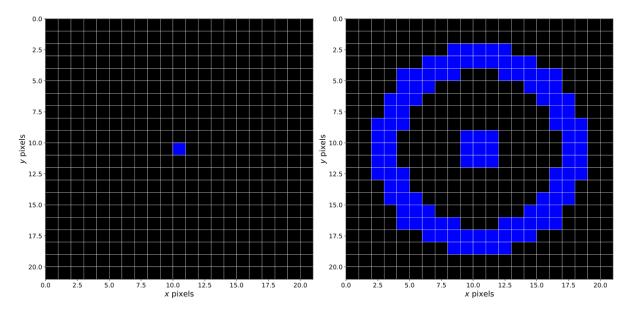
## **Gravitational Lensing Figures**

by Willem Davison



**Figure 1.** For all functions in this program, images are 2 dimensional arrays with each value of the array representing the brightness of the corresponding pixel. The source image, on the left, is a single central pixel in a 21x21 pixels image. This plot is lensed with an  $r_c$  value of 0.7 to produce the plot on the right. This plot varies slightly from the test case due to differences in pixels sampling positions. The function will therefor be tested in another way as shown in Figure 2 to ensure its accuracy.

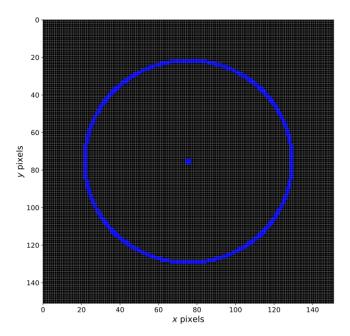
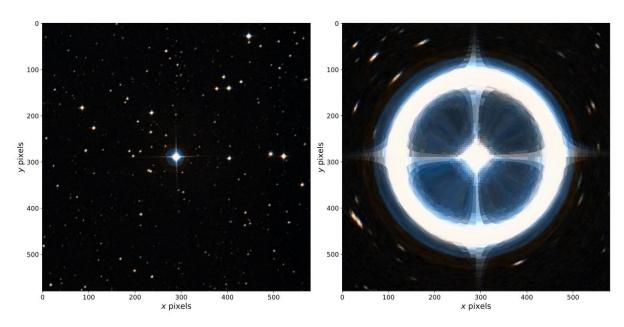


Figure 2. This figure is produced almost identically to the lensed image in Figure 1 with the only difference being a resolution of  $151 \times 151$  pixels used instead of  $21 \times 21$ . To test the accuracy of the program, the equation  $r = \sqrt{1 - r_c^2}$  is used to see if the Einstein ring produced is of the correct size. The used value of  $r_c$ , 0.7, gives an expected radius ratio of 0.714. The pixel separation from the outermost ring pixels is 108. Our experimental value of radius ratio is therefore  $\frac{108}{151} = 0.715$ . These values (to higher orders of accuracy) give a relative uncertainty of 0.152%. This small uncertainty confirms the program to be working as expected and to be producing accurate plots.



**Figure 3.** In this function, an uploaded image, shown to the left, is lensed. This is done by converting the image into an rxrx3 array then splitting this array into 3 rxr arrays of brightness values for each colour red, green, and blue. Each colour array is then lensed. The red, green, and blue lensed arrays are recombined to create another rxrx3 array which can be plotted to produce the image on the right. Decoding an image into RGB arrays means that lensing of uploaded images is very simple as they can be lensed through the same function used to make Figures 1 & 2. For this figure, the value of  $r_c$  is 0.8.

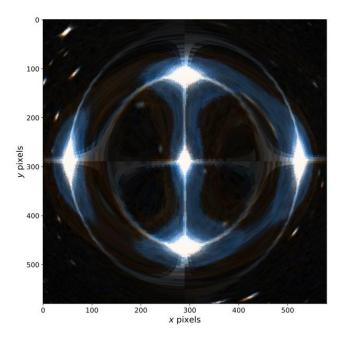
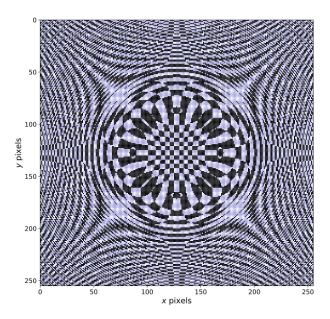
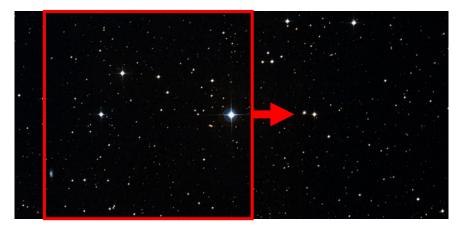


Figure 4. This figure demonstrates the effect of lensing with an elliptical lens object. The source image is the same as the source used in Figure 3 but the value of  $r_c$  is 0.7. A ellipticity of  $\epsilon$ =0.1 is used. This is a more accurate simulation of what observed gravitational lensing should look like as elliptical galaxies are the most abundant type of galaxy found in our universe. The value of  $\epsilon$  used is a reasonable estimate of ellipticity of a lens galaxy. Comparing this figure to the lensed image in Figure 3, we can see how an Einstein cross is formed instead of an Einstein ring due to the change in ellipticity.



**Figure 5.** A good way to show the general shape of the lensing produced is to lens a chessboard like source as shown above. In this figure, a resolution of  $255 \times 255$  is used with an  $r_c$  value of 0.8. The regularity of the chessboard pattern makes this plot an effective way to show the distortions of the lensing. It is interesting to note the central part of this image as being almost perfectly chessboard-like, the most central square being magnified by a factor of 25. This effect can be seen, to a lesser extent, in other plots such as in Figure 3



**Figure 6.** This image demonstrates what is being shown in the GIF file, MovingLens.gif, within my submitted folder. A rectangular source image is taken and the same function as used in Figure 3 is used to create many frames of a GIF. This is done by looping through the program and increasing the x pixel starting point each time. The images produced are then combined to make the GIF. This simulates lensing from a massive object with the background source moving to the left relative to the lens object. Once I was content with my lensing tool, I decided to work on studying the effect of a moving lens. In comparison to our lifetimes, astronomical objects move incredibly slowly and so effects like this could not be observed. The question: "What would a 10-million-year video of a galaxy's lensing look like?" is what lead me to the development of this figure.

## VaryingElip.gif

Figure 7. This figure refers to the file named VaryingElip.gif found within my submitted folder. In Figure 5, lensing with an object of constant ellipticity is shown but to best showcase the effect of ellipticity I believe varying the value is needed. By varying ellipticity, we can directly see comparison between a large and small value of  $\varepsilon$ . This simulation could not compare to a real-life example as ellipticity is an almost constant property of astronomical objects such as galaxies. The production of this GIF works on the same principles as in Figure 6 but instead of varying lens position, the ellipticity of the lens is varied. The value of  $\varepsilon$  is varied from -1 to 1 to show the entire range of values.

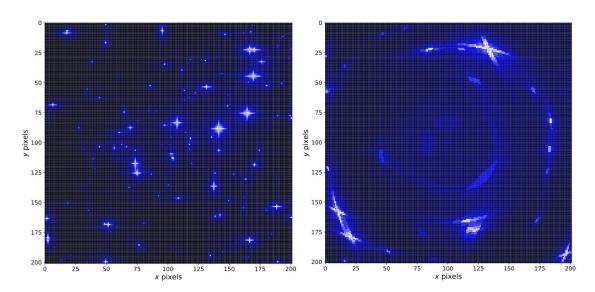


Figure 8. This figure shows the two images produce by the final function of this program. This function produces a source image of a random distribution of galaxies, shown on he left, with random ellipticities and brightnesses (exponentially decaying radially for each galaxy). This is done by randomly selecting values x brightness, y brightness and position for each galaxy. The effective brightness of every galaxy on each pixel is then stacked to find a final value of brightness for that pixel. The image on the right is of the source image lensed through values  $\epsilon$ =0 and  $r_c$ =0.95. This plot effectively showcases lensing on distant galaxies which may not be perfectly in line with lens objects. Both plots are of size 201x201 pixels and 150 galaxies were created to produce the source image.