

# EL SOLUCIONARIO

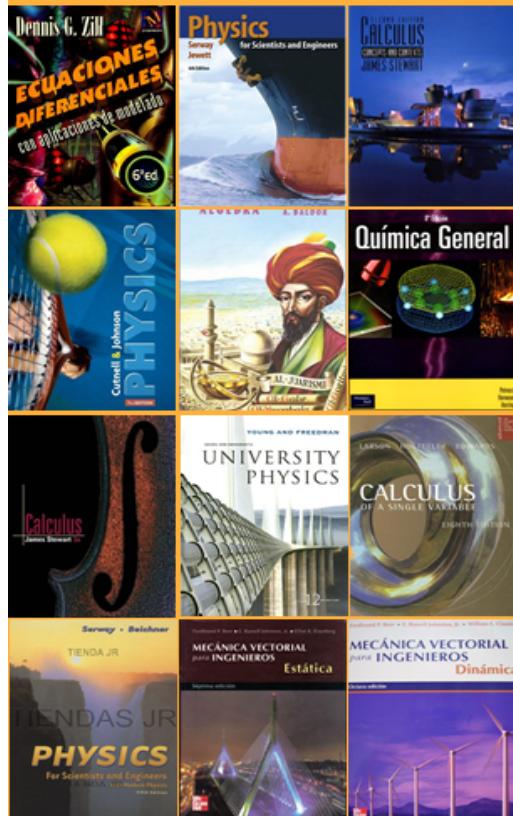
# EL SOLUCIONARIO



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## EL SOLUCIONARIO



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# **STUDENT SOLUTIONS MANUAL**

**INTERNATIONAL EDITION**

**FOR**

**DIGITAL AND ANALOG  
COMMUNICATION SYSTEMS**  
**7TH EDITION, INTERNATIONAL EDITION**

**LEON W. COUCH, II**

**2006**



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## PREFACE and ACKNOWLEDGEMENTS

This **Student Solutions Manual** for *Digital and Analog Communication Systems*, 7th Edition (Internation Version) **contains complete solutions for the problems in the 7th Edition that are marked with a ★.**

Within the textbook you will often see a computer symbol. This designates that files with MATLAB and MATHCAD computer solutions are available. However, within the Solutions Manual itself, a MATHCAD printed solution is shown. (MATHCAD solutions are shown since they clearly display the algorithms used and the output takes up less space.)

MATHCAD files and MATLAB M files for these problems can be downloaded from the Internet Web Sites maintained by the author.

These websites are located at

<http://lcouch.us>

or

<http://www.couch.ece.ufl.edu>

In the textbook, a computer symbol is used to indicate that both MATLAB and MATHCAD solutions are provided for that material; although, for the student, only those homework problems marked with a ★ are available. (For the instructor, MATLAB and MATHCAD solutions are given for all material marked with a computer symbol. These are provided to the instructor only, for download from from Prentice Hall website located at <http://prenhall.com>).

This solutions manual was prepared by Leon W. Couch, II, with the help and valuable suggestions of many undergraduate and graduate electrical engineering students at the University of Florida. Their assistance is greatly appreciated. Several graduate students worked out solutions or contributed problems with solutions; they are:

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Thanks also to Ronald F. Smith who wrote the original code for many of the MATLAB M files.

The author values your comments and suggestions. Also, for future editions, new problems and problems with computer solutions are welcomed. Please send them to:

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# Chapter 1

1-6

Let  $p_1$  = prob. of sending a binary 1

(a)  $p_2$  = prob. of sending a binary 0 =  $1 - p_1$

$$H = \sum_{i=1}^2 p_i I_i = p_1 \log_2 \left( \frac{1}{p_1} \right) + (1-p_1) \log_2 \left( \frac{1}{1-p_1} \right)$$

$$H = \frac{1}{\ln 2} \left[ -p_1 \ln(p_1) - (1-p_1) \ln(1-p_1) \right]$$

$$\frac{\partial H}{\partial p_1} = 0 \Rightarrow -(\ln p_1 + 1) - ((-1) \ln(1-p_1) + \frac{1-p_1}{1-p_1} (-1)) = 0$$

$$\Rightarrow -\ln p_1 + \ln(1-p_1) + 1 = 0$$

$$\text{or } \ln \left( \frac{1-p_1}{p_1} \right) = 0 = \ln 1$$

$$\text{thus } \frac{1}{p_1} - 1 = 1 \Rightarrow p_1 = \frac{1}{2} = p_2$$

$$(b) H_{\max} = \frac{1}{2} \log_2 2 + (1-\frac{1}{2}) \log_2 2 = \underline{\underline{1 \text{ bit}}}$$

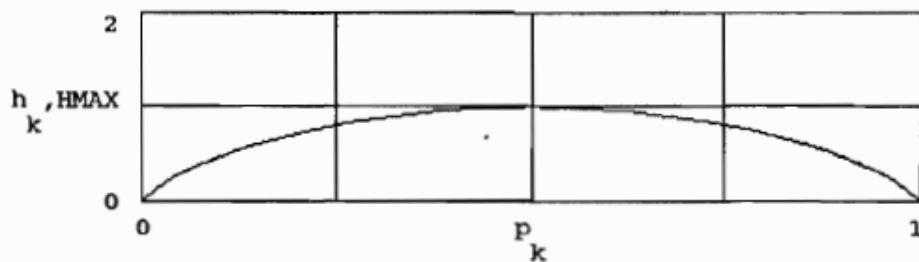
## Math CAD Solution

LET  $p$  = The probability for sending a binary 1, then the probability for sending a binary 0 is  $(1-p)$ . From the entropy formula for  $H(p)$ , we can draw the figure of  $H(p)$ , and from this figure, we can find the maximum entropy and the  $p$ .

$$H(p) = (p * \ln(p) + (1-p) * \ln(1-p)) / (-\ln(2))$$

$$k \equiv 0 \dots 50 \quad p_k := \frac{p}{50} \quad HMAX := 1$$

$$h_k := \frac{-1}{\ln(2)} \left[ p_k \ln \left[ p_k \right] + \left[ 1 - p_k \right] \cdot \ln \left[ 1 - p_k \right] \right]$$



From the above figure, we know the maximum entropy is 1 where the probability for sending 1 or 0 is 0.5.

1-8

$$M = 10 \quad P_j = \frac{1}{10} \quad j = 1, 10 \quad R = \frac{H}{T} = 2 \frac{b}{s}$$

$$H = -\frac{10(0.1) \ln .1}{\ln 2} = 3.322 \text{ bits}$$

$$T = \frac{H}{R} = \frac{3.322 \text{ bits}}{2 \text{ bits/sec}} = \underline{\underline{1.661 \text{ sec.} = T}}$$

1-11

(a) chars := 110 Number of characters available

$b := \text{ceil} \left[ \frac{\log(\text{chars})}{\log(2)} \right]$  Number of bits required to represent a character

$$\implies b = 7 \text{ bits}$$

(b) B := 3200 Hz Channel bandwidth  
SNRdB := 20 dB Signal to noise ratio

$$\frac{\text{SNRdB}}{10} \implies \text{SNR} = 100 \text{ (Absolute power ratio)}$$

$$C := B \cdot \left[ \frac{\log(1 + \text{SNR})}{\log(2)} \right]^4 \implies C = 2.131 \cdot 10^4 \text{ Channel capacity (bits/sec)}$$

$$C := \frac{C}{b} \implies C = 3.044 \cdot 10^3 \text{ Channel capacity (chars/sec)}$$

(c) Assuming equally likely characters,  
information content of each character is:

$$P := \frac{1}{\text{chars}} \quad \text{Probability of each character}$$

$$\log \left[ \frac{1}{P} \right]$$

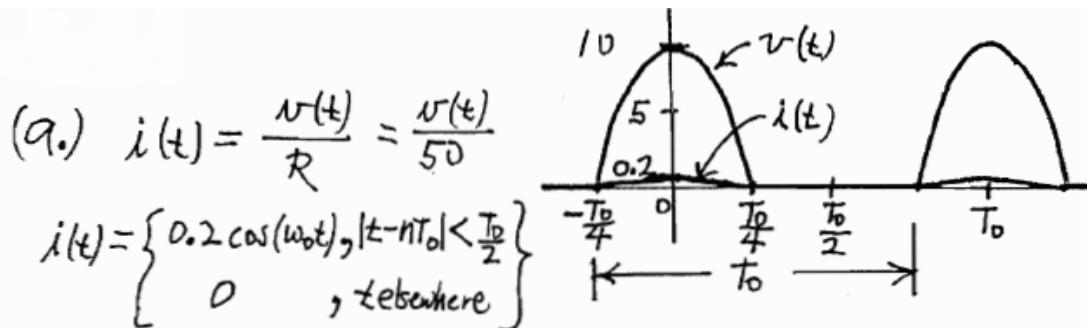
$$I := \frac{\log(2)}{\log(2)} \implies I = 6.781 \text{ bits}$$

## Chapter 2

2-1

$$\begin{aligned}
 v(t) &= A \sin \omega_0 t ; \quad V_{rms}^2 = \langle v^2(t) \rangle = \frac{1}{2} [1 + \cos(2\omega_0 t)] \\
 \langle v^2(t) \rangle &= \frac{1}{T_0} \int_0^{T_0} A^2 \sin^2 \omega_0 t dt = \frac{A^2}{T_0} \int_0^{T_0} [1 - \cos^2(\omega_0 t)] dt \\
 &= \frac{A^2}{T_0} \left[ T_0 - \frac{T_0}{2} - \frac{1}{2} \int_0^{T_0} \cos(2\omega_0 t) dt \right] = \frac{A^2}{T_0} \left( \frac{T_0}{2} \right) \\
 \Rightarrow V_{rms} &= \sqrt{\langle v^2(t) \rangle} = \sqrt{\frac{A^2}{2}} = \underline{\underline{\frac{A}{\sqrt{2}}}}
 \end{aligned}$$

2-3



(b.)  $V_{OC} = \langle v(t) \rangle = \frac{V_p}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \cos(\omega_0 t) dt = \frac{2V_p}{T_0} \frac{\sin(\omega_0 \frac{T_0}{4})}{\omega_0}$

$$= \frac{2V_p}{T_0} \frac{\sin\left(\frac{2\pi}{T_0} \frac{T_0}{4}\right)}{\frac{2\pi}{T_0}} = \frac{2}{2\pi} V_p \sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow V_{OC} = \frac{V_p}{\pi} = \frac{10}{\pi} = \underline{\underline{3.183 \text{ Volts}}}$$

$$\Rightarrow I_{OC} = \frac{V_p}{\pi} = \frac{0.2}{\pi} = \underline{\underline{0.064 \text{ Amps}}}$$

2-3 (Continued)

$$\begin{aligned}
 (c.) V_{rms}^2 &= \langle v^2(t) \rangle = \frac{1}{T_0} \int_0^{T_0/2} v^2(t) dt = \frac{V_p^2}{T_0} \int_{-T_0/4}^{T_0/4} \cos^2 \omega_b t dt \\
 \Rightarrow V_{rms}^2 &= \frac{V_p^2}{T_0} \int_{-T_0/4}^{T_0/4} \frac{1}{2} [1 + \cos(2\omega_b t)] dt = \frac{V_p^2}{2T_0} \left[ \frac{t}{2} + \frac{\sin(2\omega_b t)}{2\omega_b} \right] \Big|_{-T_0/4}^{T_0/4} \\
 &= \frac{V_p^2}{2T_0} \cdot \frac{3T_0}{4} = \frac{V_p^2}{4} = V_{rms}^2 \\
 \Rightarrow V_{rms} &= \frac{V_p}{2} = \frac{10}{2} = \underline{\underline{5 \text{ volts rms}}} \\
 I_{rms} &= \frac{I_0}{2} = \frac{0.2}{2} = \underline{\underline{0.1 \text{ amper}}}
 \end{aligned}$$

(d)  $P = \langle p(t) \rangle = V_{rms} I_{rms} = (5)(0.1) = \underline{\underline{0.5 \text{ watts}}}$

2-7

$$\begin{aligned}
 (a.) P_{in} &= \frac{V_{rms}^2}{R_{in}} = \frac{(3.5 \times 10^{-6})^2}{300} = \underline{\underline{4.083 \times 10^{-14} \text{ W}}} \\
 (b.) dB_m &= 10 \log_{10} \left( \frac{P}{10^{-3}} \right) = 10 \log_{10} \left( \frac{4.083 \times 10^{-14}}{10^{-3}} \right) = \underline{\underline{-103.9 \text{ dBm}}} \\
 (c.) P_{in} &\approx \frac{V_{rms}^2}{75} = 4.08 \times 10^{-14} \\
 \Rightarrow V_{rms} &= \sqrt{75 (4.08 \times 10^{-14})} = \underline{\underline{1.75 \mu \text{volts}}}
 \end{aligned}$$

2-10

$$\begin{aligned}
 W(f) &= \int_{-\infty}^{\infty} w(t) e^{j\omega t} dt = \int_{-\infty}^{\infty} e^{-(\alpha+j\omega)t} dt \\
 &= \frac{e^{-(\alpha+j\omega)t}}{-(\alpha+j\omega)} \Big|_1^{\infty} = \frac{e^{-\alpha} e^{-j2\pi f}}{\alpha + j2\pi f} = W(f)
 \end{aligned}$$

2-13

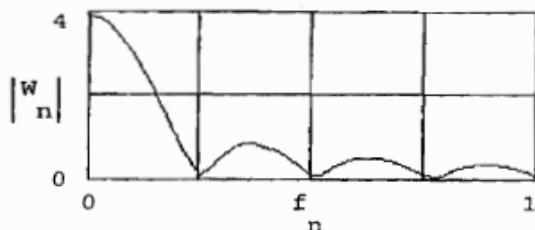
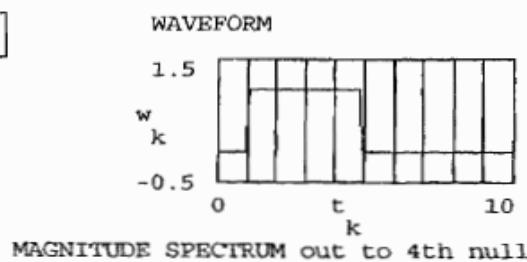
$$\begin{aligned}
 S(f) &= \int_{-\infty}^{\infty} s(t) e^{j\omega t} dt = \int_0^{T_0} A t e^{-j\omega t} dt \\
 &= A \left[ e^{j\omega t} \left( \frac{t}{-j\omega} + \frac{1}{\omega^2} \right) \right] \Big|_0^{T_0} \\
 &\quad \text{↑ } \boxed{\int x e^{ax} dx = e^{ax} \left[ \frac{x}{a} - \frac{1}{a^2} \right]} \\
 &= A \left\{ e^{-j\omega T_0} \left( \frac{T_0}{-j\omega} + \frac{1}{\omega^2} \right) - \frac{1}{\omega^2} \right\} \\
 &= \frac{A}{(2\pi f)^2} \left\{ e^{-j2\pi f T_0} - 1 \right\} + \frac{A T_0 e^{-j2\pi f T_0}}{-j2\pi f} \\
 \Rightarrow S(f) &= \frac{-A}{(2\pi f)^2} + A e^{-j2\pi f T_0} \left( \frac{1}{(2\pi f)^2} + j \frac{T_0}{2\pi f} \right)
 \end{aligned}$$

## 2-18

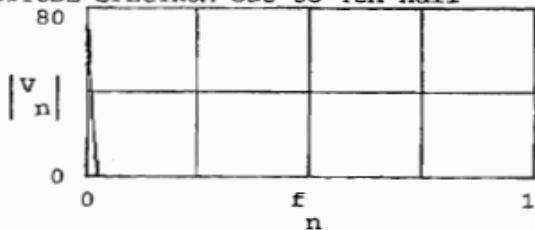
```
(a.)
M := 8          M
N := 2          N = 256   k := 0 .. N - 1      T := 40
T
dt := -          t := k dt - 10
N          k
w_k := Φ[t_k - 1.0] - Φ[t_k - 5.0]
w_0 = 0          dt = 0.156
f4 := 4

n := 0 .. N - 1
W := dt [√N] icfft(w)
f_n := n / T     fs := 1 / dt
W_0 = 3.906     fs = 6.4
f_1 = 0.025     f4 = 4
```

```
(b.) v_k := 2.0
v := dt [√N] icfft(v)
v_0 = 80
```

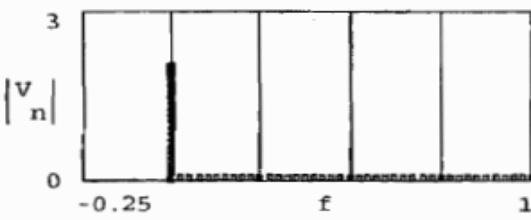


MAGNITUDE SPECTRUM out to 4th null

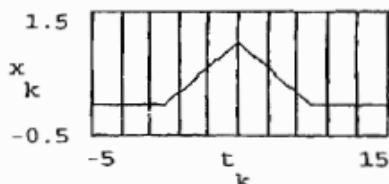
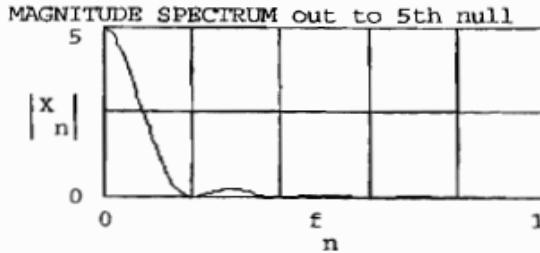


NOTE: The FFT cannot give the correct amplitude value for a delta function since the delta function has an infinite amplitude. However the area under the FFT result that approximates the delta function will be approximately the correct weight for the delta function. The value for the weight of the delta function may be calculated directly via the FFT by using (2-187). This is shown below.

```
v := [1 / √N] icfft(v)
v_0 = 2      --Weight of δ
```



```
(c.)
x_k := 0.2 [t_k [Φ[t_k] - Φ[t_k - 5]] - [t_k - 10] [Φ[t_k - 5] - Φ[t_k - 10]]]
X := dt [√N] icfft(x)
```

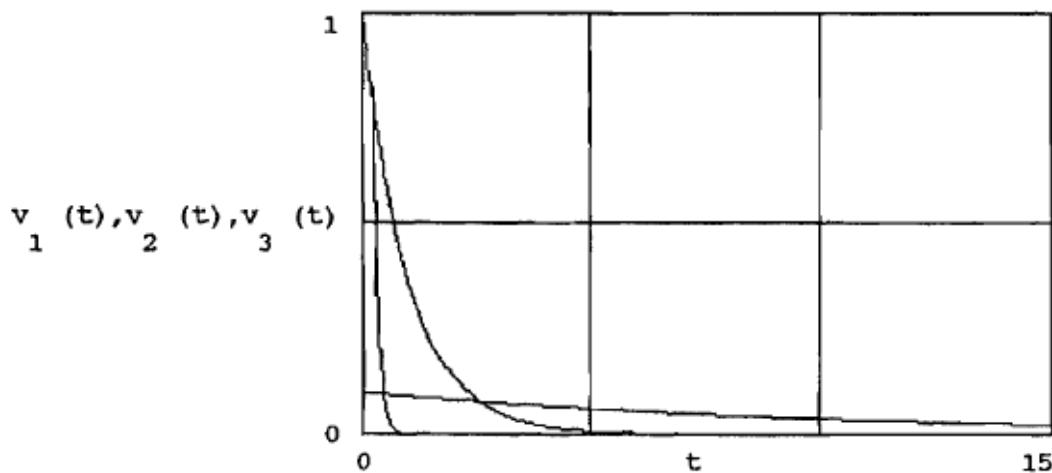


X\_0 = 5

2-25

(a)  $t := 0, 0.05 \dots 15$

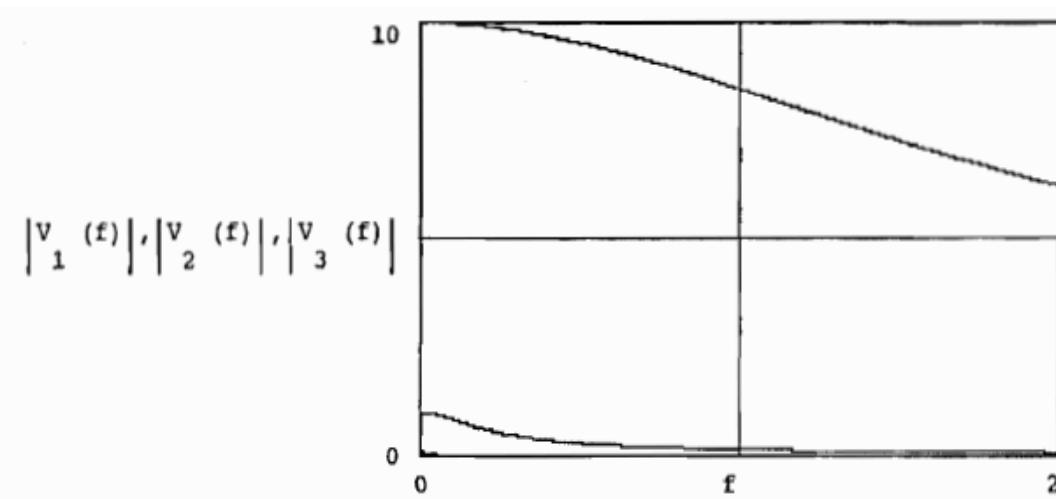
$$v_1(t) := 0.1 \cdot e^{-0.1 \cdot t} \quad v_2(t) := e^{-t} \quad v_3(t) := 10 \cdot e^{-10 \cdot t}$$



(b)  $f := 0, 0.001 \dots 2$

$$V_1(f) := \frac{0.1}{1 + j \cdot 20 \pi \cdot f} \quad V_2(f) := \frac{1}{1 + j \cdot 2 \pi \cdot f} \quad V_3(f) := \frac{10}{1 + j \cdot 0.2 \cdot \pi \cdot f}$$

$$V_1(0) = 0.1 \quad V_2(0) = 1 \quad V_3(0) = 10$$



2-29

$$\begin{aligned}
 w(t) &= w_1(t)w_2(t) \\
 W(f) &= \int_{-\infty}^{\infty} w(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} w_1(t) w_2(t) e^{-j\omega t} dt \\
 &= \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} W_1(\lambda) e^{j2\pi\lambda t} d\lambda \right] w_2(t) e^{-j2\pi f t} dt \\
 &= \int_{-\infty}^{\infty} W_1(\lambda) \underbrace{\int_{-\infty}^{\infty} w_2(t) e^{-j2\pi(f-\lambda)t} dt}_{W_2(f-\lambda)} d\lambda = W(f)
 \end{aligned}$$

2-34

$$(a.) \int_{-\infty}^{\infty} \frac{\sin 4\lambda}{4\lambda} \delta(t-\lambda) d\lambda = \underline{\underline{\frac{\sin(4t)}{4t}}}$$

$$(b.) \int_{-\infty}^{\infty} (\lambda^3 - 1) \delta(2-\lambda) d\lambda = 2^3 - 1 = \underline{\underline{7}}$$

2-40

$$s(t) = A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2)$$

## 2-40 (Continued)

$$\phi_2 = 0 \text{ for simplicity}$$

$$(a.) \omega_1 = \omega_2 ; \phi_1 = \phi_2 = 0$$

$$s(t) = (A_1 + A_2) \cos \omega_1 t$$

$$s_{\text{rms}}(t) = \left[ (A_1 + A_2)^2 \frac{1}{T} \int_0^T \cos^2(\omega_1 t) dt \right]^{1/2}$$

$$= (A_1 + A_2) \left[ \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{1}{2} + \frac{1}{2} \cos(2\theta) \right) d\theta \right]^{1/2}$$

$$\begin{aligned} \omega t &= \theta \\ dt &= \frac{d\theta}{\omega_1} = \frac{d\theta T}{2\pi} \end{aligned}$$

$$= (A_1 + A_2) \left[ \frac{1}{2\pi} \left( \frac{1}{2} \right) 2\pi \right]^{1/2} = \underline{\underline{\frac{(A_1 + A_2)}{\sqrt{2}}}}$$

$$(b.) \omega_1 = \omega_2 ; \phi_1 = \phi_2 + \pi/2 = \pi/2$$

$$s(t) = A_1 \cos(\omega_1 t + \pi/2) + A_2 \cos(\omega_1 t)$$

$$= A_1 (0 - \sin \omega_1 t \sin \pi/2) + A_2 \cos \omega_1 t$$

$$\begin{aligned} &\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B) \\ &= -A_1 \sin \omega_1 t + A_2 \cos \omega_1 t \end{aligned}$$

$$\begin{aligned} \langle s^2(t) \rangle &= \langle A_1^2 \sin^2(\omega_1 t) \rangle - \langle A_1 A_2 \sin(\omega_1 t) \cos(\omega_1 t) \rangle \\ &\quad + \langle A_2^2 \cos^2(\omega_1 t) \rangle = \frac{A_1^2 + A_2^2}{2} \end{aligned}$$

$$\therefore s_{\text{rms}}(t) = \underline{\underline{\frac{\sqrt{A_1^2 + A_2^2}}{\sqrt{2}}}}$$

## 2-40 (Continued)

$$(c.) \omega_1 = \omega_2 ; \phi_1 = \phi_2 + \pi = \pi$$

$$\begin{aligned} s(t) &= A_1 \cos(\omega t + \pi) + A_2 \cos \omega t \\ &= (A_2 - A_1) \cos \omega t \end{aligned}$$

$$\underline{\underline{s(t)_{\text{rms}} = \frac{(|A_2 - A_1|)}{\sqrt{2}}}} \quad \text{from (a.) above}$$

$$(d.) \omega_1 = 2\omega_2 ; \phi_1 = \phi_2 = 0$$

$$s(t) = A_1 \cos(2\omega_2 t) + A_2 \cos(\omega_2 t)$$

$$\begin{aligned} \langle s^2(t) \rangle &= \langle A_1^2 \cos^2(2\omega_2 t) \rangle \rightarrow 0 \\ &+ A_1 A_2 \langle \cos(2\omega_2 t) \cos(\omega_2 t) \rangle \\ &+ \langle A_2^2 \cos^2(\omega_2 t) \rangle \end{aligned}$$

$$\underline{\underline{\therefore s(t)_{\text{rms}} = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}}}$$

$$(e.) \omega_1 = 2\omega_2 ; \phi_1 = \phi_2 + \pi = \pi$$

$$\begin{aligned} s(t) &= A_1 \cos(2\omega_2 t + \pi) + A_2 \cos(\omega_2 t) \\ &= -A_1 \cos(2\omega_2 t) + A_2 \cos(\omega_2 t) \end{aligned}$$

$$\langle s^2(t) \rangle = \left( \frac{A_1^2 + A_2^2}{2} \right)$$

$$\underline{\underline{s(t)_{\text{rms}} = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}}}$$

2-43

Use (2-110) and (2-112)

$$(a.) \quad c_n = f_0 P(f) \Big|_{f=nf_0}$$

$$\text{where } P(f) = \mathcal{F}[p(t)] = \int_{-\infty}^{\infty} p(t) e^{-j\omega t} dt$$

$$\text{For } f=0 \quad P(0) = \int_0^T A t dt = \frac{At^2}{2} \Big|_0^T = \frac{AT^2}{2}, \quad f=0$$

For  $f \neq 0$

$$P(f) = \int_0^T A t e^{j\omega t} dt$$

$$\begin{aligned} \text{Let } u &= At \quad dv = e^{-j\omega t} \\ du &= Adt \quad v = \frac{e^{-j\omega t}}{-j\omega} \end{aligned}$$

$$\begin{aligned} P(f) &\stackrel{\text{I}}{=} \frac{At e^{-j\omega t}}{-j\omega} \Big|_0^T + \frac{A}{j\omega} \left\{ \int_0^T e^{-j\omega t} dt \right\} \\ &= \frac{jAT e^{-j\omega T}}{\omega} + \frac{A}{\omega^2} (e^{-j\omega T} - 1) \\ P(f) &= \frac{A [e^{-j\omega T} + j\omega T e^{-j\omega T} - 1]}{\omega^2}, \quad f \neq 0 \end{aligned}$$

$$c_n = \frac{1}{T_0} P\left(\omega = \frac{n2\pi}{T_0}\right) = f_0 P\left(\omega = n2\pi f_0\right)$$

$$c_n = \begin{cases} \frac{AT^2}{2T_0}, & n=0 \\ \frac{A [e^{-j2\pi nf_0 T} (1 + jn2\pi f_0 T) - 1]}{T_0 \omega^2}, & n \neq 0 \end{cases}$$

$$(b.) \quad x_n = \operatorname{Re}\{c_n\}; \quad y_n = \operatorname{Im}\{c_n\}$$

$$c_n = A \left\{ [\cos(n2\pi f_0 T) - j\sin(n2\pi f_0 T)] \cdot \frac{[1 + jn2\pi f_0 T] - 1}{T_0 \omega^2} \right\}$$

2-43 (Continued)

$$x_n = \begin{cases} \frac{AT^2}{2T_0}, & n=0 \\ A \left\{ \frac{\cos(n2\pi f_0 T) + n2\pi f_0 T \sin(n2\pi f_0 T) - 1}{T_0 \omega^2} \right\}, & n \neq 0 \end{cases}$$

$$y_n = \begin{cases} 0, & n=0 \\ A \left\{ \frac{n2\pi f_0 T \cos(n2\pi f_0 T) - \sin(n2\pi f_0 T)}{T_0 \omega^2} \right\}, & n \neq 0 \end{cases}$$

(C.)

$$|x_n| = \begin{cases} c_0, & n=0 \\ 2\sqrt{x_n^2 + y_n^2}, & n \geq 1 \end{cases}, \quad \phi_n = \begin{cases} 0, & n=0 \\ \tan^{-1}\left(\frac{y_n}{x_n}\right), & n \geq 1 \end{cases}$$

2-52

$$x(t) = e^{-400\pi t} \longleftrightarrow X(f) = \frac{1}{400\pi + j2\pi f}$$

$$\text{Energy in } x(t) = E_x = \int_{-\infty}^{\infty} x^2(t) dt = \int_0^{\infty} e^{-800\pi t} dt = \frac{1}{800\pi} \text{ Joules}$$

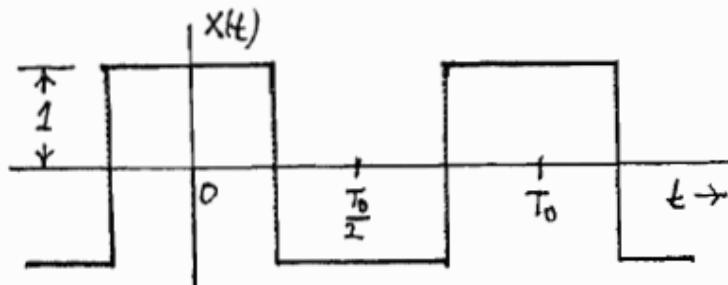
$$E_{out} = \frac{1}{2} E_x = \frac{1}{1600\pi} = 2 \int_0^B |X(f)|^2 df = 2 \int_0^B \frac{1}{400^2 + f^2} df = \frac{1}{2\pi} \left[ \frac{1}{200} \tan^{-1}\left(\frac{B}{200}\right) \right]$$

$$\Rightarrow \frac{400\pi f}{1600\pi} = \tan^{-1}\left(\frac{B}{200}\right) \Rightarrow \frac{B}{200} = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\Rightarrow \underline{B = 200 \text{ Hz}}$$

2-57

Let the input square wave be represented by the Fourier Series:



$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j n \omega_0 t}$$

where  $c_n = \begin{cases} \frac{2 \sin(n\pi)}{n\pi}, & n \neq 0 \\ 0, & n = 0 \end{cases}$

for the waveform shown above.

Then the output waveform is, using (2-140),

$$y(t) = \sum_{-\infty}^{\infty} H(nf_0) c_n e^{j n \omega_0 t} = \sum_{-\infty}^{\infty} d_n e^{j n \omega_0 t}$$

where  $d_n \triangleq H(nf_0) c_n$ ,  $H(f) = \frac{1}{1 + j(\frac{f}{f_1})}$

and  $f_1 = 1,500 \text{ Hz}$  for the RC low-pass filter.

## 2-57 (Continued)

We also know that  $d_{-n} = d_n^*$  since  $x(t)$  is real and the impulse response of the filter is real.

Now reduce the output Fourier Series to a form that can be easily plotted. Using (2-103),

$$y(t) = D_0 + \sum_{n=1}^{n=\infty} D_n \cos(n\omega_0 t + \phi_n)$$

where  $D_0 = 0$  since  $C_0 = 0$

and  $D_n = 2|d_n| = 2|H(nf_0)c_n|, n > 0$

or  $\Rightarrow D_n = 2 \left| \frac{1}{1 + j \left( \frac{n f_0}{f_1} \right)} \right| \begin{cases} \frac{2}{n\pi}, n = \text{odd} \\ 0, n = \text{even} \end{cases} = \frac{4}{\sqrt{1 + \left( \frac{n f_0}{f_1} \right)^2 (n\pi)}}, n = \text{odd}$

$$\phi_n = \angle d_n = \angle 2H(nf_0)c_n = -\tan^{-1} \left( \frac{n f_0}{f_1} \right) + \pi \left( \frac{1 - \sin \left( \frac{n\pi}{2} \right)}{2} \right), n = \text{odd}$$

$$\Rightarrow y(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} D_n \cos(n\omega_0 t + \phi_n)$$

The following MathCAD program plots this  $y(t)$ .

```

f0 := 300          f1 := 1500          n := 1, 3 . 11
                                         t := 0, 0.00005 . 0.004

D_n := 
$$\frac{4}{n\pi \sqrt{1 + \left[ n \frac{f_0}{f_1} \right]^2}}$$


```

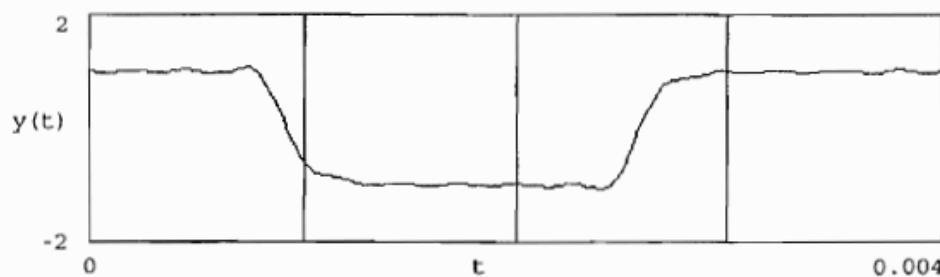
2-57 (Continued)

$$\phi_n := \pi \left[ \frac{1 - \sin\left(n \frac{\pi}{2}\right)}{2} \right] - \text{atan}\left[n \frac{f_o}{f_l}\right]$$

$$y(t) := \sum_n D_n \cos[n 2 \pi f_o t + \phi_n]$$

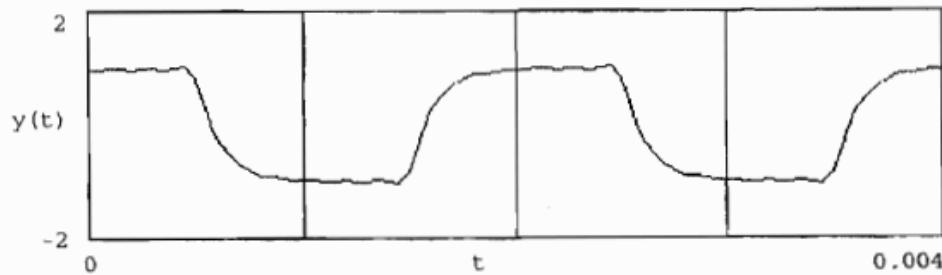
(a)

$$f_o = 300$$



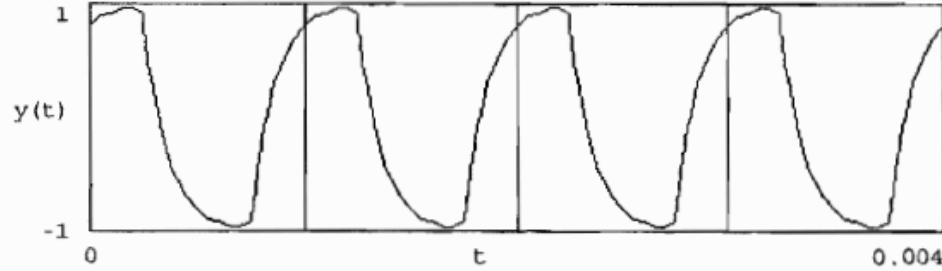
(b)

$$f_o = 500$$



(c)

$$f_o = \frac{1}{10} \cdot 3$$



2-60

$$\omega_0 = 2\pi f_0 = 500 \Rightarrow f_0 = \frac{500}{2\pi}$$

$$f_s > 2f_0 = \frac{2(500)}{2\pi} = \frac{500}{\pi}$$

$$(a.) T_s = \frac{1}{f_s} \leq \frac{\pi}{500} = \underline{6.28 \text{ msec}}$$

$$(b.) N = \frac{1 \text{ sec}}{6.28 \times 10^{-3} \text{ sec/sample}} = \underline{160 \text{ samples}}$$

2-62

$$M := 6 \quad N := 2^M \quad N = 64 \quad k := 0 .. N - 1 \quad Tl := 10 \quad T := 1$$

$$dt := \frac{Tl}{N} \quad t_k := k dt$$

NOTE: In FFT time domain, points for negative time are the same as those measured from the end of the data span-length  $Tl$  for positive time.

$$w_k := \text{if}\left[t_k < T, \left[\frac{-1}{T}\right] \left[\frac{t_k - T}{T}\right], 0\right] + \text{if}\left[t_k > (Tl - T), \frac{k}{T}, 0\right]$$

$$w_0 = 1 \quad dt = 0.156$$

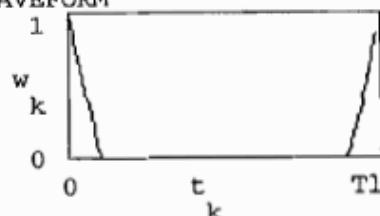
$$n := 0 .. N - 1$$

$$w := dt \sqrt{N} \text{ icfft}(w)$$

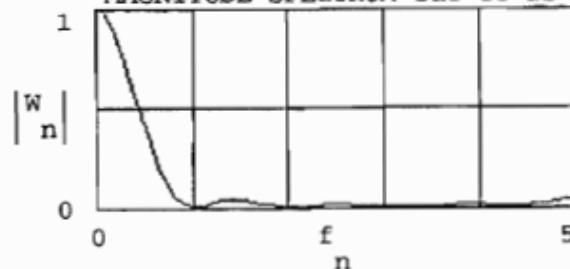
$$f_n := \frac{n}{Tl} \quad fs := \frac{1}{dt}$$

$$f_1 = 0.1 \quad fs = 6.4$$

WAVEFORM



MAGNITUDE SPECTRUM out to fs



2-68

$$s(t) = \Lambda\left(\frac{t}{T_0}\right) \leftrightarrow S(f) = T_0 [S_a(\pi f T_0)]^2$$

Table 2-2

(a.) Using results in 2-61 (1.) above  $\underline{\underline{B_{qfs} = \infty}}$

$$(b.) S(f_{3dB}) = \frac{T_0}{V_2} = T_0 [S_a(\pi f_{3dB} T_0)]^2$$

$$\Rightarrow \pi f_{3dB} T_0 \approx (2)^{1/4} \Rightarrow \underline{\underline{B_{3dB} = f_{3dB} = \frac{1.19}{\pi T_0} = 0.38/T_0}}$$

$$(c.) B_{eq} = \frac{1}{|H(f_0)|^2} \int_{-\infty}^{\infty} |H(f)|^2 df = \frac{1}{T_0^2} \int_{-\infty}^{\infty} T_0^2 [S_a(\pi f T_0)]^4 df$$

$$= \frac{1}{\pi^2 T_0} \left(\frac{\pi}{3}\right) = \frac{1}{3 T_0} \quad \underline{\underline{B_{eq} = \frac{1}{3 T_0}}}$$

$$(d.) \underline{\underline{B_{2cm-4dB} = \frac{1}{T_0}}} \quad \left( \begin{array}{l} \text{Similar to} \\ \text{2-90 (4) above.} \end{array} \right)$$

# Chapter 3

3-6

$$(a.) f_s = 2B = 2(100) = \underline{\underline{200 \text{ samples/sec}}}$$

(b.) Using the results given in prob. 3-7.

$$n \geq 3.32 \log_{10} \left( \frac{50}{P} \right) = 3.32 \log_{10} \left( \frac{50}{0.1} \right) \\ = 8.96$$

$$\underline{n = 9 \text{ bits/word}}$$

$$(c.) \underline{\underline{R = \left( \frac{n \text{ bits}}{\text{word}} \right) \left( \frac{f_s \text{ words}}{\text{sec}} \right) = 200(9) = 1.8 \frac{k \text{ bits}}{\text{sec}}}}$$

(d.) For binary PCM  $D = R$

$$\text{eq. (3-74)} \quad D = \frac{2B}{1+r}, \text{ for } B_{\min}, r=0$$

$$\Rightarrow B = \frac{D}{2} = \underline{\underline{900 \text{ Hz}}}$$

3-8

$$(a.) f_s \geq 2B_{\text{analog}} = 2(20 \text{ kHz}) = \underline{\underline{40 \frac{\text{h.samples}}{\text{sec}}}}$$

For 8x oversampling of the recovered PCM signal  
(used to increase  $f_s$  8x and simplify LPF requirements)

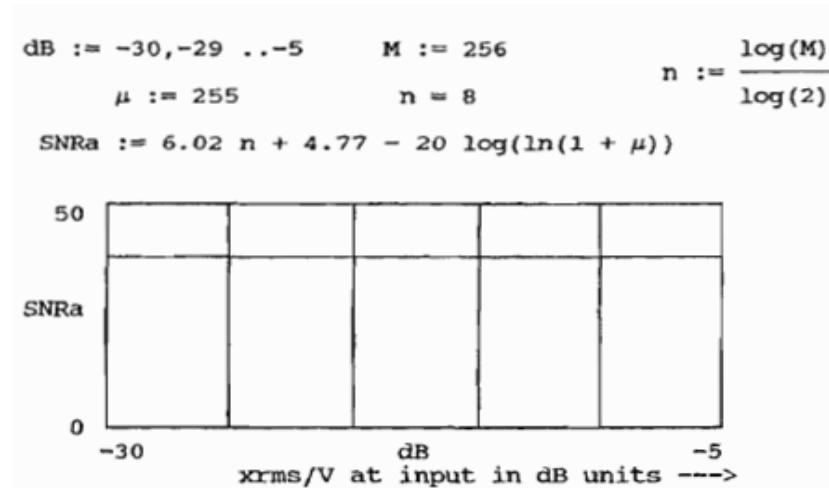
$$\Rightarrow f_{8x} = 8f_s = 320 \text{ h.samples/sec}$$

$$B_{\text{null}} = R = n f_{8x} = \left( 16 \frac{\text{bits}}{\text{sample}} \right) \left( 320 \frac{\text{h.samples}}{\text{sec}} \right) = \underline{\underline{5.12 \text{ MHz}}}$$

(b) Using (3-18)

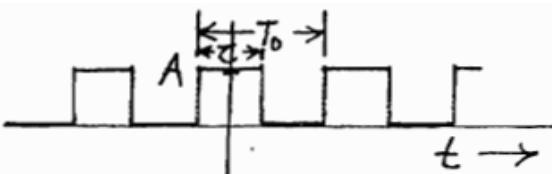
$$\left( \frac{S}{N} \right)_{\text{peak}} = 6.02n + 4.77 \text{ dB} = 6.02(15) + 4.77 = \underline{\underline{94.77 \text{ dB}}}$$

3-14



3-17

For alternating data the waveform is periodic where  $T_0 = 2T_b$ .



From (2-109) the spectrum is

$$W(f) = \sum_{-\infty}^{\infty} C_n \delta(f - n f_0)$$

where  $C_n = \frac{1}{T_0} \int_a^{a+T_0} w(t) e^{-j2\pi n f_0 t} dt = \frac{1}{T_0} \int_{-\frac{T}{2}}^{\frac{T}{2}} A e^{-j2\pi n f_0 t} dt$

$$= \frac{A}{T_0} \left[ \frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \right]_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{2A}{T_0} \left( \frac{e^{-j2\pi n f_0 \frac{T}{2}} + e^{j2\pi n f_0 \frac{T}{2}}}{-2j2\pi n f_0} \right)$$

$$\Rightarrow C_n = \frac{2A}{T_0} \frac{\sin(\pi n f_0 Z_2)}{\pi n f_0 Z_2} = \frac{A}{Z_2} \left( \frac{\pi}{T_0} \right) \frac{\sin\left(\frac{\pi}{Z_2} Z_2\right)}{\left(\frac{\pi}{Z_2} Z_2\right)}$$

$$\Rightarrow W(f) = \sum_{n=-\infty}^{\infty} \frac{A}{Z_2} \left( \frac{\sin\left(\frac{\pi}{Z_2} Z_2\right)}{\left(\frac{\pi}{Z_2} Z_2\right)} \right) \delta\left(f - \frac{n}{Z_2}\right) \quad (A)$$

where  $Z_2 = \frac{1}{T_0} = \text{bit rate}$

3-17 (Continued)

(a) Using (A) for NRZ signaling with  $\tau = T_b$

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin(\frac{n\pi}{2})}{(\frac{n\pi}{2})} \right| \delta(f - \frac{n}{2}R) \quad \begin{matrix} \text{Unipolar} \\ \text{NRZ} \\ (\text{alternating} \\ \text{data}) \end{matrix}$$

If the data are a sequence of four "1"s followed by four "0"s, the waveform would have the same shape except  $T_0$  would be 4 times as large.

i.e.  $T_0 = 8T_b$ .

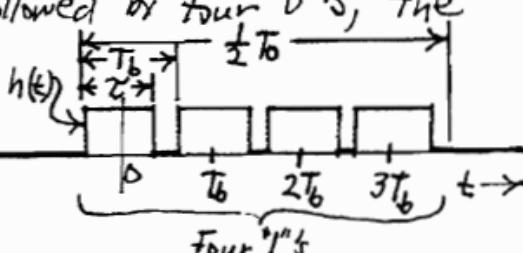
$$\Rightarrow |W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin(\frac{n\pi}{2})}{(\frac{n\pi}{2})} \right| \delta(f - \frac{n}{8}R) \quad \begin{matrix} \text{Unipolar NRZ} \\ \{1\}^4 \text{ do } \{0\}^4 \end{matrix}$$

(b) Using (A) for RZ signaling with  $\tau = \frac{3}{4}T_b$ ,

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{3}{8} A \left| \frac{\sin(\frac{3}{8}n\pi)}{(\frac{3}{8}n\pi)} \right| \delta(f - \frac{n}{2}R) \quad \begin{matrix} \text{Unipolar RZ} \\ (\text{alternating data}) \end{matrix}$$

For RZ with four "1"s followed by four "0"s, the periodic waveform would appear as shown where  $T_0 = 8T_b$ . The mathematical calculations are simplified if (2-112) is used

$$c_h = f_0 H(nf_0)$$



where  $h(t)$  is the basic waveform that is repeated to create the periodic waveform (as shown in the figure).

$h(t)$  consists of the superposition of four rectangular pulses. Using the time delay theorem of Table 2-1

## 3-17 (Continued)

and the rectangular pulse spectrum of Table 2-2

$$H(f) = A \frac{\sin(\pi f T_b)}{\pi f T_b} \left[ 1 + e^{-j\omega T_b} + e^{j\omega 2T_b} + e^{-j\omega 3T_b} \right]$$

Or

$$C_n = \frac{A}{8T_b} \frac{\sin\left(\frac{n\pi}{8}\frac{T_b}{T_b}\right)}{\left(\frac{n\pi}{8}\frac{T_b}{T_b}\right)} \left[ 1 + e^{j\frac{n\pi}{4}} + e^{j\frac{3n\pi}{4}} + e^{-j\frac{3n\pi}{4}} \right]$$

$$\boxed{f = n f_0 = \frac{n}{T_b} = \frac{n}{8T_b}}$$

For RZ with  $\tau = \frac{3}{4}T_b$ , this becomes

$$C_n = \frac{3}{32} A \left( \frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)} \right) \left[ 1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{3n\pi}{2}} + e^{-j\frac{3n\pi}{4}} \right]$$

Thus, the spectrum for Unipolar RZ with four alternate "1" and "0"s is

$$|W(f)| = \sum_{n=-\infty}^{\infty} \frac{3}{32} A \left| \frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)} \right| \left| 1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{3n\pi}{2}} + e^{-j\frac{3n\pi}{4}} \right| \delta(f - \frac{n}{8R})$$


---

3-18

(a) Substituting (3-40) into (3-36a) the PSD for Polar RZ signaling is

$$P(f) = \frac{A^2}{T_b} |F(f)|^2$$

where the pulse shape,  $f(t)$ , is shown in the figure. Thus,

$$F(f) = \mathcal{F}[f(t)] = \mathcal{Z} \frac{\sin(\pi f T_b)}{(\pi f T_b)}$$

and  $P(f) = \frac{A^2 \gamma^2}{T_b} \left[ \frac{\sin(\pi f T_b)}{(\pi f T_b)} \right]^2$

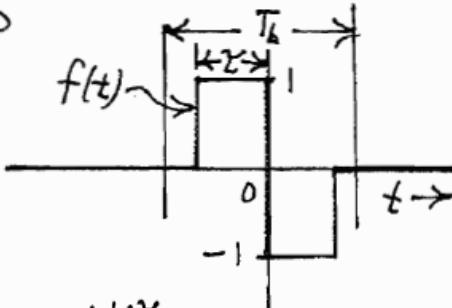
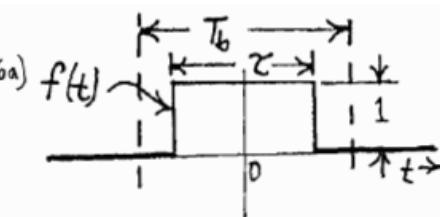
for the case of  $\gamma = \frac{1}{2} T_b$ , this becomes

$$\underline{P(f) = \frac{A^2 T_b}{4} \left[ \frac{\sin(\frac{\pi}{2} f T_b)}{(\frac{\pi}{2} f T_b)} \right]^2}$$

The first-null bandwidth is  $B_{\text{null}} = \frac{2}{T_b} = 2R$   
and the bandwidth efficiency is  $\eta = \frac{1}{2} (\text{bit/sec})/\text{Hz}$ .

(b) Equation (3-36) can also be used to evaluate the PSD for RZ Manchester signaling where the pulse shape is shown in the figure.

$$F(f) = \mathcal{Z} \left( \frac{\sin(\pi f T_b)}{(\pi f T_b)} \right) \left[ e^{j \omega_m \frac{T_b}{2}} - e^{-j \omega_m \frac{T_b}{2}} \right]$$



3-18 (Continued)

$$\Rightarrow F(f) = j 2 \pi \left( \frac{\sin(\pi f z)}{(\pi f z)} \right) \sin\left(\frac{\omega z}{2}\right)$$

Using (3-40) and (3-36), the PSD for Manchester signaling is

$$P(f) = \frac{4 A^2 z^2}{T_b} \left[ \frac{\sin(\pi f z)}{(\pi f z)} \right]^2 \left[ \sin(\pi f z) \right]^2$$

If  $z = \frac{1}{4} T_b$ , this becomes

$$\underline{P(f) = \frac{1}{4} A^2 T_b \left[ \frac{\sin\left(\frac{\pi}{4} f T_b\right)}{\left(\frac{\pi}{4} f T_b\right)} \right]^2 \left[ \sin\left(\frac{\pi}{4} f T_b\right) \right]^2}$$

The first-null bandwidth is  $B_{null} = \frac{4}{T_b} = 4R$   
and the spectral efficiency is  $\eta = \frac{1}{4} (6 bits/sec)/Hz$ .

3-20

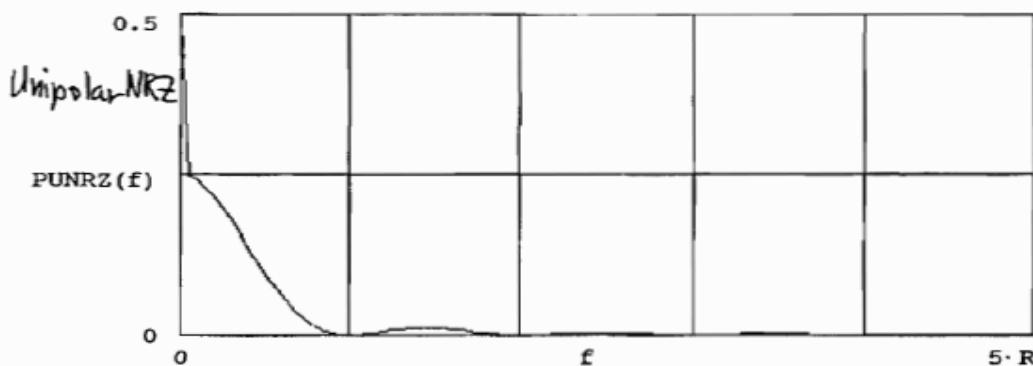
$$A := 1 \quad R := 1 \quad f := 0, 0.05 .. 5 \quad Tb := \frac{1}{R}$$

$$Sa(x) := \text{if}[x \neq 0, \frac{\sin(x)}{x}, 1]$$

The PSD for Unipolar NRZ is given by (3-39b) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-39b) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

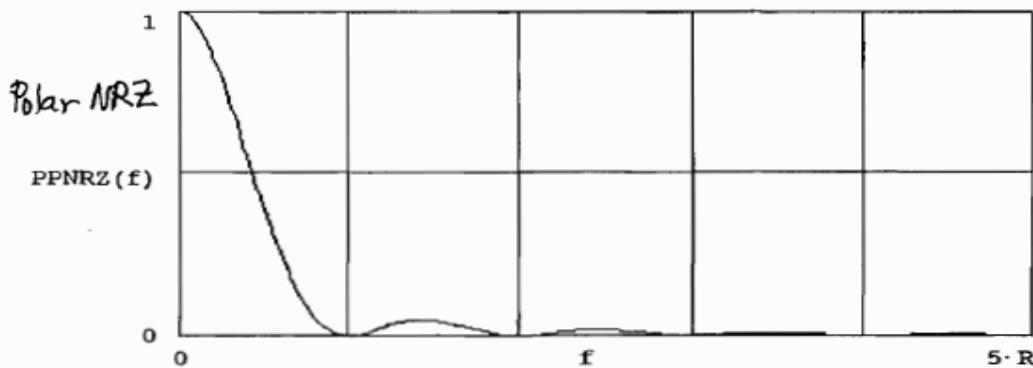
$$PUNRZc(f) := \left[ \frac{2}{4} \cdot \frac{Tb}{A} \right] \cdot (Sa(\pi f \cdot Tb))^2 \quad PUNRZd(f) := \text{if}[f \neq 0, 0, \frac{A}{4}]$$

$$PUNRZ(f) := PUNRZc(f) + PUNRZd(f)$$



Use (3-41) for Polar NRZ spectrum:

$$PPNRZ(f) := \left[ \frac{2}{4} \cdot \frac{Tb}{A} \right] \cdot (Sa(\pi f \cdot Tb))^2$$



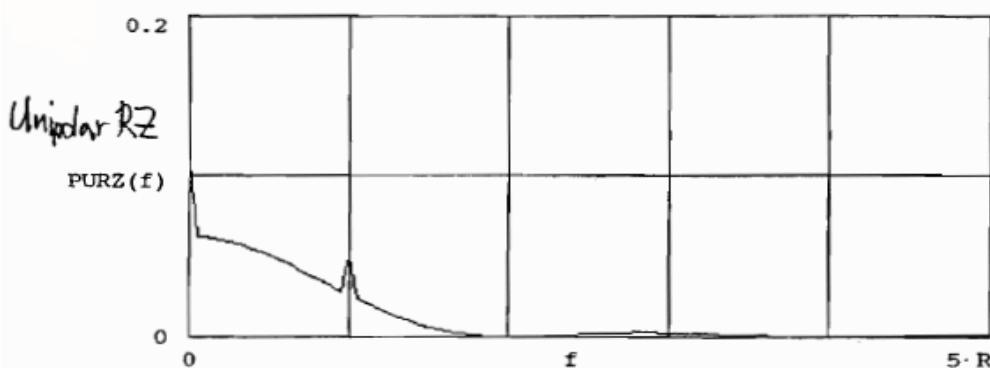
The PSD for Unipolar RZ is given by (3-43) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-43) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

$$PURZc(f) := \left[ \frac{2}{16} \cdot \frac{Tb}{A} \right] \cdot \left[ Sa\left[\pi f \cdot \frac{Tb}{2}\right]\right]^2$$

$$PURZd(f) := \text{if}\left[\text{mod}(f, R) \neq 0, 0, \frac{A}{16} \cdot \left[ Sa\left[\pi f \cdot \frac{Tb}{2}\right]\right]^2\right]$$

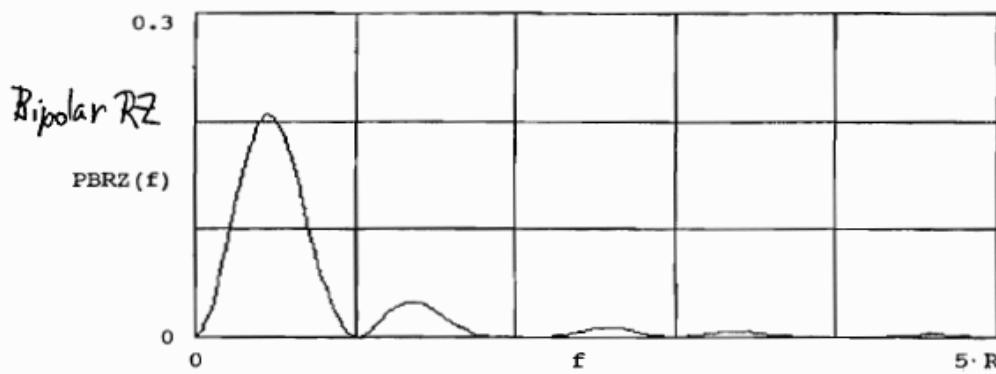
### 3-20 (Continued)

$$PURZ(f) := PURZc(f) + PURZd(f)$$



Using (3-45) the PSD for Bipolar RZ is:

$$PBRZ(f) := A^2 \left[ \frac{Tb}{4} \right] \cdot \left[ \text{Sa} \left[ \pi f \cdot \frac{Tb}{2} \right] \right]^2 (\sin(\pi f Tb))^2$$

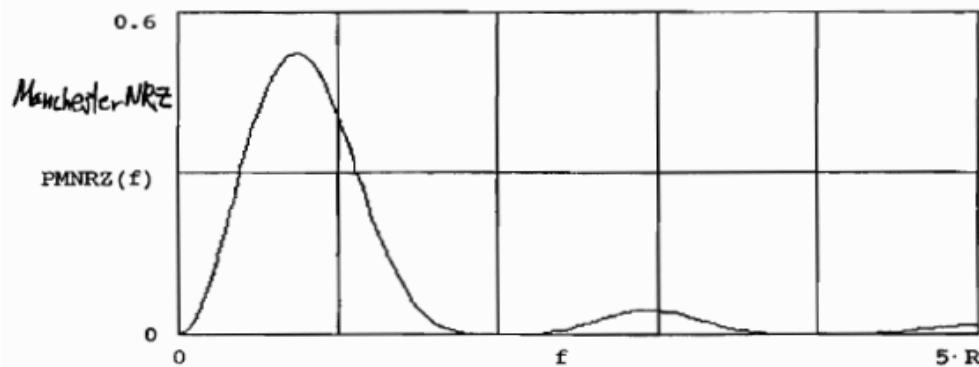


$$A := 1 \quad R := 1 \quad f := 0, 0.05 \dots 0.5$$

$$\text{Sa}(x) := \text{if}[x \neq 0, \frac{\sin(x)}{x}, 1] \quad Tb := \frac{1}{R}$$

Use (3-46c) for the Manchester NRZ spectrum:

$$PMNRZ(f) := A^2 \cdot Tb \left[ \text{Sa} \left[ \pi f \cdot \frac{Tb}{2} \right] \right]^2 \left[ \sin \left[ \pi f \cdot \frac{Tb}{2} \right] \right]^2$$



3-27

$$L = 8 = 2^l \Rightarrow l = 3$$

$$(a) D = \frac{R}{l} = \frac{9600 \text{ bits/sec}}{3 \text{ bits/symbol}} = \underline{\underline{3.2 \text{ k symbol/sec}}}$$

$$(b) D = \frac{2R}{1+r} = \frac{2(2.4k)}{1+r} = 3.2k \Rightarrow \underline{\underline{r = 0.5}}$$

3-35

$$M = 16 = 2^4 \Rightarrow h = 4$$

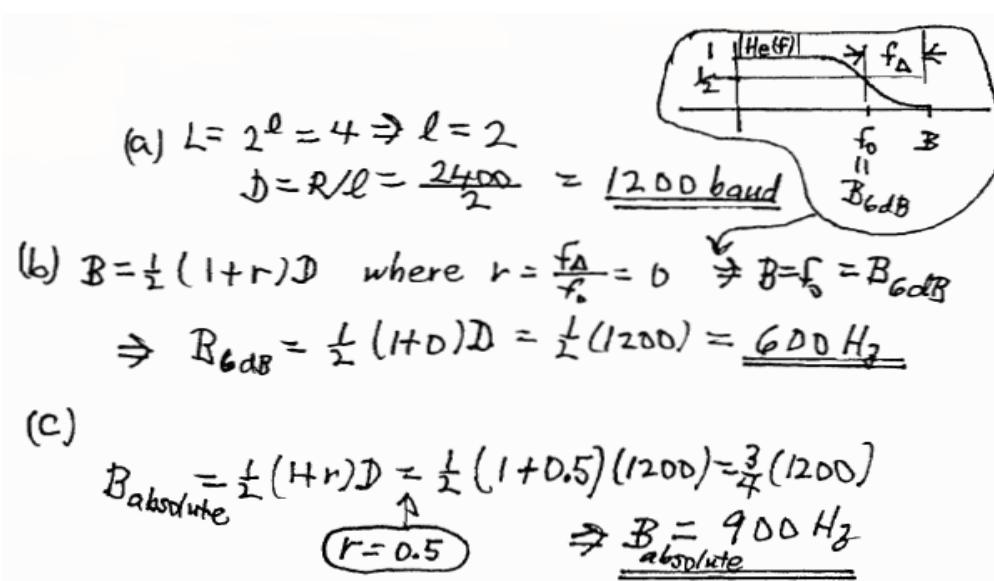
$$(a) \text{ Binary PCM } \Rightarrow l = 1, R = n f_s = 4 f_s = D$$

$$D = \frac{2R}{1+r} = \frac{2(4k f_s)}{1+0.5} = \underline{\underline{5.33 \text{ k bits/sec}}}$$

$$(b) \text{ From (a)} \quad f_s = \frac{D}{4} = \frac{5.33k}{4} = 1.33k \text{ Hz}$$

$$B_{\text{analog max}} = \frac{f_s}{2} = \frac{1.33k}{2} = \underline{\underline{667 \text{ Hz}}}$$

3-36



3-41

(a.) From (3-84)

$$\delta = \frac{2\pi f_a A}{f_s} ; \quad f_a = 3.4 \text{ kHz} \quad \& \quad A = \frac{1}{2}$$

We need to determine the  $f_s$  which the channel can support. Assuming that a  $r=0$  roll-off factor is used, then

$$f_s = D = 2B = 2(1 \text{ MHz}) = 2 \times 10^6 \frac{\text{Samples}}{\text{sec}}$$

$$\Rightarrow s = \frac{2\pi (3.4k) (\frac{1}{2})}{2 \times 10^6} = 0.00534$$

(Note: The channel has to be equalized with a Nyquist filter.)

(b.)

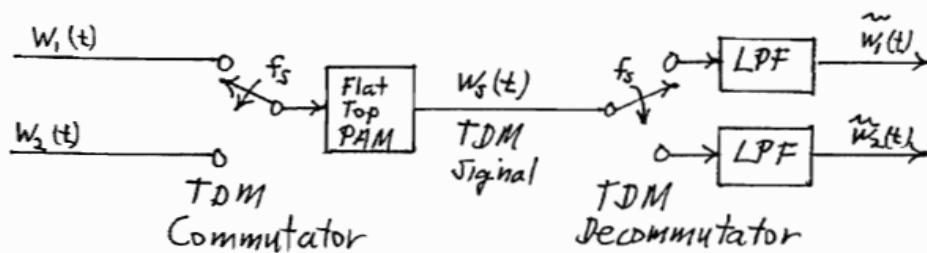
$$\delta = \frac{2\pi (3.4k)(\frac{1}{2})}{25 \times 10^3} = \underline{\underline{0.427}}$$

(Note: No Channel equalization required.)

3-44

(a) Each analog signal has a highest frequency of  $B = 3 \text{ kHz}$

⇒ The minimum sampling frequency for each analog signal is  $f_s = 2B = 6 \text{ kHz}$



(b) Referring to (3-8), the sampled TDM signal is

$$w_S(t) = \sum_{k=-\infty}^{\infty} w_1(kT_s) h_1(t-kT_s) + \sum_{k=-\infty}^{\infty} w_2(kT_s) h_2(t-kT_s)$$

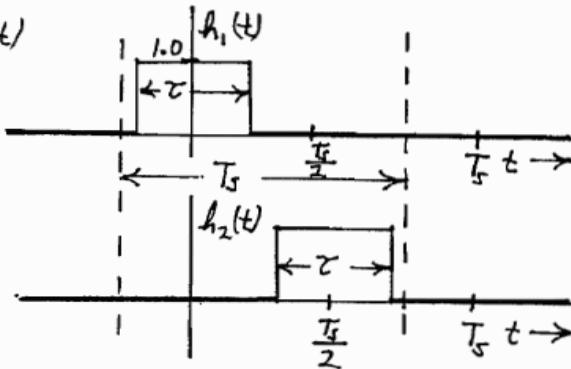
where  $h_1(t)$  and  $h_2(t)$  are shown in the figure and  $\tau \leq \frac{T_s}{2}$  and  $f_s \geq 2B$ .

Following the same procedure as described in (3-8) thru (3-13), the spectrum of the TDM instantaneously sampled (flat-topped) PAM signal is

$$W_S(f) = \frac{1}{T_s} H_1(f) \sum_{k=-\infty}^{\infty} W_1(f-kf_s) + \frac{1}{T_s} H_2(f) \sum_{k=-\infty}^{\infty} W_2(f-kf_s)$$

where  $H_1(f) = \tau \left( \frac{\sin(\pi f \tau)}{\pi f \tau} \right)$  and  $H_2(f) = \tau \left( \frac{\sin(\pi f \tau)}{\pi f \tau} \right) e^{-j2\pi f \frac{\tau}{2}}$

$$\Rightarrow W_S(f) = f_s \tau \frac{\sin(\pi f \tau)}{\pi f \tau} \sum_{k=-\infty}^{\infty} \Pi \left( \frac{f-kf_s}{2B} \right) + 2B\tau \left( \frac{\sin(\pi f \tau)}{\pi f \tau} \right) e^{j\pi f \tau} \sum_{k=-\infty}^{\infty} \Lambda \left( \frac{f-kf_s}{B} \right)$$

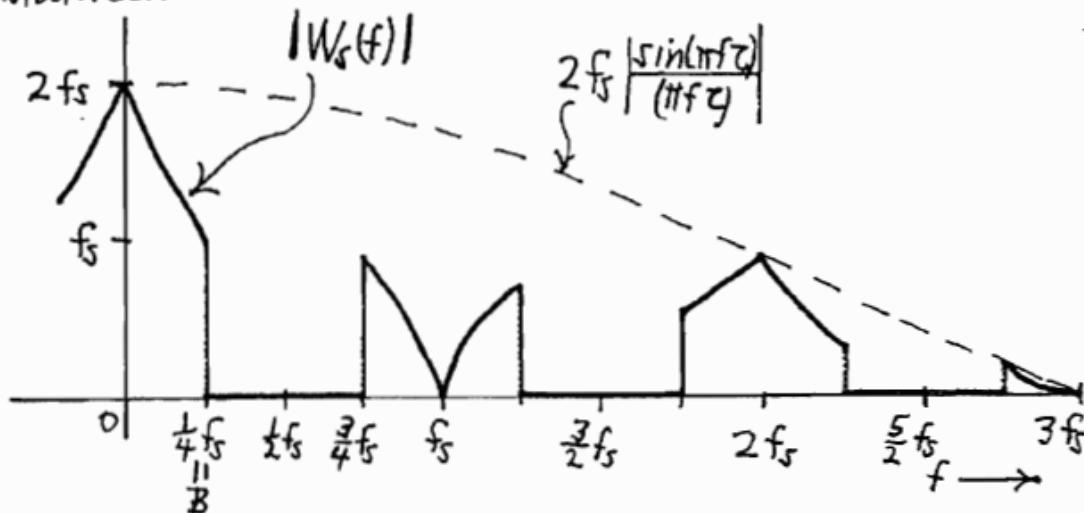


3-44

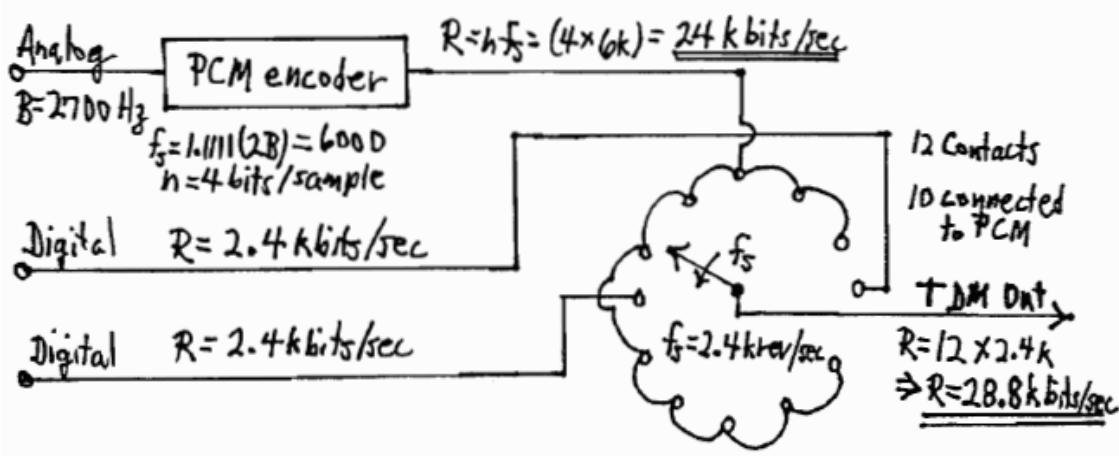
Thus,

$$|W_s(f)| = f_s \left| \frac{\sin(\pi f t)}{(\pi f t)} \right| \sum_{k=-\infty}^{\infty} \left| \pi \left( \frac{f-kf_s}{2B} \right) + e^{j\pi f t} \right| \delta \left( \frac{f-kf_s}{B} \right)$$

For the sketch, let the parameters be the same as those shown in Fig. 3-6. Let  $T_s = 1/3$ ,  $f_s = 4B$ . Using a programmable calculator, the following sketch is obtained.



3-46



## Chapter 4

4-2

Using (2-26) with the help of Sec. A-5,  
 $G(f) = A_c M(f) = 50 [-j\delta(f-1000) + j\delta(f+1000)]$

Substituting this into (4-15) and using  $\delta(-f) = \delta(f)$ ,  
the voltage spectrum of this DSB-SC signal is

$$S(f) = -j25\delta(f-f_c-1000) + j25\delta(f-f_c+1000) \\ -j25\delta(f+f_c-1000) + j25\delta(f+f_c+1000)$$

4-3



#### 4-3 (Continued)

$$(b) \quad v_2(t) = \operatorname{Re} \{ g_2(t) e^{j\omega_c t} \}$$

where  $g_2(t) = \frac{1}{2} g_1(t) * h(t) \leftrightarrow G_2(f) = \frac{1}{2} G_1(f) K(f)$

and  $h(t) = \mathcal{F}^{-1}[K(f)]$

$$\text{Also, } H(f) = \frac{1}{2} [K(f-f_c) + K^*(-f-f_c)]$$

$$\Rightarrow K(f) = \begin{cases} 2, & |f| < B_T/2 \\ 0, & f \text{ elsewhere} \end{cases}$$

Evaluate  $h(t)$ :

$$h(t) = \int_{-\frac{B_T}{2}}^{\frac{B_T}{2}} 2 e^{j2\pi ft} dt = 2 B_T \frac{\sin(\pi B_T t)}{(\pi B_T t)}$$

$$\text{Know that } g_1(t) = A \pi\left(\frac{t}{T}\right) = A \begin{cases} 1, & |t| < T/2 \\ 0, & t \text{ elsewhere} \end{cases}$$

$$\Rightarrow g_2(t) = \frac{1}{2} g_1(t) * h(t) = \frac{1}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} A/2 B_T \frac{\sin[\pi B_T(t-\lambda)]}{[\pi B_T(t-\lambda)]} d\lambda$$

$$\text{Let } \lambda_1 = \pi B_T(t-\lambda) \Rightarrow d\lambda_1 = -\pi B_T d\lambda$$

$$\Rightarrow g_2(t) = A B_T \int_{\pi B_T(t+\frac{T}{2})}^{\pi B_T(t-\frac{T}{2})} \frac{\sin \lambda_1}{\lambda_1} \left( -\frac{1}{\pi B_T} d\lambda_1 \right)$$

$$= \frac{A}{\pi} \left[ - \int_{\pi B_T(t+\frac{T}{2})}^0 \frac{\sin \lambda_1}{\lambda_1} d\lambda_1 - \int_0^{\pi B_T(t-\frac{T}{2})} \frac{\sin \lambda_1}{\lambda_1} d\lambda_1 \right]$$

$$\Rightarrow g_2(t) = \frac{A}{\pi} \left[ + \operatorname{Si}[\pi B_T(t+\frac{T}{2})] - \operatorname{Si}[\pi B_T(t-\frac{T}{2})] \right]$$

$$\text{and } v_2(t) = \operatorname{Re} \{ g_2(t) e^{j\omega_c t} \}$$

#### 4-3 (Continued)

$$\text{Thus, } v_2(t) = \frac{A}{\pi} \left\{ \text{Si} \left[ \pi B_T \left( t + \frac{T}{2} \right) \right] - \text{Si} \left[ \pi B_T \left( t - \frac{T}{2} \right) \right] \right\} \cos(\omega_c t)$$

(C.) When  $B_T = \frac{4}{\pi}$

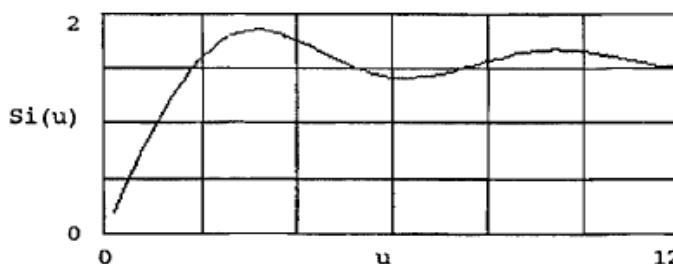
$$v_2(t) = \frac{A}{\pi} \left\{ \text{Si} \left[ 2\pi \left( \frac{2t}{\pi} + 1 \right) \right] - \text{Si} \left[ 2\pi \left( \frac{2t}{\pi} - 1 \right) \right] \right\} \cos(\omega_c t)$$

This is plotted with the help of the Si(u) function. (See p. 232 of Abramowitz and Stegun for a description of the Si(u) function.)

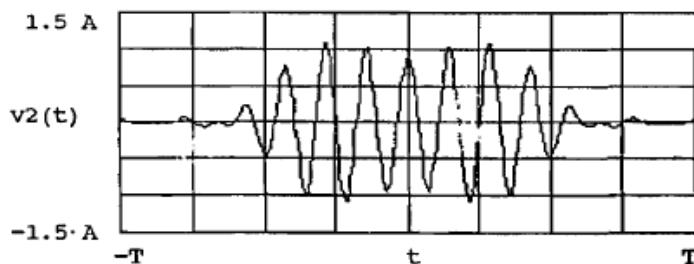
Using MathCAD we get:

$A := 1 \quad T := 1 \quad \omega := 2 \pi \cdot 7 \quad u := 0, 0.2 \dots 12$

$$\text{Si}(u) := \int_{0.001}^u \frac{\sin(x)}{x} dx \quad t := -T, -T + 0.01 \dots T$$



$$v_2(t) := \left[ \frac{A}{\pi} \cdot \left[ \text{Si} \left[ 2\pi \left[ 2 \cdot \frac{t}{T} + 1 \right] \right] - \text{Si} \left[ 2\pi \left[ 2 \cdot \frac{t}{T} - 1 \right] \right] \right] \right] \cos(\omega t)$$



4-6

$$(a.) s(t) = \operatorname{Re} \{ 500 e^{j\omega_c t} \} + \operatorname{Re} \left\{ -j100 e^{j(\omega_c + \omega_a)t} + j100 e^{j(\omega_c - \omega_a)t} \right\}$$

$$s(t) = \operatorname{Re} \left\{ 500 \left[ 1 - j(2j) \frac{100}{500} \left( \frac{e^{j\omega_a t} - e^{-j\omega_a t}}{2j} \right) \right] e^{j\omega_c t} \right\}$$

$$= \operatorname{Re} \left\{ 500 \left[ \underbrace{1 + \frac{2}{5} \sin(\omega_a t)}_{1 + m(t)} \right] e^{j\omega_c t} \right\} \quad \text{AM}$$

$$\Rightarrow g(t) = 500 + 200 \sin(\omega_a t), \quad m(t) = 0.4 \sin(\omega_a t)$$

$$(b.) x(t) = \operatorname{Re} \{ g(t) \} = \underline{500 + 200 \sin(\omega_a t)}, \quad y(t) = \operatorname{Im} \{ g(t) \} = \underline{0}$$

$$(c.) R(t) = |g(t)| = \underline{500 + 200 \sin(\omega_a t)}, \quad \theta(t) = \angle g(t) = \underline{0^\circ}$$

$$(d.) P = \frac{1}{T} \langle |g(t)|^2 \rangle = \frac{1}{100} \left[ (500)^2 + 2 \times 10^5 \langle \sin^2(\omega_a t) \rangle + 200^2 \sin^2(\omega_a t) \right]$$

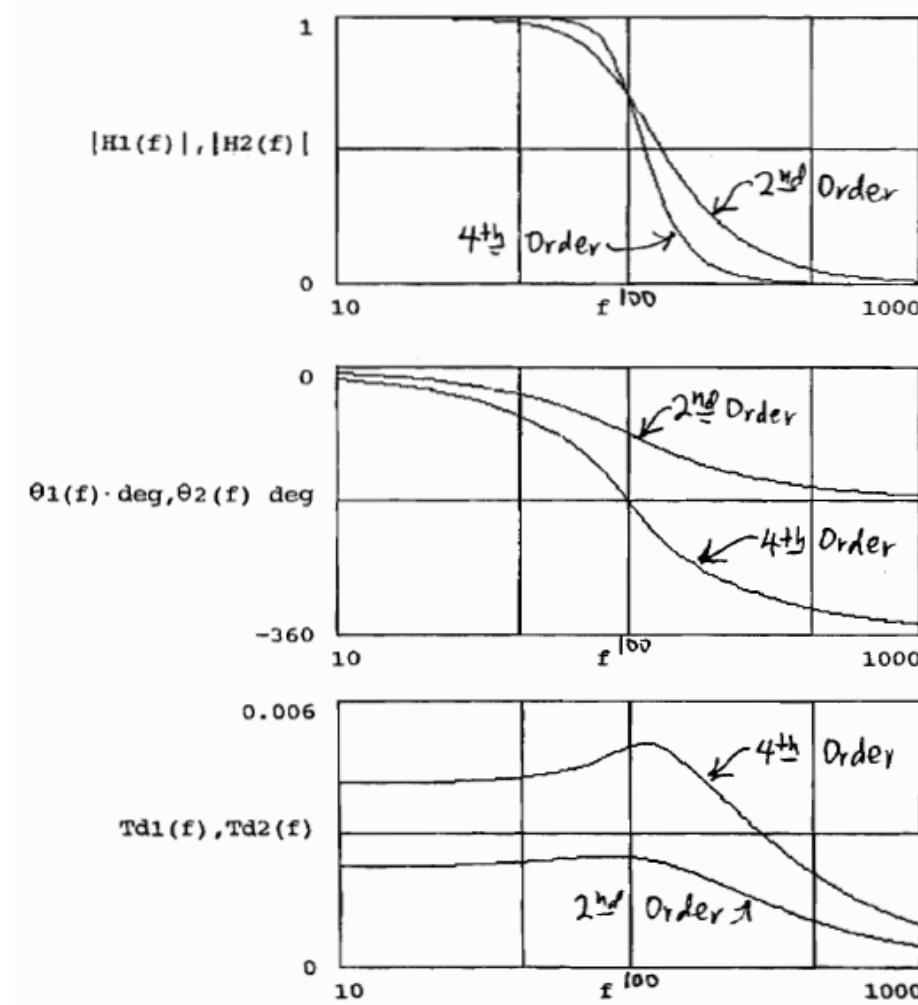
$$\Rightarrow \underline{P = 2,700 \text{ watts}}$$

4-9

```

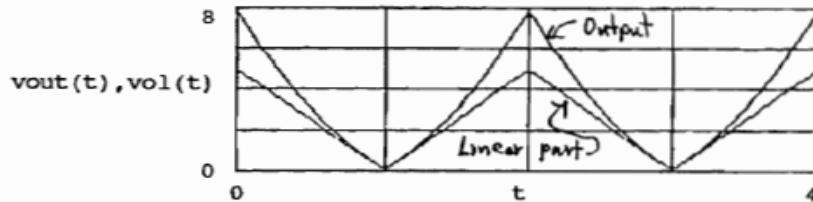
B := 100          f := 10,20 ..1000      rad = 1      deg =  $\frac{\text{rad}}{\pi} \cdot 180$ 
                                                1
H1(f) :=  $\frac{1}{1 - \left[ \frac{f}{B} \right]^2 + j \sqrt{2} \frac{f}{B}}$ 
θ1(f) := arg(H1(f))           Td1(f) :=  $\frac{1}{-2 \cdot \pi \cdot f} \cdot \theta_1(f)$ 
H2(f) :=  $\frac{1}{\left[ 1 - \left[ \frac{f}{B} \right]^2 + 0.765j \frac{f}{B} \right] \left[ 1 - \left[ \frac{f}{B} \right]^2 + 1.848j \frac{f}{B} \right]}$ 
θ(f) := arg(H2(f))
θ2(f) := if(f < 100, θ(f), θ(f) - 2 · π)
Td2(f) :=  $\frac{1}{-2 \cdot \pi \cdot f} \cdot \theta_2(f)$ 

```



## 4-12

```
t := 0, 0.1 .. 4           m := 1 .. 6
(a.) vin(t) :=  $\frac{1}{2} + \frac{4}{\pi} \cdot \left[ \sum_m \left[ \frac{\cos((2 \cdot m - 1) \cdot \pi \cdot t)}{(2m - 1)^2} \right] \right]$ 
Note: vin(t) is the Fourier series for a triangle waveform.
vout(t) := 5 · vin(t) + 1.5 · (vin(t))2 + 1.5 · (vin(t))3
vol(t) := 5 · vin(t)      -----Linear part of the output.
```



```
(b.)
M := 5           M
N := 2           N = 32           k := 0 .. N - 1           T := 2
T           dt := 0.063           t := k · dt
dt :=  $\frac{T}{N}$            k
```

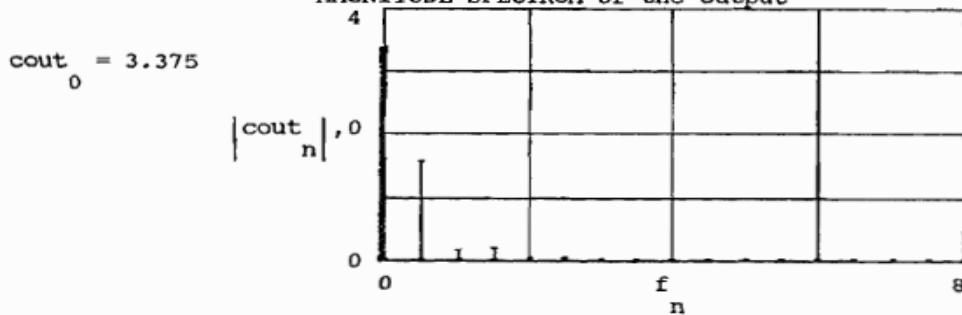
```
vo := vout(k · dt)           vol := vol(k · dt)           n := 0 .. N - 1
```

Since the signal is periodic, the spectrum will consist of delta functions (which can't be plotted directly since the delta function has an infinite value). However the weights of the delta functions are finite and can be plotted. The weights may be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

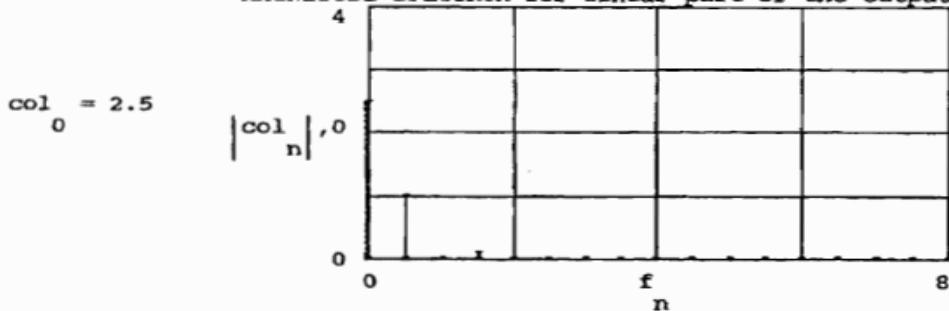
$$c_{out} := \left[ \frac{1}{\sqrt{N}} \right] \cdot \text{icfft}(v_o) \quad c_{ol} := \left[ \frac{1}{\sqrt{N}} \right] \cdot \text{icfft}(v_{ol})$$

$$f := \frac{n}{n \cdot T} \quad f = 0.5 \quad f_s := \frac{1}{dt} \quad f_s = 16$$

MAGNITUDE SPECTRUM of the output



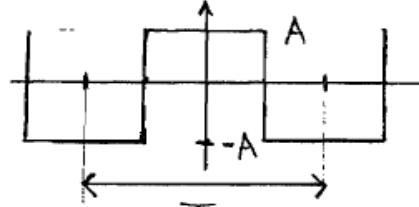
MAGNITUDE SPECTRUM for linear part of the output.



4-15

The output is a square wave as shown

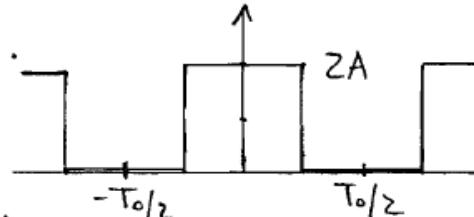
$$v(t) = \sum_{n=0}^{\infty} v_n \cos(n\omega_0 t)$$



where, using (2-96),

$$b_n = 0 \quad \text{and} \quad a_n = v_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} v(t) \cos(n\omega_0 t) dt$$

Since we are only interested in  $v_n$  for  $n \geq 1$ , we can shift the DC level of the waveform to make the integral easier to evaluate and yet the  $v_n$  will remain the same for  $n \neq 0$ .



$$v_n = \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} 2A \cos(n\omega_0 t) dt$$

$$= \frac{2}{T_0} (2A) \left. \frac{\sin(n\omega_0 t)}{n\omega_0} \right|_{-T_0/4}^{T_0/4} = \frac{4A}{n\pi} 2 \sin\left(\frac{n\pi}{2}\right)$$

$$\Rightarrow v_n^2 = \begin{cases} 0, & n = \text{even} \\ \left(\frac{4A}{n\pi}\right)^2, & n = \text{odd} \end{cases}$$

Using (4-47)

$$\text{THD \%} = \sqrt{\frac{\sum_{n=2}^{\infty} v_n^2}{v_1^2}} \times 100 = \sqrt{\frac{\sum_{n=3}^{\infty} \left(\frac{4A}{n\pi}\right)^2}{\left(\frac{4A}{\pi}\right)^2}}$$

## 4-15 (Continued)

$$\text{THD \%} = \sqrt{\sum_{\substack{n=3 \\ n=\text{odd}}}^{\infty} \frac{1}{n^2}} \times 100 = \underline{\underline{48.3\%}}$$

Check :

Using programmable calculator

$$\begin{aligned} \text{THD \%} &= \sqrt{\frac{\text{Total Power} - \left(\frac{V_1}{\sqrt{2}}\right)^2}{\left(\frac{V_1}{\sqrt{2}}\right)^2}} \times 100 \\ &= \sqrt{\frac{A^2 - \left(\frac{4A}{\pi\sqrt{2}}\right)^2}{\left(\frac{4A}{\pi\sqrt{2}}\right)^2}} \times 100 \\ &= \sqrt{\frac{\pi^2 - 8}{8}} (100) = \sqrt{0.233} (100) = \underline{\underline{48.3\%}} \end{aligned}$$

4-18

$$\begin{aligned} s(t) &= A_c [m(t) \cos \omega_c t + \hat{m}(t) \sin \omega_c t] \\ &= \operatorname{Re} \{ A_c [m(t) + j \hat{m}(t)] e^{j \omega_c t} \} \\ \Rightarrow g(t) &= A_c [m(t) + j \hat{m}(t)] = R(t) \angle \theta(t) \end{aligned}$$

Output of Envelope Detector is

$$v_{\text{out}}(t) = k R(t) = k |g(t)|$$

$$\Rightarrow v_{\text{out}}(t) = k A_c \sqrt{m^2(t) + \hat{m}^2(t)} \neq k m(t)$$

The output is distorted.

4-21

$$\frac{d\theta_e(t)}{dt} = \frac{d\theta_i(t)}{dt} - k_d k_v \theta_e(t) * f(t)$$

$$\Rightarrow s\theta_e(s) = s\theta_i(s) - k_d k_v \theta_e(s) F(s)$$

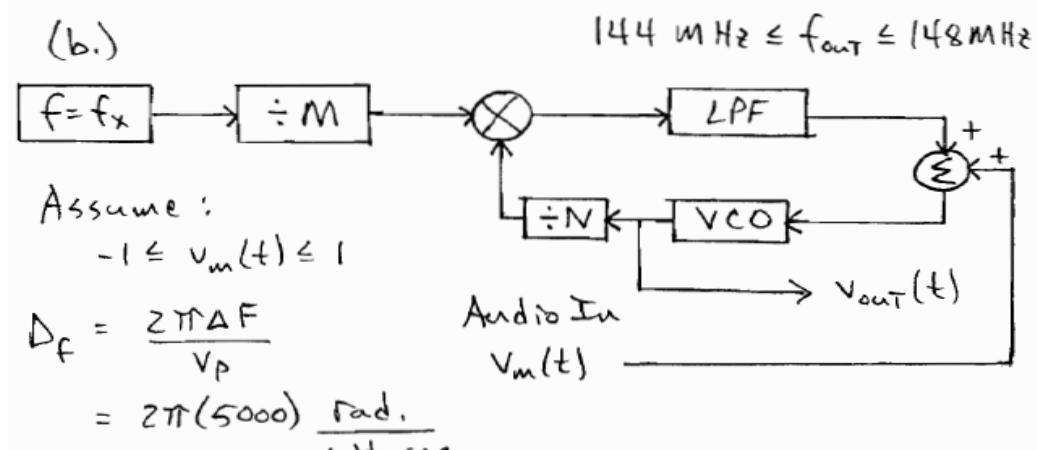
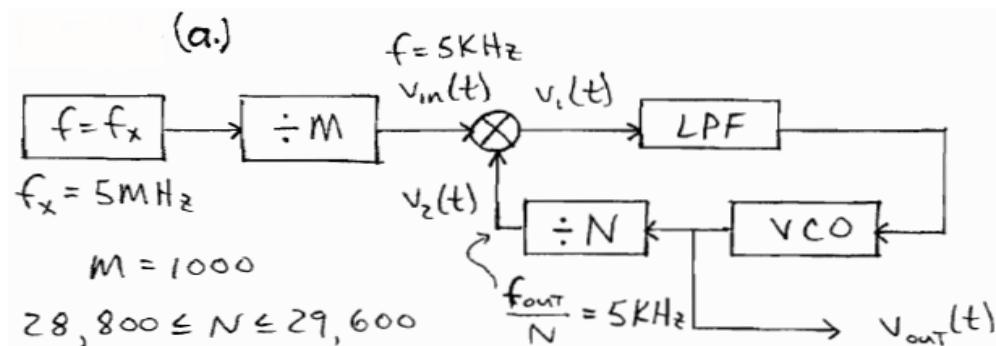
or  $\theta_e(s) = \frac{s\theta_i(s)}{s + k_d k_v F(s)}$

Final value theorem:

$$\lim_{t \rightarrow \infty} \theta_e(t) = \lim_{s \rightarrow 0} [\theta_e(s)] = \lim_{s \rightarrow 0} \frac{s^2 \theta_i(s)}{s + k_d k_v F(s)}$$

$$\Rightarrow \text{If } F(s) \neq 0 \Rightarrow \lim_{t \rightarrow \infty} \theta_e(t) = 0$$

4-23



4-26

$$(a.) f_{L_0} = 96.9 + 10.7 = \underline{107.6 \text{ MHz}}$$

(b.) RF: Flat bandpass over 96.81 MHz to 96.99 MHz and reject image frequency of 118.3 MHz

IF: Flat bandpass over 10.61 MHz to 10.79 MHz and reject adjacent channel signals on each side of this bandpass

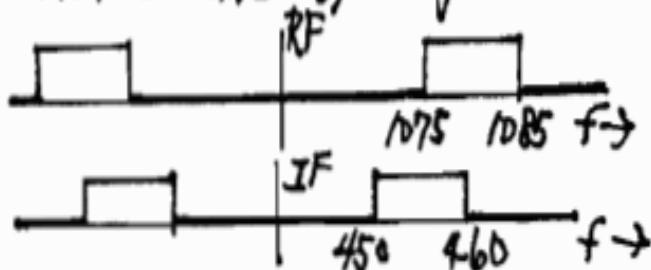
$$(c.) f_{\text{image}} = f_c + 2f_{\text{if}} = 96.9 + 2(10.7) = \underline{118.3 \text{ MHz}}$$

4-30

(a.) RF filter  $\Rightarrow f_c \pm \frac{\Delta}{2}$ ; IF filter  $\Rightarrow f_{\text{if}} \pm \frac{\Delta}{2}$  where  $\Delta = 10 \text{ kHz}$  be since the AM channel spacing is 10 kHz.

$$\text{RF: } 1080 \pm 5 \text{ kHz}$$

$$\text{IF: } 455 \pm 5 \text{ kHz}$$



$$(b.) f_{\text{image}} = f_c + 2f_{\text{if}} = 1990 \text{ kHz}$$

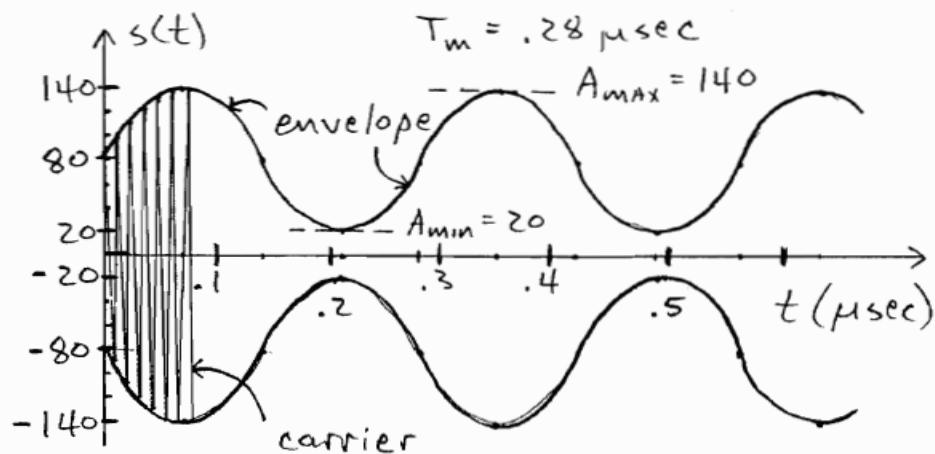
# Chapter 5

5-2

$$(a.) \quad m(t) = -0.2 + 0.6 \sin \omega_c t$$

$$f_m = f_c = 3.57 \text{ MHz} ; \quad A_c = \underline{\underline{100}}$$

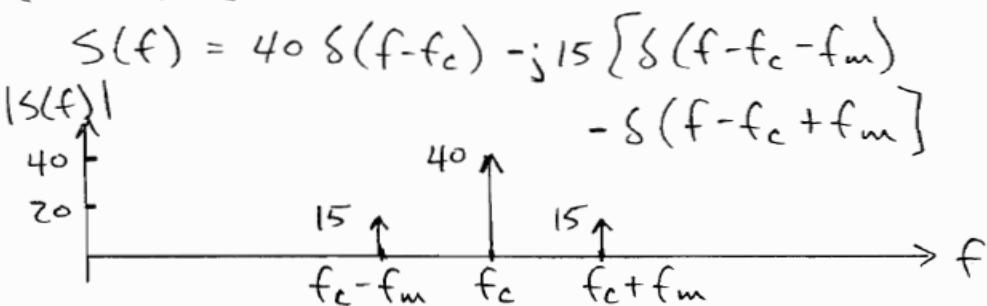
$$s(t) = 100 (0.8 + 0.6 \sin \omega_c t) \cos \omega_c t$$



$$(b.) \quad \% \text{ pos. mod.} = \frac{A_{\max} - A_c}{A_c} (100) = \frac{140 - 100}{100} (100) \\ = \underline{\underline{40\%}}$$

$$\% \text{ neg. mod.} = \frac{A_c - A_{\min}}{A_c} (100) = \frac{100 - 20}{100} (100) \\ = \underline{\underline{80\%}}$$

$$(c.) \quad f > 0$$



5-4

From (5-5a) given

$$\% \text{ Pos. Mod.} = \frac{A_{\max} - A_c}{A_c} (100) \downarrow = 120$$

where :

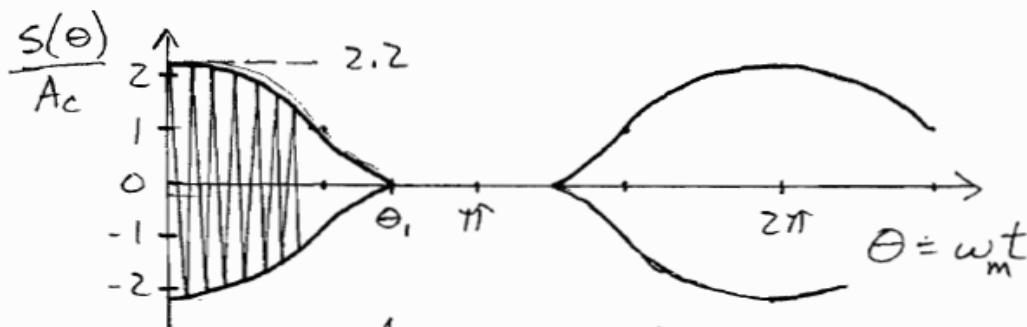
$$s(t) = \begin{cases} A_c \{1 + A_m \cos \omega_m t\} \cos \omega_c t; & m(t) \geq -1 \\ 0 & ; m(t) < -1 \end{cases}$$

$$m(t) = A_m \cos \omega_m t; \quad A_{\max} = A_c [1 + A_m]$$

$$\frac{A_{\max} - A_c}{A_c} = \underline{\underline{A_m = 1.2}}$$

$$g(t) = \begin{cases} A_c \{1 + 1.2 \cos \omega_m t\}, & 1.2 \cos \omega_m t \geq -1 \\ 0 & , 1.2 \cos \omega_m t < -1 \end{cases}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{j n \omega_m t} \Rightarrow G(f) = \sum_{-\infty}^{\infty} c_n \delta(f - n f_m)$$



$$A_m \cos \Theta_1 = -1$$

$$\text{Aside : } \Theta_1 = \cos^{-1}\left(\frac{-1}{1.2}\right) = \underline{\underline{146.4^\circ}}$$

$$\begin{aligned} c_n &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t) e^{-j n \omega_m t} dt \\ &= \frac{A_c}{2\pi} \int_{\Theta_1}^{\Theta_1} [1 + A_m \cos \theta] e^{-j n \theta} d\theta \end{aligned}$$

## 5-4 (Continued)

$$c_n = \frac{A_c}{2\pi} \left[ \frac{e^{-jn\theta}}{-jn} \Big|_{-\theta_1}^{\theta_1} + A_m \left\{ \int_{-\theta_1}^{\theta_1} (\cos\theta) e^{-jn\theta} d\theta \right\} \right]$$

$$\uparrow = \frac{A_c}{2\pi} \left[ \frac{2}{n} \left( \frac{e^{jn\theta_1} - e^{-jn\theta_1}}{jz} \right) + A_m \frac{e^{-jn\theta_1}}{(-jn)^2 + 1} \right]$$

Using Sec. A-5

where  $a = -jn$

$$\cdot (-jn \cos\theta + \sin\theta) \Big|_{-\theta_1}^{\theta_1} \Big]$$

$$= \frac{A_c}{2\pi} \left[ \frac{2 \sin n\theta_1}{n} + A_m \left\{ \frac{e^{-jn\theta_1} (-jn \cos\theta_1 + \sin\theta_1)}{1-n^2} \right. \right.$$

$$\left. \left. - \frac{e^{jn\theta_1} (-jn \cos\theta_1 - \sin\theta_1)}{1-n^2} \right\} \right]$$

$$= \frac{A_c}{2\pi} \left[ 2\theta_1 \left( \frac{\sin n\theta_1}{n\theta_1} \right) + A_m \left\{ jn(zj) \left( \frac{e^{jn\theta_1} - e^{-jn\theta_1}}{2j} \right) \cos\theta_1 \right. \right.$$

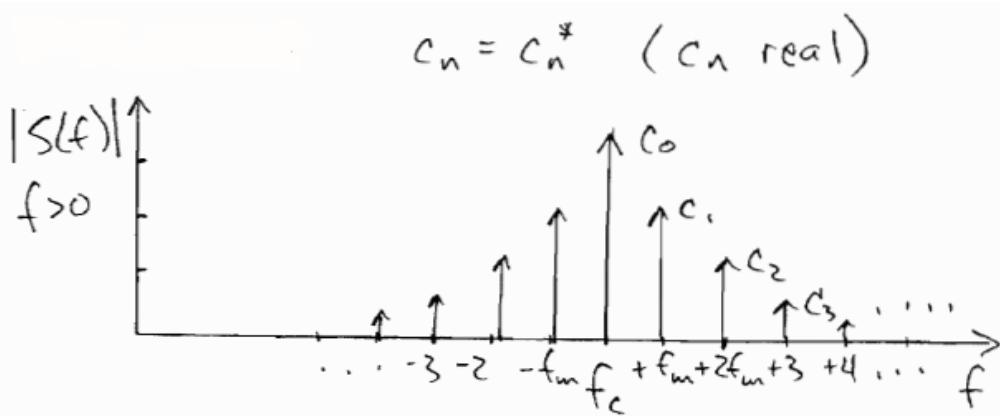
$$\left. \left. + \frac{2 \left( \frac{e^{jn\theta_1} + e^{-jn\theta_1}}{2} \right) \sin\theta_1}{1-n^2} \right\} \right]$$

$$c_n = \frac{A_c}{2\pi} \left[ 2\theta_1 \left( \frac{\sin(n\theta_1)}{n\theta_1} \right) + 2A_m \left\{ \frac{\cos(n\theta_1) \sin\theta_1}{1-n^2} \right. \right.$$

$$\left. \left. - \frac{n \sin(n\theta_1) \cos\theta_1}{1-n^2} \right\} \right]$$

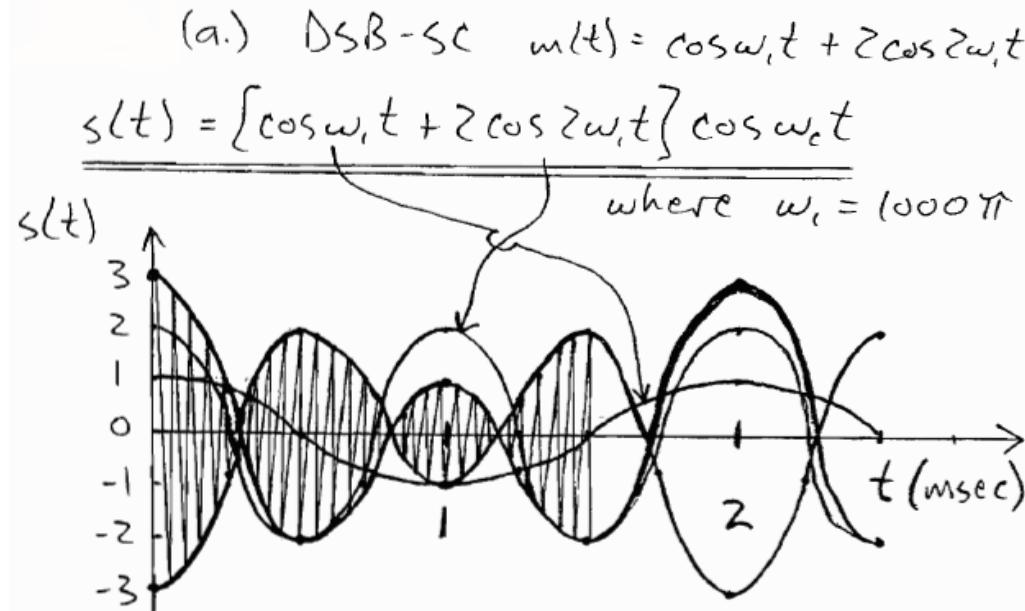
$$S(f) = \frac{1}{2} \left[ \sum_{-\infty}^{\infty} c_n \delta(f-f_c-nf_m) + \sum_{-\infty}^{\infty} c_n^* \delta(-f-f_c-nf_m) \right]$$

## 5-4 (Continued)



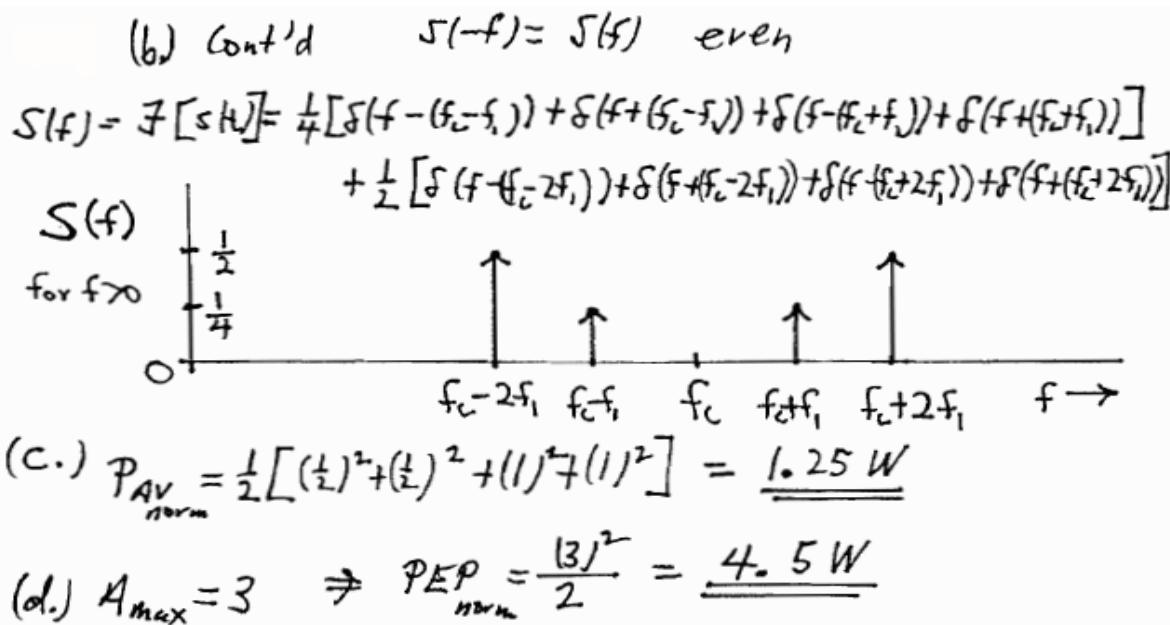
$$\begin{aligned} c_0 &= \frac{A_c}{2\pi} \left[ 2\Theta_i (\text{rad.}) + 2A_m \sin \Theta_i \right] \\ &= \frac{A_c}{2\pi} \left[ 2(2.56) + 2(1.2)(.553) \right] \end{aligned}$$

## 5-5



$$\begin{aligned} (b.) \quad s(t) &= \frac{1}{2} \left[ \cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t \right] \\ &\quad + \cos(\omega_c - 2\omega_i)t + \cos(\omega_c + 2\omega_i)t \end{aligned}$$

### 5-5 (Continued)



### 5-10

$$m(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & t \text{ elsewhere} \end{cases}$$

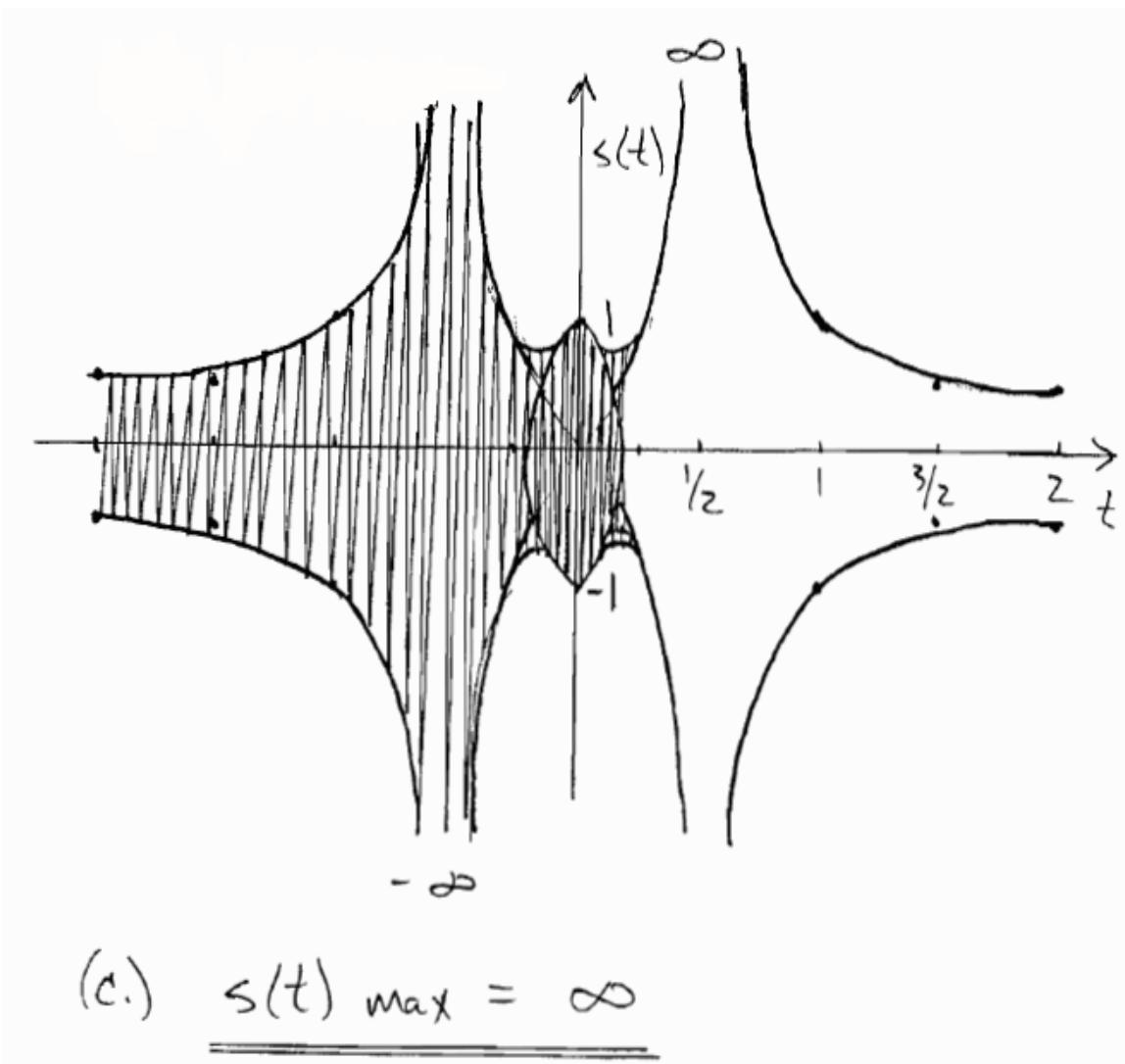
(a.)  $\hat{m}(t) = m(t) * \frac{1}{\pi t}$   
 $= \int_{-1/2}^{1/2} \frac{1}{\pi} \frac{1}{t-\lambda} d\lambda = \frac{-1}{\pi} \int_{t+1/2}^{t-1/2} \frac{1}{\lambda} d\lambda,$   
 $\lambda_1 = t - \lambda$   
 $d\lambda_1 = -d\lambda$   
 $= \int_{t-1/2}^{t+1/2} \frac{1}{\pi} \frac{1}{\lambda_1} d\lambda_1 = \frac{1}{\pi} (\ln |\lambda_1|) \Big|_{t-1/2}^{t+1/2}$   
 $\hat{m}(t) = \frac{1}{\pi} \ln \left[ \frac{|t+\frac{1}{2}|}{|t-\frac{1}{2}|} \right]$

(b.) For USSB:

$$s(t) = m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t$$

$$= \pi(t) \cos \omega_c t - \frac{1}{\pi} \ln \left[ \frac{|t+\frac{1}{2}|}{|t-\frac{1}{2}|} \right] \sin \omega_c t$$

5-10 (Continued)



5-11

Note:  $T$  has units of Hz.

$$(a) m(t) = \frac{\sin(\pi T t)}{\pi T t} \longleftrightarrow M(f) = \frac{1}{T} \Pi\left(\frac{\pi f}{T}\right) = \frac{1}{T} \left[ \Pi\left(\frac{f-\frac{T}{2}}{T/2}\right) + \Pi\left(\frac{f+\frac{T}{2}}{T/2}\right) \right]$$

$$\Rightarrow \hat{m}(t) = M(f) \begin{cases} -j, f > 0 \\ j, f < 0 \end{cases} = \frac{1}{T} \left[ -j \Pi\left(\frac{f-T/4}{T/2}\right) + j \Pi\left(\frac{f+T/4}{T/2}\right) \right]$$

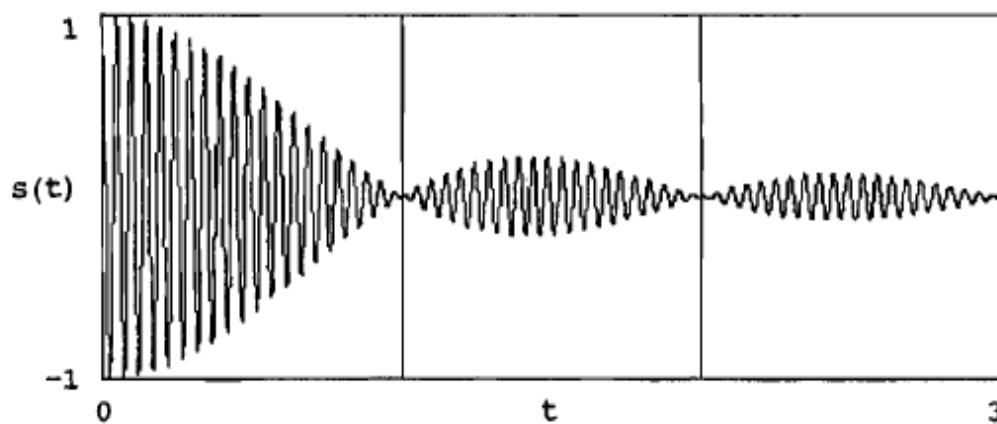
$$\begin{aligned} \hat{m}(t) &= -j \frac{1}{2} \frac{\sin(\pi \frac{T}{2} t)}{\pi \frac{T}{2} t} e^{j 2\pi \frac{T}{4} t} + j \frac{1}{2} \frac{\sin(\pi \frac{T}{2} t)}{\pi \frac{T}{2} t} e^{-j 2\pi \frac{T}{4} t} \\ &= \frac{\sin(\pi \frac{T}{2} t)}{\pi \frac{T}{2} t} \frac{e^{j \pi \frac{T}{2} t} - e^{-j \pi \frac{T}{2} t}}{2j} = \frac{\sin(\pi \frac{T}{2} t)}{\pi \frac{T}{2} t} \sin(\pi \frac{T}{2} t) = \frac{\sin^2(\pi \frac{T}{2} t)}{\pi \frac{T}{2} t} \end{aligned}$$

$$(b) \quad t := 10^{-7}, 0.002 \dots 3 \quad T := 2$$

$$\frac{f}{c} := 20 \quad \omega_c := 2 \cdot \pi \cdot \frac{f}{c}$$

$$m(t) := \frac{\sin(\pi \cdot T \cdot t)}{\pi \cdot T \cdot t} \quad m_h(t) := \frac{\left[ \sin\left(\frac{\pi}{2} \cdot \frac{T}{2} \cdot t\right) \right]^2}{\frac{T}{2} \cdot t}$$

$$s(t) := m(t) \cdot \cos\left[\omega_c \cdot t\right] - m_h(t) \cdot \sin\left[\omega_c \cdot t\right]$$



5-18

$$(a.) \Theta(t) = D_p m_p(t) = 20 \cos \omega_i t$$

$$\Rightarrow m_p(t) = \frac{20}{D_p} \cos \omega_i t = \underline{\underline{0.2 \cos(2000\pi t)}}$$

$$m_p(t)_{\text{peak}} = \underline{\underline{0.2 \text{ v}}} ; f_m = \underline{\underline{1 \text{ kHz}}}$$

$$(b.) \Theta(t) = D_f \int_{-\infty}^t m_f(\lambda) d\lambda = 20 \cos \omega_i t$$

$$\Rightarrow m_f(t) = \frac{20}{D_f} \frac{d}{dt} [\cos \omega_i t]$$

$$= \frac{-20}{10^6} (2000\pi) \sin \omega_i t$$

$$m_f(t) = \underline{\underline{-1257 \sin \omega_i t}}$$

$$m_f(t)_{\text{peak}} = \underline{\underline{-1257 \text{ v}}} ; f_m = \underline{\underline{1 \text{ kHz}}}$$

$$(c.) P_{AV} = \frac{V_{rms}^2}{R} = \frac{(500)^2}{2(50)} = \underline{\underline{2.5 \text{ kW}}}$$

$$PEP = \underline{\underline{2.5 \text{ kW}}}$$

5-22

$$(a.) P_T = \frac{1}{2} \langle |g(t)|^2 \rangle = \frac{A_c^2}{2} = \frac{(100)^2}{2} = \underline{\underline{5,000 \text{ watts}}}$$

$$(b.) P_T = \frac{1}{2} \langle |g(t)|^2 \rangle = \frac{1}{2} \left\langle \sum_{n=-N}^N |c_n|^2 \right\rangle = \frac{1}{2} \sum_{n=-N}^N |c_n|^2 = \frac{1}{2} A_c^2 \sum_{n=-N}^N J_n(\beta)$$

Using (5-55)                      Using (5-57)

$$\Rightarrow P_T = \frac{1}{2} A_c^2 \left[ J_0^2(\beta) + 2 \sum_{n=1}^N J_n^2(\beta) \right]$$

$$\text{where } 2Nf_m = B_T \Rightarrow N = \frac{B_T}{2f_m} = \frac{56 \text{ Hz}}{2(8)} = 3.5 \Rightarrow \text{Use } N=3$$

$$\text{Also, } A = \frac{\Delta F}{f_m} = \frac{k_d A_m}{f_m} = \frac{(8 \text{ Hz/volt})(5 \text{ volt})}{8 \text{ Hz}} = 5.0$$

Using MathCAD:  $A_c := 100$        $\beta := 5.0$        $N := 3$        $n := 1 .. N$

$$P_T := 0.5 \cdot A_c^2 \left[ J_0(\beta)^2 + 2 \cdot \sum_n J_n(n, \beta)^2 \right]$$

$$\underline{\underline{P_T = 2583.485 \text{ watts (normalized for } 1\Omega\text{)}}$$

5-26

$$s(t) = \operatorname{Re} \{ q(t) e^{j\omega_c t} \} = \operatorname{Re} \{ 10 e^{j\theta(t)} e^{j\omega_c t} \}$$

$$\theta(t) = \beta_m(t)$$

$$\beta = 45^\circ \left( \frac{\pi \text{ rad}}{180^\circ} \right) = 0.785 = \frac{\pi}{4}$$

$$q(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t}$$

$$c_n = \frac{1}{T_m} \int_{-T_m/2}^{T_m/2} q(t) e^{-jn\omega_m t} dt = \frac{10}{T_m} \left[ \int_{-T_m/2}^{-T_m/4} e^{j\theta} e^{-jn\omega_m t} dt + \int_{-T_m/4}^{T_m/4} e^{j\beta} e^{-jn\omega_m t} dt + \int_{T_m/4}^{T_m/2} e^{j\theta} e^{-jn\omega_m t} dt \right]$$

$$c_n = \frac{10}{T_m} \left[ \int_{T_m/4}^{T_m/2} e^{jn\omega_m t} dt + \int_{-T_m/4}^{T_m/4} e^{j\beta} e^{-jn\omega_m t} dt + \int_{T_m/4}^{T_m/2} e^{-jn\omega_m t} dt \right]$$

$$= \frac{20}{T_m} \int_{T_m/4}^{T_m/2} \left( \frac{e^{jn\omega_m t} + e^{-jn\omega_m t}}{2} \right) dt + \frac{10 e^{j\beta}}{T_m} \left. \frac{e^{jn\omega_m t}}{-jn\omega_m} \right|_{-T_m/4}^{T_m/4}$$

$$= \frac{20}{T_m} \left[ \frac{\sin(n\omega_m t)}{n\omega_m} \Big|_{T_m/4}^{T_m/2} + e^{j\beta} \frac{e^{j\pi/2} - e^{-j\pi/2}}{2j n\omega_m} \right]$$

$$= \frac{20}{T_m} \left[ \frac{\sin(\frac{n\pi}{2}) - \sin(\frac{n\pi}{2})}{n\omega_m} + e^{j\beta} \frac{\sin(\frac{n\pi}{2})}{n\omega_m} \right]$$

$$c_n = 5(e^{j\beta} - 1) \left[ \frac{\sin(\frac{n\pi}{2})}{\frac{n\pi}{2}} \right], \quad \beta = \frac{\pi}{4}$$

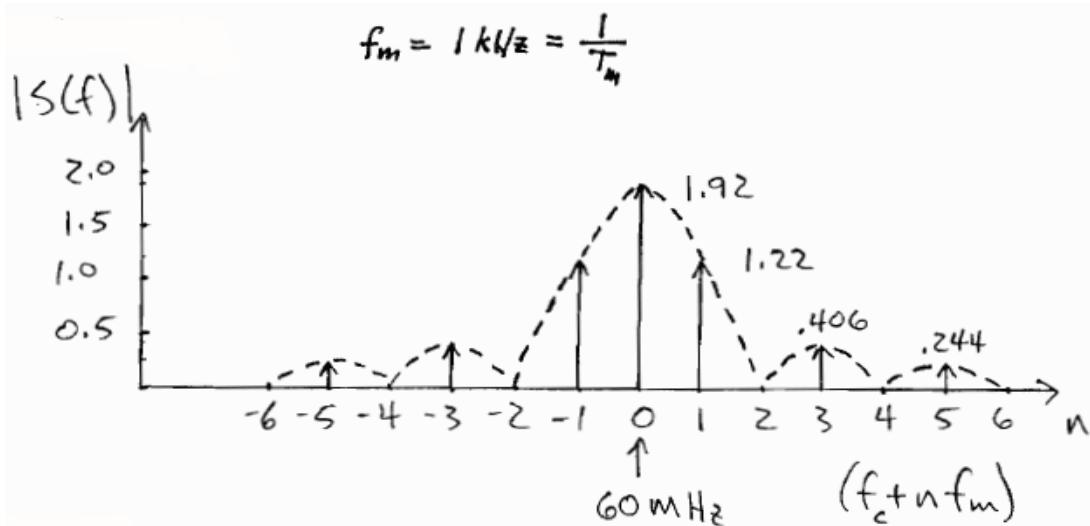
$$|c_n| = 5 \sqrt{(cos\beta - 1)^2 + (sin\beta)^2} \left| \frac{\sin(\frac{n\pi}{2})}{\frac{n\pi}{2}} \right|$$

$$\Rightarrow |c_n| = 3.83 \left| \frac{\sin(\frac{n\pi}{2})}{\frac{n\pi}{2}} \right|$$


---


$$S'(f) = \frac{1}{2} \left[ \sum_{n=-\infty}^{\infty} c_n \delta(f - f_c - nf_m) + \sum_{n=-\infty}^{\infty} c_n^* \delta(f + f_c + nf_m) \right]$$

## 5-26 (Continued)



## 5-30

NBFM

$$\Theta(t) = D_f \int_0^t m(\lambda) d\lambda$$

(a.)

$m(t)$

$$T_m = 10 \text{ msec}$$

$$\Delta \Theta = D_f \int_0^{T_m/2} m(t) dt = D_f \int_0^{T_m/2} 5 dt = D_f 5 t \Big|_0^{T_m/2} = D_f 5 \frac{T_m}{2} = \frac{10 \pi}{180} \left( \frac{2}{5 T_m} \right) = \frac{20 \pi}{5 (180) 10^{-2}} = 6.98 \frac{\text{rad}}{\text{V.sec}}$$

$$\Rightarrow D_f = \frac{10 \pi}{180} \left( \frac{2}{5 T_m} \right) = \frac{20 \pi}{5 (180) 10^{-2}} = 6.98 \frac{\text{rad}}{\text{V.sec}}$$

$$(5-6) \Rightarrow \Delta F = \frac{D_f V_p}{2\pi} = \frac{6.98(5)}{2\pi} = 5.55 \text{ Hz}$$

(b.) From (5-26) and (5-27)

$$S'(f) = \frac{A_c}{2} \left\{ \delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi} \frac{M(f-f_c)}{f-f_c} - \frac{D_f}{2\pi} \frac{M(f+f_c)}{f+f_c} \right\}$$

$$M(f) = \mathbb{E}[m(t)] = \sum_{n=-\infty}^{\infty} c_n \delta(f-nf_m) \text{ where } f_m = \frac{1}{T_m} = 100 \text{ Hz}$$

---


$$S'(f) = \frac{A_c}{2} \left\{ \delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \left( \frac{c_n}{nf_m} \right) \left[ \delta(f-f_c-nf_m) - \delta(f+f_c-nf_m) \right] \right\}$$


---

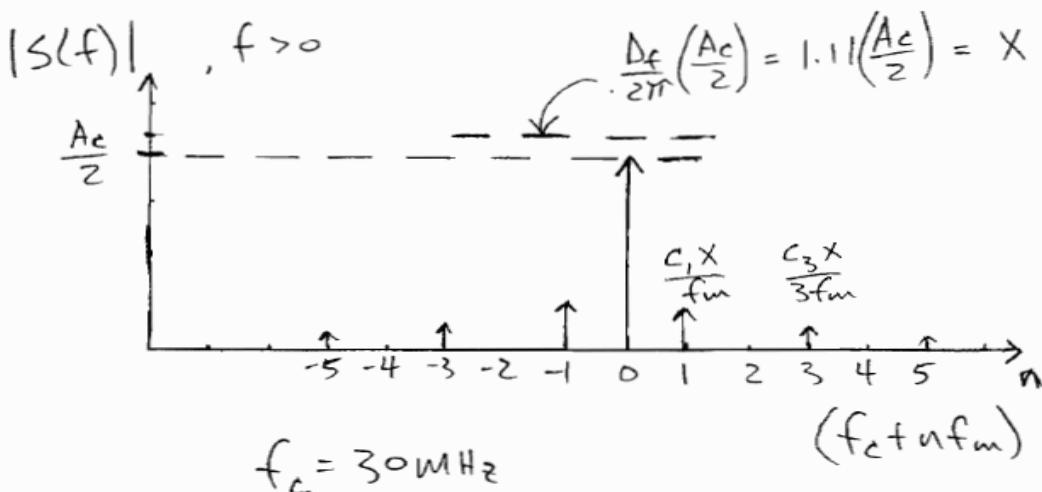
## 5-30 (Continued)

Aside : Evaluate  $c_n$ 

$$\begin{aligned}
 c_n &= \frac{1}{T_m} \int_0^{T_m} m(t) e^{-j n \omega_m t} dt \\
 &= \frac{1}{T_m} \left[ \int_0^{T_m/2} 5 e^{-j n \omega_m t} dt + \int_{T_m/2}^{T_m} (-5) e^{-j n \omega_m t} dt \right] \\
 &= \frac{5}{T_m} \left[ \frac{e^{-j n \omega_m t}}{-j n \omega_m} \Big|_0^{T_m/2} - \frac{e^{-j n \omega_m t}}{-j n \omega_m} \Big|_{T_m/2}^{T_m} \right] \\
 &= \frac{5}{T_m} \left[ \frac{e^{-j n \omega_m \frac{T_m}{2}} - e^{j 0}}{-j n \omega_m} - e^{-j n \omega_m T_m} + e^{-j n \omega_m \frac{T_m}{2}} \right] \\
 &= \frac{5}{T_m} \left[ \frac{2 e^{-j n \pi} - 1 - e^{-j n \pi}}{-j n \omega_m} \right] \\
 &= 10 \left[ \frac{1 - e^{-j n \pi}}{j n \omega_m T_m} \right] = 10 e^{-j \frac{n \pi}{2}} \left[ \frac{e^{j \frac{n \pi}{2}} - e^{-j \frac{n \pi}{2}}}{j 2 \pi n} \right] \\
 &= \frac{10}{2} e^{-j \frac{n \pi}{2}} \left[ \frac{\sin\left(\frac{n \pi}{2}\right)}{n \pi / 2} \right] \\
 c_n &= 5 (-j)^n \left[ \frac{\sin\left(\frac{n \pi}{2}\right)}{n \pi / 2} \right], n \neq 0; c_0 = 0
 \end{aligned}$$

### 5-30 (Continued)

$$\underline{c_n = 0, n = \text{even}}$$

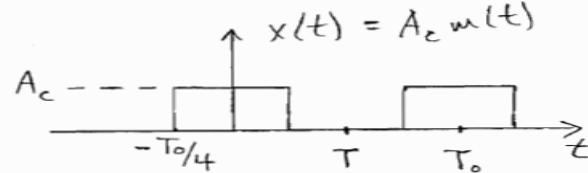


### 5-33

$$(a.) s(t) = x(t) \cos \omega_c t, \text{ where}$$

$$T = \frac{1}{R} = \frac{1}{24000}$$

$$T_o = 2T$$



OOK:

$$S(f) = \frac{1}{2} [x(f-f_c) + x^*(-f-f_c)]$$

$$X(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f-nf_o); x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

$$c_n = \frac{1}{T_o} \int_{-T_o/4}^{T_o/4} A_c e^{-jn\omega_o t} dt = \frac{A_c}{T_o} \frac{e^{-jn\omega_o T_o/4}}{-jn\omega_o} \Big|_{-T_o/4}^{T_o/4}$$

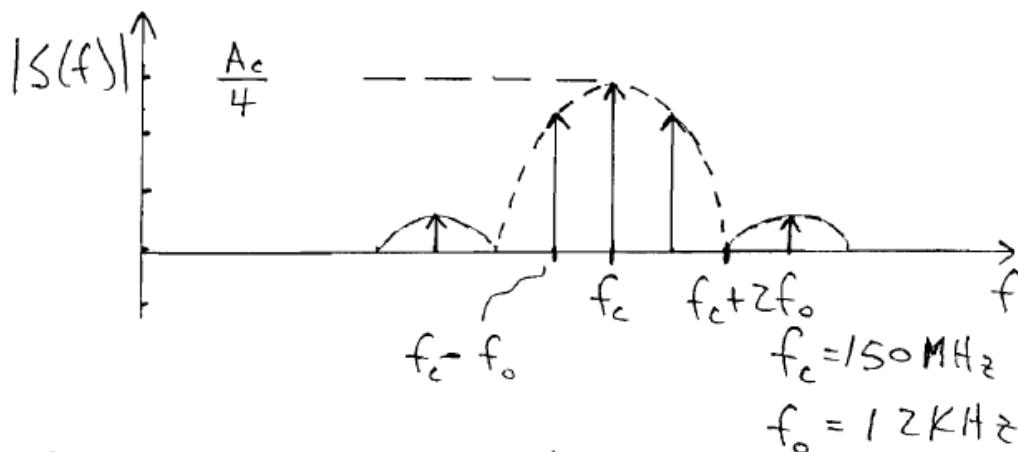
$$= \frac{A_c}{T_o} \frac{e^{-jn\pi/2} - e^{jn\pi/2}}{-jn2\pi/T_o} = \frac{A_c}{2} \frac{\sin(n\pi/2)}{n\pi/2}$$

$$X(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} \left[ \frac{\sin(n\pi/2)}{n\pi/2} \right] \delta(f-nf_o)$$

$$f_o = \frac{1}{T_o} = \frac{1}{2T} = \frac{f}{2}$$

$$\underline{s(f) = \frac{1}{2} [x(f-f_c) + x^*(-f-f_c)]}$$

5-33 (Continued)

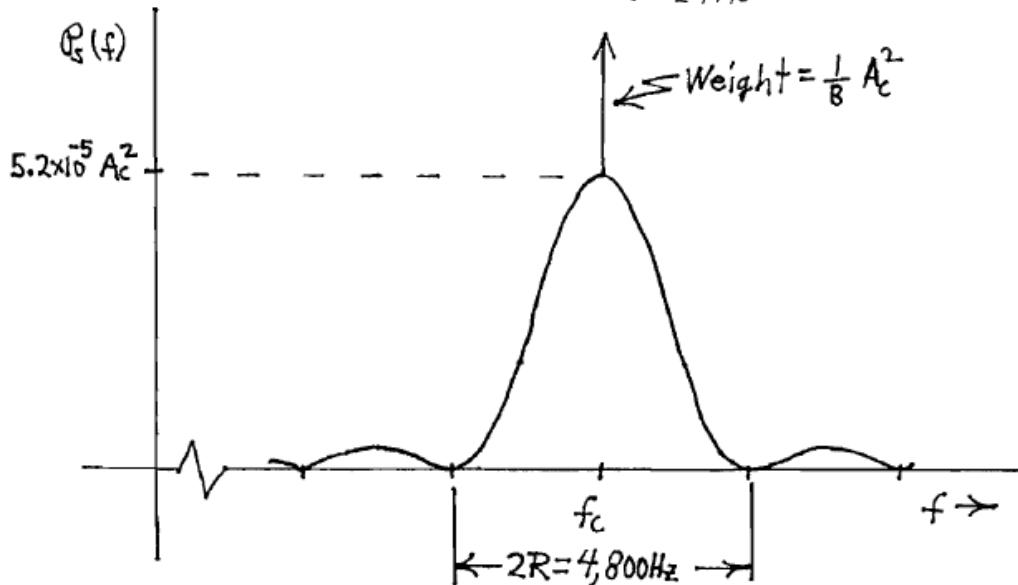


$$B\omega_T = 2(2f_o) = 2R = \underline{48 \text{ kHz}}$$

(c.) Using (5-2b) and (5-72)

$$P_s(f) = \frac{1}{4} \frac{A_c^2}{2} \left[ \delta(f-f_c) + T_b \left( \frac{\sin \pi(f-f_c)T_b}{\pi(f-f_c)T_b} \right)^2 + \delta(f+f_c) + T_b \left( \frac{\sin \pi(f+f_c)T_b}{\pi(f+f_c)T_b} \right)^2 \right]$$

$$\text{where } f_c = 150 \text{ MHz} \text{ and } T_b = \frac{1}{R} = \frac{1}{24 \times 10^3} = 0.0417 \text{ msec}$$



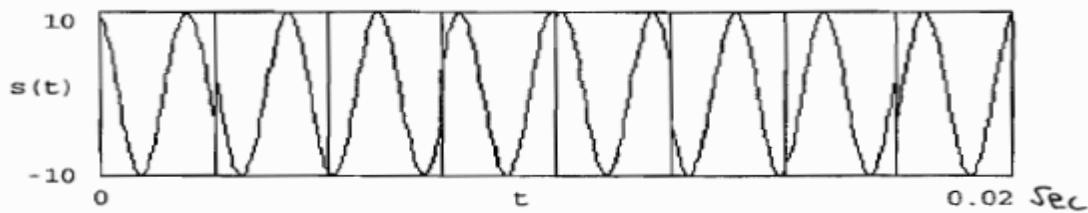
The null-to-null bandwidth is the same for both (b) and (c). Both have sinc/x type spectral envelope.

5-34

```

k := 0 .. 7      T := 0.0025      t := 0,0.00008 .. 8 T
a :=
k
1
-1
-1
1
-1
1
1
-1
ω := 1000 π
c
rect(t) := if[|t -  $\frac{T}{2}| \leq \frac{T}{2}, 1, 0]
m(t) :=  $\sum_k [a_k rect(t - k T)]$$ 
```

(a) SET INDEX  
 $h := 0.2$        $N := 64$        $n := 1 .. N - 1$        $j := 0 .. \frac{N}{2}$   
 $D := \frac{h \pi}{2}$        $Ts := \frac{0.02}{N}$        $t_1 := n Ts$   
 $s(t) := 10 \cos[\omega_c t + \frac{D}{p} m(t)]$



```

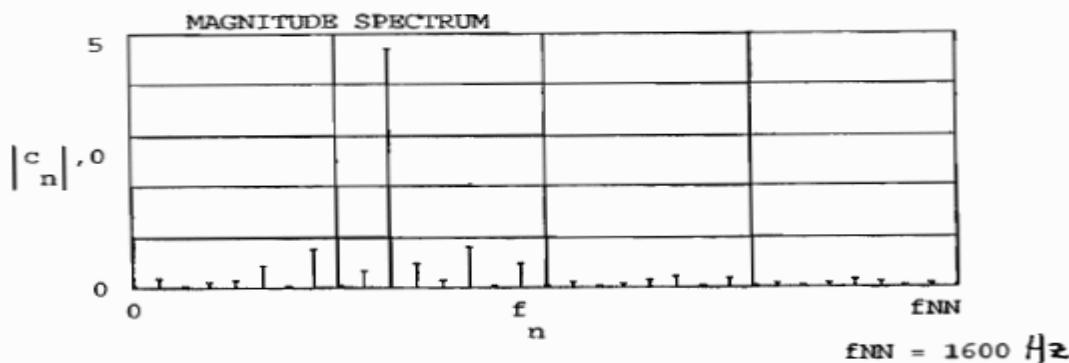
M := 6      M
N := 2      N = 64      k := 0 .. N - 1      T0 := 8 T
dt :=  $\frac{T0}{N}$       dt = 0      t_k := k dt
ss_k := s(k * dt)      n := 0 .. N - 1      NN := 2^{M-1}

```

Assume that the signal is periodic with period  $T_0=8T$ . The spectrum can be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

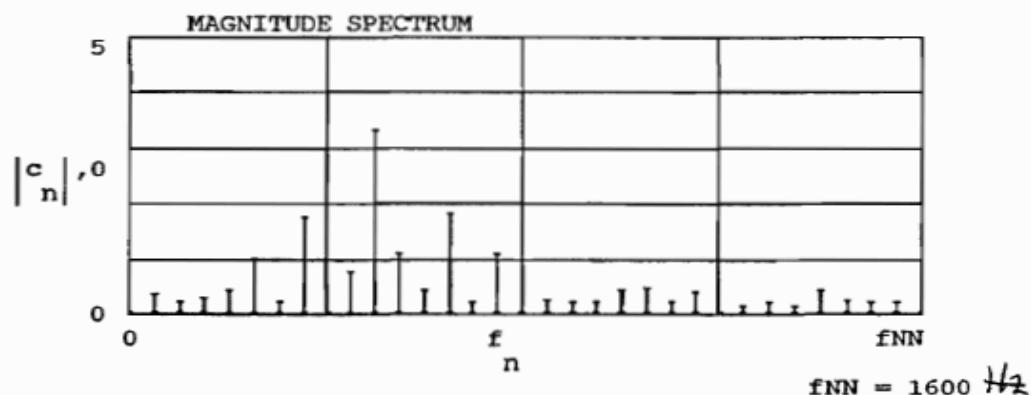
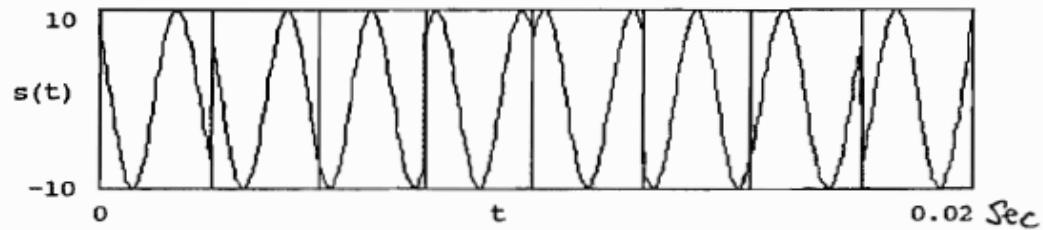
```

c :=  $\left[ \frac{1}{\sqrt{N}} \right] iFFT(ss)      fs := \frac{1}{dt}      fs = 3200 Hz
f_n :=  $\frac{n}{T0}      f_1 = 50      f_{NN} := NN f_1      f_{NN} = 1600$$ 
```

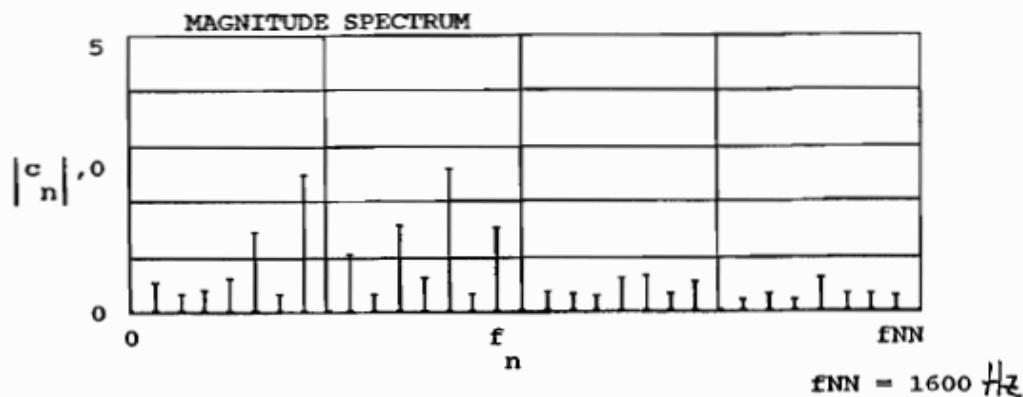
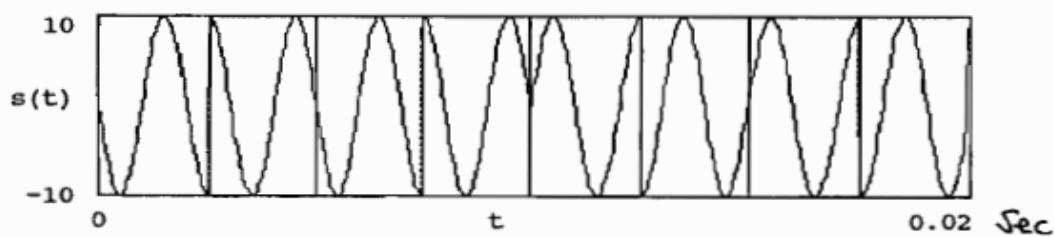


5-34 (Continued)

(b.)  $h = 0.5$



(c.)  $h = 1.0$



5-41

Use (5-106)  $B_{\pi} = (1+r)\frac{R}{l}$  where  $l=2$  for QPSK

$$(a.) \Rightarrow 24 = (1+r)\frac{30}{2} \Rightarrow (1+r) = \frac{2(24)}{30} = 1.6$$

or  $r = 0.6$

(b.) Max R allowed is when  $r=0$

$$\Rightarrow R_{\max} = \frac{2B_{\pi}}{l} = 2(24) = 48 \text{ Mb/sec}$$

$\Rightarrow No.$  A roll-off factor, r, could not be found support 50Mb/s QPSK signaling

5-49

From the description of  $\pi/4$  QPSK in Sec. 5-10,  
use the table shown  
at the right

Input Bits	$\Delta\theta$
11	+45°
01	+135°
00	-135°
10	-45°

Data	10	11	01	00	10	10	10
$\Delta\theta$	-45°	+45°	+135°	-135°	-45°	-45°	-45°

(b) From (5-106)

$$B_{\pi} = \left(\frac{1+r}{l}\right)R = \left(\frac{1+0.5}{2}\right)R = \frac{\frac{3}{2}}{2}R = \frac{3}{4}R$$

$r=0.5, l=2$

$$B_{\pi} = \frac{3}{4}R = \frac{3}{4}(1.5) = \underline{1.13 \text{ Mb/sec}}$$

$R=1.5 \text{ Mb/s}$

## 5-52

```
i := 0 ..14    n := 0 ..14    t := -5,-4.99 ..5    T := 1      a := -1
a := -1      a := -1      a := 1      a := 1      a := 1      a0 := -1
1           2           3           4           5           6
a := -1      a := -1      a := 1      a := -1      a := -1      a := 1
7           8           9           10          11          12
a := -1      a := -1
13          14
h(t) := φ(t) - φ(t - 1)           n1 := 0,2 ..14           n2 := 1,3 ..13
tt := 0,0.2 ..15
```

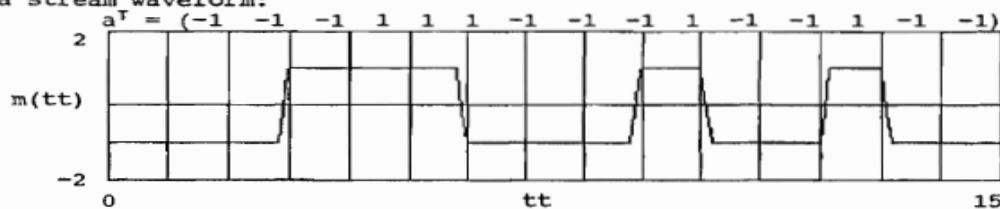
$$hx(t) := \sum_n a_n \cdot h(t - nT)$$

$$x(tt) := \sum_{n1} a_{n1} \cdot hx(tt - n1T)$$

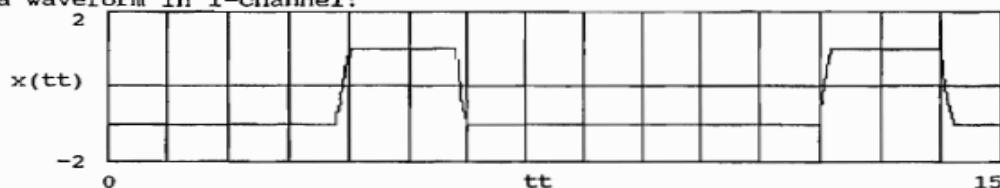
$$y(tt) := \sum_{n2} a_{n2} \cdot hx(tt - n2T) \sin\left[\pi \frac{tt - n2T}{2}\right]$$

$$xx(tt) := \sum_{n1} a_{n1} \cdot hx(tt - n1T) \cos\left[\pi \frac{tt - n1T - 1}{2}\right]$$

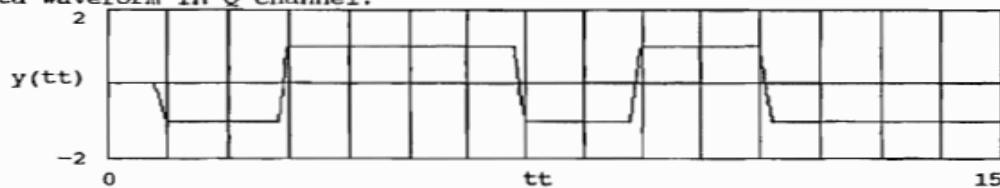
Data stream waveform:



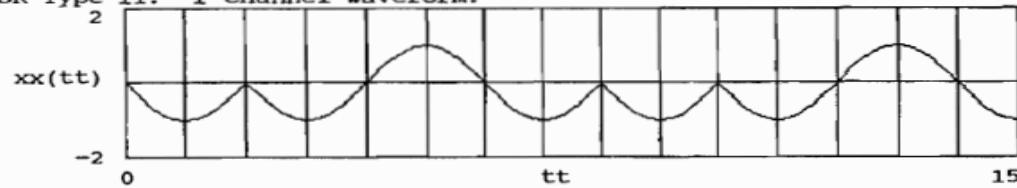
Data waveform in I-channel:



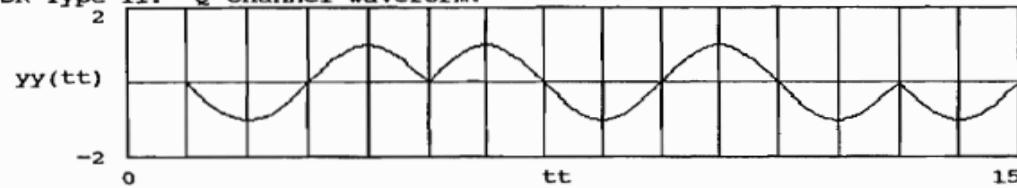
Data waveform in Q-channel:



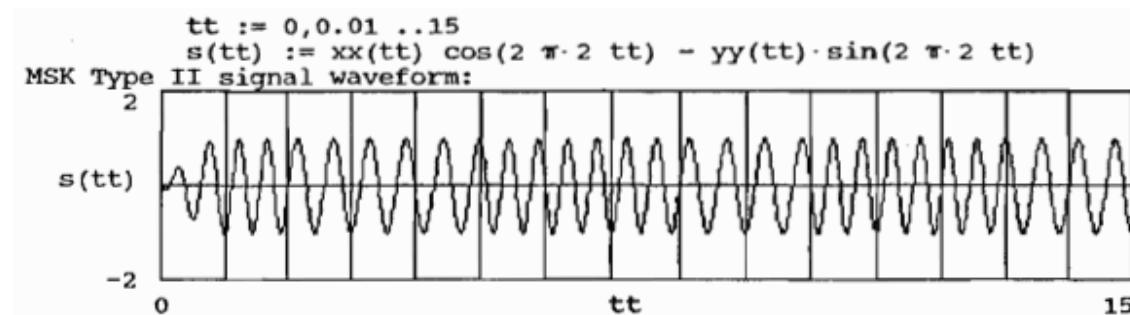
MSK Type II: I-channel waveform:



MSK Type II: Q-channel waveform:



## 5-52 (Continued)



## 5-54

$$(a) H(f) = e^{-(\frac{f}{B})^2 \left(\frac{\ln 2}{2}\right)} = \left[ e^{-\pi \left( f \sqrt{\frac{\ln 2}{2}} \frac{1}{B} \right)^2} \right] \Rightarrow h(t) = \sqrt{\frac{2\pi}{\ln 2}} e^{-\pi \left( t \sqrt{\frac{2\pi}{\ln 2}} \right)^2}$$

$\pi \left( \frac{t}{\pi} \right) \xrightarrow{h(\lambda)} h_s(t) \quad (\pi(-t) = \pi(t)) \quad \text{Table 2-2}$

$$h_s(t) = \pi \left( \frac{t}{\pi} \right) * h(t) = \int h(\lambda) \pi \left( \frac{t-\lambda}{\pi} \right) d\lambda \stackrel{\text{Convolution}}{=} \int h(\lambda) \pi \left( \frac{\lambda-t}{\pi} \right) d\lambda$$

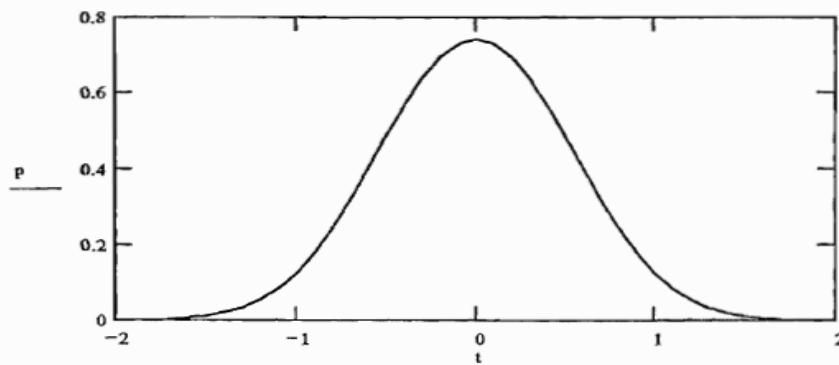
$$= \int_{t-\frac{\pi}{2}}^{t+\frac{\pi}{2}} \left( \sqrt{\frac{2\pi}{\ln 2}} \right) B e^{-\frac{2\pi^2}{\ln 2} \lambda^2 B^2} d\lambda$$

$$\Rightarrow h_s(t) = \left( \sqrt{\frac{2\pi}{\ln 2}} \right) (B\pi) \int_{\frac{t}{\pi} - \frac{1}{2}}^{\frac{t}{\pi} + \frac{1}{2}} e^{-\frac{2\pi^2}{\ln 2} (B\pi)^2 x^2} dx = p(t)$$

Let  $\lambda = \pi x$   
 $d\lambda = \pi dx$   
 $x = \frac{1}{\pi} \lambda$

$$(b) BTb := 0.3 \quad Tb := 1 \quad N := 20 \quad dt := \frac{2 \cdot Tb}{N} \quad n := 0, 1..2 \cdot N \quad t_n := (n - N) \cdot dt$$

$$p(t) := \left[ \sqrt{2 \cdot \frac{\pi}{\ln(2)}} \cdot BTb \cdot \int_{\frac{t}{Tb} - 0.5}^{\left( \frac{t}{Tb} \right) + 0.5} e^{-\left[ \frac{(2) \cdot \left( \frac{\pi^2}{\ln(2)} \cdot (BTb)^2 \right) x^2}{2} \right]} dx \right] \quad p_n := p(t_n)$$



5-60

(a) Referring to Fig 5-42a, the FSK signal is

$$v_1(t) = \cos[\omega_c t + \theta(t)] \text{ where } \theta(t) = D_f \int_{t_0}^t m(\lambda) d\lambda$$

The output of the FH spreader is

$$\begin{aligned} v_2(t) &= A_c \cos[\omega_c t + \theta(t)] \cos[\omega_i t] \\ &= \frac{A_c}{2} \cos[(\omega_c - \omega_i)t + \theta(t)] + \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)] \end{aligned}$$

The output of the BPF is the sum frequency part of  $v_2(t)$ ,

$$\Rightarrow s(t) = \underline{\frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)]}$$

(b) Referring to Fig 5-42b the signal out of the FH spreader

$$\begin{aligned} v_5(t) &= s(t) 2 \cos(\omega_i t) = A_c \cos[(\omega_c + \omega_i)t + \theta(t)] \cos(\omega_i t) \\ &= \underbrace{\frac{A_c}{2} \cos[\omega_c t + \theta(t)]}_{\text{diff term}} + \underbrace{\frac{A_c}{2} \cos[(\omega_c + 2\omega_i)t + \theta(t)]}_{\text{sum term}} \end{aligned}$$

$\Rightarrow$  The output of the BPF is  $\underline{v_6(t) = \frac{A_c}{2} \cos[\omega_c t + \theta(t)]}$  which is FSK.

# Chapter 6

6-2

(a.)

$$\begin{aligned}\overline{x(t)} &= \int_0^{\pi/2} \frac{2}{\pi} \cdot A \cos(\omega_0 t + \theta) d\theta \\ &= \frac{2A}{\pi} \left[ \sin(\omega_0 t + \theta) \right]_0^{\pi/2} \\ &= \frac{2A}{\pi} \left[ \sin(\omega_0 t + \pi/2) - \sin \omega_0 t \right] \\ &= \frac{2A}{\pi} \left[ \cos \omega_0 t - \sin \omega_0 t \right] \\ &= \frac{2\sqrt{2}A}{\pi} \left[ \cos\left(\frac{\pi}{4}\right) \cos \omega_0 t - \sin\left(\frac{\pi}{4}\right) \sin \omega_0 t \right] \\ &\quad \text{↑} \\ \cos \frac{\pi}{4} &= \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4} \\ &= \underline{\underline{\frac{2\sqrt{2}A}{\pi} \cos\left(\omega_0 t + \frac{\pi}{4}\right)}}\end{aligned}$$

(b.)  $\overline{x(t)}$  is a function of time $\therefore x(t)$  is not stationary.

6-6

$$\text{Ergodicity} \Rightarrow \langle \{\cdot\} \rangle = \overline{\{\cdot\}}$$

$$\begin{aligned}
 \text{(a.) } P &= \overline{n^2(t)} = \overline{\{n_1(t) + n_2(t)\}^2} \\
 &= \overline{n_1^2(t)} + 2 \overline{n_1(t)n_2(t)} + \overline{n_2^2(t)} \\
 &= \overline{n_1^2(t)} + \overline{n_2^2(t)} \quad 0 \quad \text{orthogonal} \\
 &= 5 + 10 = \underline{\underline{15 \text{ Watts}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b.) } P &= \overline{n^2(t)} = \overline{n_1^2(t)} + 2 \overline{n_1(t)n_2(t)} + \overline{n_2^2(t)} \\
 &\quad \text{uncorrelated} \quad \overline{\overline{n_1(t)}} \overline{\overline{n_2(t)}}
 \end{aligned}$$

$$P = 5 + 2(-2)(1) + 10 = \underline{\underline{11 \text{ Watts}}}$$

$$\begin{aligned}
 \text{(c.) } P &= \overline{n^2(t)} = \overline{\{n_1(t) + n_2(t)\}^2} \\
 &= \overline{n_1^2(t)} + 2 \overline{n_1(t)n_2(t)} + \overline{n_2^2(t)} \\
 &\quad 2 R_{n_1 n_2}(0) \\
 &= 5 + 2(2) + 10 = \underline{\underline{19 \text{ Watts}}}
 \end{aligned}$$

6-8

(a.)  $\sin \omega_0 t$     ①  $R(t) \neq R(-t)$   $\times$   
NO

(b.)  $\frac{\sin \omega_0 t}{\omega_0 t}$     ①  $R(t) = R(-t)$   
YES    ②  $R(0) \geq |R(t)|$

YES    ③  $\mathcal{F}\left\{\frac{\sin \omega_0 t}{\omega_0 t}\right\} = \text{Non-negative rectangle}$

(c.)  $\cos \omega_0 t + \delta(t)$     ①  $R(t) = R(-t)$

YES    ②  $R(0) \geq |R(t)|$

③  $\mathcal{F}\{\cos \omega_0 t + \delta(t)\} = \frac{1}{2} \delta(f-f_0) + \frac{1}{2} \delta(f+f_0) + 1$   
 $\geq 0$

(d.)  $e^{-at/t}$  where a < 0    ①  $R(t) = R(-t)$

NO    ②  $R(0) \neq |R(t)| \times$

6-10

$$R_x(z) = 4e^{-z^2} + 3$$

(a.)  $P_x(f) = \mathcal{F}[R_x(z)] = \mathcal{F}[4e^{-z^2}] + \mathcal{F}[3] = \mathcal{F}\left[4e^{-\pi(\frac{f}{\sqrt{\pi}})^2}\right] + \mathcal{F}[3]$

$\Rightarrow P_x(f) = \frac{4\sqrt{\pi} e^{-\pi(f/\sqrt{\pi})^2} + 3\delta(f)}{\text{Using Table 2-2}} = \frac{4\sqrt{\pi} e^{-(\pi f)^2} + 3\delta(f)}{}$

(b.) This is a low-pass spectrum.  $\Rightarrow$  Use (r-97) and (r-98).

$$\overline{f^2} = \frac{\int_{-\infty}^{\infty} f^2 P_x(f) df}{\int_{-\infty}^{\infty} P_x(f) df} = \frac{\frac{4}{\pi} \int_0^{\infty} f^2 \frac{1}{\sqrt{\pi}} e^{-f^2/2(\frac{1}{\sqrt{\pi}})^2} df + 3 \int_0^{\infty} \delta(f) df}{4 \left( \int_0^{\infty} \frac{1}{\sqrt{\pi}} e^{-f^2/2(\frac{1}{\sqrt{\pi}})^2} df \right) + 3} = \frac{1}{1} = \frac{1}{1}$$

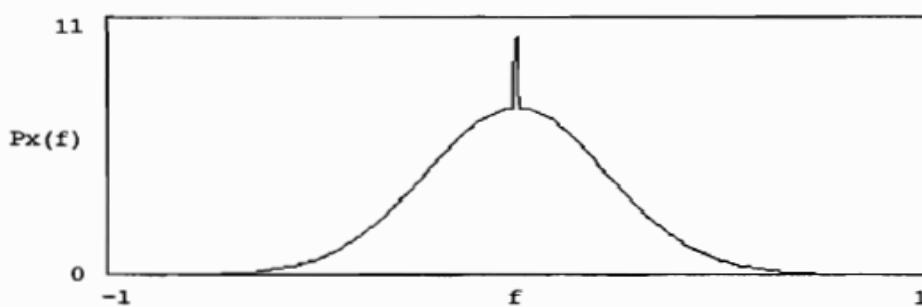
$$\Rightarrow \overline{f^2} = \frac{4 \left( \frac{1}{\sqrt{\pi}} \right)^2}{4 + 3} = \frac{4}{7} \cdot \frac{1}{2\pi^2} = \frac{2}{7\pi^2}$$

Thus,  $R_{rms} = \sqrt{\overline{f^2}} = \sqrt{\frac{2}{7\pi^2}} = \sqrt{\frac{2}{7}} \cdot \frac{1}{\pi} = \underline{0.170 \text{ Hz}}$

$\delta(0)$  cannot be plotted since it is infinity. Consequently, following the usual convention in EE, plot the WEIGHT of  $\delta(f)$  instead at  $f=0$ .

$$f := -1, -0.99, \dots, 1 \quad \delta(f) := \text{if}(f \approx 0, 1, 0)$$

$$P_x(f) := 4 \sqrt{\pi} \cdot e^{-\left[\left(\pi \cdot f\right)^2\right]} + 3 \cdot \delta(f) \quad P_x(0) = 10.09$$



$$Brms := \sqrt{\frac{2}{7} \left[ \frac{1}{\pi} \right]} \quad Brms = 0.17$$

6-14

$$(a.) \quad x_{rms} = \sqrt{\overline{x^2(t)}}$$

$$\begin{aligned}\overline{x^2(t)} &= R_x(0) = \sum_{-\infty}^{\infty} P_x(f) df \\ &= \int_{-B}^{0} \frac{1}{B} (B+f) df + \int_0^B \frac{1}{B} (B-f) df \\ &= \frac{1}{B} \left[ \left( Bf + \frac{f^2}{2} \right) \Big|_{-B}^0 + \left( Bf - \frac{f^2}{2} \right) \Big|_0^B \right] \\ &= \frac{1}{B} \left[ -\left( -B^2 + \frac{B^2}{2} \right) + \left( B^2 - \frac{B^2}{2} \right) \right] = \frac{1}{B} [2B^2 - B^2] \\ &= B \quad \Rightarrow \quad x_{rms} = \underline{\sqrt{B}}\end{aligned}$$

$$(b.) \quad P_x(f) = \frac{1}{B} \Pi\left(\frac{f}{B}\right) * \frac{1}{B} \Pi\left(\frac{f}{B}\right)$$

$$R_x(\tau) = \mathcal{F}^{-1}[P_x(f)] = \frac{1}{B} \left\{ \mathcal{F}^{-1}[\Pi(\frac{f}{B})] \right\}^2$$

Table 2-1 - Multiplication property of  $\mathcal{F}\{\cdot\}$

$$R_x(\tau) = \frac{1}{B} \left\{ \mathcal{F}^{-1}[\Pi(\frac{f}{B})] \right\}^2$$

Table 2-2

$$\hookrightarrow = \frac{1}{B} \left\{ B \frac{\sin(\pi B \tau)}{\pi B \tau} \right\}^2$$

$$R_x(\tau) = B \left[ \frac{\sin(\pi B \tau)}{\pi B \tau} \right]^2$$

6-16

$$(a.) \quad P_g(f) = |H(f)|^2 P_n(f) = \left| \frac{K}{j2\pi f} \right|^2 \frac{N_0}{2}$$

$$\underline{P_g(f)} = \frac{N_0 K^2}{8\pi^2 f^2}$$

$$(b.) \quad y_{rms}^2 = R_g(0) = \int_{-\infty}^{\infty} P_g(f) df$$

$$= \int_{-\infty}^{\infty} \frac{N_0 K^2}{8\pi^2 f^2} df = 2 \int_{0}^{\infty} \left[ \frac{N_0 K^2}{8\pi^2} \right] \frac{1}{f^2} df$$

$$= \frac{N_0 K^2}{4\pi^2} \left[ -\frac{1}{f} \right] \Big|_0^{\infty} = \frac{N_0 K^2}{4\pi^2} \left[ \frac{-1}{\infty} + \frac{1}{0} \right] = \underline{\underline{\infty}}$$

A practical integrator will have a large (i.e. finite) output.

6-18

$$\text{From (6-95): } \left( \frac{S}{N} \right)_{\text{out}} = \frac{2A_0^2 RC}{N_0 [1 + (2\pi f_0 RC)^2]}$$

$$\text{Let } RC = z, \quad 2\pi f_0 = \omega_0$$

$$\Rightarrow \left( \frac{S}{N} \right)_{\text{out}} = \frac{2A_0^2 z}{N_0 [1 + (\omega_0 z)^2]}$$

$$\text{For } \max \left[ \left( \frac{S}{N} \right)_{\text{out}} \right], \text{ set } \frac{d \left[ \left( \frac{S}{N} \right)_{\text{out}} \right]}{dz} = 0$$

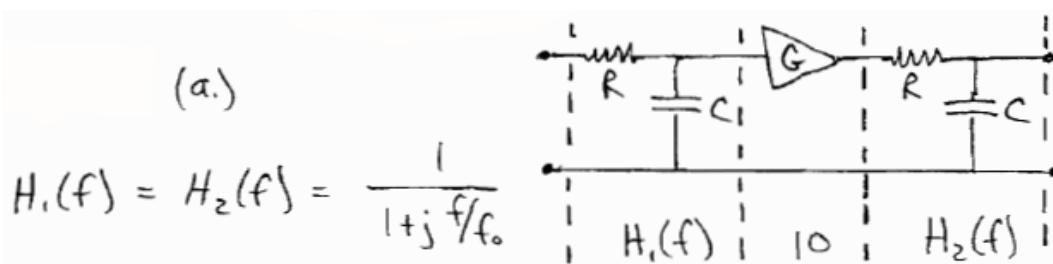
$$\Rightarrow \frac{d \left[ \left( \frac{S}{N} \right)_{\text{out}} \right]}{dz} = \frac{[1 + (\omega_0 z)^2] 2A_0^2 - 2A_0^2 z (2)(\omega_0 z) \omega_0}{N_0 [1 + (\omega_0 z)^2]^2}$$

$$\text{Set numerator} = 0 \Rightarrow 2A_0^2 [1 + \omega_0^2 z^2 - 2z \omega_0^2 z] = 0$$

$$\Rightarrow [1 - \omega_0^2 z^2] = 0 \Rightarrow \omega_0^2 z^2 = 1 \Rightarrow z^2 = \frac{1}{\omega_0^2} \Rightarrow z = \frac{1}{\omega_0}$$

$$\text{Thus, } \underline{RC = \frac{1}{2\pi f_0}} \text{ for } \max \left( \frac{S}{N} \right)_{\text{out}}$$

6-22



where  $f_0 = \frac{1}{2\pi RC}$ , and  $G = 10$

$$H(f) = H_1(f)\{10\} H_2(f) = \frac{10}{[1 + j f/f_0]^2}$$

$$|H(f)| = \frac{10}{\sqrt{1 + (2f/f_0)^2 - (f/f_0)^2}} = \frac{10}{\sqrt{[1 - (f/f_0)^2]^2 + (2f/f_0)^2}}$$

Using a programmable calculator, find the value of  $f = f_c$ , such that :

$$|H(f_c)| = \sqrt{\frac{10}{2}} \Rightarrow f_c = 0.690 f_0 ; f_0 = \frac{1}{2\pi RC}$$

6-25

(a.)  $x_1 + y_2$  uncorrelated  $\Rightarrow$  Independent

when  $R_{xy}(t) = \overline{x(t_1)y(t_2)} = 10 \sin(2\pi t) = 0 = \overline{x_1 y_2}$

$\Rightarrow 2\pi t = \pm n\pi \Rightarrow$  These r.v.'s are independent

only when  $t_2 - t_1 = \tau = \pm \frac{n}{2} \quad ; \quad n = 0, 1, \dots$

## 6-25 (Continued)

$$10 \sin(2\pi f) = 10 \sin[2\pi(t_2 - t_1)] = \overline{x(t_1)y(t_2)}$$

This cannot be expressed as  $\overline{x(t_1)} \overline{y(t_2)}$

$\therefore x(t)$  and  $y(t)$  are not indep.

## 6-27

(a.) Evaluate  $\overline{x^2(t)} = R_x(0)$

$$\overline{x^2(t)} = \overline{A_0^2 \cos^2(\omega_0 t + \theta)} = \frac{A_0^2}{2} \left\{ 1 + \overline{\cos(2\omega_0 t + 2\theta)} \right\}$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2} \int_0^{\pi/2} \cos(2\omega_0 t + 2\theta) \frac{2}{\pi} d\theta$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2} \left. \frac{\sin(2\omega_0 t + 2\theta)}{2} \right|_0^{\pi/2}$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2\pi} \left[ \sin(2\omega_0 t + \pi) - \sin(2\omega_0 t) \right]$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2\pi} [-2 \sin(2\omega_0 t)]$$

This is a function  
of  $t \therefore x(t)$  not  
W.S.S.

Using Sec. A-1

## 6-27 (Continued)

$$\begin{aligned}
 & \text{(b.)} \\
 \text{Eqn. (6-42)} \quad P_x(f) &= \lim_{T \rightarrow \infty} \left[ \frac{|X_T(f)|^2}{T} \right] \\
 \text{where } X_T(f) &= \int_{-T/2}^{T/2} x(t) e^{-j\omega t} dt \\
 &= \int_{-T/2}^{T/2} A_0 \cos(\omega_0 t + \theta) e^{-j\omega t} dt \\
 &= A_0 \int_{-T/2}^{T/2} \frac{e^{j(\omega_0 t + \theta)} + e^{-j(\omega_0 t + \theta)}}{2} e^{-j\omega t} dt \\
 &= \frac{A_0}{2} e^{j\theta} \int_{-T/2}^{T/2} e^{j(\omega_0 - \omega)t} dt + \frac{A_0}{2} e^{-j\theta} \int_{-T/2}^{T/2} e^{-j(\omega_0 + \omega)t} dt \\
 &= \frac{A_0}{2} e^{j\theta} \frac{e^{j(\omega_0 - \omega)t}}{j(\omega_0 - \omega)} \Big|_{-T/2}^{T/2} + \frac{A_0}{2} e^{-j\theta} \frac{e^{-j(\omega_0 + \omega)t}}{-j(\omega_0 + \omega)} \Big|_{-T/2}^{T/2} \\
 &= A_0 \left[ e^{j\theta} \frac{e^{j(\omega_0 - \omega)T/2} - e^{-j(\omega_0 - \omega)T/2}}{2j(\omega_0 - \omega)} \right. \\
 &\quad \left. + e^{-j\theta} \frac{e^{j(\omega_0 + \omega)T/2} - e^{-j(\omega_0 + \omega)T/2}}{2j(\omega_0 + \omega)} \right] \\
 &= A_0 e^{j\theta} \frac{\sin(\omega_0 - \omega)T/2}{(\omega_0 - \omega)} + A_0 e^{-j\theta} \frac{\sin(\omega_0 + \omega)T/2}{(\omega_0 + \omega)}
 \end{aligned}$$

Let  $x_1 = (\omega_0 - \omega)T/2$  and  $x_2 = (\omega_0 + \omega)T/2$

$$= \frac{A_0 T}{2} \left[ e^{j\theta} \frac{\sin x_1}{x_1} + e^{-j\theta} \frac{\sin x_2}{x_2} \right]$$

$$\begin{aligned}
 \frac{|X_T(f)|^2}{T} &= \frac{X_T(f) X_T^*(f)}{T} = \\
 &= \frac{\left( \frac{A_0 T}{2} \right)^2}{T} \left[ e^{j\theta} \frac{\sin x_1}{x_1} + e^{-j\theta} \frac{\sin x_2}{x_2} \right] \left[ e^{-j\theta} \frac{\sin x_1}{x_1} + e^{j\theta} \frac{\sin x_2}{x_2} \right]
 \end{aligned}$$

## 6-27 (Continued)

$$\frac{\overline{|X_T(f)|^2}}{T} = \frac{A_0^2 T}{4} \left[ \left( \frac{\sin x_1}{x_1} \right)^2 + e^{j2\theta} \left( \frac{\sin x_1}{x_1} \right) \left( \frac{\sin x_2}{x_2} \right) + e^{-j2\theta} \left( \frac{\sin x_1}{x_1} \right) \left( \frac{\sin x_2}{x_2} \right) + \left( \frac{\sin x_2}{x_2} \right)^2 \right]$$

Aside:

$$\overline{e^{j2\theta}} = \int_0^{\pi/2} e^{j2\theta} \cdot \frac{2}{\pi} d\theta = j^2/\pi$$

$$\overline{e^{-j2\theta}} = \int_0^{\pi/2} e^{-j2\theta} \cdot \frac{2}{\pi} d\theta = -j^2/\pi$$

$$\frac{\overline{|X_T(f)|^2}}{T} = \frac{A_0^2}{4} \left[ \frac{q}{\pi} \left( \frac{\sin \pi T(f-f_0)}{\pi T(f-f_0)} \right)^2 + \frac{T\pi}{\pi} \left( \frac{\sin \pi T(f+f_0)}{\pi T(f+f_0)} \right)^2 \right]$$

From Sec. A-8

$$\delta(x) = \lim_{a \rightarrow \infty} \left[ \frac{q}{\pi} \left( \frac{\sin ax}{ax} \right)^2 \right]$$

$$\Rightarrow P_x(f) = \frac{A_0^2}{4} [\delta(f-f_0) + \delta(f+f_0)]$$

$$(c.) \quad \overline{x(t)} = A_0 \overline{\cos(\omega_0 t + \theta)} = A_0 \cdot 0 = 0$$

$$R_x(\tau) = \overline{x(t)x(t+\tau)} \\ = A_0^2 \cos(\omega_0 t + \theta) \cos(\omega_0 t + \omega_0 \tau + \theta)$$

$$= \frac{A_0^2}{2} \cos \omega_0 \tau + \frac{A_0^2}{2} \overline{\cos(2\omega_0 t + \omega_0 \tau + 2\theta)} \rightarrow 0$$

$$= \frac{A_0^2}{2} \cos \omega_0 \tau ; \text{ not a function of } t$$

$\therefore x(t)$  is W.S.S.

6-30

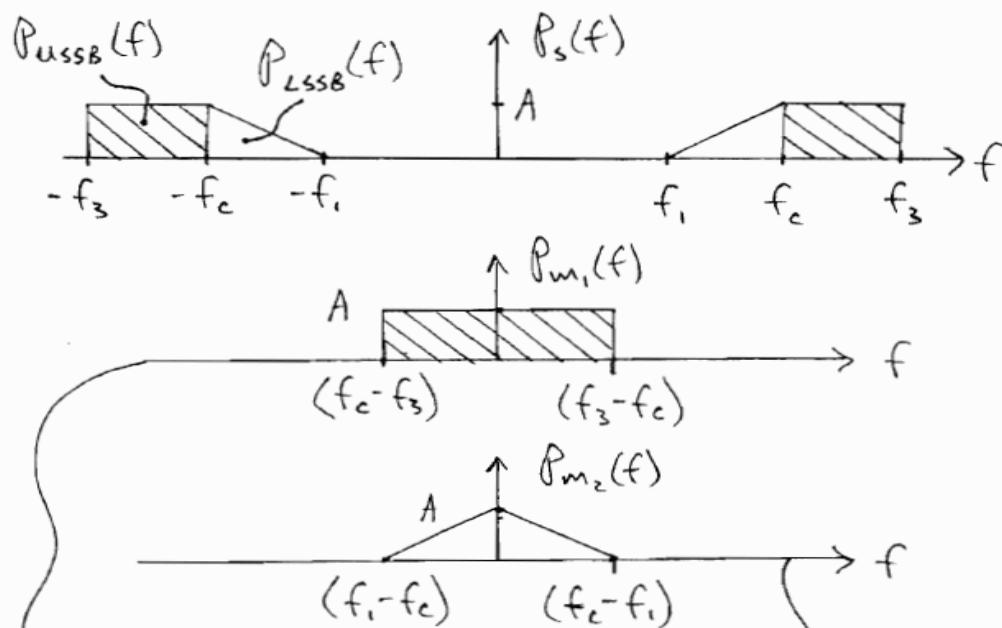
$$\begin{aligned}s(t) &= x(t) \cos(\omega_c t + \theta_c) \\ &\quad - y(t) \sin(\omega_c t + \theta_c) \\ &= s_{\text{USSB}}(t) + s_{\text{LSSB}}(t)\end{aligned}$$

where

$$\begin{aligned}s_{\text{USSB}}(t) &= m_1(t) \cos(\omega_c t + \theta_c) - \hat{m}_1(t) \sin(\omega_c t + \theta_c) \\ s_{\text{LSSB}}(t) &= m_2(t) \cos(\omega_c t + \theta_c) + \hat{m}_2(t) \sin(\omega_c t + \theta_c)\end{aligned}$$

$$\Rightarrow s(t) = [m_1(t) + m_2(t)] \cos(\omega_c t + \theta_c) - [\hat{m}_1(t) - \hat{m}_2(t)] \sin(\omega_c t + \theta_c)$$

$$\Rightarrow \underline{x(t)} = \underline{m_1(t) + m_2(t)} ; \underline{y(t)} = \underline{\hat{m}_1(t) - \hat{m}_2(t)}$$



$$\underline{P_{m_1}(f)} = A \pi \left( \frac{f}{2(f_3 - f_c)} \right)$$

$$\underline{P_{m_2}(f)} = A \Delta \left( \frac{f}{f_c - f_1} \right)$$

## 6-32

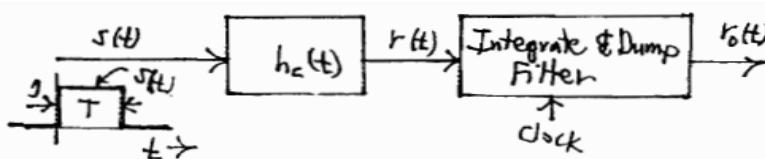
From example 6-9 :

$$P_v(f) = \frac{1}{4} [P_x(f-f_c) + P_x(-f-f_c)]$$

Where, from solution to problem 6-18. :

$$P_x(f) = T_b \left[ \frac{1 - \cos(\pi f T_b)}{\pi f T_b} \right]^2 = P_x(-f)$$

## 6-36



$$H_c(f) = \frac{B}{B+jf} = \frac{1}{1 + j\left(\frac{f}{B}\right)}$$

Let  $a = 2\pi B$

$$\Rightarrow h_c(t) = \begin{cases} 2\pi B e^{-2\pi B t}, & t > 0 \\ 0, & t < 0 \end{cases} = \begin{cases} a e^{-at}, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$r(t) = s(t) * h_c(t) = \int_0^t s(\lambda) h_c(t-\lambda) d\lambda = \begin{cases} \int_0^t a e^{-a(t-\lambda)} d\lambda, & 0 < t < \pi \\ \int_0^\pi a e^{-a(t-\lambda)} d\lambda, & t > \pi \end{cases}$$

$$\Rightarrow r(t) = \begin{cases} 1 - e^{-at}, & 0 < t < \pi \\ e^{-at}[e^{a\pi} - 1], & t > \pi \end{cases}$$

$$r_o(t) = \begin{cases} \int_0^t r(\lambda) d\lambda, & 0 < t < \pi \\ \int_\pi^t r(\lambda) d\lambda, & \pi < t < 2\pi \end{cases} = \begin{cases} \int_0^t [1 - e^{-a\lambda}] d\lambda, & 0 < t < \pi \\ ((e^{a\pi} - 1) / a) \int_\pi^t e^{-a\lambda} d\lambda, & \pi < t < 2\pi \end{cases}$$

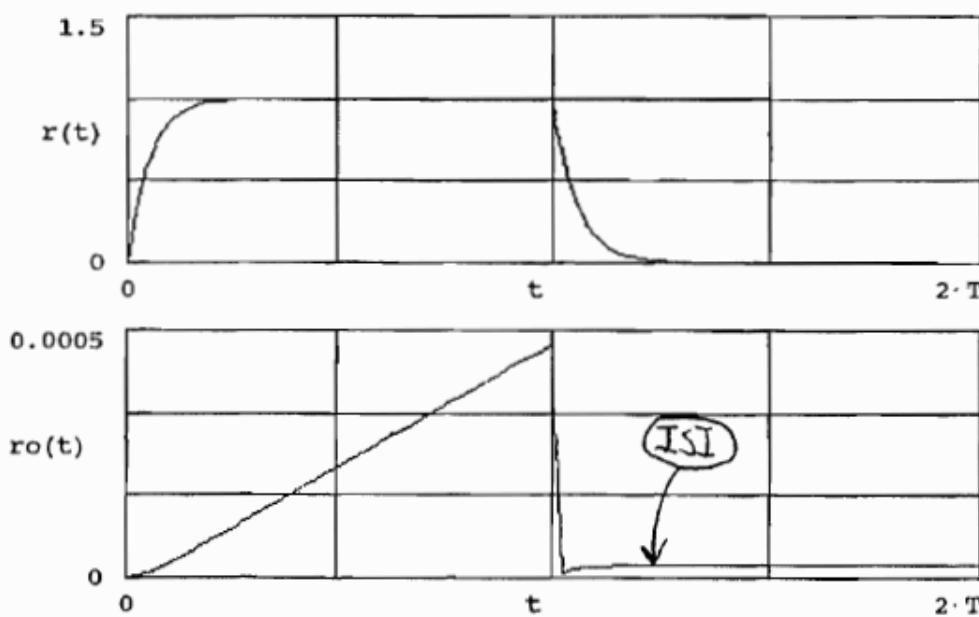
$$\Rightarrow r_o(t) = \begin{cases} t + \frac{1}{a} (e^{-at} - 1), & 0 < t < \pi \\ \frac{1}{a} (e^{a\pi} - 1)(e^{-at} - 1), & \pi < t < 2\pi \end{cases}$$

(a.)

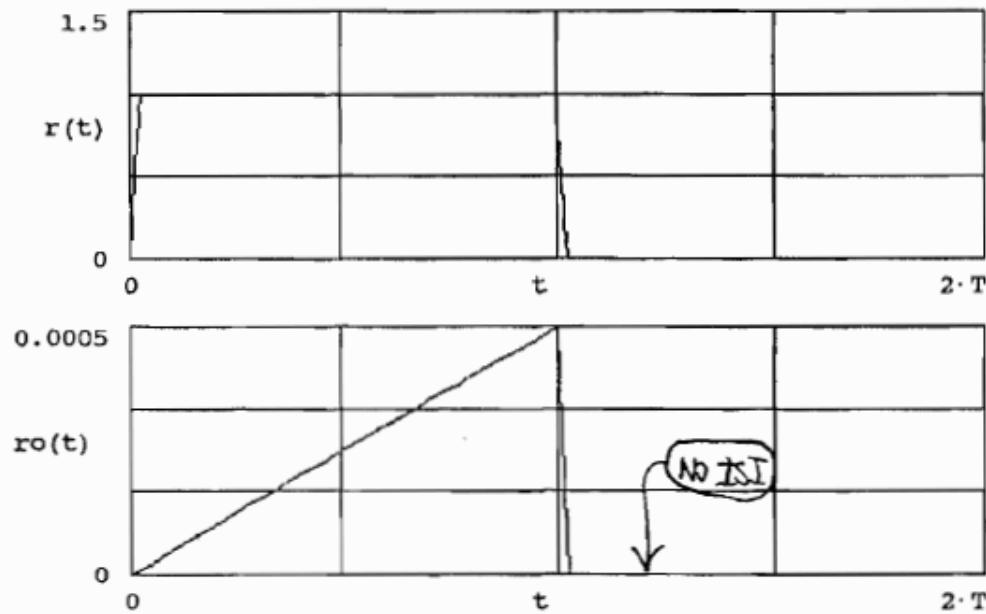
```

R := 2000      T := 1      B := 6000      a := 2 * pi * B      t := 0, T / 40 .. 2 * T
      R          R
r(t) := if[t ≤ T, 1 - e^{-a * t}, [e^{a * T} - 1] * e^{-a * t}]
r_o(t) := if[t ≤ T, t + (-e^{-a * t} - 1) / a, (-e^{-a * T} - 1) / a * [e^{-a * T} - e^{-a * t}]]
```

### 6-36 (Continued)



(b.) For all pass channel  $\Rightarrow$  Let  $B \rightarrow \infty$ , use  $B=100,000$ .  
 Then, results are as follows.



# Chapter 7

7-1

$$(a.) \quad r_o = \begin{cases} A + n_o, & s_1 \text{ sent} \\ -A + n_o, & s_2 \text{ sent} \end{cases}$$

$$\Rightarrow f(r_o | s_1) = \frac{1}{\sqrt{2} \Delta_o} e^{-\frac{\sqrt{2}|r_o - A|}{\Delta_o}}$$

$$f(r_o | s_2) = \frac{1}{\sqrt{2} \Delta_o} e^{-\frac{\sqrt{2}|r_o + A|}{\Delta_o}}$$

Using (7-8)

$$P_e = \frac{1}{2} \int_{-\infty}^T \frac{1}{\sqrt{2} \Delta_o} e^{-\frac{\sqrt{2}|r_o - A|}{\Delta_o}} dr_o + \frac{1}{2} \int_T^{\infty} \frac{1}{\sqrt{2} \Delta_o} e^{-\frac{\sqrt{2}|r_o + A|}{\Delta_o}} dr_o$$

$m_{r_{o1}} = A$ ,  $m_{r_{o2}} = -A$ , the source probabilities are equally likely, and the conditional probabilities have symmetrical shapes about  $\pm A$ .

Thus  $V_T = 0$ .

$$\therefore P_e = \frac{1}{2\sqrt{2}\Delta_o} \left[ \int_{-\infty}^0 e^{-\frac{\sqrt{2}|r_o - A|}{\Delta_o}} dr_o + \int_0^{\infty} e^{-\frac{\sqrt{2}|r_o + A|}{\Delta_o}} dr_o \right]$$

$$= \frac{1}{2\sqrt{2}\Delta_o} \left[ \int_{-\infty}^0 e^{\frac{\sqrt{2}(r_o - A)}{\Delta_o}} dr_o + \int_0^{\infty} e^{\frac{-\sqrt{2}(r_o + A)}{\Delta_o}} dr_o \right]$$

Let  $x_1 = \frac{\sqrt{2}(r_o - A)}{\Delta_o}$  and  $x_2 = \frac{-\sqrt{2}(r_o + A)}{\Delta_o}$

$$dx_1 = \frac{\sqrt{2}}{\Delta_o} dr_o \quad dx_2 = \frac{-\sqrt{2}}{\Delta_o} dr_o$$

$$= \frac{1}{2\sqrt{2}\Delta_o} \left[ \int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_1 \left( \frac{\Delta_o}{\sqrt{2}} dx_1 \right)} + \int_{-\sqrt{2}A/\Delta_o}^{\infty} e^{x_2 \left( \frac{-\Delta_o}{\sqrt{2}} dx_2 \right)} \right]$$

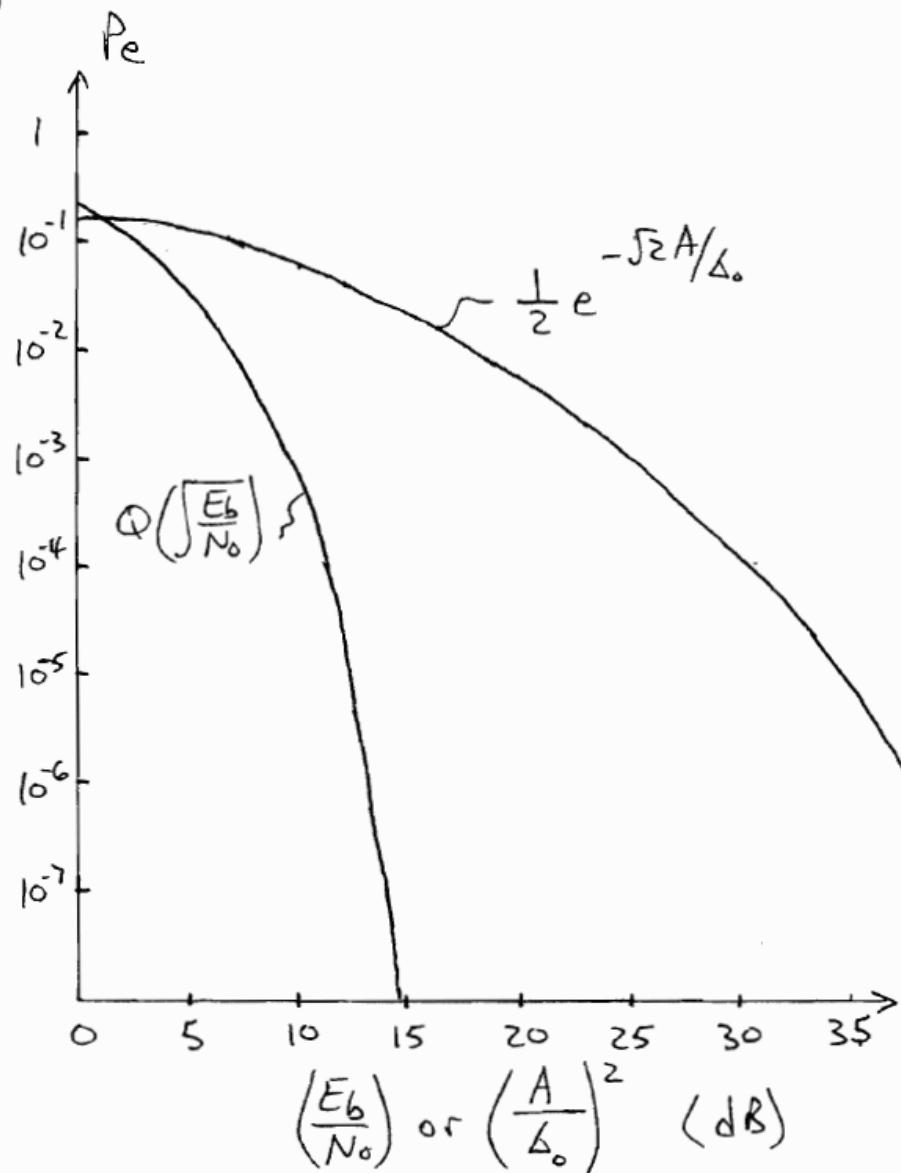
$$= \frac{1}{4} \left[ \int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_1 dx_1} + \int_{-\infty}^{-\sqrt{2}A/\Delta_o} e^{x_2 dx_2} \right]$$

7-1 (Continued)

$$P_e = \frac{1}{2} \left[ \int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^x dx \right] = \frac{1}{2} e^x \Big|_{-\infty}^{-\sqrt{2}A/\Delta_0}$$

$$= \frac{1}{2} \left[ e^{-\sqrt{2}A/\Delta_0} - e^{-\infty} \right] = \underline{\frac{1}{2} e^{-\sqrt{2}A/\Delta_0}} = P_e$$

(b.)



$P_e$  much larger for Laplacian Noise.

7-4

$$\left(\frac{S}{N}\right)_{in} = \frac{\frac{E_b}{T_b}}{\left(\frac{N_0}{2}\right)(2B_{eq})} = \frac{E_b R}{N_0 B_{eq}} \Rightarrow \frac{E_b}{N_0} = \frac{B_{eq}}{R} \left(\frac{S}{N}\right)_{in}$$

Aside:

$$B_{eq} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df}{2 |H(0)|^2} = \frac{\left(\frac{2\pi}{N_0}\right)^2 \int_{-\infty}^{\infty} |\mathcal{N}'(f)|^2 df}{2 \left(\frac{2\pi}{N_0}\right)^2 |\mathcal{N}'(0)|^2} = \frac{\int_{-\infty}^{\infty} |\mathcal{N}'(f)|^2 df}{2 |\mathcal{N}'(0)|^2}$$

Using (6-155) f<sub>0</sub>-MF:  $H(f) = \frac{K \mathcal{N}'(f) e^{-j\omega t_0}}{N_0/2}$

$$\Rightarrow B_{eq} = \frac{T_b^2 \int_{-\infty}^{\infty} \left(\frac{\sin(\pi T_b f)}{\pi T_b f}\right)^2 df}{2 T_b^2 1^2} = \frac{1}{2\pi T_b} \int_{-\infty}^{\infty} \left(\frac{\sin x}{x}\right)^2 dx = \frac{1}{2\pi T_b} = \frac{1}{2R}$$

$\mathcal{N}(t) = \pi \left(\frac{t}{T_b}\right) \Leftrightarrow \mathcal{N}'(f) = T_b \frac{\sin(\pi T_b f)}{\pi T_b f}$

Let  $x = \pi T_b f$   
 $dx = \pi T_b df$

$$\Rightarrow \frac{E_b}{N_0} = \frac{\frac{1}{2} R}{R} \left(\frac{S}{N}\right)_{in} = \frac{1}{2} \left(\frac{S}{N}\right)_{in} = \frac{E_b}{N_0}$$

Using (7-246):

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{1}{2} \left(\frac{S}{N}\right)_{in}}\right)$$

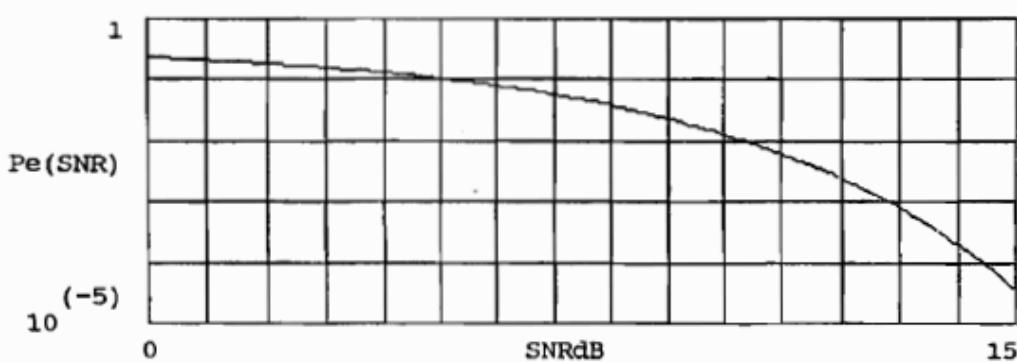
SNRdB := 0, 0.1 .. 15

Q(x) := 1 - cnorm(x)

P<sub>e</sub>(SNR) := Q<sub>0.5 · SNR(SNRdB)</sub>

$$\frac{SNRdB}{10}$$

$$SNR(SNRdB) := 10$$



7-6

(a.) For derivation of  $P_e$ , follow the same procedure as used in the solution for Prob. 7-7.

$$B_{eq} = \int_0^{\infty} \frac{|H(f)|^2}{|H(0)|^2} df \stackrel{f=0}{=} \int_0^{\infty} \frac{1}{1 + \left(\frac{f}{f_0}\right)^4} df = f_0 \int_0^{\infty} \frac{1}{1 + x^4} dx = \frac{f_0 \pi}{2\sqrt{2}}$$

$\text{Let } x = f/f_0$        $\text{Using Sec. A-3}$

$$S_{01} = \int_0^T S_{01}(T-\lambda) h(\lambda) d\lambda = \int_0^T A [\sqrt{2} w_0 e^{-(w_0 \sqrt{2})\lambda} \sin(\frac{w_0}{\sqrt{2}}\lambda)] d\lambda$$

or

$$S_{01} = \int_0^T 2A e^{-x} \sin(x) dx = A [1 - e^{-\sqrt{2}\pi} (\sin(\sqrt{2}\pi) + \cos(\sqrt{2}\pi))] \quad \text{Using Sec. A-5}$$

$\text{Let } x = \frac{w_0}{\sqrt{2}}\lambda$

$$\Rightarrow S_{01} = 1.01447 A$$

$$\sigma_0^2 = \frac{N_0}{2} \quad B_{eq} = N_0 B_{eq} = \frac{N_0 \pi f_0}{2\sqrt{2}} = \frac{N_0 \pi}{2\sqrt{2} T}$$

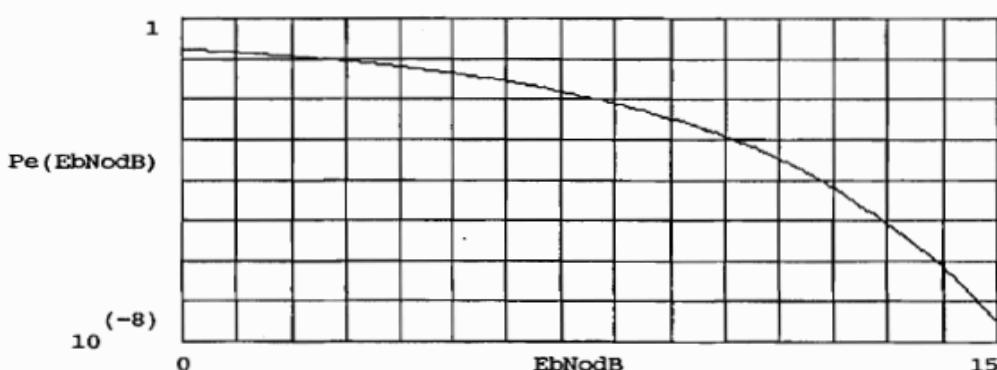
Using (7-17) where  $S_{01} = -S_{02}$ ,

$$P_e = Q\left(\sqrt{\frac{S_{01}^2}{\sigma_0^2}}\right) = Q\left(\sqrt{\frac{(1.01447)^2 A^2}{\frac{N_0 \pi}{2\sqrt{2} T}}}\right) = Q\left(\sqrt{\frac{2\sqrt{2} (1.01447)^2 A^2 \pi}{\pi N_0}}\right)$$

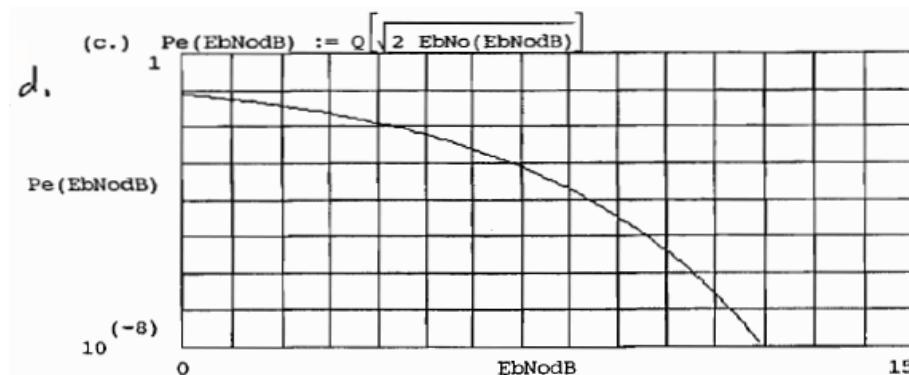
$$\Rightarrow P_e = Q\left(\sqrt{\frac{2\sqrt{2} (1.01447)^2}{\pi} \left(\frac{E_b}{N_0}\right)}\right) = Q\left(\sqrt{0.92656 \left(\frac{E_b}{N_0}\right)}\right)$$

(b.)	EbNodB := 0, 0.1 .. 15	EbNodB
	Q(x) := 1 - cnorm(x)	10
	EbNo(EbNodB) := 10	

$$P_e(EbNodB) := Q\left[\sqrt{0.92656 \cdot EbNo(EbNodB)}\right]$$



## 7-6 (Continued)



7-9

(a.) Referring to the solution for SA 7-3,

$$P_e = Q\left(\frac{\sqrt{A^2}}{\sqrt{4 N_0 B}}\right) \quad (7-24a)$$

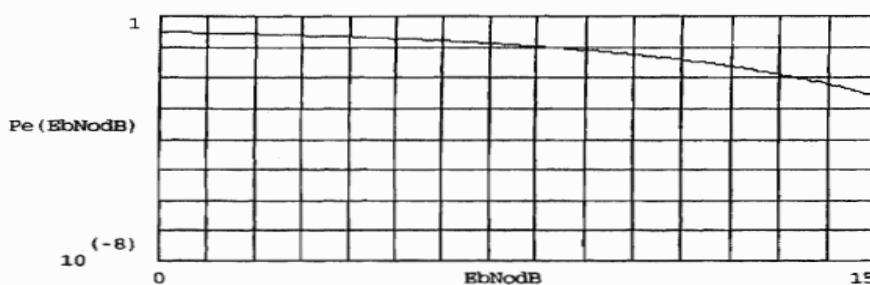
where  $B = \frac{1}{T} = 2R$  and  $E_b = \left(\frac{A^2}{2}\right)T = \frac{A^2}{2R}$

Thus,  $P_e = Q\left(\sqrt{\frac{A^2}{8N_0 R}}\right) = Q\left(\sqrt{\frac{A^2}{4N_0 2R}}\right) = Q\left(\sqrt{\frac{E_b}{4N_0}}\right)$

EbNodB := 0, 0.1 .. 15  
 $Q(x) := 1 - \text{cnorm}(x)$   
 $EbNo(EbNodB) := 10$

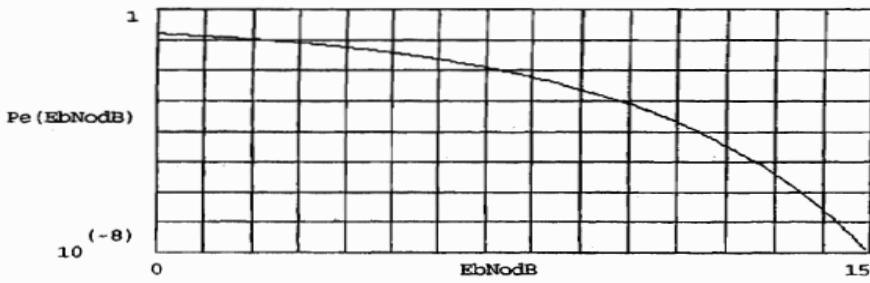
(a.)

$$Pe(EbNodB) := Q\left[\sqrt{0.25 \cdot EbNo(EbNodB)}\right] \quad \text{---- LPF with ISI Result}$$



(b.)

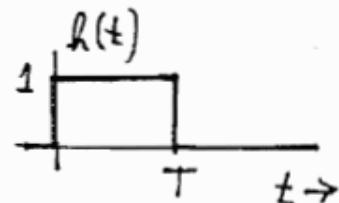
$$Pe(EbNodB) := Q\left[\sqrt{EbNo(EbNodB)}\right] \quad \text{Matched Filter Result -- (7-24b)}$$



7-11

(a) The impulse response is:

$$h(t) = \delta_{\text{oi}}(T-t) = \underline{\underline{\delta_{\text{oi}}(t)}}$$



$$H(f) = \int_{-\infty}^{\infty} h(t) e^{j\omega t} dt = \int_0^T e^{-j\omega t} dt = \frac{e^{-j\omega t}}{-j\omega} \Big|_0^T = \frac{e^{-j\omega T} - e^{j\omega 0}}{-j\omega}$$

$$\Rightarrow H(f) = e^{-j\omega T/2} \left[ \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\pi f} \right] = T e^{-j\pi f T} \left[ \underline{\underline{\frac{\sin(\pi f T)}{\pi f T}}} \right]$$

See Fig. 6-17

$$(b) B_{\text{eq}} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df}{|H(0)|^2} = \frac{T^2 \int_0^{\infty} \left[ \frac{\sin(\pi f T)}{\pi f T} \right]^2 df}{T^2}$$

$$= \int_0^{\infty} \left( \frac{\sin x}{x} \right)^2 \left( \frac{1}{\pi T} dx \right) = \frac{1}{\pi T} \left( \frac{\pi}{2} \right) = \frac{1}{2T} = \underline{\underline{B_{\text{eq}}}}$$

Let  $x = \pi f T$ ;  $dx = \pi T df$

Using Sec. A-5

7-13

From (7-8)

$$P_e = P(1) \int_{-\infty}^{V_T} f(r_0 | s_1) dr_0 + P(0) \int_{V_T}^{\infty} f(r_0 | s_0) dr_0$$

where  $r_0 = \begin{cases} A + n_0, & \text{for a binary 1 sent} \\ -A + n_0, & \text{" " 0 " } \end{cases}$

Thus

$$P_e = P(1) \int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi}\Delta} e^{-\frac{(r_0-A)^2}{2\Delta^2}} dr_0 + P(0) \int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi}\Delta} e^{-\frac{(r_0+A)^2}{2\Delta^2}} dr_0$$

$\left( \begin{array}{l} \text{Let } \lambda_1 = -(r_0 - A)/\Delta ; \lambda_2 = (r_0 + A)/\Delta \\ d\lambda_1 = -\frac{1}{\Delta} dr_0 \quad d\lambda_2 = \frac{1}{\Delta} dr_0 \end{array} \right)$

$$\Rightarrow P_e = P(1) \int_{(-V_T+A)/\Delta}^{-\lambda_1/2} \frac{1}{\sqrt{2\pi}} e^{-\lambda_1^2/2} d\lambda_1 + P(0) \int_{(\lambda_2/2)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda_2^2/2} d\lambda_2$$

$$\text{or } P_e = P(1) Q\left[\frac{-V_T+A}{\Delta}\right] + P(0) Q\left[\frac{V_T+A}{\Delta}\right]$$

$$= P(1) Q\left[\sqrt{\frac{(-V_T+A)^2 2T}{N_0}}\right] + P(0) Q\left[\sqrt{\frac{(V_T+A)^2 2T}{N_0}}\right]$$

$\Delta^2 = \frac{N_0}{2T}$

To Check : Let  $P(1) = P(0) = \frac{1}{2}$ ;  $V_T = 0$

$$P_e = \frac{1}{2} Q\left[\sqrt{\frac{A^2 2T}{N_0}}\right] + \frac{1}{2} Q\left[\sqrt{\frac{A^2 2T}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2A^2 T}{N_0}}\right]; \text{ This checks with eqn. (7-26b)}$$

7-16

Referring to Fig. 7-7

(a.) Let  $s_1(t) = A \cos \omega_c t$ ,  $s_2(t) = -A \cos \omega_c t$

$$n(t) = x(t) (\cos \omega_c t - y(t) \sin \omega_c t)$$

$$\text{Coherent reference} = 2 \cos(\omega_c t + \theta_e)$$

$$\Rightarrow r_o(t) = \frac{1}{2} A \cos \theta_e + n_o(t) = s_{o_1}(t) + n_o(t)$$

$$\frac{n_o^2(t)}{N_o B} = \frac{x^2(t)}{N_o B} \cos^2 \theta_e + 2 \frac{x(t) y(t)}{N_o B} \cos \theta_e \sin \theta_e + \frac{y^2(t)}{N_o B} \sin^2 \theta_e$$

$$\Rightarrow \frac{n_o^2(t)}{N_o B} = \frac{1}{2} N_o B (\cos^2 \theta_e + \sin^2 \theta_e) = \frac{1}{2} N_o B$$

$\boxed{x^2 = y^2 = 2 N_o B}$

Using (7-17):  $H(f) = \text{LPF}$

$$P_e = Q \left[ \sqrt{\frac{(s_{o_1} - s_{o_2})^2}{4 \sigma_o^2}} \right] = Q \left[ \sqrt{\frac{4 A^2 \cos^2 \theta_e}{8 N_o B}} \right] = Q \left[ \sqrt{\frac{A^2 \cos^2 \theta_e}{2 N_o B}} \right]$$

Corresponds to (7-36)

Where  $\begin{cases} + \text{ is used when } s_{o_1} > s_{o_2} \Rightarrow |\theta_e| < \frac{\pi}{2} \\ - \text{ is used when } s_{o_2} > s_{o_1} \Rightarrow \frac{\pi}{2} < |\theta_e| < \pi \end{cases}$

(b.) If  $H(f)$  is matched to the output of the multiplier, then

$$P_e = Q \left( \sqrt{\frac{E_d}{2 N_o}} \right) \quad \text{using (7-20)}$$

where  $N'_o = 2 N_o$  and

$$E_d = \int_0^T [s_{o_1}(t) - s_{o_2}(t)]^2 dt = \int_0^T 4 A^2 \cos^2 \theta_e dt$$

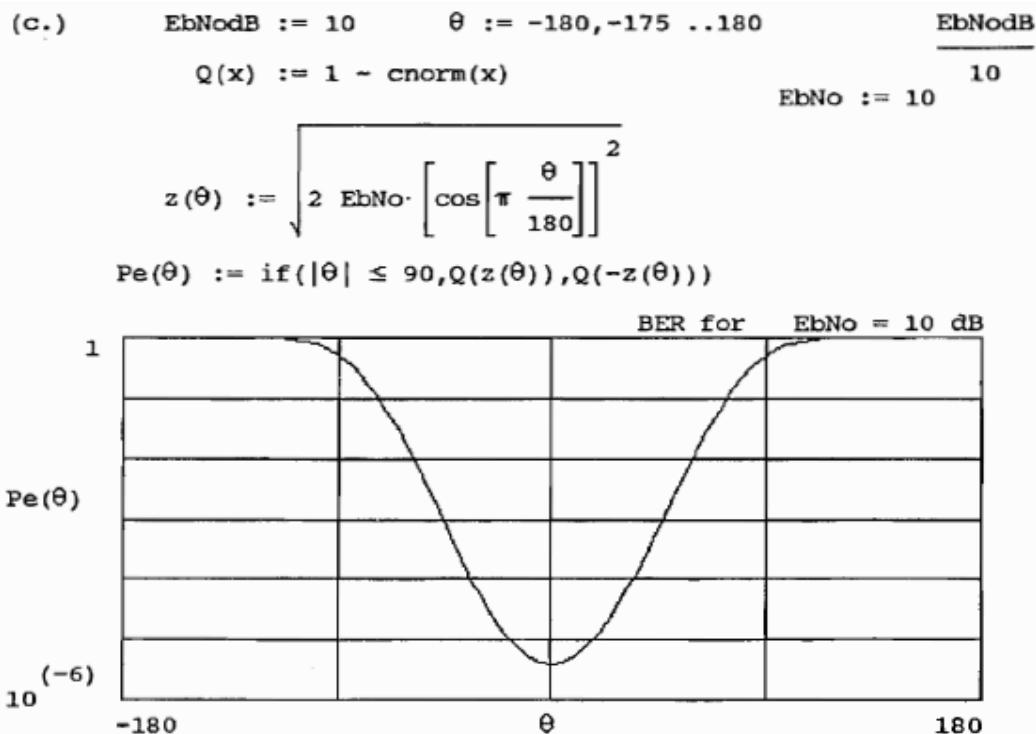
$$\text{or } E_d = 4 A^2 \pi \cos^2 \theta_e = 8 E_b \cos^2 \theta_e$$

$$\Rightarrow P_e = Q \left[ \sqrt{\frac{2(E_b)}{N_o} \cos^2 \theta_e} \right]$$

$E_b = \frac{1}{2} E_s + \frac{1}{2} E_{s_2} = \frac{1}{2} (\frac{1}{2} A^2 T) + \frac{1}{2} (\frac{1}{2} A^2 T) = \frac{1}{2} A^2 T$

Corresponds to (7-38)

## 7-16 (Continued)



## 7-17

(a.) Overall  $P_e = 10(P_e)_i = \underline{\underline{5 \times 10^{-7}}}$

(b.) When repeaters were used, the  $E_b/N_0$  at the input to each was described by:

$$P_e = Q\left[\sqrt{2}\left(\frac{E_b}{N_0}\right)\right] = 5 \times 10^{-8} \approx \sqrt{2\pi\left(\frac{E_b}{N_0}\right)} e^{-\frac{1}{2}} \quad \text{Sec.A-10}$$

$$\Rightarrow \frac{E_b}{N_0} = \underline{\underline{14.2}}$$

Now with 10 amplifiers, the Rx input consists of the BPSK signal with  $E_b$  energy / bit plus a noise level 10 times that present before (since the line from one amp to the next contributes a PSD of  $N_0/2$ , and there are 10 such lines). Thus  $\left(\frac{E_b}{N_0}\right)' = \frac{14.2}{10} = \underline{\underline{1.42}}$

$$\therefore P_e' = Q\left(\sqrt{2}(1.42)\right) = Q(1.69) \approx Q(1.7) = \underline{\underline{4.4 \times 10^{-2}}}$$

7-18

$$(a.) \quad B_T = 2700 - 300 = 2400 \text{ Hz}$$

The largest bit rate that can be accommodated w/o I.S.I. is (Table 7-1.)

$$\underline{B = R = 2400 \text{ bits/sec}}$$

(b) From Table 7.1 for BPSK:

$$P_e = Q\left(\sqrt{2}\left(\frac{E_b}{N_0}\right)\right) = \frac{1}{\sqrt{4\pi}\left(\frac{E_b}{N_0}\right)} e^{-\frac{E_b}{N_0}}, \text{ for each repeater}$$

(Sec. A-10)

$$\text{But } \left(\frac{S}{N}\right) = \frac{P_s}{N_0 B_T} = \frac{P_s}{N_0 R} = \frac{P_s T}{N_0} = \frac{E_b}{N_0} = \frac{S}{N} = 15 \text{ dB}$$

$B_T = R$        $R = \frac{1}{T}$

$$\Rightarrow \frac{E_b}{N_0} = 15 \text{ dB} = 31.6$$

$$\therefore P_e = \frac{1}{4\pi(31.6)} e^{-31.6} = 9.26 \times 10^{-16} / \text{repeater}$$

$$\text{There are } n = \frac{600 \text{ mi.}}{50 \text{ mi/rep}} = 12 \text{ repeaters (including } R_x \text{)}$$

$$\therefore \text{Overall } (P_e) = n P_e = 12 (9.26 \times 10^{-16}) = \underline{\underline{1.11 \times 10^{-14}}}$$

Note: If there are n repeaters, there is an error at the end of the line only if there are an odd number of errors along the line (for the bit in question).

$$\Rightarrow P(\text{K errors}) = \binom{n}{k} p_e^k (1-p_e)^{n-k}; \quad \binom{n}{k} = \frac{n!}{(n-k)! k!}$$

$$\Rightarrow \text{Overall } Pe = \sum_{\substack{k=1 \\ k \text{ odd}}}^n P(k \text{ errors}) \approx nPe \quad \text{If } nPe \ll 1$$

7-21

From Table 7-1 for FSK w/ noncoherent detection:

$$P_e = \frac{1}{2} e^{-\frac{1}{2} \left( \frac{E_b}{N_{\text{total}}} \right)}$$

$$N_{\text{total}} = K(T_0 + T_{\text{eff}}) = K(T_0 + (F-1)T_0) = KFT_0$$

$$T_{\text{eff}} = (F-1)T_0 = (1.38 \times 10^{-23}) / (10^{6/10}) \approx 290$$

$$\frac{E_b}{N_{\text{total}}} = \frac{P_s T}{N_{\text{tot}}} = \frac{P_s}{N_{\text{tot}} R} = \frac{V_s^2 / R_A}{KFT_0 R} = 28.53$$

$$R = 1/T$$

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(28.53)} = \underline{\underline{3.2 \times 10^{-7}}}$$

7-24

See Table 7-1.

(a.) For QPSK the largest  $R$  and min  $P_e$  are obtained.

$$R = 2B = 2(2700 - 300) = \underline{\underline{4800 \text{ b/s}}}$$

$$\frac{S}{N} = \frac{P_s}{\frac{N_0}{2}(2B)} = \frac{2P_s}{N_0 R} = \frac{2P_s T}{N_0} = \frac{2E_b}{N_0}$$

$$B = \frac{1}{2}R \quad R = \frac{1}{T}$$

$$\Rightarrow \frac{E_b}{N_0} = \frac{1}{2} \left( \frac{S}{N} \right) = \frac{1}{2} (10^{2.5}) = 158.1 \Rightarrow 22 \text{ dB}$$

$$P_e = Q\left(\sqrt{2}\left(\frac{E_b}{N_0}\right)\right) \ll 10^{-5} \text{ for } \frac{E_b}{N_0} = 22 \text{ dB}$$

Figure 7-14.

## 7-24 (Continued)

(b.)

$$C = B \log_2 \left( 1 + \frac{S}{N} \right) = 2400 \log_2 \left( 1 + 10^{2.5} \right)$$

$$= \frac{2400}{\ln 2} \ln (1 + 316.23) = 1.99 \times 10^4 = \underline{\underline{19,900 \text{ b/s}}}$$

## 7-25

$$(a.) R = \left( 8K \frac{\text{samples}}{\text{sec}} \right) \left( 8 \frac{\text{bits}}{\text{sample}} \right) = 64 K \text{ b/s}$$

$$\frac{S}{N} = \frac{P_s}{N_0 B} \stackrel{\uparrow}{=} \frac{P_s T_b}{N_0} = \frac{E_b}{N_0} = 10^{2.5} = 6.3$$

$$\boxed{B = R = \frac{1}{T_b}} \leftarrow \text{Table 7-1. for DPSK :}$$

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{N_0}\right)} = \frac{1}{2} e^{-6.3} = \underline{\underline{9.18 \times 10^{-4}}}$$

(b.) Using (7-70) with  $m = 2^8 = 256$  :

$$\left( \frac{S}{N} \right)_{\text{out}} = \frac{3m^2}{1 + 4(m^2 - 1)P_e} = \frac{3(256)^2}{1 + 4[(256)^2 - 1]9.18 \times 10^{-4}}$$

$$= 813.6 \Rightarrow \underline{\underline{29.1 \text{ dB}}}$$

7-27

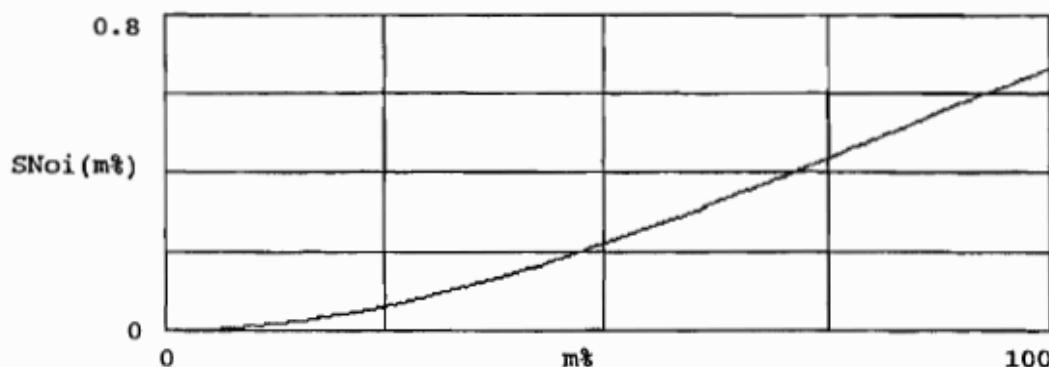
$$SN_{oi} \triangleq \frac{(S/N)_{out}}{(S/N)_{in}} = \frac{2\overline{m^2}}{1 + \overline{m^2}}$$

$$\text{Let } m(t) = \left(\frac{m\%}{100}\right) \cos \omega_m t \Rightarrow \overline{m^2} = \sum \left(\frac{m\%}{100}\right)^2$$

$$\Rightarrow SN_{oi} = \frac{\left(\frac{m\%}{100}\right)^2}{1 + \sum \left(\frac{m\%}{100}\right)^2}$$

$$m\% := 0, 5 \dots 100$$

$$SN_{oi}(m\%) := \frac{\left[\frac{m\%}{100}\right]^2}{1 + 0.5 \left[\frac{m\%}{100}\right]^2}$$



7-28

$$m(t) = 0.4 \sin \omega_m t \Rightarrow \overline{m^2} = \frac{[0.4]^2}{2} = 0.08$$

For AM, with product detector, use (7-90).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \frac{\overline{m^2}}{1 + \overline{m^2}} = \frac{0.08}{1.08} = 0.0741 \Rightarrow -11.3 \text{ dB}$$

Also get same result for env. det when  $(S/N)$  is large.

For DSB-SC, use (7-98).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = 1 \Rightarrow 0 \text{ dB} \Rightarrow \text{The AM system is inferior by } 11.3 \text{ dB}$$

7-31

Using Carson's Rule:

$$\beta_{IF} = 2(\beta_f + 1)B \Rightarrow 25\text{kHz} = 2(\beta_f + 1)5\text{kHz} \Rightarrow \beta_f = 1.5$$

$$f_i = 2.1\text{ kHz} ; \frac{B}{f_i} = \frac{5}{2.1} \gg 1 \therefore (7-139) \text{ is not valid}$$

$$\text{Eqn. (7-124a.) } s_o(t) = \frac{KDF}{2\pi} m(t) = \frac{KB\beta_f}{V_p} m(t)$$

$$\overline{s_o^2(t)} = K^2 B^2 \beta_f^2 \left( \frac{m}{V_p} \right)^2 ; \left( \frac{m}{V_p} \right)^2 = \frac{1}{2} \text{ for sinusoid}$$

$$\text{Eqn. (7-136) } \overline{[\tilde{u}_o(t)]^2} = 2 \left( \frac{K}{A_c} \right)^2 N_o f_i^3 \left[ \frac{B}{f_i} - \tan^{-1} \left( \frac{B}{f_i} \right) \right]$$

$$\therefore \left( \frac{S}{N} \right)_o = \frac{K^2 B^2 \beta_f^2 \left( \frac{m}{V_p} \right)^2}{2 \left( \frac{K}{A_c} \right)^2 N_o f_i^3 \left[ \frac{B}{f_i} - \tan^{-1} \left( \frac{B}{f_i} \right) \right]}$$

$$\text{Eqn. (7-128) } \left( \frac{S}{N} \right)_{in} = \frac{A_c^2}{4 N_o (\beta_f + 1) B}$$

$$\begin{aligned} \left( \frac{S}{N} \right)_o &= \frac{2 \left( \frac{B}{f_i} \right)^3 \beta_f^2 (\beta_f + 1) \left( \frac{m}{V_p} \right)^2}{\left[ \frac{B}{f_i} - \tan^{-1} \left( \frac{B}{f_i} \right) \right]} = \frac{2 (13.5 \times 2.25 \times 2.5) \frac{1}{2}}{1.21} \\ \left( \frac{S}{N} \right)_{in} &= 62.9 \end{aligned}$$

$$N_{total} = kFT_o ; F = 10^{1.2} = 15.8$$

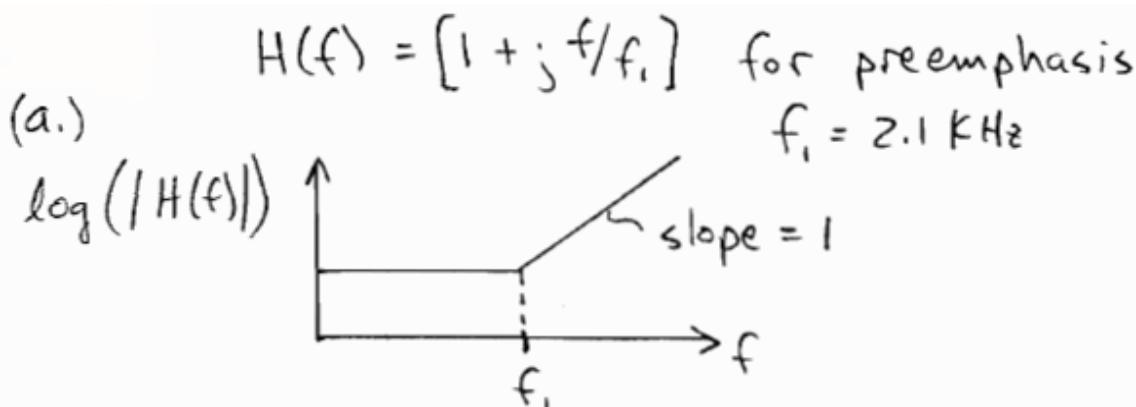
$$\left( \frac{S}{N} \right)_{in} = \frac{P_s}{\frac{N_{tot}}{2} (2B_{IF})} = \frac{P_s}{kFT_o B_{IF}} = \frac{P_s}{1.57 \times 10^{-15}}$$

$$\Rightarrow P_s = \left( \frac{S}{N} \right)_o \left( \frac{1}{62.9} \right) (1.57 \times 10^{-15})$$

$$= 10^{3.5} \left( \frac{1}{62.9} \right) (1.57 \times 10^{-15}) = 7.89 \times 10^{-14} \text{ W}$$

$$10 \log_{10} \left( \frac{7.89 \times 10^{-14}}{10^{-3}} \right) = \underline{-101 \text{ dBm}} = P_{s \min}$$

7-35



At  $f = 15 \text{ kHz}$  the gain is:

$$|H(f)| = \left| 1 + j \left( \frac{15}{2.1} \right) \right| = \sqrt{1 + \left( \frac{15}{2.1} \right)^2} = 7.21$$

At  $f = 1 \text{ kHz}$ :

$$|H(f)| = \left| 1 + j \left( \frac{1}{2.1} \right) \right| = \sqrt{1 + \left( \frac{1}{2.1} \right)^2} = 1.10$$

$$\Delta F = 75 \text{ kHz} \left( \frac{7.21}{1.10} \right) = \underline{\underline{488 \text{ kHz}}}$$

$$\% \text{ mod} = \frac{488}{75} (100) = \underline{\underline{651 \% \text{ mod}}}$$

(b.) The amplitudes of the high frequency audio components are much smaller than those of the low frequency components. For example, if the components at 15 kHz are more than  $20 \log_{10} \left( \frac{7.21}{1.1} \right) = 16.3 \text{ dB}$  below the 1 kHz components, there is no problem.

# Chapter 8

8-1

Assuming that the 150 G.Lite subscribers have VF service over the DSL lines, as well as VF service to the 300 VF subscriber lines, we get

$$300 \times 64 \text{ kb/s} + 150 \times 64 \text{ kb/s} + 150 \times 1,500 \text{ kb/s} \\ = 253,800 \text{ kb/s} = \underline{\underline{253.8 \text{ Mb/s}}}$$

8-7

$$P_{Tx} = 0.1 \text{ W}, f_c = 2.6 \text{ GHz}, 3.28 \text{ ft/meter}$$

$$(a.) G_A = \frac{7A}{\lambda^2} \approx \frac{7.0\pi \left(\frac{2}{3.28}\right)^2}{\left(\frac{3 \times 10^8}{2 \times 10^9}\right)^2} = \underline{\underline{363.14}} = \underline{\underline{2.5.6 \text{ dB}}} \\ A = \pi r^2, \lambda = c/f \quad 10 \log(363.4) = 25.6$$

$$(b.) P_{EIRP} = P_{Tx} G_{AT} = 0.1 (363.4) = \underline{\underline{36.3 \text{ W}}}$$

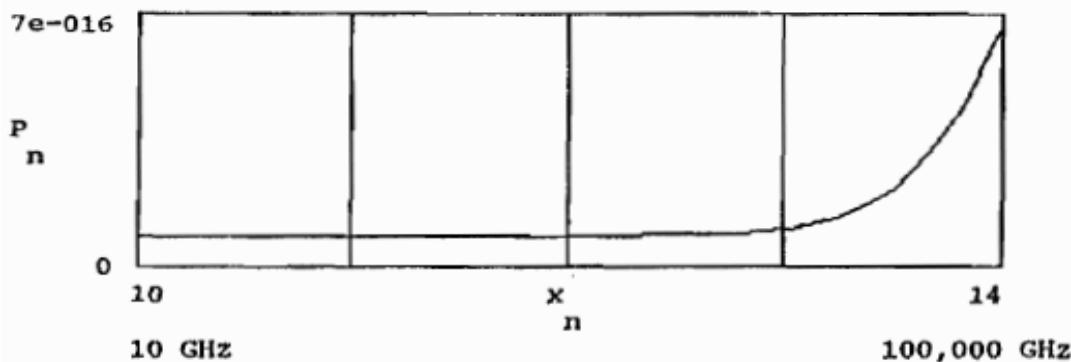
$$(c.) P_{Rx} = P_{Tx} G_{AT} G_{RR} \left(\frac{\lambda}{4\pi d}\right)^2 = (0.1)(363.4)^2 \left[ \frac{(0.15)(3.28)}{4\pi(15)(5280)} \right]$$

$$\Rightarrow P_{Rx} = 3.23 \times 10^{-9} \text{ W}$$

$$\text{or } (P_{Rx})_{dBm} = 10 \log \left( \frac{3.23 \times 10^{-9}}{10^{-3}} \right) = \underline{\underline{-54.9 \text{ dBm}}}$$

8-8

$$\begin{aligned}
 n &:= 100 \dots 140 & h &:= 6.6252 \cdot 10^{-34} & k &:= 1.381 \cdot 10^{-23} \\
 T &:= 300 & & & & \\
 R &:= 10000 & x := \frac{n}{10} & & & \\
 & & & f := \frac{x}{n} & & \\
 P_n &:= 2 \cdot R \left[ \frac{\frac{h \cdot f}{n}}{2} + \frac{\frac{h \cdot f}{n}}{\frac{h \cdot f}{n}} \right] & & & & \\
 & & & e^{\frac{k \cdot T}{n}} - 1 & &
 \end{aligned}$$



8-13

$$\begin{aligned}
 (a.) \quad T_{\text{eff}} &= T_0 (F-1) \\
 &= 290 (10^{16} - 1) = \underline{\underline{129^\circ K}}
 \end{aligned}$$

$$\begin{aligned}
 (b.) \quad P_{a_{\text{out}}} &= K T_{IN} \beta G_a \\
 &= (1.38 \times 10^{-23}) (30^\circ + 129^\circ) (10 \times 10^6) (10^3) \\
 &= \underline{\underline{2.2 \times 10^{-11} W}} \\
 &= 10 \log_{10} \left( \frac{2.2 \times 10^{-11}}{10^{-3}} \right) = \underline{\underline{-76.6 \text{ dBm}}}
 \end{aligned}$$

8-15

From Table 7-1 for FSK w/ noncoherent detection:

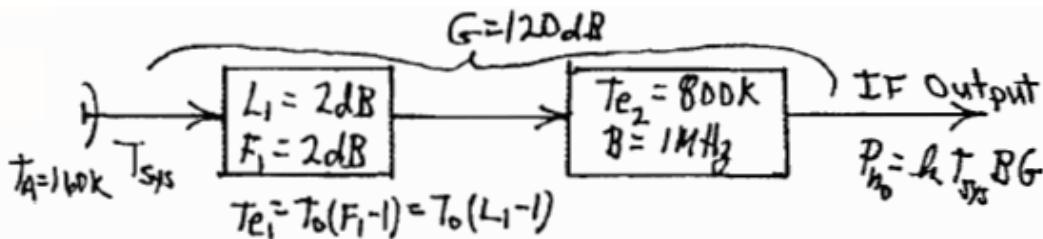
$$P_e = \frac{1}{2} e^{-\frac{1}{2} \left( \frac{E_b}{N_{total}} \right)}$$

$$\begin{aligned} N_{total} &= k(T_0 + T_{eff}) = k(T_0 + (F-1)T_0) = kFT_0 \\ &\quad \text{( } T_{eff} = (F-1)T_0 \text{ )} \\ &= (1.38 \times 10^{-23}) (10^{6/10}) (290) \end{aligned}$$

$$\frac{E_b}{N_{total}} = \frac{P_s T}{N_{total}} = \frac{P_s}{N_{tot} R} = \frac{V_r^2 / R_R}{kFT_0 R} = 28.53$$

$$P_e = \frac{1}{2} e^{-\frac{1}{2} (28.53)} = \underline{\underline{3.2 \times 10^{-7}}}$$

8-20



$$T_{sys} = T_A + T_e = T_A + \left( T_{e1} + \frac{T_{e2}}{G_1} \right) = T_A + \left( T_{e1} + T_{e2} L_1 \right)$$

$$\Rightarrow T_{sys} = T_A + \underbrace{\left[ T_o(L_1 - 1) + T_{e2} L_1 \right]}_{T_e}. \text{ Also } T_e = T_o(F - 1) \Rightarrow F = \frac{T_e}{T_o} + 1$$

(a) Compute the system noise temperature  $T_s$  evaluated at the antenna input of the waveguide:

$$\begin{aligned} T_a &:= 160 \text{ K} & T_o &:= 290 \text{ K} & T_{e2} &:= 800 \text{ K} \\ L_1 &:= 10 & G &:= 10^{12} & B &:= 10^6 \text{ Hz} \\ T_e &:= T_o \cdot (L_1 - 1) + T_{e2} L_1 & T_s &:= T_a + T_e & T_s &= 1597.534 \text{ K} \end{aligned}$$

(b) Noise figure  $F$ :

$$F := \frac{T_e}{T_o} + 1 \quad F = 5.957 \quad F_{dB} := 10 \cdot \log(F) \quad F_{dB} = 7.75$$

(c) The available output noise power  $P_{no}$ :

$$P_{no} := 1.38 \cdot 10^{-23} \cdot T_s \cdot B \cdot G \quad P_{no} = 0.022 \text{ Watt}$$

8-22

Assume 4 GHz downlink

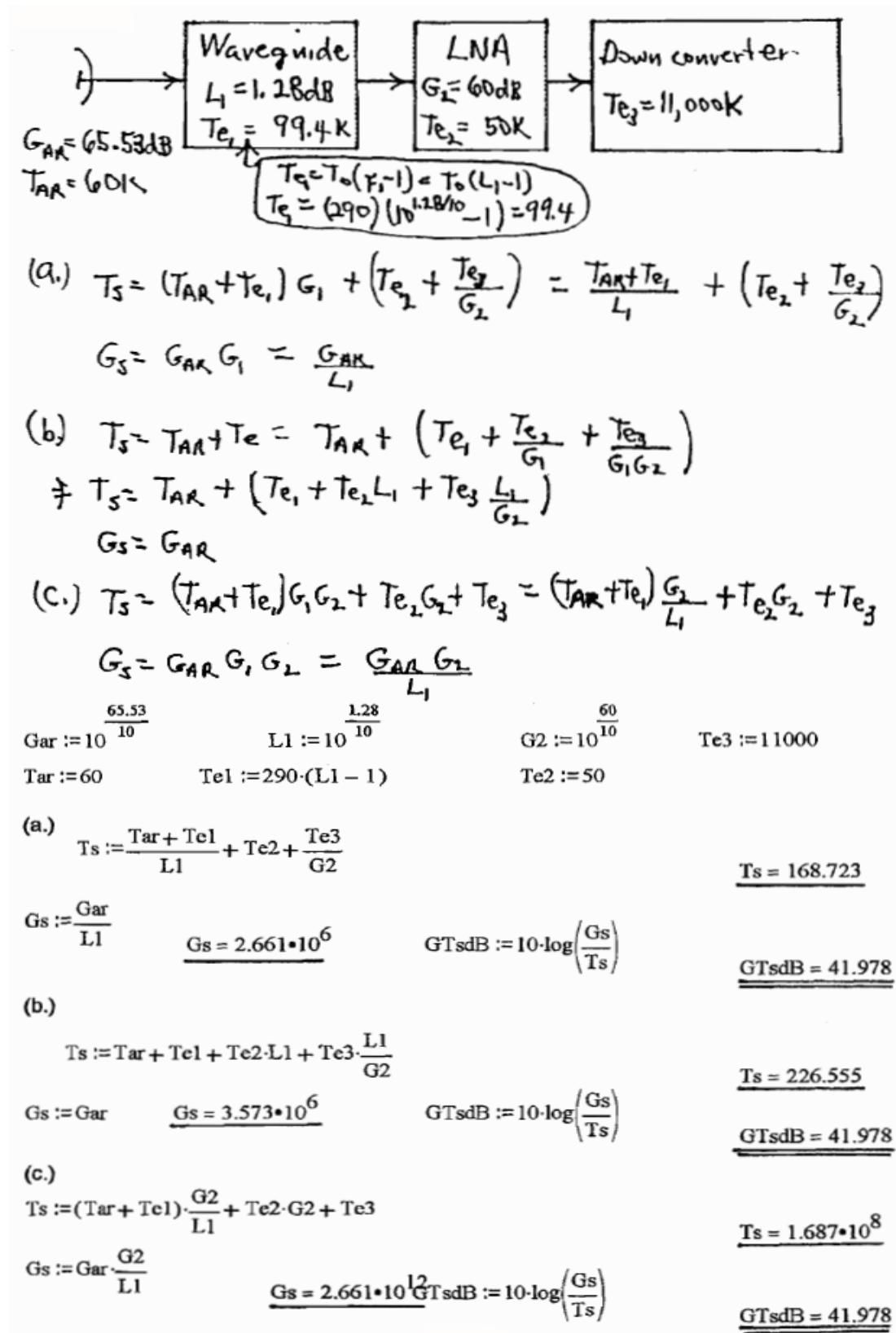
$$\frac{GAR}{T_{sys}} = 10^4 = \frac{4\pi n [\pi (15)^2]}{85 \left(\frac{3 \times 10^8}{4 \times 10^9}\right)^2} = 1.86 \times 10^4 n$$

$$\Rightarrow n = 54 \% \text{ for } 30 \text{ m antenna}$$

$$10^4 = 1.86 \times 10^4 n \left[ \frac{(12.5)^2}{(15)^2} \right] = 1.29 \times 10^4 n$$

$$\Rightarrow n = 77.4 \% \text{ for } 25 \text{ m antenna}$$

8-25



8-27

$$f_c = 2 \text{ GHz} ; \lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^9} = .15 \text{ m}$$

$$\frac{E_b}{N_0} = \frac{P_{Tx} G_{AT} G_{FS} G_{AR}}{K T_{sys} R}$$

$$G_{AT} = \frac{E_b}{N_0} \frac{k T_{sys} R}{P_{Rx} G_{FS} G_{AR}}$$

$$G_{FS} = \left( \frac{\lambda}{4\pi d} \right)^2 \cdot L_{\text{incidental}} = \left( \frac{0.15}{4\pi (7.5 \times 10^2)} \right)^2 (10^{-0.2}) = 1.6 \times 10^{-30}$$

$$G_{AR} = 7\pi \left( \frac{r}{\lambda} \right)^2 = 7\pi \left( \frac{32}{0.15} \right)^2 = 1 \times 10^6$$

$$G_{AT} = \frac{10^{0.9588} (1.38 \times 10^{-25}) (16)(300)}{10 (1.6 \times 10^{-30})(10^6)} = 4.03 \times 10^4 \xrightarrow{\text{parabolic}} \frac{7A}{1^2} = \frac{7A}{(0.15)^2}$$

$$\Rightarrow A = 129.44 \text{ m}^2 = \pi r^2$$

$$\pm r = \sqrt{\frac{129.44}{\pi}} = 6.42 \text{ m} \Rightarrow D = 2r = 12.84 \text{ m} \quad \begin{matrix} \text{parabolic ant} \\ (\text{rather large}) \end{matrix}$$

8-29

Using (8-47) and (8-8)

$$P_{dBm}(d) = P_{dBm}(d_0) - 10n \log\left(\frac{d}{d_0}\right)$$

$$\text{where } P_{dBm}(d_0) = (P_T)_{dBm} + (G_{TA})_{dBm} - 20 \log\left(\frac{4\pi d_0}{\lambda}\right) + (G_{AR})_{dBm}$$

$$= 40 + 18 - 20 \log\left(\frac{4\pi (0.25 \text{ miles})(5280 \text{ ft})}{3 \times 10^8 / 1.8 \times 10^9} \right) + 0$$

$$\Rightarrow P_{dBm}(d_0) = -31.64 \text{ dBm} \rightarrow 89.64$$

Distance (miles)	Power Received (dBm)				
	0.25	1	2	5	10
n=2 (free space)	-31.6	-43.7	-49.7	-57.7	-63.7
n=3	-31.6	-49.7	-58.7	-70.6	-79.7
n=4	-31.6	-55.7	-67.8	-83.7	-95.7

8-32

The available bandwidth is

$$B_T = \frac{1}{2}f_b = \frac{1}{2}(15.734) = 7.867 \text{ kHz}$$

Referring to Fig. 8-33, the spectrum of the professional subcarrier (PSC) needs to roll off fast enough so that it does not interfere with the SAP FM subcarrier and the DSB-SC stereo subcarrier. Since the SAP subcarrier is FM, the capture effect of the FM detector will suppress the PSC interference when the PSC spectrum is, say, 10 to 15 dB below the SAP spectrum (in the SAP band). Referring to Fig. 5-33, it is seen that these interference specifications are easily satisfied if the first-null bandwidth criterion is used. Thus, for the M-ary signal class,

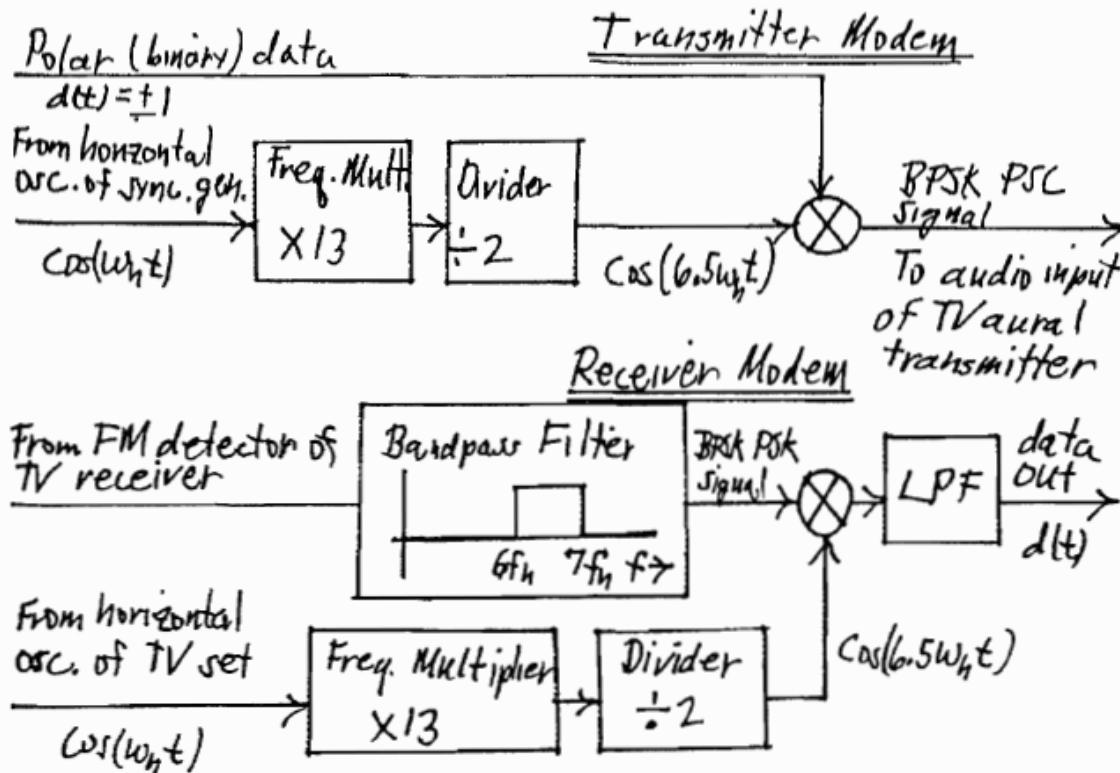
8-32 (Continued)

$$B_T = 2 \left( \frac{R}{\ell} \right) \leq 7.867 \text{ or } \ell \geq \left( \frac{2R}{7.867} \right)$$

(a) For  $R = 1.2 \text{ kb/s}$ ,  $\ell \geq \frac{2(1.2)}{7.867} = 0.305$ .

$\Rightarrow$  Choose  $\ell = 1 \Rightarrow$  Use BPSK signaling

A  $R = 1200 \text{ b/s}$  BPSK system is shown below.



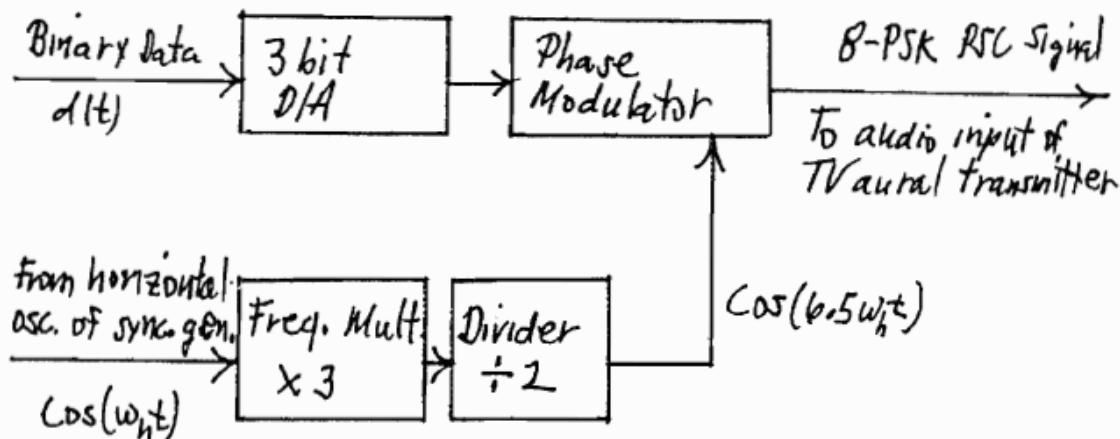
(b) For  $R = 9.6 \text{ kb/s}$ ,  $\ell \geq \frac{2(9.6)}{7.867} = 2.44$   
 $\Rightarrow$  Choose  $\ell = 3 \Rightarrow M = 2^3 = 8$

$\Rightarrow$  Use 8-PSK

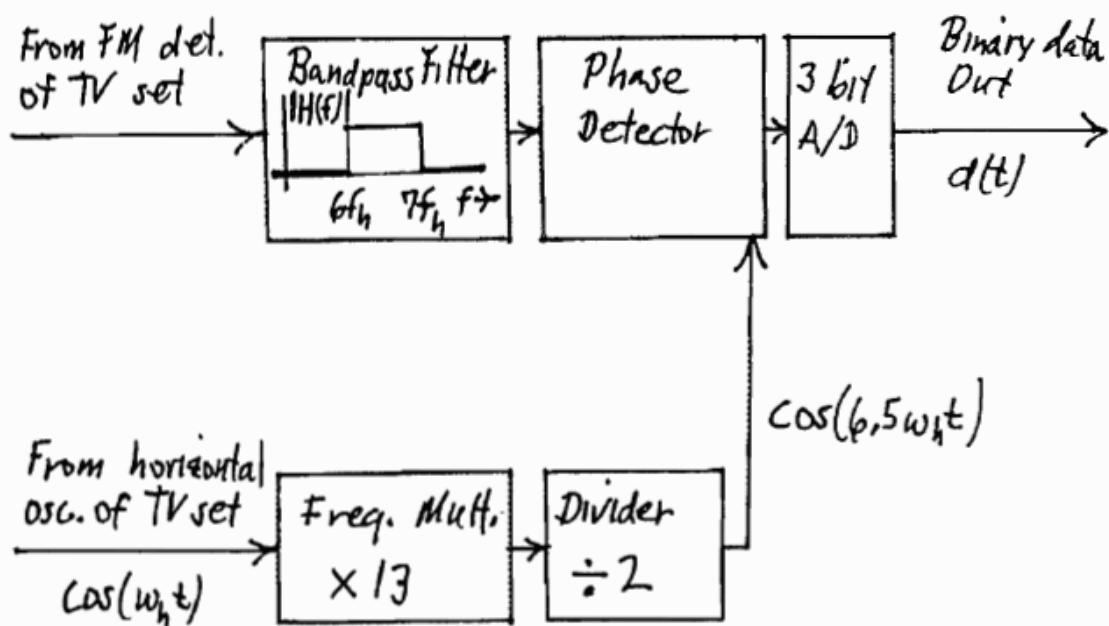
8-32 (Continued)

A  $R = 9600 \text{ b/s}$  8-PSK system is shown below

### Transmitter Modem



### Receiver Modem



8-34

$$R_{\text{with Coding}} = (R_{\text{without Coding}}) \left( \frac{1}{R_{\text{TCM}}} \right) \left( \frac{1}{R_{\text{RS}}} \right) = 19.39 \left( \frac{3}{2} \right) \left( \frac{207}{187} \right)$$

$$\Rightarrow R_{\text{with Coding}} = 32.20 \text{ Mb/s}$$

$$\Rightarrow D_{\text{with coding}} = \frac{R_{\text{with coding}}}{l} = \frac{32.2}{3} = 10.73 \text{ Mbaud}$$

$l=3$  for 8 levels

The segment sync replaces the payload sync at the beginning of each segment. One segment of training data is added after 3/2 segments.

Thus,

$$D_{\text{overall}} = (D_{\text{with coding}}) \left( \frac{3/2+1}{3/2} \right) = 10.73 \left( \frac{3/2+1}{3/2} \right) = \underline{\underline{10.76 \text{ Mbaud}}}$$

## Appendix B

B-1

$$P(1) = \frac{n_1}{n} = \frac{1428}{1428 + 2668} = \underline{\underline{0.3486}}$$

B-3

$$\begin{aligned} \text{(a.) } P(1+3+5) &= P(1) + P(3) + P(5) = \frac{3}{6} = \underline{\underline{\frac{1}{2}}} \\ \text{(b.) } P(4/E) &= \frac{P(4 \cdot E)}{P(E)} = \frac{P(4)}{P(E)} = \frac{1/6}{1/2} = \underline{\underline{\frac{1}{3}}} \end{aligned}$$

B-8

$$\begin{aligned} \text{(a.) } \int_{-\infty}^{\infty} f(x) dx &= 1 = \int_0^{\infty} k e^{-bx} dx \\ &= \frac{k e^{-bx}}{-b} \Big|_0^{\infty} = \frac{k}{b} [e^{-\infty} - e^0] = \frac{k}{b} = 1 \Rightarrow \underline{\underline{k=b}} \end{aligned}$$

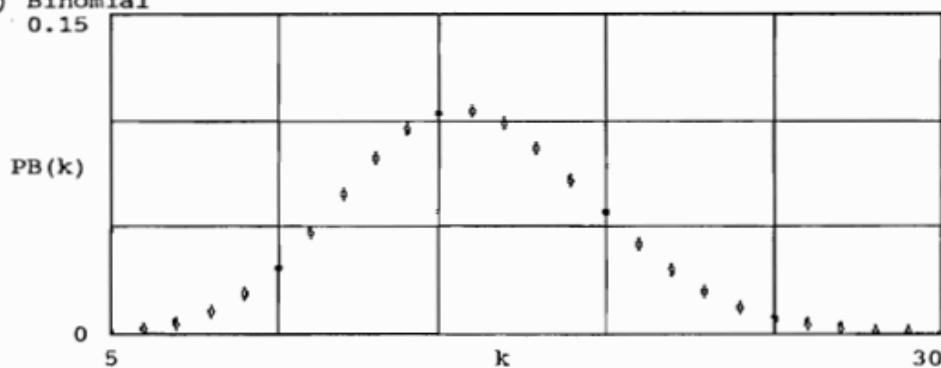
$$\begin{aligned} \text{(b.) } f(x) &= b e^{-bx} \\ m = \bar{x} &= \int_{-\infty}^{\infty} x f(x) dx = \int_0^{\infty} b x e^{-bx} dx \\ &= b \left[ \frac{-x e^{-bx}}{b} \Big|_0^{\infty} + \int_0^{\infty} \frac{e^{-bx}}{b} dx \right] = b \left[ \frac{e^{-bx}}{-b^2} \Big|_0^{\infty} \right] \\ &\quad \boxed{\begin{array}{l} \text{Let } u=x, \frac{du}{dx}=1 \\ \frac{dv}{dx}=e^{-bx}, v=-e^{-bx}/b \end{array}} \\ \Rightarrow m &= b \left[ \frac{-1}{b} (e^{-\infty} - e^0) \right] = \underline{\underline{\frac{1}{b} = m}} \end{aligned}$$

$$\begin{aligned} \text{(c.) } \sigma^2 &= \bar{x}^2 - (\bar{x})^2 \text{ where } \bar{x}^2 = \int_{-\infty}^{\infty} x^2 b e^{-bx} dx \\ \text{Using Sec. A-5 } \bar{x}^2 &= b e^{-bx} \left[ \frac{x^2}{b} - \frac{2x}{b^2} - \frac{2}{b^3} \right] \Big|_0^{\infty} = -b e^0 \left( \frac{2}{b^3} \right) = \frac{2}{b^2} \\ \Rightarrow \sigma^2 &= \frac{2}{b^2} - \left( \frac{1}{b} \right)^2 = \underline{\underline{\frac{1}{b^2} = \sigma^2}} \end{aligned}$$

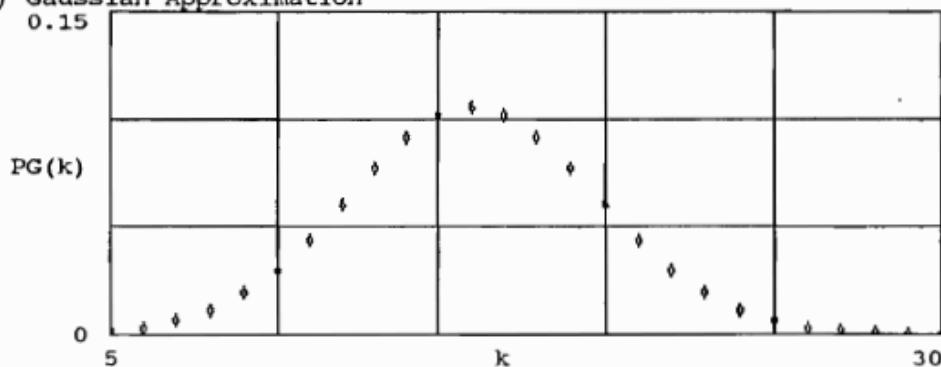
## B-12

$$\begin{aligned}
 n &:= 160 & p &:= 0.1 & q &:= 1 - p & \lambda &:= n \cdot p \\
 m &:= n \cdot p & \sigma &:= \sqrt{n \cdot p \cdot q} & k &:= 0 \dots 2 \cdot \lambda & m &= 16 \\
 PB(k) &:= \frac{n!}{k! (n-k)!} p^k q^{n-k} & \text{Binomial} & \\
 PP(k) &:= e^{-\lambda} \frac{\lambda^k}{k!} & \text{Poisson} & \\
 PG(k) &:= \frac{1}{\sigma \sqrt{2 \cdot \pi}} e^{\frac{-(k-m)^2}{2 \cdot \sigma^2}}
 \end{aligned}$$

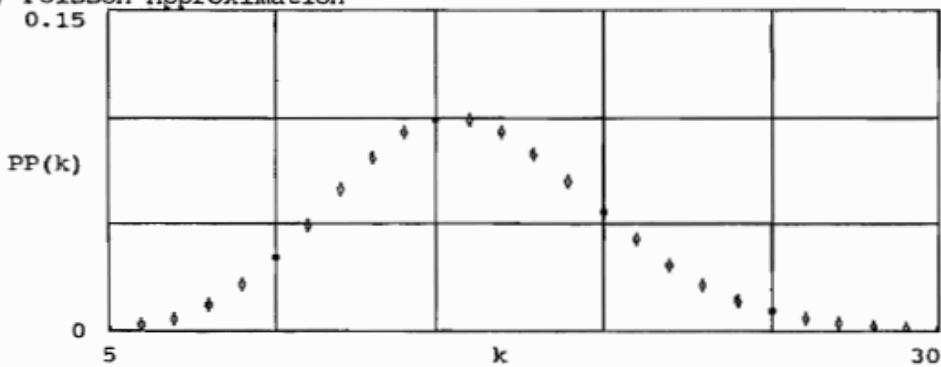
(a.) Binomial



(b.) Gaussian Approximation



(c.) Poisson Approximation



B-16

Let  $f(x) = \text{pdf of } \rho \text{ values where}$   
 $x = \rho \text{ value}$

$\text{set}$

$$\Rightarrow \int_{\bar{x}}^{1.1\bar{x}} f(x) dx \stackrel{\downarrow}{=} .95$$

$$\Rightarrow \int_{\bar{x}}^{1.1\bar{x}} \frac{1}{\sqrt{2\pi\Delta}} e^{-\frac{(x-\bar{x})^2}{2\Delta^2}} dx = .95$$

(Let  $x_1 = x - \bar{x}$ ;  $dx_1 = dx$ )

$$= \int_{-.1\bar{x}}^{.1\bar{x}} \frac{1}{\sqrt{2\pi\Delta}} e^{-x_1^2/2\Delta^2} dx_1 = .95 = 1 - 2Q\left(\frac{.1\bar{x}}{\Delta}\right)$$

$$\Rightarrow Q\left(\frac{.1\bar{x}}{\Delta}\right) = \frac{1 - .95}{2} = .025 \stackrel{\downarrow}{\Rightarrow} \frac{.1\bar{x}}{\Delta} = 1.96$$

$$\Rightarrow \Delta = \frac{.1\bar{x}}{1.96} = \frac{(.1)(1000)}{1.96} = \underline{\underline{51.0 \text{ ohms}}} = \sigma$$

(A-10)

B-25

The input is  $x(t) = A \sin \omega_m t$  where  $A=8$ .  
 The output consists of a quantized sinusoid similar to that shown in Fig. 3-8b. The PDF of the output,  $y(t)$ , will consist of  $\delta$  functions at the quantized values.

Thus,  $f(y) = \sum_{k=1}^M p_k \delta(y - y_k)$

where  $M=8$ , the step size is  $\delta = \frac{2A}{M} = \frac{16}{8} = 2$ , and the quantized values are:  $y_k = \frac{(2k-M-1)}{2} \delta$

$$p_k = \int_{y_k - \delta/2}^{y_k + \delta/2} f_x(x) dx = \int_{y_k - \delta/2}^{y_k + \delta/2} \frac{1}{\pi \sqrt{A^2 - x^2}} dx = \int_{y_k - \delta/2}^{y_k + \delta/2} \frac{1}{\pi \sqrt{1 - (\frac{x}{A})^2}} dx$$

*(Using (B-67))*

$$= \frac{1}{\pi} \left[ \sin^{-1}\left(\frac{x}{A}\right) \right]_{y_k - \delta/2}^{y_k + \delta/2} = \frac{1}{\pi} \left[ \sin^{-1}\left(\frac{y_k + \delta/2}{A}\right) - \sin^{-1}\left(\frac{y_k - \delta/2}{A}\right) \right]$$

*(Using (A-29))*

or

$$p_k = \frac{1}{\pi} \left[ \sin^{-1}\left(\frac{(2k-M-1)\delta}{2A}\right) - \sin^{-1}\left(\frac{(2k-M-1-1)\delta}{2A}\right) \right]$$

$$p_k = \frac{1}{\pi} \left[ \sin^{-1}\left(\frac{(2k-M)\frac{2A}{M}}{2A}\right) - \sin^{-1}\left(\frac{(2k-M-2)\frac{2A}{M}}{2A}\right) \right] = \frac{1}{\pi} \left[ \sin^{-1}\left(\frac{2k-M}{M}\right) - \sin^{-1}\left(\frac{2k-M-2}{M}\right) \right]$$

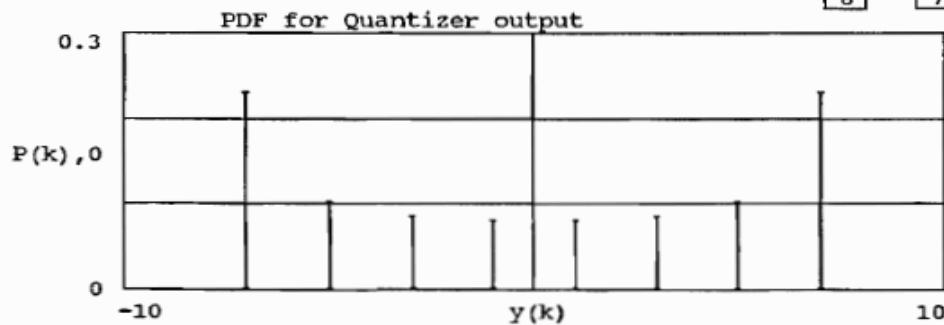
$$A := 8 \quad M := 8 \quad \delta := 2 \quad \frac{A}{M} \quad \delta = 2 \quad \text{--Step size}$$

$$k := 1 \dots M$$

$$y(k) := (2k - M - 1) 0.5 \cdot \delta$$

$$p(k) := \frac{1}{\pi} \left[ \sin\left(\frac{2k - M}{M}\right) - \sin\left(\frac{2k - M - 2}{M}\right) \right]$$

k	y(k)
1	-7
2	-5
3	-3
4	-1
5	1
6	3
7	5
8	7



B-27

$$\begin{aligned}
 \bar{y} &= \int_{-\infty}^{\infty} y f(y) dy = \int_0^{\infty} \frac{y}{\sqrt{2\pi} B\Delta} e^{-y^2/2B^2\Delta^2} dy \\
 &\quad + \frac{1}{2} \int_{-\infty}^{\infty} y \delta(y) dy \\
 &= \left( \frac{-B\Delta}{\sqrt{2\pi}} \right) \int_0^{\infty} e^{-y^2/2B^2\Delta^2} \left( \frac{-y}{B^2\Delta^2} \right) dy \\
 &\stackrel{\uparrow}{=} \left( \frac{-B\Delta}{\sqrt{2\pi}} \right) \int_0^{\infty} e^z dz = \left( \frac{-B\Delta}{\sqrt{2\pi}} \right) e^z \Big|_0^{\infty} \\
 &= \underline{\underline{\frac{B\Delta}{\sqrt{2\pi}}}} = \bar{y}
 \end{aligned}$$

Let  $z = \frac{-y^2}{2B^2\Delta^2}$   
 $dz = \frac{-y}{B^2\Delta^2} dy$

B-34

$$\begin{aligned}
 mx &:= \begin{bmatrix} 2 \\ -1 \end{bmatrix} & Cx &:= \begin{bmatrix} -2 \\ 5 \\ -2 \\ \hline \sqrt{5} \\ \sqrt{5} \end{bmatrix} & T &:= \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ \hline 1 & 1 \end{bmatrix}
 \end{aligned}$$

(a.) Compute the mean vector for y:

$$my := T mx$$

$$\underline{\underline{my = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}}}$$

(b.) Compute the covariance matrix, Cy:

$$Cy := T \cdot Cx \cdot T^T$$

$$\underline{\underline{Cy = \begin{bmatrix} 5.106 & 3.382 \\ 3.382 & 4.356 \end{bmatrix}}}$$

## B-34 (Continued)

(c.) Compute the correlation coefficient for  $y_1$  and  $y_2$ :

$$\rho := \frac{c_{y_{0,1}}}{\sqrt{c_{y_{0,0}}} \sqrt{c_{y_{1,1}}}}$$

$\rho = 0.717$   
 ~~~~$c_{y_{0,1}}$~~~~



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