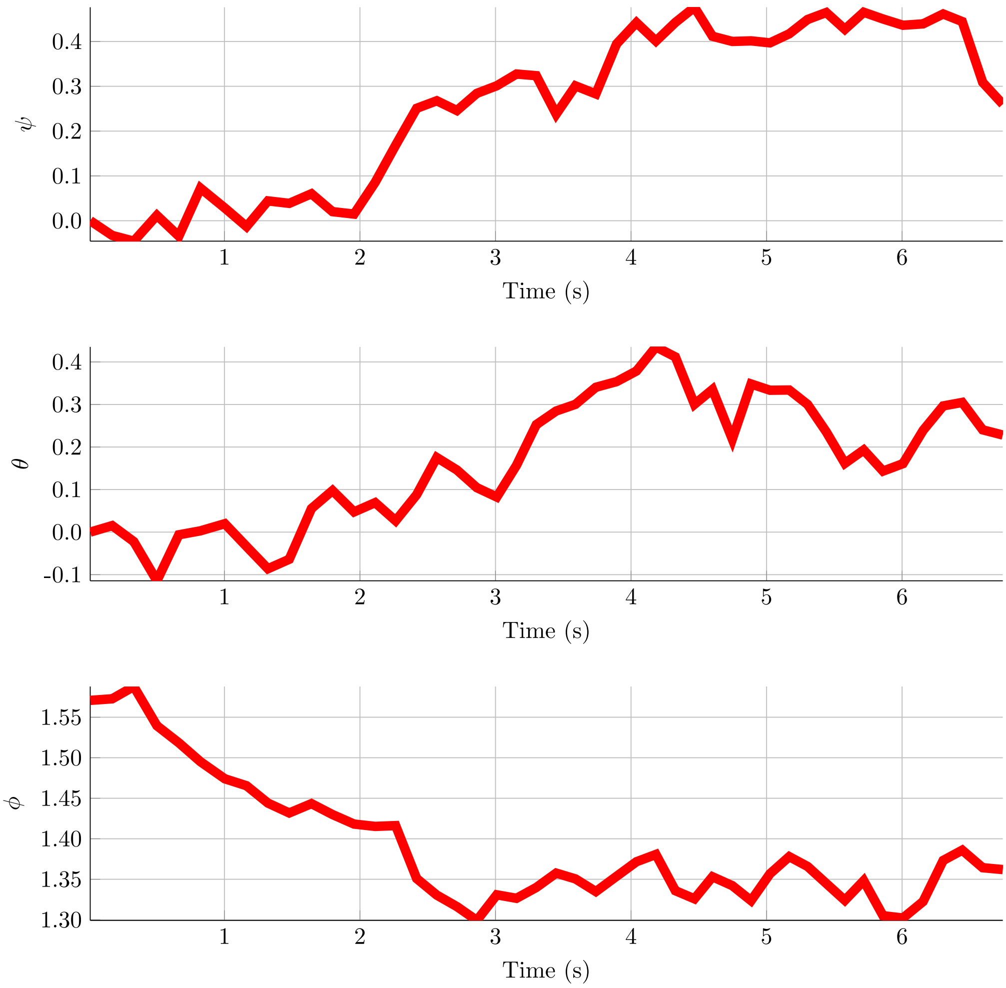



Figure 0.1: integrated $w_b$ data using Euler's forward integration to get the Euler angles in the body frame

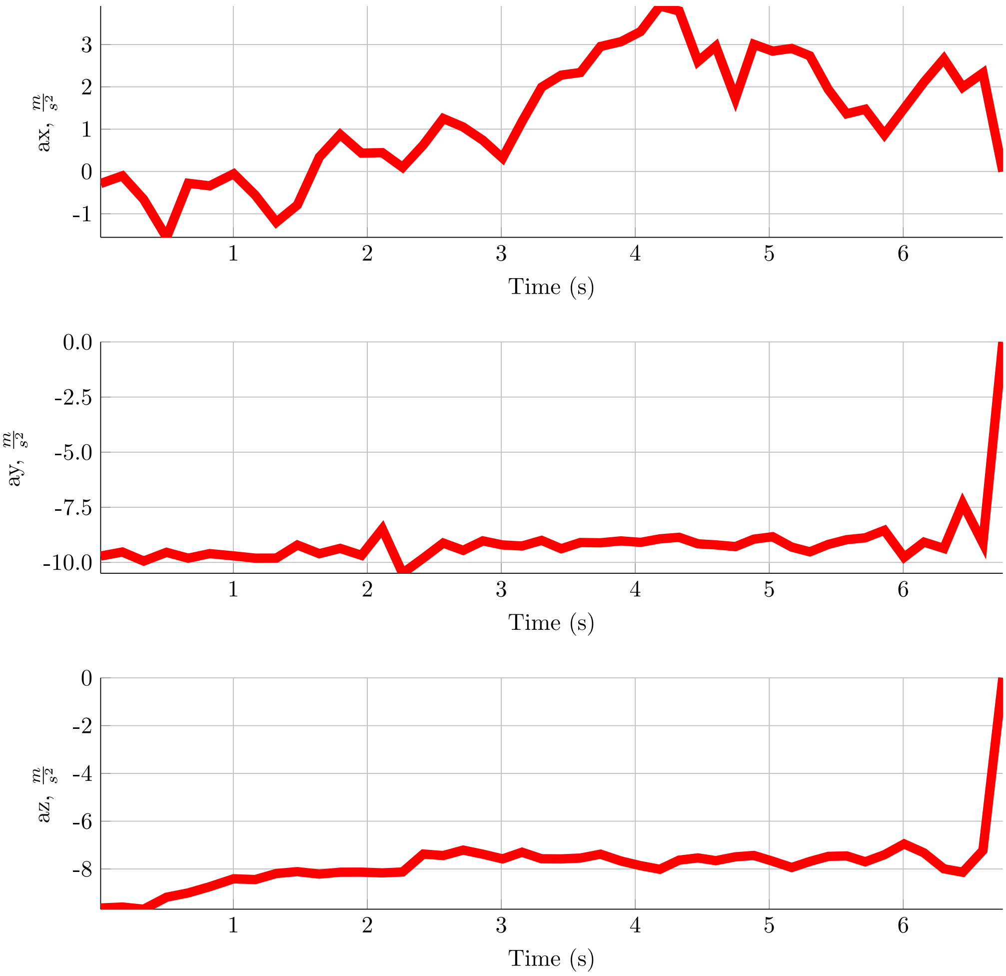



Figure 0.2: accleration in inertial frame

## PROBLEM 4.18, PART B

As can be seen in Fig. 0.3,Fig. 0.4,Fig. 0.5, and Fig. 0.6 both the velocity and position are not well captured using inertial navigation when compared to the Vicon system.

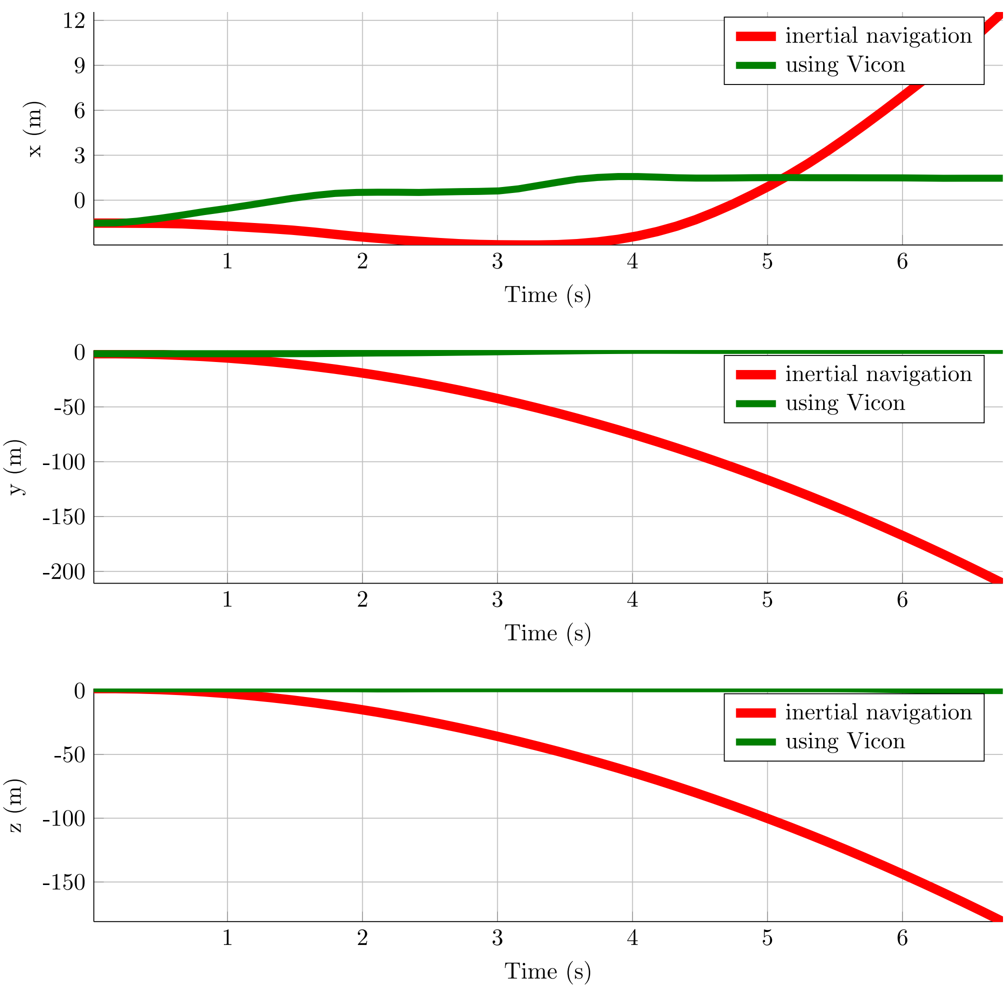

12
9
6
3
0
x (m)
1
2
3
4
5
6
Time (s)
inertial navigation
using Vicon
0
-50
-100
-150
-200
y (m)
1
2
3
4
5
6
Time (s)
inertial navigation
using Vicon
0
-50
-100
-150
z (m)
1
2
3
4
5
6
Time (s)
inertial navigation
using Vicon

Figure 0.3

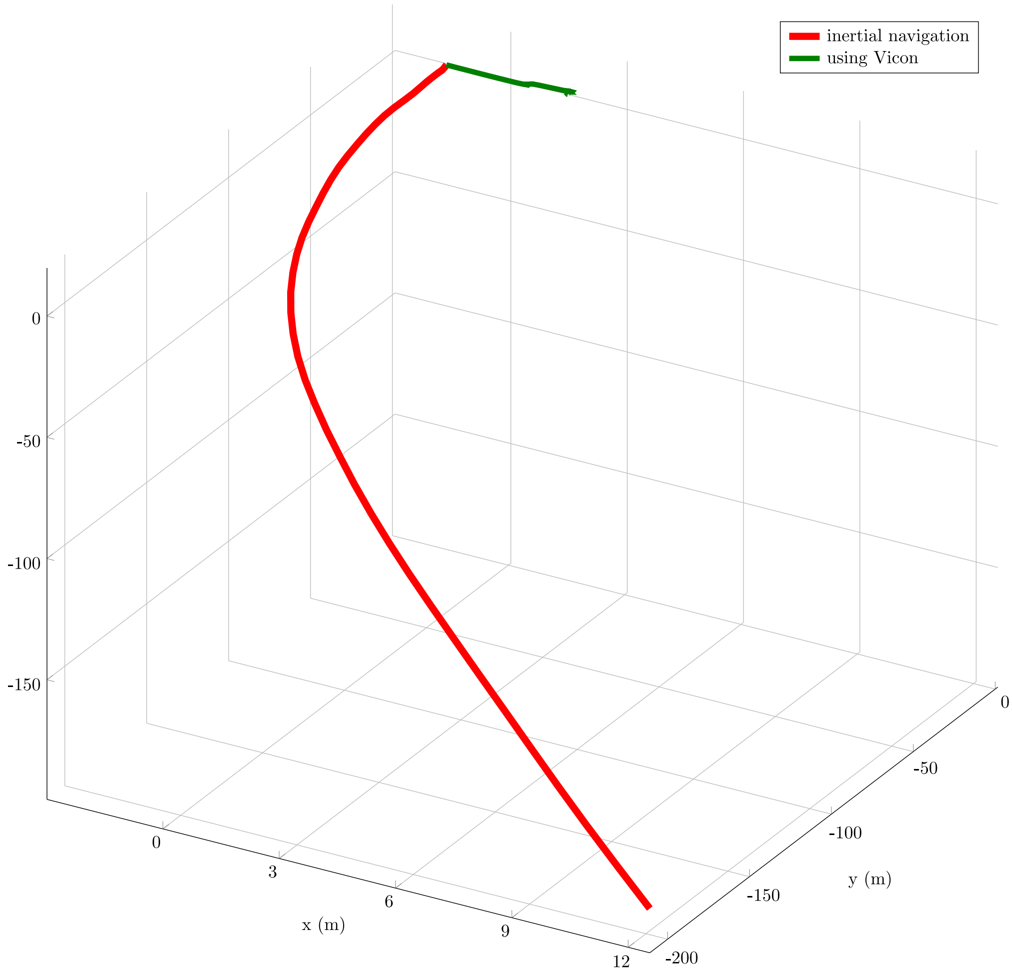

inertial navigation
using Vicon
0
-50
-100
-150
0
3
6
9
12
x (m)
0
-50
-100
-150
-200
y (m)

Figure 0.4

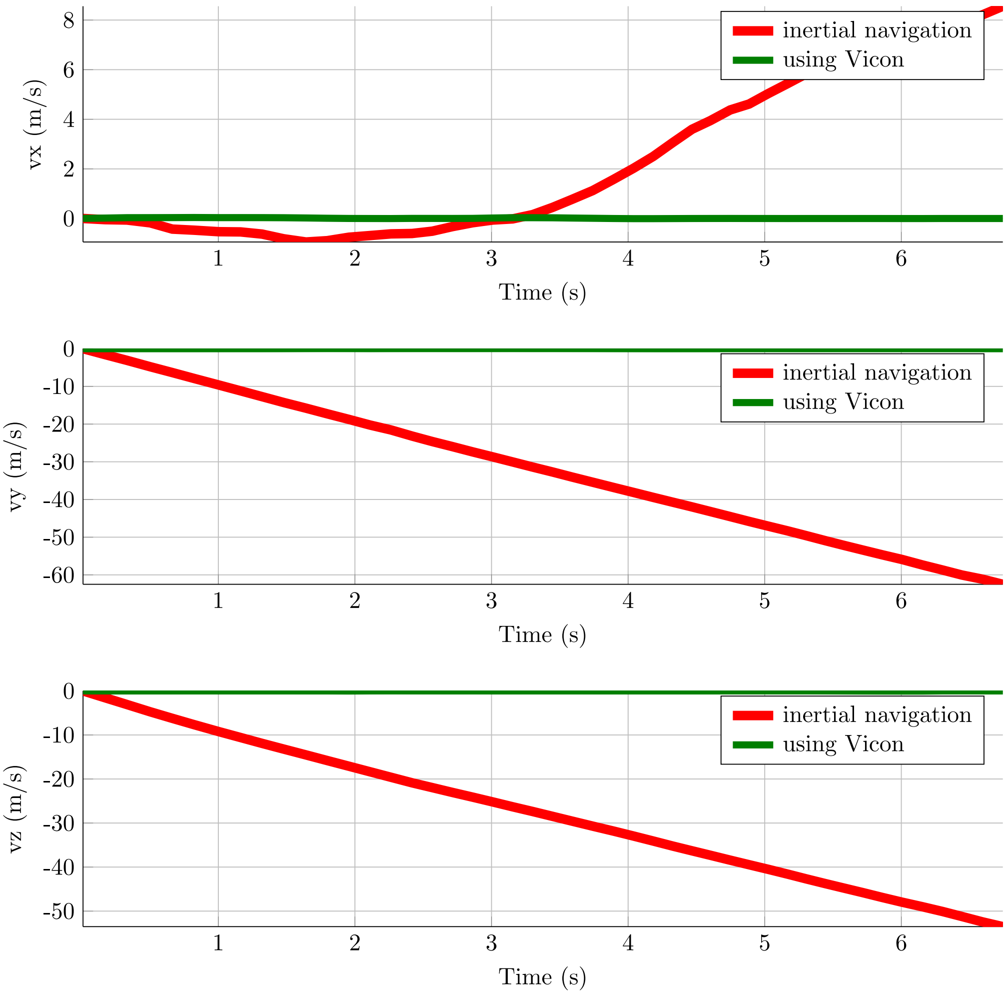

inertial navigation
using Vicon
vx (m/s)
Time (s)
inertial navigation
using Vicon
vy (m/s)
Time (s)
inertial navigation
using Vicon
vz (m/s)
Time (s)

Figure 0.5

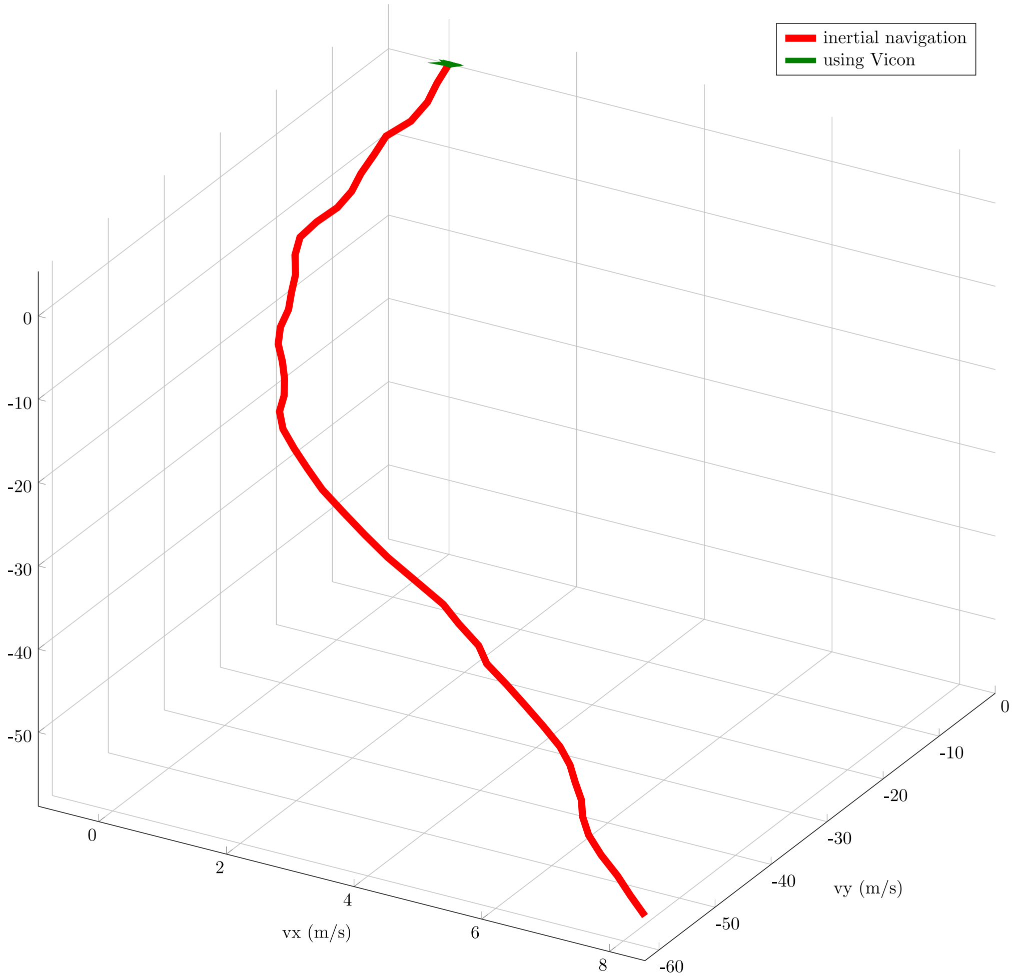

inertial navigation
using Vicon
0
-10
-20
-30
-40
-50
0
2
4
6
8
vx (m/s)
0
-10
-20
-30
-40
-50
-60
vy (m/s)

Figure 0.6

## Problem 4.18, Part c

PROB 4.8
PART (C)

## GAUSS-MARKOV

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t)$$
$$y(t) = C(t)x(t) + V(t)$$

$$\underline{X}_k = [x^I \ V_x^I \ y^I \ V_y^I \ z^I \ V_z^I]^T$$
$$\underline{U}_k = [a_x^I \ a_y^I \ a_z^I]^T$$
$$\underline{Z}_k = [x^I \ y^I \ z^I]^T$$
$$\epsilon_p = [\epsilon_x \ \epsilon_y \ \epsilon_z]^T \rightarrow R_p = \begin{bmatrix} 10^{-4} & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix}$$
$$\epsilon_a = [\epsilon_{ax} \ \epsilon_{ay} \ \epsilon_{az}]^T \rightarrow R_a = \begin{bmatrix} 0.025 & 0 & 0 \\ 0 & 0.025 & 0 \\ 0 & 0 & 0.025 \end{bmatrix} \delta(t)$$

with,
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \ \& \ B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$W = \overbrace{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\omega} \quad \epsilon_A = \omega \epsilon_a$$

PROB 4.8
PART (C)
PS#2

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\underbrace{V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{V} \epsilon_p = V \epsilon_p$$

$$R_V = V R_p V^T \qquad \& \quad R_W = W R_a W^T$$

FOR THE RECURSIVE NAVIGATOR:

SEE CODE!

IN APPENDIX, PART C

The Kalman Filter does a nice job, all of the plots are zoomed in because the inertial n

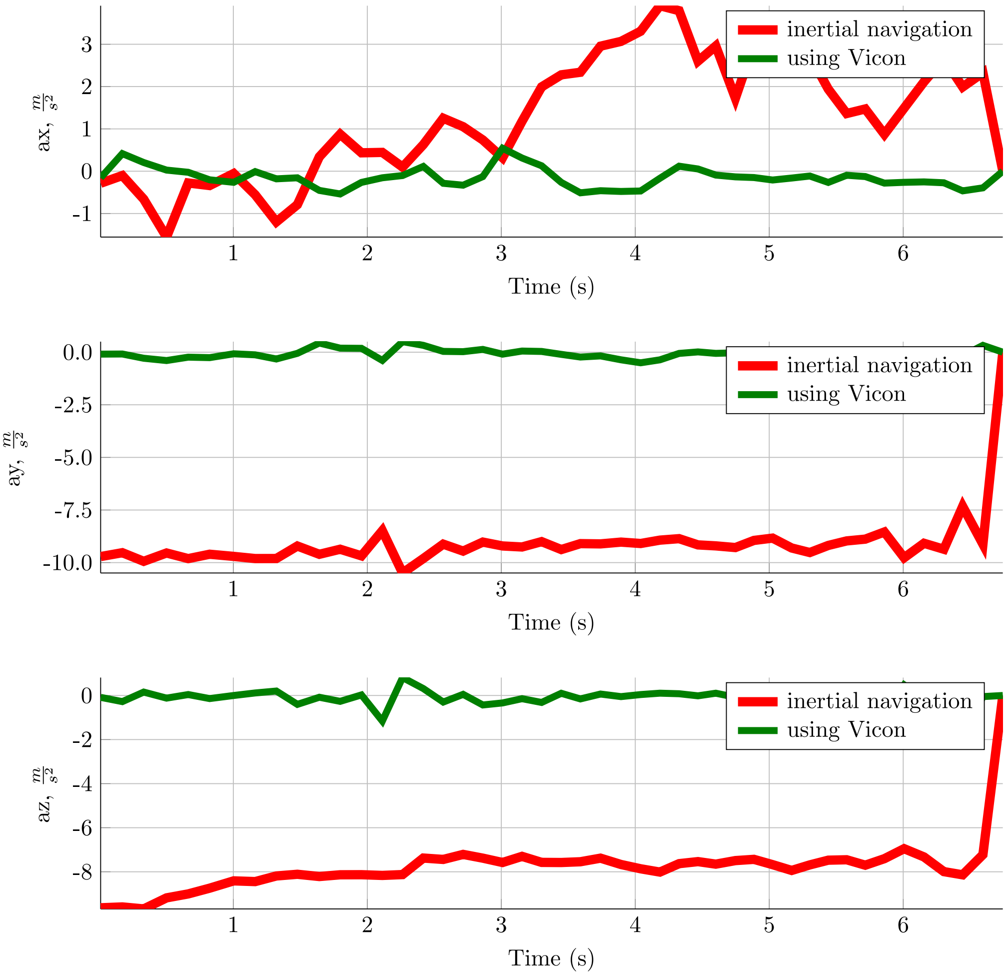
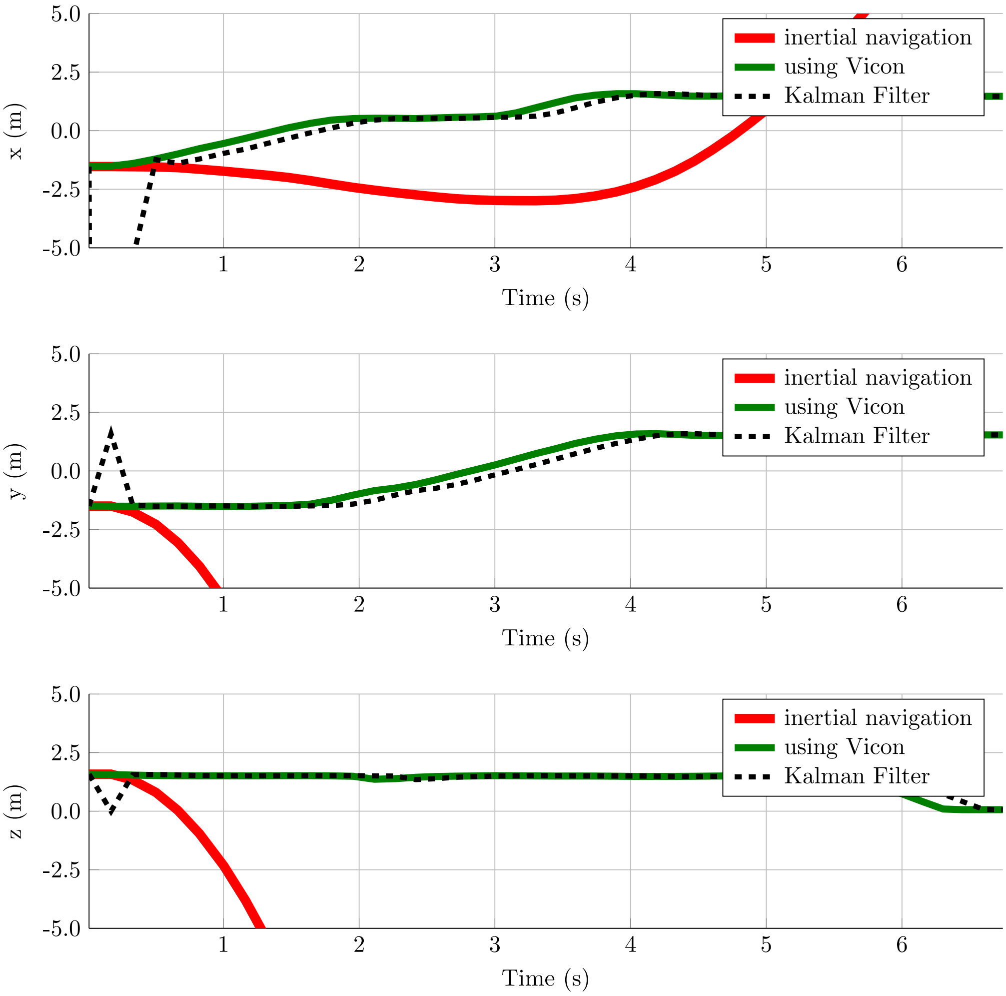
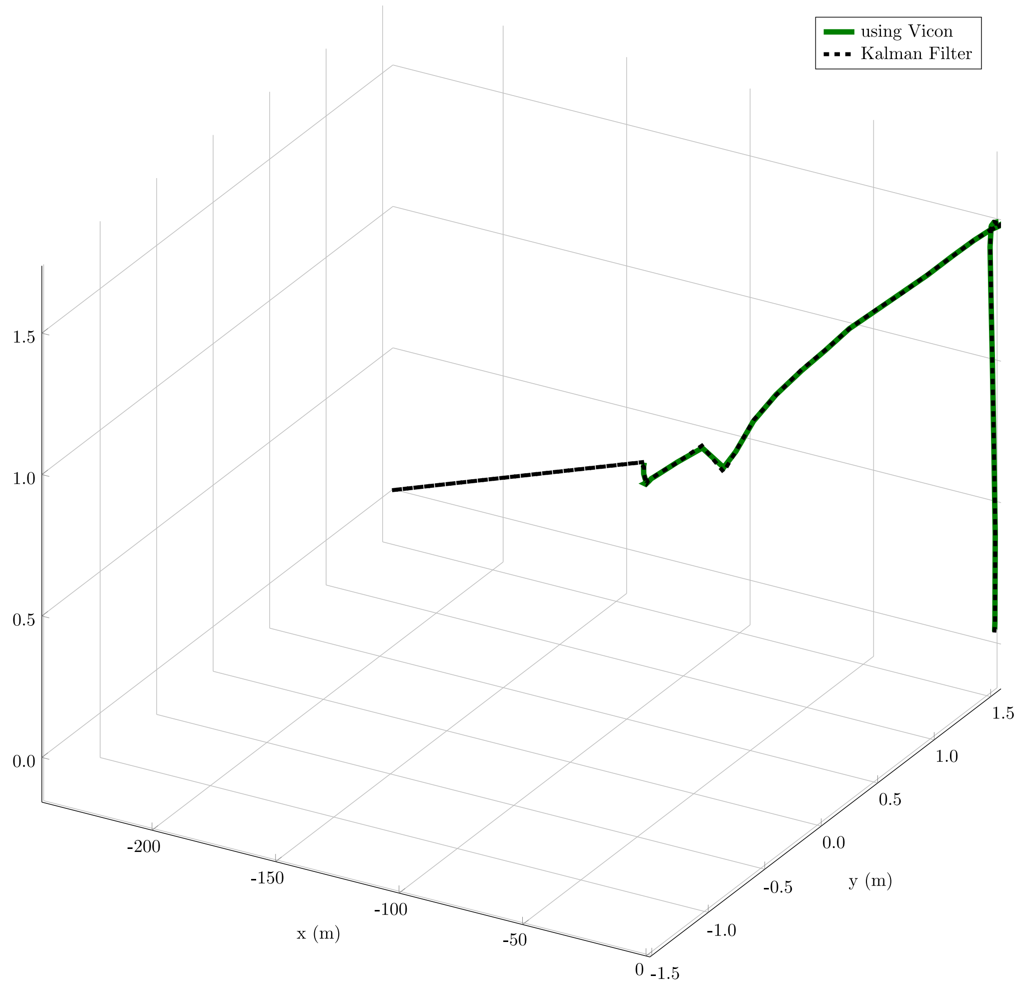



Figure 0.7



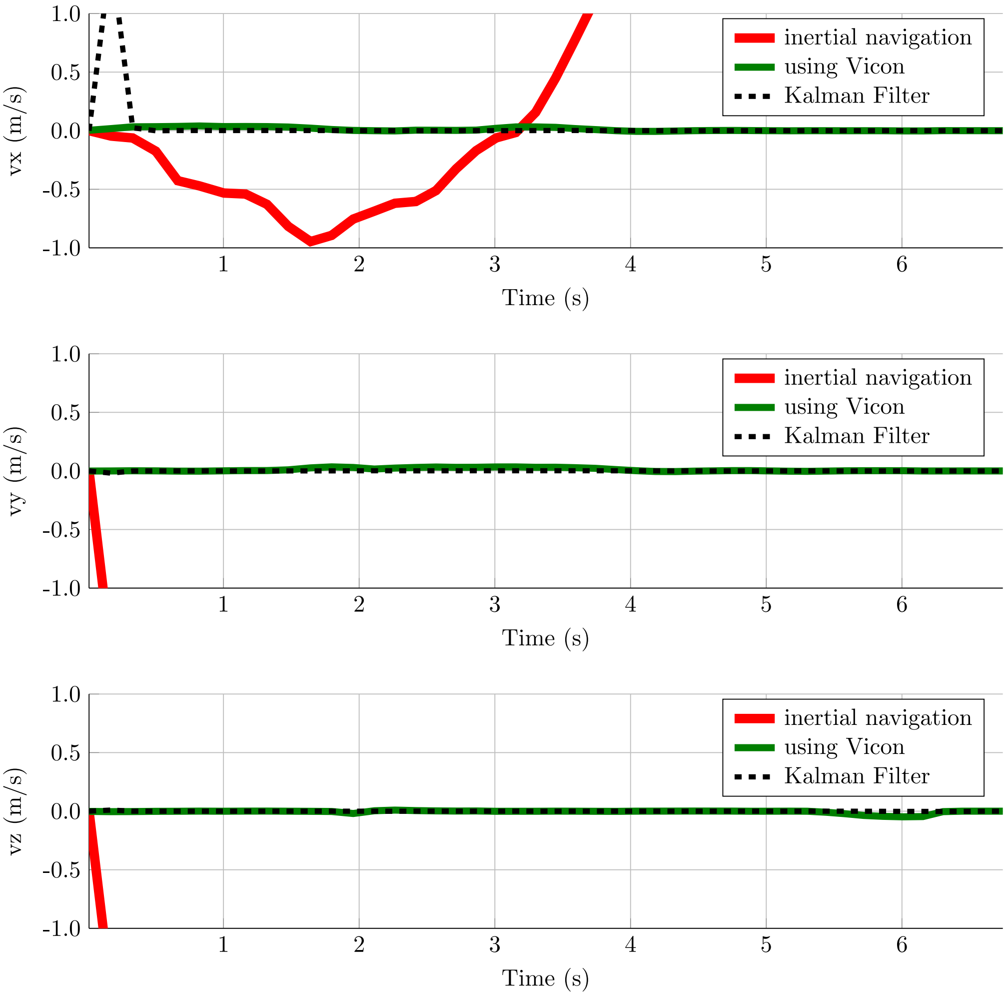
x (m)
5.0
2.5
0.0
-2.5
-5.0
1
2
3
4
5
6
Time (s)
inertial navigation
using Vicon
Kalman Filter
y (m)
5.0
2.5
0.0
-2.5
-5.0
1
2
3
4
5
6
Time (s)
inertial navigation
using Vicon
Kalman Filter
z (m)
5.0
2.5
0.0
-2.5
-5.0
1
2
3
4
5
6
Time (s)
inertial navigation
using Vicon
Kalman Filter


Figure 0.8



using Vicon
Kalman Filter
1.5
1.0
0.5
0.0
-200
-150
-100
-50
0
x (m)
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
y (m)


Figure 0.9



inertial navigation
using Vicon
Kalman Filter
vx (m/s)
vy (m/s)
vz (m/s)
Time (s)


Figure 0.10

# Problem 4.18, Part d i

Using this much higher covariance significantly decreases performance.

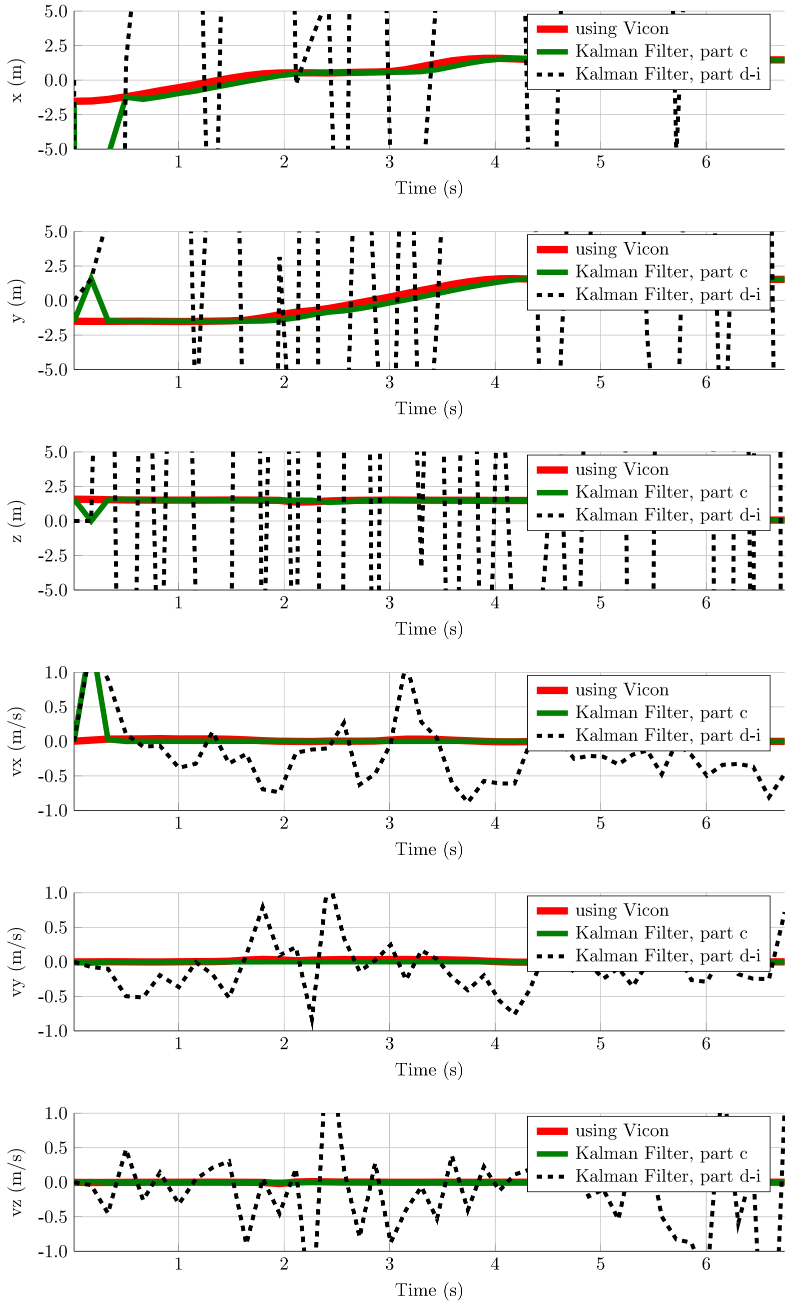



Figure 0.11

## PROBLEM 4.18, PART D II

While case ii is better than case i, it is not really any better than the original Filter that was designed.

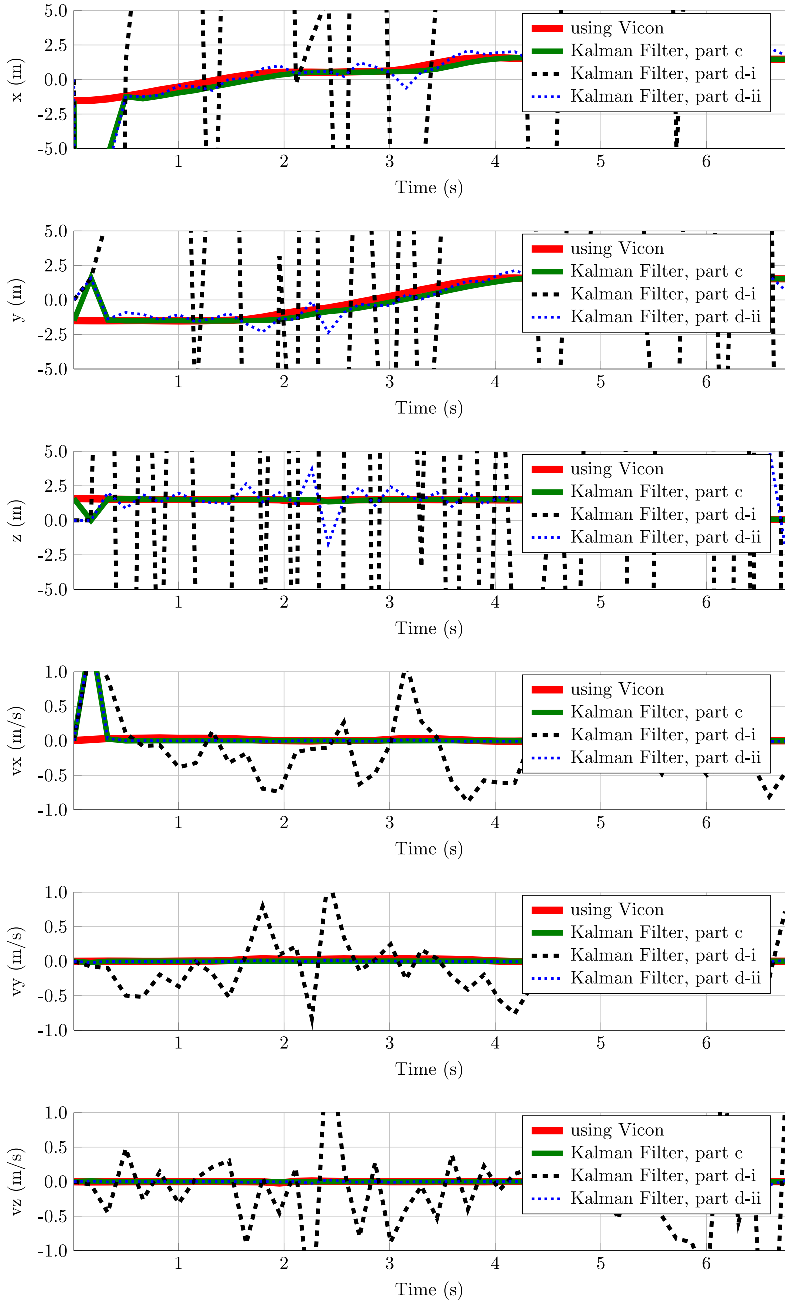



Figure 0.12

# PROBLEM 4.18, PART E I AND E II

Less frequent updates has a significant impact on the x and y states.

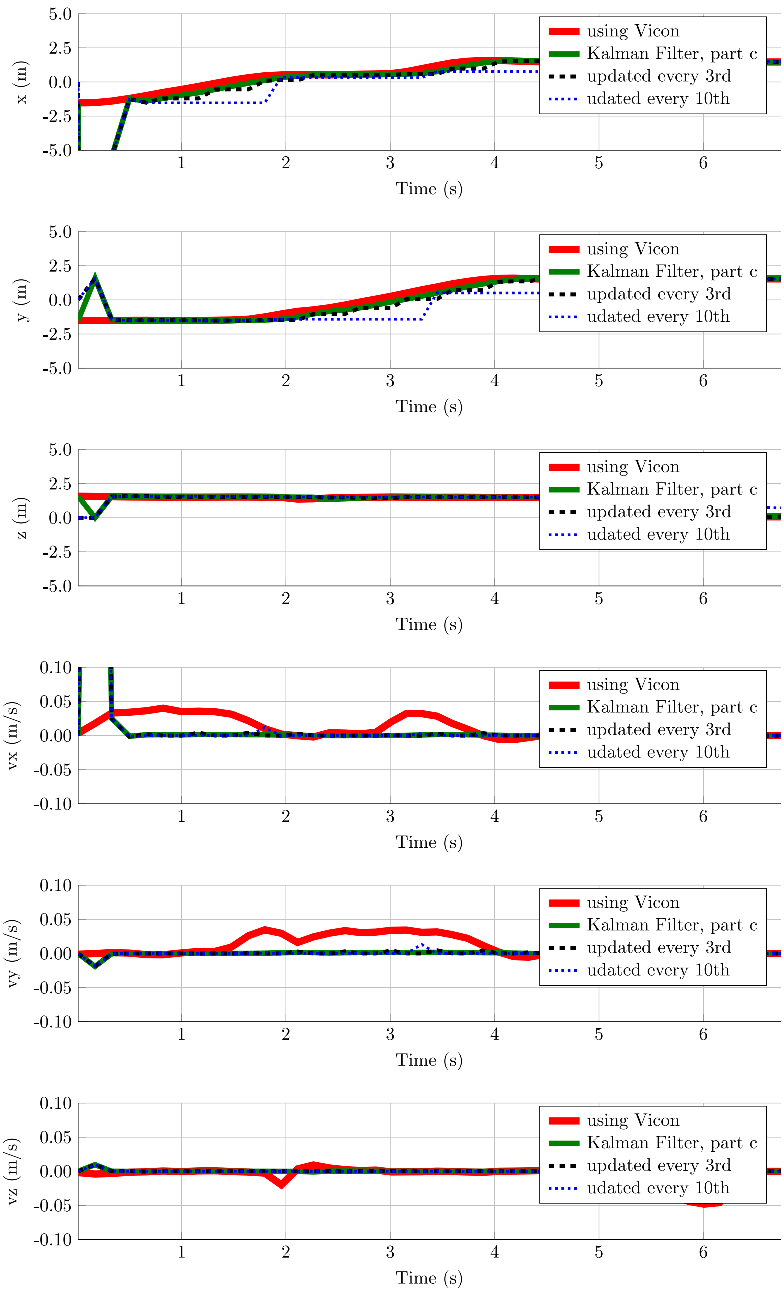



Figure 0.13

## Problem 4.18, Julia code

```
using Plots
pgfplots()
using LaTeXStrings
PGFPlots.pushPGFPlotsPreamble("\\usepackage{amssymb}")
using Interpolations
using OrdinaryDiffEq
using DiffEqBase
using DataFrames

d = readtable("data.csv")

# extract data
t = d[:t]/1000; # convert to seconds
xi = d[:xi]
yi = d[:yi]
zi = d[:zi]
q_x = d[:q_x]
q_y = d[:q_y]
q_z = d[:q_z]
q_w = d[:q_w]
ax_b = d[:ax_b]
ay_b = d[:ay_b]
az_b = d[:az_b]
wx_b = d[:wx_b]
wy_b = d[:wy_b]
wz_b = d[:wz_b]

# misc variables
L = length(wz_b)
s1 = ""
l1 = (4,:red,:solid)

#s2 = "asmptotic observer"
l2 = (3,:green,:solid)

#s3 = "input"
l3 = (2.2,:black,:dash)

l4 = (1.5,:blue,:dot)

# rotation matrix, https://en.wikipedia.org/wiki/Conversion_between_quaternions_and_Eul
```

```
function R(q_x,q_y,q_z,q_w)
   [1 - 2*(q_y^2 + q_z^2)        2*(q_x*q_y - q_w*q_z)      2*(q_w*q_y + q_x*q_z);
    2*(q_x*q_y + q_w*q_z)        1-2*(q_x^2 + q_z^2)        2*(q_z*q_y - q_w*q_x);
    2*(q_x*q_z - q_w*q_y)        2*(q_z*q_y + q_w*q_x)      1-2*(q_y^2 + q_x^2)];
end

# direction cosine matrices, 3-2-1 sequence (psi,theta,phi)
# [x,y,z] = Rz(psi)*Ry(theta)*Rz(phi)[X;Y;Z]
function Rz(psi)
 [cos(psi) -sin(psi) 0;
  sin(psi)  cos(psi) 0;
     0         0     1]
end

function Ry(theta)
 [cos(theta)      0    sin(theta);
      0           1        0;
  -sin(theta)     0      cos(theta)]
end

function Rx(phi)
 [1        0         0;
  0        cos(phi) -sin(phi);
  0        sin(phi)  cos(phi)]
end

##################
# part a)
psi_b = zeros(L); psi_b[1] = 0;
theta_b = zeros(L); theta_b[1] = 0;
phi_b = zeros(L); phi_b[1] = pi/2;

# integrated w_b data  using Euler's forward integration to get the Euler angles in the
for i in 1:L-1
 psi_b[i+1] = psi_b[i] + wx_b[i]*(t[i+1] - t[i])
 theta_b[i+1] = theta_b[i] + wy_b[i]*(t[i+1] - t[i])
 phi_b[i+1] = phi_b[i] + wz_b[i]*(t[i+1] - t[i])
end

# find accleration in inertial frame
ax = zeros(L)
ay = zeros(L)
az = zeros(L)
```

```
for i in 1:L-1
 ab = [ax_b[i];ay_b[i];az_b[i]]
 a = Rx(phi_b[i])*Ry(theta_b[i])*Rz(psi_b[i])*ab + [0;0;-9.8]
 ax[i] = a[1]; ay[i] = a[2]; az[i] = a[3];
end

# angular velocity plots
p1 = plot(t,psi_b,line=l1,label=s1)
yaxis!(string(" ",L"$\psi$"))
xaxis!("Time (s)")

p2 = plot(t,theta_b,line=l1,label=s1)
yaxis!(string(" ",L"$\theta$"))
xaxis!("Time (s)")

p3 = plot(t,phi_b,line=l1,label=s1)
yaxis!(string(" ",L"$\phi$"))
xaxis!("Time (s)")

plot(p1,p2,p3,layout=@layout([a;b;c]),size=[600,600])

savefig(string("figs/p3_a",".",:svg));

# inertial acceleration plots
p1 = plot(t,ax,line=l1,label=s1)
yaxis!(string("ax, ",L"$\frac{m}{s^2}$"))
xaxis!("Time (s)")

p2 = plot(t,ay,line=l1,label=s1)
yaxis!(string("ay, ",L"$\frac{m}{s^2}$"))
xaxis!("Time (s)")

p3 = plot(t,az,line=l1,label=s1)
yaxis!(string("az, ",L"$\frac{m}{s^2}$"))
xaxis!("Time (s)")

pap = plot(p1,p2,p3,layout=@layout([a;b;c]),size=[600,600])

savefig(string("figs/p3_a2",".",:svg));

#################
# part b)
X0 = [-1.53; -1.5; 1.58]
V0 = [0; 0; 0]
```

```
x_in = zeros(L); x_in[1] = X0[1];
y_in = zeros(L); y_in[1] = X0[2];
z_in = zeros(L); z_in[1] = X0[3];
vx_in = zeros(L); vx_in[1] = V0[1];
vy_in = zeros(L); vy_in[1] = V0[2];
vz_in = zeros(L); vz_in[1] = V0[3];

# integrate using inertial navigation
for i in 1:L-1
 vx_in[i+1] = vx_in[i] + ax[i]*(t[i+1] - t[i])
 vy_in[i+1] = vy_in[i] + ay[i]*(t[i+1] - t[i])
 vz_in[i+1] = vz_in[i] + az[i]*(t[i+1] - t[i])
 x_in[i+1] = x_in[i] + vx_in[i]*(t[i+1] - t[i])
 y_in[i+1] = y_in[i] + vy_in[i]*(t[i+1] - t[i])
 z_in[i+1] = z_in[i] + vz_in[i]*(t[i+1] - t[i])
end

# differentiate Vicon position data
vxi = zeros(L)
vyi = zeros(L)
vzi = zeros(L)

for i in 1:L-1
 vxi[i] = (xi[i+1] - xi[i])*(t[i+1] - t[i])
 vyi[i] = (yi[i+1] - yi[i])*(t[i+1] - t[i])
 vzi[i] = (zi[i+1] - zi[i])*(t[i+1] - t[i])
end

s1 = "inertial navigation"
s2 = "using Vicon"

# position
p1 = plot(t,x_in,line=l1,label=s1)
plot!(t,xi,line=l2,label=s2)
yaxis!("x (m)")
xaxis!("Time (s)")

p2 = plot(t,y_in,line=l1,label=s1)
plot!(t,yi,line=l2,label=s2)
yaxis!("y (m)")
xaxis!("Time (s)")

p3 = plot(t,z_in,line=l1,label=s1)
```

```
plot!(t,zi,line=l2,label=s2)
yaxis!("z (m)")
xaxis!("Time (s)")

plot(p1,p2,p3,layout=@layout([a;b;c]),size=[600,600])

savefig(string("figs/p3_a3",".",:svg));

plot(x_in,y_in,z_in,line=l1,label=s1,cbar=false)
plot!(xi,yi,zi,line=l2,label=s2,cbar=false,size=[800,800])
xaxis!("x (m)")
yaxis!("y (m)")
savefig(string("figs/p3_a4",".",:svg));

# velocity
p1 = plot(t,vx_in,line=l1,label=s1)
plot!(t,vxi,line=l2,label=s2)
yaxis!("vx (m/s)")
xaxis!("Time (s)")

p2 = plot(t,vy_in,line=l1,label=s1)
plot!(t,vyi,line=l2,label=s2)
yaxis!("vy (m/s)")
xaxis!("Time (s)")

p3 = plot(t,vz_in,line=l1,label=s1)
plot!(t,vzi,line=l2,label=s2)
yaxis!("vz (m/s)")
xaxis!("Time (s)")

plot(p1,p2,p3,layout=@layout([a;b;c]),size=[600,600])

savefig(string("figs/p3_a5",".",:svg));

plot(vx_in,vy_in,vz_in,line=l1,label=s1,cbar=false)
plot!(vxi,vyi,vzi,line=l2,label=s2,cbar=false,size=[800,800])
xaxis!("vx (m/s)")
yaxis!("vy (m/s)")
savefig(string("figs/p3_a6",".",:svg));

#################
# part c)
# calculate a using quaternion data from Vicon system
axi = zeros(L)
```

```
ayi = zeros(L)
azi = zeros(L)

for i in 1:L-1
 ab = [ax_b[i];ay_b[i];az_b[i]]
 a = R(q_x[i],q_y[i],q_z[i],q_w[i])*ab + [0;0;-9.8]
 axi[i] = a[1]; ayi[i] = a[2]; azi[i] = a[3];
end

# inertial acceleration plots
p1 = plot(t,ax,line=l1,label=s1)
plot!(t,axi,line=l2,label=s2)
yaxis!(string("ax, ",L"$\frac{m}{s^2}$"))
xaxis!("Time (s)")

p2 = plot(t,ay,line=l1,label=s1)
plot!(t,ayi,line=l2,label=s2)
yaxis!(string("ay, ",L"$\frac{m}{s^2}$"))
xaxis!("Time (s)")

p3 = plot(t,az,line=l1,label=s1)
plot!(t,azi,line=l2,label=s2)
yaxis!(string("az, ",L"$\frac{m}{s^2}$"))
xaxis!("Time (s)")

plot(p1,p2,p3,layout=@layout([a;b;c]),size=[600,600])

savefig(string("figs/p3_a7",".",:svg));

# Kalman Filter
v = eye(3)
w = [0 0 0;
      1 0 0;
      0 0 0;
      0 1 0;
      0 0 0;
      0 0 1]
Ra = eye(3)*10.0^(-4)
Rp = eye(3)*0.025
Rw = w*Rp*w'
Rv = v*Ra*v'

x_k = zeros(L); x_k[1] = X0[1];
y_k = zeros(L); y_k[1] = X0[2];
```

```
z_k = zeros(L); z_k[1] = X0[3];
vx_k = zeros(L); vx_k[1] = V0[1];
vy_k = zeros(L); vy_k[1] = V0[2];
vz_k = zeros(L); vz_k[1] = V0[3];

x = zeros(L,6); x[1,1:3] = X0; x[1,4:6] = V0;
P = zeros(L,6,6)
for i in 1:L-1
 dt = d[:t][i+1] - d[:t][i]
 A = [1  dt 0  0  0  0;
      0  1  0  0  0  0;
      0  0  1  dt 0  0;
      0  0  0  1  0  0;
      0  0  0  0  1  dt;
      0  0  0  0  0  1]

 B = [0   0  0;
      dt  0  0;
      0   0  0;
      0   dt 0;
      0   0  0;
      0   0  dt]

 C = [1  dt 0 0  0  0;
      0 0  1 dt  0  0;
      0 0  0 0  1  dt]

 # time update (predict)
 u = [axi[i];ayi[i];azi[i]]
 x[i+1,:] = A*x[i,:] + B*u
 P[i+1,:,:] = A*P[i,:,:]*A' + Rw

 # measurment update
 y = [xi[i];yi[i];zi[i]]
 K = P[i+1,:,:]*C'*inv((C*P[i+1,:,:]*C' + Rv))
 x[i+1,:] = x[i+1,:] + K*(y-C*x[i+1,:])
 P[i+1,:,:] = (eye(6) - K*C)*P[i+1,:,:]

 # save results
 x_k[i+1] = x[i+1,1]
 vx_k[i+1] = x[i+1,2]

 y_k[i+1] = x[i+1,3]
 vy_k[i+1] = x[i+1,4]
```

```
  z_k[i+1] = x[i+1,5]
  vz_k[i+1] = x[i+1,6]
end

s1 = "inertial navigation"
s2 = "using Vicon"
s3 = "Kalman Filter"

# position
p1 = plot(t,x_in,line=l1,label=s1)
plot!(t,xi,line=l2,label=s2)
plot!(t,x_k,line=l3,label=s3)
yaxis!("x (m)")
xaxis!("Time (s)")
ylims!(-5,5);

p2 = plot(t,y_in,line=l1,label=s1)
plot!(t,yi,line=l2,label=s2)
plot!(t,y_k,line=l3,label=s3)
yaxis!("y (m)")
xaxis!("Time (s)")
ylims!(-5,5);

p3 = plot(t,z_in,line=l1,label=s1)
plot!(t,zi,line=l2,label=s2)
plot!(t,z_k,line=l3,label=s3)
yaxis!("z (m)")
xaxis!("Time (s)")
ylims!(-5,5);

plot(p1,p2,p3,layout=@layout([a;b;c]),size=[600,600])

savefig(string("figs/p3_a8",".",:svg));

#plot(x_in,y_in,z_in,line=l1,label=s1,cbar=false)
plot(xi,yi,zi,line=l2,label=s2,cbar=false)
plot!(x_k,y_k,z_k,line=l3,label=s3,cbar=false,size=[800,800])
xaxis!("x (m)")
yaxis!("y (m)")
savefig(string("figs/p3_a9",".",:svg));

# velocity
p1 = plot(t,vx_in,line=l1,label=s1)
```

```
plot!(t,vxi,line=l2,label=s2)
plot!(t,vx_k,line=l3,label=s3)
yaxis!("vx (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

p2 = plot(t,vy_in,line=l1,label=s1)
plot!(t,vyi,line=l2,label=s2)
plot!(t,vy_k,line=l3,label=s3)
yaxis!("vy (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

p3 = plot(t,vz_in,line=l1,label=s1)
plot!(t,vzi,line=l2,label=s2)
plot!(t,vz_k,line=l3,label=s3)
yaxis!("vz (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

plot(p1,p2,p3,layout=@layout([a;b;c]),size=[600,600])

savefig(string("figs/p3_a10",".",:svg));

#################
# part d) i
Ra =[2.5  0   0;
     0    2.5 0;
     0    0   2.5]

s1 = "using Vicon"
s2 = "Kalman Filter, part c"
s3 = "Kalman Filter, part d-i"
s4 = "Kalman Filter, part d-ii"

# Kalman Filter
v = eye(3)
w = [0 0 0;
     1 0 0;
     0 0 0;
     0 1 0;
     0 0 0;
     0 0 1]
#Ra = eye(3)*10.0^(-4)
```

```
Rp = eye(3)*0.025
Rw = w*Rp*w'
Rv = v*Ra*v'

x_k2 = zeros(L); x_k[1] = X0[1];
y_k2 = zeros(L); y_k[1] = X0[2];
z_k2 = zeros(L); z_k[1] = X0[3];
vx_k2 = zeros(L); vx_k[1] = V0[1];
vy_k2 = zeros(L); vy_k[1] = V0[2];
vz_k2 = zeros(L); vz_k[1] = V0[3];

x = zeros(L,6); x[1,1:3] = X0; x[1,4:6] = V0;
P = zeros(L,6,6)
for i in 1:L-1
 dt = d[:t][i+1] - d[:t][i]
 A = [1  dt 0  0  0  0;
      0  1  0  0  0  0;
      0  0  1  dt 0  0;
      0  0  0  1  0  0;
      0  0  0  0  1  dt;
      0  0  0  0  0  1]

 B = [0   0  0;
      dt  0  0;
      0   0  0;
      0   dt 0;
      0   0  0;
      0   0  dt]

 C = [1 dt 0 0  0  0;
      0 0  1 dt 0  0;
      0 0  0 0  1  dt]

 # time update (predict)
 u = [axi[i];ayi[i];azi[i]]
 x[i+1,:] = A*x[i,:] + B*u
 P[i+1,:,:] = A*P[i,:,:]*A' + Rw

 # measurment update
 y = [xi[i];yi[i];zi[i]]
 K = P[i+1,:,:]*C'*inv((C*P[i+1,:,:]*C' + Rv))
 x[i+1,:] = x[i+1,:] + K*(y-C*x[i+1,:])
 P[i+1,:,:] = (eye(6) - K*C)*P[i+1,:,:]
```

```
  # save results
  x_k2[i+1] = x[i+1,1]
  vx_k2[i+1] = x[i+1,2]

  y_k2[i+1] = x[i+1,3]
  vy_k2[i+1] = x[i+1,4]

  z_k2[i+1] = x[i+1,5]
  vz_k2[i+1] = x[i+1,6]
end

# position
p1 = plot(t,xi,line=l1,label=s1)
plot!(t,x_k,line=l2,label=s2)
plot!(t,x_k2,line=l3,label=s3)
yaxis!("x (m)")
xaxis!("Time (s)")
ylims!(-5,5);

p2 = plot(t,yi,line=l1,label=s1)
plot!(t,y_k,line=l2,label=s2)
plot!(t,y_k2,line=l3,label=s3)
yaxis!("y (m)")
xaxis!("Time (s)")
ylims!(-5,5);

p3 = plot(t,zi,line=l1,label=s1)
plot!(t,z_k,line=l2,label=s2)
plot!(t,z_k2,line=l3,label=s3)
yaxis!("z (m)")
xaxis!("Time (s)")
ylims!(-5,5);

# velocity
p4 = plot(t,vxi,line=l1,label=s1)
plot!(t,vx_k,line=l2,label=s2)
plot!(t,vx_k2,line=l3,label=s3)
yaxis!("vx (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

p5 = plot(t,vyi,line=l1,label=s1)
plot!(t,vy_k,line=l2,label=s2)
plot!(t,vy_k2,line=l3,label=s3)
```

```
yaxis!("vy (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

p6 = plot(t,vzi,line=l1,label=s1)
plot!(t,vz_k,line=l2,label=s2)
plot!(t,vz_k2,line=l3,label=s3)
yaxis!("vz (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

plot(p1,p2,p3,p4,p5,p6,layout=@layout([a;b;c;d;e;f]),size=[600,1000])

savefig(string("figs/p3_all",".",:svg));

##################
# part d) ii
Ra = eye(3)*2.5^(-4)

s1 = "using Vicon"
s2 = "Kalman Filter, part c"
s3 = "Kalman Filter, part d-i"
s4 = "Kalman Filter, part d-ii"

# Kalman Filter
v = eye(3)
w = [0 0 0;
     1 0 0;
     0 0 0;
     0 1 0;
     0 0 0;
     0 0 1]
#Ra = eye(3)*10.0^(-4)
Rp = eye(3)*0.025
Rw = w*Rp*w'
Rv = v*Ra*v'

x_k3 = zeros(L); x_k[1] = X0[1];
y_k3 = zeros(L); y_k[1] = X0[2];
z_k3 = zeros(L); z_k[1] = X0[3];
vx_k3 = zeros(L); vx_k[1] = V0[1];
vy_k3 = zeros(L); vy_k[1] = V0[2];
vz_k3 = zeros(L); vz_k[1] = V0[3];
```

```
x = zeros(L,6); x[1,1:3] = X0; x[1,4:6] = V0;
P = zeros(L,6,6)
for i in 1:L-1
 dt = d[:t][i+1] - d[:t][i]
 A = [1  dt 0  0  0  0;
      0  1  0  0  0  0;
      0  0  1  dt 0  0;
      0  0  0  1  0  0;
      0  0  0  0  1  dt;
      0  0  0  0  0  1]

 B = [0   0   0;
      dt  0   0;
      0   0   0;
      0   dt  0;
      0   0   0;
      0   0   dt]

 C = [1  dt 0 0  0  0;
      0 0  1 dt 0  0;
      0 0  0 0  1  dt]

 # time update (predict)
 u = [axi[i];ayi[i];azi[i]]
 x[i+1,:] = A*x[i,:] + B*u
 P[i+1,:,:] = A*P[i,:,:]*A' + Rw

 # measurment update
 y = [xi[i];yi[i];zi[i]]
 K = P[i+1,:,:]*C'*inv((C*P[i+1,:,:]*C' + Rv))
 x[i+1,:] = x[i+1,:] + K*(y-C*x[i+1,:])
 P[i+1,:,:] = (eye(6) - K*C)*P[i+1,:,:]

 # save results
 x_k3[i+1] = x[i+1,1]
 vx_k3[i+1] = x[i+1,2]

 y_k3[i+1] = x[i+1,3]
 vy_k3[i+1] = x[i+1,4]

 z_k3[i+1] = x[i+1,5]
 vz_k3[i+1] = x[i+1,6]
end
```

```
# position
p1 = plot(t,xi,line=l1,label=s1)
plot!(t,x_k,line=l2,label=s2)
plot!(t,x_k2,line=l3,label=s3)
plot!(t,x_k3,line=l4,label=s4)
yaxis!("x (m)")
xaxis!("Time (s)")
ylims!(-5,5);

p2 = plot(t,yi,line=l1,label=s1)
plot!(t,y_k,line=l2,label=s2)
plot!(t,y_k2,line=l3,label=s3)
plot!(t,y_k3,line=l4,label=s4)
yaxis!("y (m)")
xaxis!("Time (s)")
ylims!(-5,5);

p3 = plot(t,zi,line=l1,label=s1)
plot!(t,z_k,line=l2,label=s2)
plot!(t,z_k2,line=l3,label=s3)
plot!(t,z_k3,line=l4,label=s4)
yaxis!("z (m)")
xaxis!("Time (s)")
ylims!(-5,5);

# velocity
p4 = plot(t,vxi,line=l1,label=s1)
plot!(t,vx_k,line=l2,label=s2)
plot!(t,vx_k2,line=l3,label=s3)
plot!(t,vx_k3,line=l4,label=s4)
yaxis!("vx (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

p5 = plot(t,vyi,line=l1,label=s1)
plot!(t,vy_k,line=l2,label=s2)
plot!(t,vy_k2,line=l3,label=s3)
plot!(t,vy_k3,line=l4,label=s4)
yaxis!("vy (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

p6 = plot(t,vzi,line=l1,label=s1)
plot!(t,vz_k,line=l2,label=s2)
```

```
plot!(t,vz_k2,line=l3,label=s3)
plot!(t,vz_k3,line=l4,label=s4)
yaxis!("vz (m/s)")
xaxis!("Time (s)")
ylims!(-1,1);

plot(p1,p2,p3,p4,p5,p6,layout=@layout([a;b;c;d;e;f]),size=[600,1000])

savefig(string("figs/p3_a12",".",:svg));


#################
# part e) i

# Kalman Filter
v = eye(3)
w = [0 0 0;
     1 0 0;
     0 0 0;
     0 1 0;
     0 0 0;
     0 0 1]
Ra = eye(3)*10.0^(-4)
Rp = eye(3)*0.025
Rw = w*Rp*w'
Rv = v*Ra*v'

x_k2 = zeros(L); x_k[1] = X0[1];
y_k2 = zeros(L); y_k[1] = X0[2];
z_k2 = zeros(L); z_k[1] = X0[3];
vx_k2 = zeros(L); vx_k[1] = V0[1];
vy_k2 = zeros(L); vy_k[1] = V0[2];
vz_k2 = zeros(L); vz_k[1] = V0[3];

x = zeros(L,6); x[1,1:3] = X0; x[1,4:6] = V0;
P = zeros(L,6,6)
for i in 1:L-1
 dt = d[:t][i+1] - d[:t][i]
 A = [1 dt 0  0  0  0;
      0  1 0  0  0  0;
      0  0  1 dt 0  0;
      0  0  0  1  0 0;
      0  0  0  0  1 dt;
      0  0  0  0  0  1]
```

```
 B = [0    0   0;
      dt   0   0;
      0    0   0;
      0   dt   0;
      0    0   0;
      0    0   dt]

 C = [1  dt  0 0   0   0;
      0 0   1  dt  0   0;
      0 0   0 0   1   dt]

 # time update (predict)
 u = [axi[i];ayi[i];azi[i]]
 x[i+1,:] = A*x[i,:] + B*u
 P[i+1,:,:] = A*P[i,:,:]*A' + Rw

 # measurment update
 if i==1 || (i-1) % 3 == 0
  y = [xi[i];yi[i];zi[i]]
 end
 K = P[i+1,:,:]*C'*inv((C*P[i+1,:,:]*C' + Rv))
 x[i+1,:] = x[i+1,:] + K*(y-C*x[i+1,:])
 P[i+1,:,:] = (eye(6) - K*C)*P[i+1,:,:]

 # save results
 x_k2[i+1] = x[i+1,1]
 vx_k2[i+1] = x[i+1,2]

 y_k2[i+1] = x[i+1,3]
 vy_k2[i+1] = x[i+1,4]

 z_k2[i+1] = x[i+1,5]
 vz_k2[i+1] = x[i+1,6]
end


#################
# part e) ii

# Kalman Filter
v = eye(3)
w = [0 0 0;
     1 0 0;
```

```
     0 0 0;
     0 1 0;
     0 0 0;
     0 0 1]
Ra = eye(3)*10.0^(-4)
Rp = eye(3)*0.025
Rw = w*Rp*w'
Rv = v*Ra*v'

x_k3 = zeros(L); x_k[1] = X0[1];
y_k3 = zeros(L); y_k[1] = X0[2];
z_k3 = zeros(L); z_k[1] = X0[3];
vx_k3 = zeros(L); vx_k[1] = V0[1];
vy_k3 = zeros(L); vy_k[1] = V0[2];
vz_k3 = zeros(L); vz_k[1] = V0[3];

x = zeros(L,6); x[1,1:3] = X0; x[1,4:6] = V0;
P = zeros(L,6,6)
for i in 1:L-1
 dt = d[:t][i+1] - d[:t][i]
 A = [1 dt 0  0  0  0;
      0  1 0  0  0  0;
      0  0  1 dt 0  0;
      0  0  0  1  0 0;
      0  0  0  0  1 dt;
      0  0  0  0  0  1]

 B = [0   0  0;
      dt  0  0;
      0   0  0;
      0  dt  0;
      0   0  0;
      0   0  dt]

 C = [1 dt 0 0  0  0;
      0 0  1 dt 0  0;
      0 0  0 0  1  dt]

 # time update (predict)
 u = [axi[i];ayi[i];azi[i]]
 x[i+1,:] = A*x[i,:] + B*u
 P[i+1,:,:] = A*P[i,:,:]*A' + Rw

 # measurment update
```

```
  if i==1 || (i-1) % 10 == 0
   y = [xi[i];yi[i];zi[i]]
  end
  K = P[i+1,:,:]*C'*inv((C*P[i+1,:,:]*C' + Rv))
  x[i+1,:] = x[i+1,:] + K*(y-C*x[i+1,:])
  P[i+1,:,:] = (eye(6) - K*C)*P[i+1,:,:]

  # save results
  x_k3[i+1] = x[i+1,1]
  vx_k3[i+1] = x[i+1,2]

  y_k3[i+1] = x[i+1,3]
  vy_k3[i+1] = x[i+1,4]

  z_k3[i+1] = x[i+1,5]
  vz_k3[i+1] = x[i+1,6]
end

s1 = "using Vicon"
s2 = "Kalman Filter, part c"
s3 = "updated every 3rd"
s4 = "udated every 10th"

# position
p1 = plot(t,xi,line=l1,label=s1)
plot!(t,x_k,line=l2,label=s2)
plot!(t,x_k2,line=l3,label=s3)
plot!(t,x_k3,line=l4,label=s4)
yaxis!("x (m)")
xaxis!("Time (s)")
ylims!(-5,5);

p2 = plot(t,yi,line=l1,label=s1)
plot!(t,y_k,line=l2,label=s2)
plot!(t,y_k2,line=l3,label=s3)
plot!(t,y_k3,line=l4,label=s4)
yaxis!("y (m)")
xaxis!("Time (s)")
ylims!(-5,5);

p3 = plot(t,zi,line=l1,label=s1)
plot!(t,z_k,line=l2,label=s2)
plot!(t,z_k2,line=l3,label=s3)
plot!(t,z_k3,line=l4,label=s4)
```

```
yaxis!("z (m)")
xaxis!("Time (s)")
ylims!(-5,5);

# velocity
p4 = plot(t,vxi,line=l1,label=s1)
plot!(t,vx_k,line=l2,label=s2)
plot!(t,vx_k2,line=l3,label=s3)
plot!(t,vx_k3,line=l4,label=s4)
yaxis!("vx (m/s)")
xaxis!("Time (s)")
ylims!(-.1,.1);

p5 = plot(t,vyi,line=l1,label=s1)
plot!(t,vy_k,line=l2,label=s2)
plot!(t,vy_k2,line=l3,label=s3)
plot!(t,vy_k3,line=l4,label=s4)
yaxis!("vy (m/s)")
xaxis!("Time (s)")
ylims!(-.1,.1);

p6 = plot(t,vzi,line=l1,label=s1)
plot!(t,vz_k,line=l2,label=s2)
plot!(t,vz_k2,line=l3,label=s3)
plot!(t,vz_k3,line=l4,label=s4)
yaxis!("vz (m/s)")
xaxis!("Time (s)")
ylims!(-.1,.1);

plot(p1,p2,p3,p4,p5,p6,layout=@layout([a;b;c;d;e;f]),size=[600,1000])

savefig(string("figs/p3_a13",".",:svg));
```