AE584 **Name (Print):** ____________________

Fall 2017

Final "Take Home" Exam

Due 12/15/17 5:00 pm

Time Limit: None Honor Code ____________________

This exam contains 7 pages (including this cover page) and 5 problems.

You *may* use your notes (electronic form is fine), the textbook and a calculator on this exam. You *may not* use tools such as MATLAB, *unless otherwise stated in the problem.* You may *not* work with your classmates on this exam. You should compile all your answers in **a single PDF file**. Include your plots in the same file. **You do not have to submit your codes.** Do *not* post any questions related to the exam on Piazza. Mail the instructor directly, if needed.

Return your exam by Friday, December 15th, at 5pm, on Canvas. *By the act of electronic submission of your exam solutions, you pledge that you have neither given nor received unauthorized help with this exam and that you agree to abide with the Michigan CoE honor code.*

Hint: For at least one problem, you will find the book "Tactical and Strategic Missile Guidance. Sixth Edition" by Paul Zarchan particularly useful. An e-book version is available from the Library.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	40	
3	30	
4	15	
5	15	
Total:	115	

1. (15 points) No MATLAB or other form of computational/software tool is permitted.

   Consider a vehicle moving along a straight line under constant acceleration and from zero initial position and velocity. The vehicle's motion is modeled as a double integrator.

   (a) (10 points) Use the method of adjoints to evaluate the position of the vehicle at a given final time.

   (b) (5 points) Compare the result you obtained with the result from the variation-of-constants formula.

2. (40 points) No MATLAB or other form of computational/software tool is permitted.

   An airplane is flying at a constant altitude under constant velocity V. The equations of motion and initial conditions are given as:

$$\begin{aligned} \dot{x}(t) &= V\cos\psi(t), & x(0) &= 0, \\ \dot{y}(t) &= V\sin\psi(t), & y(0) &= 0, \\ \dot{\psi}(t) &= \frac{V}{R}, & \psi(0) &= 0, \end{aligned}$$

   where R is a constant, known parameter. Assume the inertial frame and the body-fixed frame at time $t = 0$ coincide.

   (a) (10 points) At what time is the aircraft orientation equal to 180 degrees relative to the inertial frame? What are the position coordinates at this time instant expressed in the inertial frame? What type of maneuver does the aircraft perform?

   (b) (10 points) Assume we are given nominal values R^0, V^0. Provide the linearized equations of the aircraft position trajectories $\delta x(t)$, $\delta y(t)$ in terms of δR, δV.

   (c) (10 points) Assume now that δR and δV are jointly Gaussian, with known covariance matrix:

$$P = \begin{bmatrix} \sigma_{RR} & \sigma_{RV} \\ \sigma_{VR} & \sigma_{VV} \end{bmatrix}$$

   Derive closed form expressions for the variances of the position coordinates $\delta x(t)$, $\delta y(t)$.

   (d) (10 points) Let us now go back to the initial system, and assume that V is constant and known, but the ratio $\frac{V}{R} = \omega$ is constant yet unknown. Consider that the measured outputs are the position coordinates $x(t)$, $y(t)$. Assume that the system dynamics and the system output are subject to additive noises $w(t)$ and $v(t)$, respectively, under the usual assumptions we made in class. Develop a continuous-time **nonlinear** estimator to obtain an estimate for ω. To this end, state all the necessary assumptions on the uncertainties present. [Hint: You might want to make ω appear in a properly defined state vector that is being estimated.]

3. (30 points) No MATLAB or other form of computational/software tool is permitted.

   An airplane starts its guided approach to landing at time $t_0 = 0$ sec (Figure 1). The equations

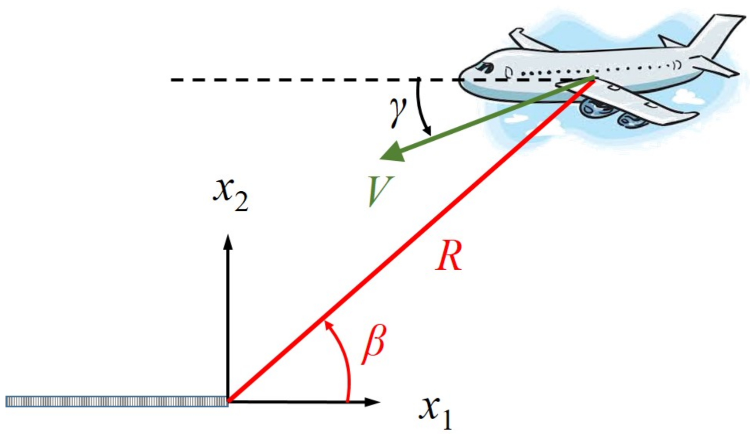

Figure 1: Problem 3: Guided Approach to Landing.

of motion are:

$$\begin{aligned} \dot{x}_1 &= -V\cos\gamma, \\ \dot{x}_2 &= -V\sin\gamma, \\ y &= x_2, \end{aligned}$$

where x_1 is the horizontal distance traveled, x_2 is the altitude, V is the velocity of the aircraft, and γ is the flight path angle of the aircraft and control input.

(a) (10 points) Write the equations of motion of the aircraft in the fundamental homing guidance form in terms of the range R and the elevation β as seen from the runway.

(b) (10 points) Write the linearized system describing the homing equations around a given nominal trajectory R^0, β^0, γ^0.

(c) (10 points) Develop navigation equations to obtain estimates of the linearized states based on the linearized output. What are the estimation error equations? How should the gains be selected so that the estimation error is asymptotically stable?

4. (30 points) MATLAB or other similar tool is permitted.

The mission for a micro aerial vehicle (MAV) is defined as to track a ground vehicle (GV) by means of vision-based measurements from its onboard camera. Let $\mathcal{W}$, $\mathcal{B}$, $\mathcal{I}$ denote the world (considered inertial) frame, MAV body-fixed frame, and image frame, respectively. $\mathcal{I}$ is fixed at the center of the MAV with its axes aligned with those of $\mathcal{B}$; therefore, the rotation matrix from $\mathcal{B}$ to $\mathcal{I}$ is the identity matrix.

The MAV camera system measures the location of the center of the GV in the $\mathcal{I}$-frame, corrupted by some additive noise v that is modeled as zero-mean, Gaussian, white process of known covariance matrix R. We use the "camera pinhole model" to transform the position coordinates of the GV from $\mathcal{I}$ to $\mathcal{B}$, as follows:

Let $x^{\mathcal{D}}$ be the MAV's position vector and $x_T^{\mathcal{D}}$ the GV's position vector in some arbitrary frame $\mathcal{D} \in \{\mathcal{I}, \mathcal{B}, \mathcal{W}\}$, then the position error vector is defined as:

$$e^{\mathcal{D}} = x_T^{\mathcal{D}} - x^{\mathcal{D}}. \tag{1}$$

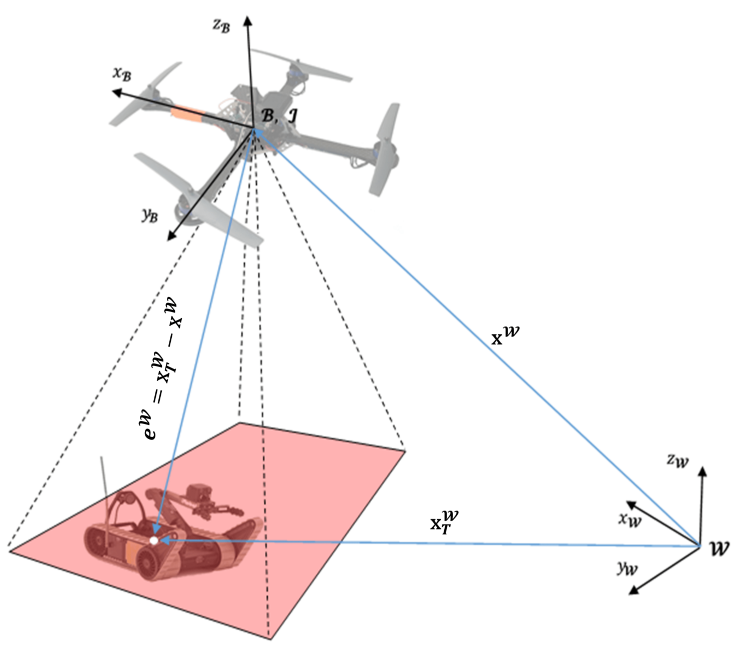

Figure 2: Problem 4. The geometry of the MAV-GV system.

Using the camera pinhole model with additive noise, the measurement equation for the position error vector e in $\mathcal{I}$ is:

$$e^{\mathcal{I}} = \lambda^{-1} K_{intr} C_{intr} e^{\mathcal{B}} + v, \tag{2}$$

where λ is the distance between MAV and the focal point (which also coincides with the absolute value of the z-axis component of $e^{\mathcal{B}}$), K_{intr} is the camera intrinsic matrix, given by:

$$K_{intr} = \begin{bmatrix} 1030.597415177913 & 0 & 361.451236121491 \\ 0 & 1030.358516382353 & 246.1238464630347 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

and

$$C_{intr} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{4}$$

Now, the equations of motion of the MAV resolved in $\mathcal{B}$ frame are given as follows:

$$\dot{e}^{\mathcal{B}} = -\omega^{\mathcal{B}\times} e^{\mathcal{B}} + {}^{\mathcal{B}}R_{\mathcal{W}} \dot{x_T}^{\mathcal{W}} - {}^{\mathcal{B}}R_{\mathcal{W}} \dot{x}^{\mathcal{W}}, \tag{5}$$

where $\omega^{\mathcal{B}\times}$ is the angular velocity matrix in Poisson's equation, and ${}^{\mathcal{B}}R_{\mathcal{W}}$ is the rotation matrix from $\mathcal{W}$-frame to $\mathcal{B}$-frame.

We further simplify (5) by considering the effect of the GV's motion as an additive disturbance w, modeled as a zero-mean, Gaussian, white noise process of known covariance Q. Then (5) becomes:

$$\dot{e}^{\mathcal{B}} = -\omega^{\mathcal{B}\times} e^{\mathcal{B}} - u + w, \tag{6}$$

where $u = {}^{\mathcal{B}}R_{\mathcal{W}}\dot{x}^{\mathcal{W}}$ and $w = {}^{\mathcal{B}}R_{\mathcal{W}}\dot{x_T}^{\mathcal{W}}$.

(a) (10 points) The MAV is flying under the full state feedback control law:

$$u = (-\omega^{\mathcal{B}\times} + K)e^{\mathcal{B}}, \tag{7}$$

where $K = \begin{bmatrix} k_v^x & 0 & 0 \\ 0 & k_v^y & 0 \\ 0 & 0 & k_v^z \end{bmatrix}$ and k_v^x=0.5, k_v^y=0.6, $k_v^z = 0.07$. The flight data are given in the attached .mat file. The columns of the matrix `D` are organized as follows:

t	x_X^{Vicon}	x_Y^{Vicon}	x_Z^{Vicon}	q_x	q_y	q_z	q_w	$z_X^{\mathcal{W}}$	$z_Y^{\mathcal{W}}$	$e_X^{\mathcal{B}}$	$e_Y^{\mathcal{B}}$	$e_Z^{\mathcal{B}}$	ξ	$e_1^{\mathcal{I}}$	$e_2^{\mathcal{I}}$

where time t is in seconds, the position variables (x_{XYZ}, z_{XY}, and e_{XYZ}) are in meters, and the camera measurements ($e_1^{\mathcal{I}}$, $e_2^{\mathcal{I}}$) are given in pixel units. Furthermore, z_Z is zero at all times in $\mathcal{W}$ frame, and $e_3^{\mathcal{I}}$ is always equal to 1. q_x, q_y, q_z, and q_w are quaternion variables. (We do not consider ξ in this problem.)

The covariance matrices of the process noise w and the measurement noise v, respectively, are given as:

$$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{8}$$

Using the provided data, measurement model and for the *closed-loop* system dynamics governing the motion of the MAV-GV system, design a Kalman filter to estimate the state of the system $e^{\mathcal{B}}$. Deduce the position trajectory $x^{\mathcal{W}}$ of the MAV in the $\mathcal{W}$ frame, and compare it with the vicon data provided in the `.mat` file. Plot the trajectories and compare the results in your answer sheet.

(b) (5 points) Following the same procedure, obtain now an estimate for the state of the system $e^{\mathcal{B}}$ for the same process noise covariance matrix Q, and for measurement noise covariance matrix $R = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. Compare the result of this estimate with the result from the previous question.

5. (15 points) MATLAB or other similar tool is permitted. [Extra Credit Problem]

Iron Man is defending Earth in a battle with an evil alien! The evil alien can shoot one destructive square along with a harmless "shadow" square. Iron Man's mission is to catch the destructive square. However, towards the end of the semester his sensing systems are not functioning that well, hence Iron Man cannot distinguish the true destructive square from the harmless shadow square unless he approaches sufficiently close to the targets. Therefore, what Iron Man decides to do is to fly under a Proportional Navigation guidance law towards the power centroid of the two squares. When the Time-to-Go between Iron Man and the power centroid is 1 sec, Iron Man can distinguish which square is the one he needs to catch.

The first square is at 1200-ft altitude while the second square is at 800-ft altitude. The power centroid is located halfway between those squares at 1000-ft altitude. Initially, the horizontal distance between Iron Man and the squares is 20,000 ft. Both squares are traveling at 1000 ft/sec. (So is the power centroid.) Iron Man is at 1000-ft altitude and is moving at 3000 ft/sec toward the power centroid. As an intelligent engineer, you know the destructive square is the one flying at 1200 ft altitude. The effective navigation constant N is 3. See also Figure 3.

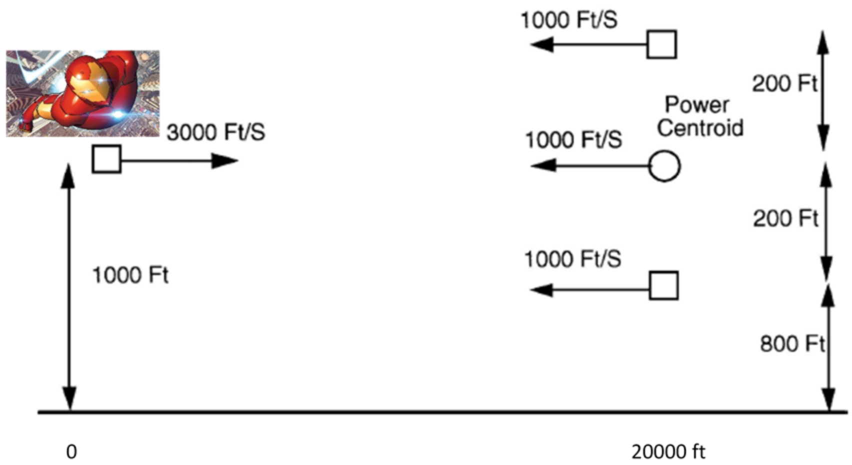

Figure 3: Problem 5. The geometry of the "Iron Man vs Squares" Battle.

(a) (3 points) Plot the position trajectories of Iron Man and the of the target that Iron Man aims at all times. (Note that initially Iron Man's target is the power centroid, and then when the Time-to-Go to the power centroid is 1 sec, Iron Man's target is the destructive square).

(b) (3 points) Plot the acceleration over time required by Iron Man so that he catches the destructive square.

(c) (3 points) Assume now that Iron Man can make the decision on which is the actual target to go at various Time-to-Go values. Plot the maximum acceleration required by Iron Man vs Time-to-Go, for this Time-to-Go varying between 0.1 sec up to 10 sec.

Hint: The closing velocity V_c is defined as the negative rate of change of the missile target separation. When the closing velocity is zero, we have a collision.

Let us now consider that Iron Man has to face an additional challenge. Due to the alien's strong electromagnetic waves, Iron Man's guidance system cannot instantaneously track the control commands, but it subject to first-order autopilot dynamics with time constant $T = 1$ sec. The control action is described in the following diagram, where λ is the line-of-sight, and D can be thought of as Iron Man's real heading angle due to the effect of the time constant. At initial time of the engagement, the value of D is same as the value of λ.

(a) (3 points) Plot the trajectory of Iron Man and the target Iron Man aims all the time (first power centroid then the destructive square).

(b) (3 points) Can Iron Man catch the destructive square in this scenario? If not, by how much does he miss the target?

Hint: When the closing velocity is negative, we have a miss.

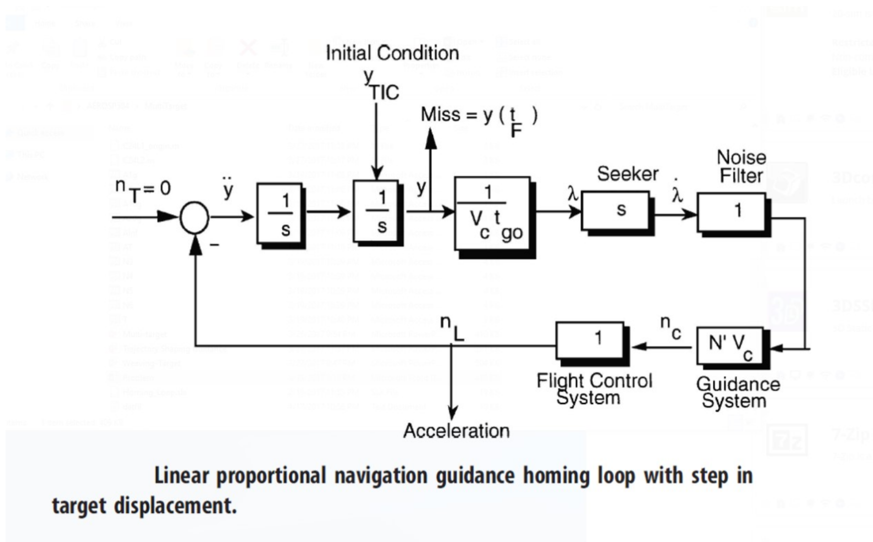

Figure 4: Problem 5. Linear Proportional Navigation Guidance Homing Loop with Step in Acceleration Displacement.

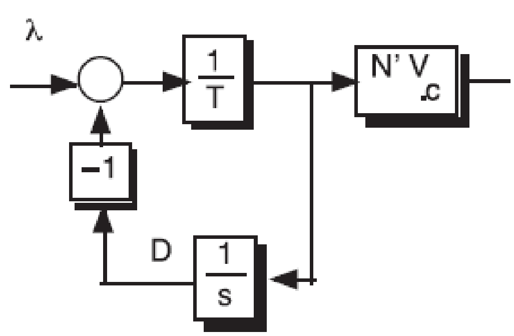

Figure 5: Problem 5. Block diagram of the autopilot dynamics.