Elihu Thomson's Jumping Ring in a Levitated Closed-Loop Control Experiment

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***Abstract*—A conducting nonmagnetic ring is levitated above a coil excited by an ac source, using the same principle as that of the well-known jumping ring experiment, originally discovered by Elihu Thomson. The ac source is amplitude modulated to ensure that the ring stays in a stationary levitated position. The circuit is analyzed using a simple electrical/magnetic treatment, and the parameters of the system are found. A control strategy is derived for the system, after it has been linearized for small perturbations about an operating point. A simple low-voltage implementation is also shown, with results.**

***Index Terms*—Controlled levitation, electromagnetic levitation, jumping ring, levitation project, single-phase linear actuator.**

I. Introduction

LEVITATION projects have always caught the imagination of students, motivating them on the almost wizardry of electrical circuitry and providing a wealth of analysis on the resulting systems that are built. They also serve as a means of introducing some of the practical application areas such as linear induction motors, Maglev transportation systems and magnetic bearings. There are generally two approaches to stationary levitation projects. The first is a mean dc voltage driven system, where magnetic poles are induced in a soft ferromagnetic ball by means of a coil. The ball is attracted to the coil and can be suspended below the coil by adjusting the amplitude of the dc voltage to the coil to counterbalance gravity and any acceleration forces on the ball [1]. The second method is an ac voltage driven system, where a current is induced in a secondary circuit by means of a primary coil. The secondary circuit is repelled from the primary or inducing coil and can be levitated above it by adjusting the amplitude or frequency of the ac voltage source to the primary coil to counterbalance gravity and any acceleration forces on the circuit [2]. The technique used in this paper is the second, where the secondary circuit is a ring levitated on the extended core of the primary or driving coil, eliminating any lateral stability problems. (Because of the lateral stability problem spheres, usually hollow, require two ac fields to maintain a stable position [3].)

While Ampère quantified the force produced from the flow of dc current in two current carrying conductors [4], Elihu Thomson discovered the force effect of ac currents

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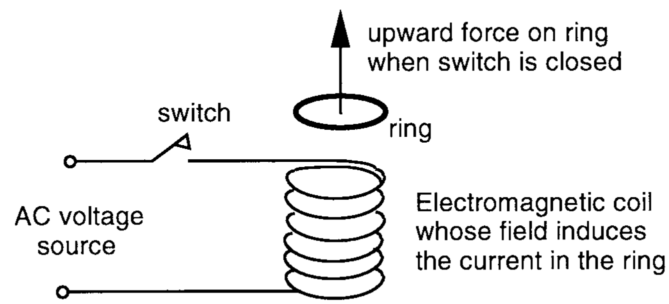


Fig. 1. Jumping ring experiment.

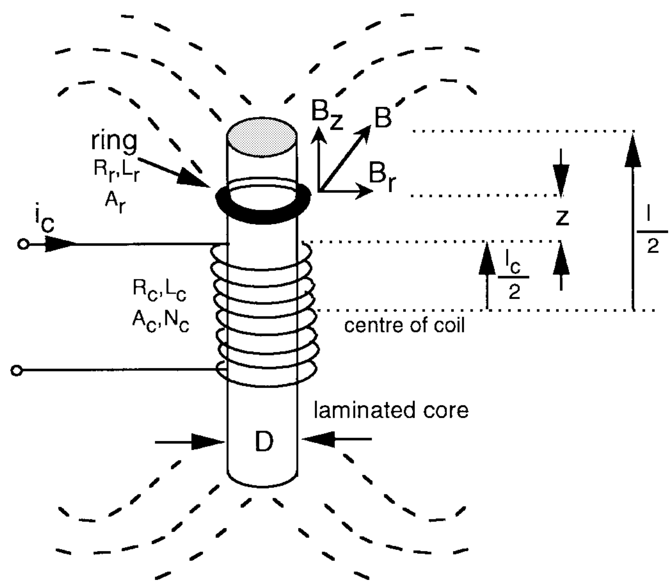


Fig. 2. Coil with core, showing parameters of the system.

[5]. The effect of the repulsion force on the ring was first demonstrated by him to the American Institute of Electrical Engineers in New York in May of 1887 and subsequently in Paris in 1889 where Fleming first saw it and borrowed the apparatus. However, Fleming first published it in 1891 [6], after his discourse at The Royal Institution of Great Britain on March 6th of the same year, where he demonstrated what is known as the jumping ring experiment, which has subsequently been used in countless courses to demonstrate the principle of electromagnetic induction, producing a force from an ac source, Lenz's law of repulsion, transformer action and pole shading. (Incidently the apparatus borrowed by Fleming was permanently acquired by the Royal Institution and put

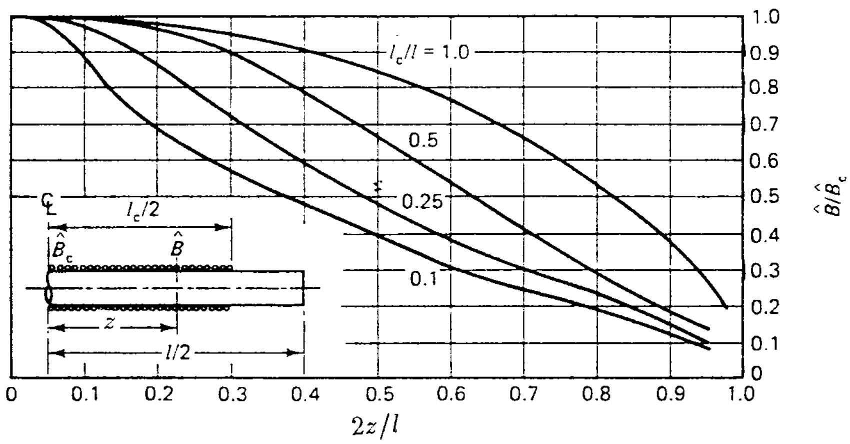


Fig. 3. Flux distribution along the core.

in a cabinet alongside the original dynamo and induction coil of Faraday.) The principle can be demonstrated by means of Fig. 1 and briefly when the switch is closed the ring, usually positioned around the extended core of the coil, is thrown upwards into the air [7].

The upward force is first calculated taking particular care to point out the significance of the phase shift produced in the secondary circuit. For a suitable topology the parameters of the system are calculated, using appropriate assumptions. By then assuming that the actual frequency of operation is sufficiently high such that the effect of the variations over a cycle can be ignored, an equation describing the dynamics of the system is produced for amplitude variation in the coil current affecting the upward force. This equation is linearized for small variations about a fixed operation point and the resulting system controlled with a suitably designed electronic circuit. Simple materials and electronic components are used in a low-power and low-voltage implementation.

II. Calculating the Force on the Ring

Consider the ring as shown in Fig. 2, a distance z up from the end of the core and with the other circuit and dimensional parameters as shown. The ring is free to move up and down on the core with zero friction between itself and the core. The core is made from laminated ferrous metal bars or rods. The ring is made from a nonmagnetic electrical conductor. Apply Ampère's Circuital Law (for the quasistatic case), namely

$$\oint_l \frac{\mathbf{B}}{\mu} \cdot dl = \int_s \mathbf{J} \cdot d\mathbf{s} = i_c$$

to the coil having a sinusoidal current $i_c = I_c \sin \omega t$ flowing in it. Then the flux density midway along the core is given as

$$B(t) = \hat{B}_c \sin \omega t$$

and directed along the axis of the core and where $\hat{B}_c$ is the peak amplitude of the flux density, dependent upon the coil parameters and the peak value of the current in the coil. Because of the nature of the open magnetic core, not all the flux passing through the center of the core will pass through the ring, as some flux lines will exit the core along the coil and between the coil and the ring. Critically some flux lines will exit through the ring itself. Let the peak value of the flux density in the core at the height z up from the top of the coil be $\hat{B}$, with a vertical component $B_z = \hat{B}_z \sin \omega t$ and a radial component $B_r = \hat{B}_r \sin \omega t$. Applying Maxwell's equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

at the ring location, the field intensity is given as

$$\mathbf{E}(b,\, t) = -\frac{\omega b \hat{B}_z}{2} \cos \omega t$$

and directed circumferentially along the ring. b is the radius of the ring. The induced voltage in the ring is given as

$$v_r = \oint_l \mathbf{E} \cdot dl = -\frac{\omega 2\pi b^2 \hat{B}_z}{2} \cos \omega t$$

from which the current i_r which flows in the ring is given as

$$i_r = \frac{v_r}{Z_r} = -\frac{\omega \pi b^2 \hat{B}_z}{|Z_r|} \cos(\omega t - \phi_r) = I_r \cos(\omega t - \phi_r)$$

where I_r has the obvious definition. Z_r is the ring impedance and ϕ_r is the ring phase shift, namely

$$|Z_r| = \sqrt{R_r^2 + (\omega L_r)^2} \quad \text{and} \quad \phi_r = \tan^{-1} \frac{L_r \omega}{R_r}.$$

L_r is the self inductance of the ring, R_r its resistance. An expression for the force acting on the ring can be calculated using the Lorentz derived equation

$$d\mathbf{f} = i_r \mathbf{dl} \times \mathbf{B}$$

or alternatively by means of the rate of change of the magnetic stored energy in the system. Looking at the former first, the upward force f over the length of the ring is given as

$$f = i_r 2\pi b \hat{B}_r \sin \omega t.$$

The average value of this force over a cycle is given as

$$F = -2\pi b I_r \hat{B}_r \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} \cos(\omega t - \phi_r) \sin \omega t \, d(\omega t)$$
$$= \pi b I_r \hat{B}_r \sin \phi_r$$

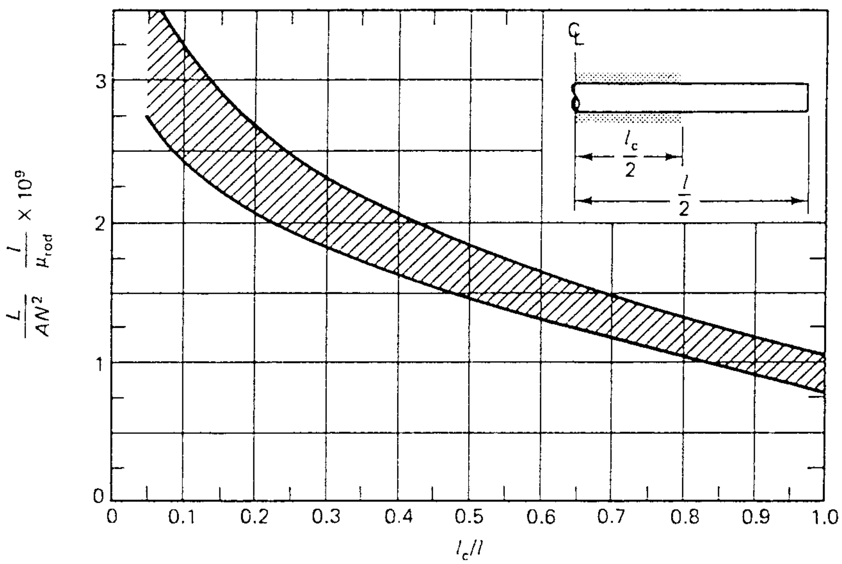


Fig. 4. Inductance of the coil as a function of l_c/l.

and is positive (as ϕ_r is positive) and so is upwards along the core axis.

The force F derived does not explicitly have the levitated distance in its expression. Instead consider the force derived from the change in stored magnetic energy, namely

$$f = i_c i_r \frac{\partial M}{\partial z} \tag{1}$$

where M is the mutual inductance between the coil and ring, (which formula assumes that only the energy in the mutual coupling between the coil and ring changes with z). The induced voltage in the ring is given as

$$v_r = -\frac{d\lambda_r}{dt} = -\frac{dMi_c}{dt} = M\omega I_c \cos \omega t$$

where λ_r is the flux linkage of the ring and M is considered constant with respect to time. The voltage v_r gives rise to a current

$$i_r = \frac{M\omega I_c}{|Z_r|} \cos(\omega t - \phi_r). \tag{2}$$

Using the foregoing in conjunction with (1) we get

$$f = \frac{M\omega I_c^2}{|Z_r|} \cos(\omega t - \phi_r) \sin \omega t \frac{\partial M}{\partial z}.$$

The mutual inductance for the circuit is not simple to calculate. However, for small perturbations about a fixed point Z it may be represented as

$$M = M_Z\left(1 - \frac{\Delta Z}{Z}\right)$$

where M_Z is a fixed value measured at a distance Z up from the coil and ΔZ is a small change measured from Z. It is noted that the rate of change of M with respect to distance is given as $-(M_Z/Z)$, a constant. And so for fixed I_c the force f at or near Z is given as

$$f = -\frac{M_Z^2\omega}{|Z_r|} \frac{I_c^2}{Z} \cos(\omega t - \phi_r) \sin \omega t$$

where we assume $\Delta Z/Z \ll 1$ and we approximate $M = M_Z$.

The average value of this force over a cycle is given as

$$\begin{aligned} F &= -\frac{M_Z^2\omega}{|Z_r|} \frac{I_c^2}{Z} \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} \cos(\omega t - \phi_r) \sin \omega t \, d(\omega t) \\ &= \frac{M_Z^2\omega}{|Z_r|} \frac{I_c^2}{Z} \frac{\sin \phi_r}{2} \\ &= K \frac{I_c^2}{Z} \end{aligned} \tag{3}$$

where $K = (M_Z^2\omega/2|Z_r|) \sin \phi_r$. The result shows that a constant dc upward force is produced, for fixed I_c and Z. It shows that it depends upon the phase shift produced by the ring. Without inductance in the ring there would be no resultant upward force. This helps to explain that there must likewise be a phase delay between the primary coil field and the secondary coil field in a single-phase induction motor in order to have a starting torque.

We note the two force formulas

$$f = 2\pi b i_r B_r \quad \text{and} \quad f = i_c i_r \frac{\partial M}{\partial z}$$

can be shown to be equal. Consider a small cylindrical volume of radius b and height ΔZ with flux entering at the bottom of flux density value $B(z)$ and exiting vertically with a value $B(z + \Delta Z)$ and radially with a value B_r. Then when a flux balance is taken

$$B_r = -\frac{b}{2} \frac{\partial B}{\partial z}.$$

Finally we note that $B = Mi_c/2\pi b$ which enables both to be shown to be identical.

III. Calculating the Parameters of the System

In order to calculate the force F in (3) we must determine the mutual inductance M_Z at the point Z and the current I_c. Clearly for the configuration given these are not simple calculations and so we use standard curves available from the

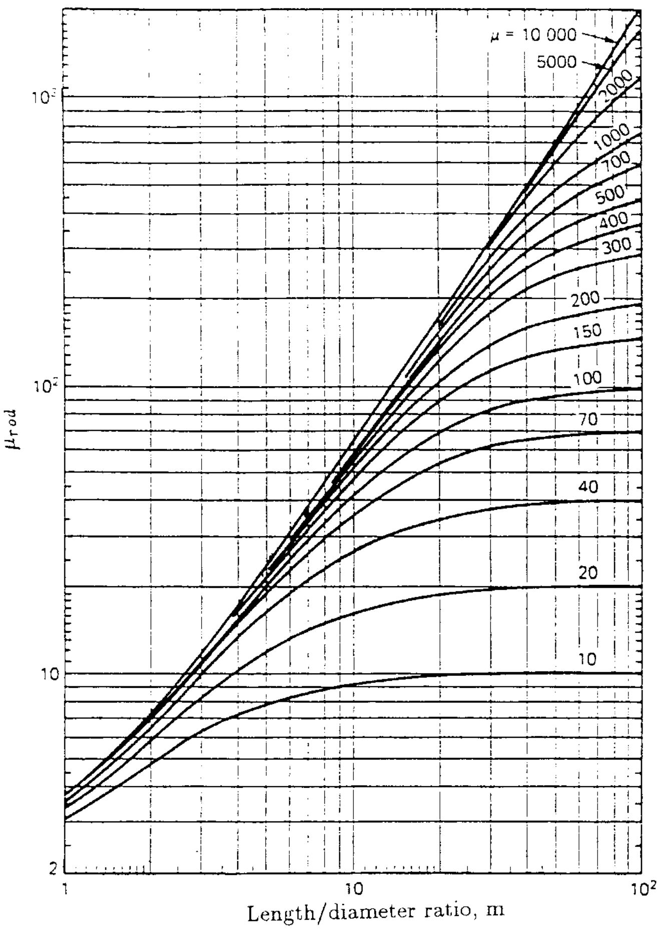


Fig. 5. μ_{rod} as a function of μ material and core dimensions.

literature. The curves used are those as given by Snelling, which are reproduced here with kind permission from publisher Butterworth–Heinemann, see Figs. 3–5, [8].

Looking at Fig. 3 and taking for example $l_c/l = 0.28$ and $2z/l = 0.43$, then the flux density $\hat{B}$ at a point z (note $z = l_c/2 + Z$ in Fig. 2) out from the center of the core is down to 0.6 of the central core value, $\hat{B}_c$, where $\hat{B}_c$ is the peak value of the flux density induced in the central core due to an applied voltage and given as a peak value $\hat{B}_c = V_c/A_c N_c \omega$ for the case of the sinusoidal applied voltage of peak amplitude V_c. A_c is the magnetic cross sectional area of the coil core and N_c its number of turns. Assuming that $\hat{B}$ is constant over the area of the ring, A_r, then the peak value of the flux passing through the ring, Φ_r, is obtained (given no saturation) and is the same as the flux linkage, as the ring consists of just one turn. From its sinusoidal variation the induced voltage is obtained. The peak current that flows in the ring is given as

$$I_r = \frac{\Phi_r \omega}{|Z_r|}.$$

The mutual inductance is subsequently given as

$$M_Z = \frac{\lambda_c}{I_r} = \frac{\hat{B}_c A_c}{I_r}$$

where λ_c is the flux linkage in the coil.

Next we wish to calculate the coil current, I_c. Fig. 4 gives a band of driving coil inductance, L_c, in the form $L_c l/A_c N_c^2 \mu_{rod} \times 10^9$, as a function of l_c/l, (with distances in mm) from which the mean band value is taken. μ_{rod} is an effective core permeability, obtained from Fig. 5, from a knowledge of the actual core μ and the parameter l/D, where D is the diameter of the core. The equation for the input voltage is

$$v_c = R_c i_c + L_c \frac{di_c}{dt} + M_Z \frac{di_r}{dt}.$$

Filling in the sinusoidal expressions for each term and performing the differentiation, we get, (with coil phase shift ϕ_e)

$$\begin{aligned} V_c \sin(\omega t + \phi_c) = {} & R_c I_c \sin \omega t + L_c I_c \omega \cos \omega t \\ & - M_Z I_r \omega \sin(\omega t - \phi_r). \end{aligned}$$

Putting the peak amplitude value of I_r, as calculated from (2) (with $M = M_Z$) into the previous equation gives

$$\begin{aligned} V_c \sin(\omega t + \phi_c) &= R_c I_c \sin \omega t + L_c I_c \omega \cos \omega t \\ &\quad - \frac{M_Z^2 I_c \omega^2}{|Z_r|} \sin(\omega t - \phi_r) \\ &= \left[R_c - \frac{M_Z^2 \omega^2}{|Z_r|} \cos \phi_r \right] I_c \sin \omega t \\ &\quad + \left[L_c \omega + \frac{M_Z^2 \omega^2}{|Z_r|} \sin \phi_r \right] I_c \cos \omega t \\ &= R_c' I_c \sin \omega t + L_c' \omega I_c \cos \omega t \end{aligned}$$

where R_c' and L_c' are the equivalent values of an R–L series circuit and given as

$$R_c' = R_c - \frac{M_Z^2 \omega^2}{|Z_r|} \cos \phi_r, \qquad L_c' = L_c + \frac{M_Z^2 \omega}{|Z_r|} \sin \phi_r$$

so the coil current and phase shift is given as

$$I_c = \frac{V_c}{\sqrt{R_c'^2 + \omega^2 L_c'^2}}, \qquad \phi_c = \tan^{-1} \frac{L_c' \omega}{R_c'}.$$

Consequently K and so the constant force F at the point Z is obtained.

Calculating the resistance of the coil and ring can be easily done knowing the resistivity of the materials used, ρ, the cross sectional areas A and lengths h and given as $R = \rho h/A$. The inductance of the ring is made up of both an external L_e and an internal L_i contribution, respectively. The external contribution can be approximated to that of a ring in air (a

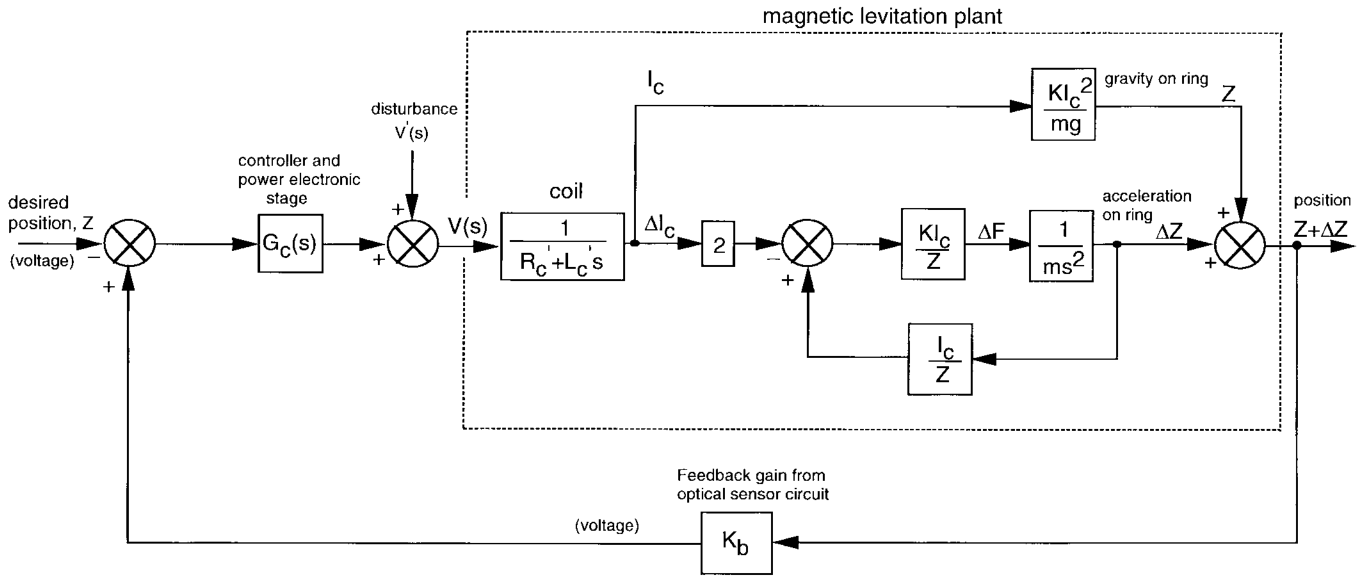


Fig. 6. Control block diagram.

minimum value), which is given as [9]

$$L_e = \frac{\mu_0 b(b-a)}{2}\int_0^{2\pi}\frac{\cos\xi\, d\xi}{\sqrt{b^2+(b-a)^2-2b(b-a)\cos\xi}}$$

where b is the mean radius of the ring and a is the radius of the wire making up the ring. This result is not integrable, as it stands. However, it may be reduced to the form

$$L_e = \mu_0\sqrt{b(b-a)}\left\{\left(\frac{2}{k}-k\right)\mathcal{K}(k)-\frac{2}{k}\mathcal{E}(k)\right\}$$

where $k^2 = 4b(b-a)/(2b-a)^2$ and

$$\mathcal{K}(k) = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\sin\theta}}$$

and

$$\mathcal{E}(k) = \int_0^{\pi/2}\sqrt{1-k^2\sin^2\theta}\, d\theta$$

and are elleptic integrals, for which tabulated values exist [10]. However, if it is assumed that $a \ll b$, then it can be expressed as

$$L_e \approx \mu_0 b\left(\ln\frac{8b}{a}-2\right).$$

The internal inductance of a length of wire per meter is given as $\mu_0/8\pi$. Then the internal inductance of the ring is given as

$$L_i = \mu_0\frac{2\pi b}{8\pi}, \quad \text{giving} \quad L_r = L_e + L_i.$$

TABLE I
PARAMETERS AND CALCULATED CIRCUIT VALUES

Given		Calculated	
l	9cm	R_r	0.485 $m\Omega$
l_c	2.5cm	L_r	0.142 μH
a	0.1079cm	Z_r	1.85 $m\Omega$
b	1.035cm	ϕ_r	$74.8°$
ω	12 566 rad/sec	I_r	1.01A
N_c	267	M_Z	17.08 μH
A_c	2.06cm^2	R_c	1.8 Ω
V_c	38V	L_c	14.1 mH
D	1.8cm	R_c'	8.3 Ω
m	0.643g	L_c'	12.2 mH
Z	0.7cm	ϕ_c	$86.9°$
		I_c	0.247A
		F	8.36mN

If the frequency of operation is high the appropriate parameters for the ring and coil can be corrected to take into account skin effect.

IV. Control Analysis

Let the force F be that force which counteracts the gravitation pull on the mass m of the ring, namely $F = mg$, where g is the gravitational constant. Consider now a small change due to a disturbance in the input voltage to the coil, V', creating a small change in the peak amplitude of the coil current I_c, namely ΔI_c, producing a small change in the average upward force F, namely ΔF. Let this force cause the ring to move up a distance ΔZ. (However, we assume K remains constant over this perturbation.) Then

$$F + \Delta F = K\frac{(I_c+\Delta I_c)^2}{(Z+\Delta Z)}.$$

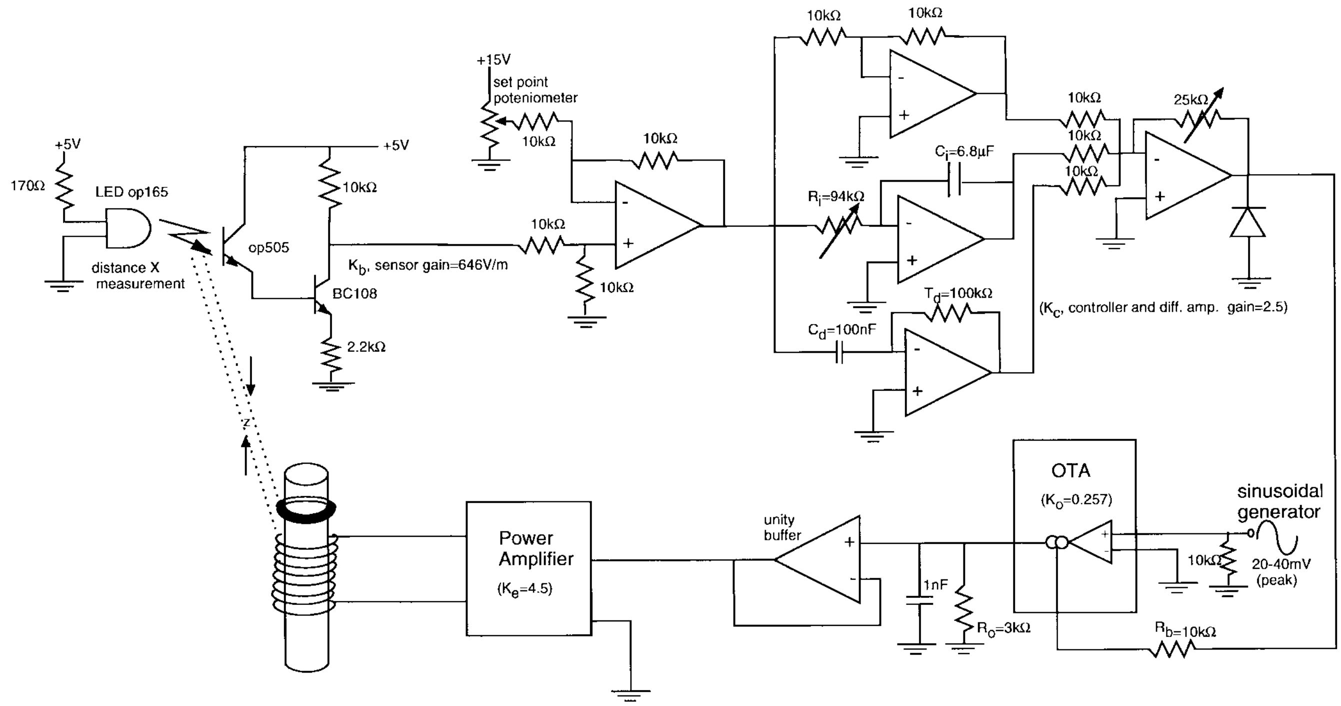


Fig. 7. Hardware circuit diagram.

Expanding out the bottom terms in a Taylor series and neglecting terms of squares, products and higher terms of perturbations, we obtain

$$F + \Delta F = K\,\frac{I_c^2}{Z} + 2K\,\frac{I_c}{Z}\,\Delta I_c - K\,\frac{I_c^2}{Z^2}\,\Delta Z$$

giving

$$\Delta F = 2K\,\frac{I_c}{Z}\,\Delta I_c - K\,\frac{I_c^2}{Z^2}\,\Delta Z.$$

This force causes the ring to accelerate in an upward direction and therefore in a direction of decreasing force and so

$$\Delta F = -m\,\frac{d^2\Delta Z}{dt^2}$$

giving

$$m\,\frac{d^2\Delta Z}{dt^2} = -2K\,\frac{I_c}{Z}\,\Delta I_c + K\,\frac{I_c^2}{Z^2}\,\Delta Z.$$

And on taking the Laplace Transform of this equation we arrive at the following transfer function:

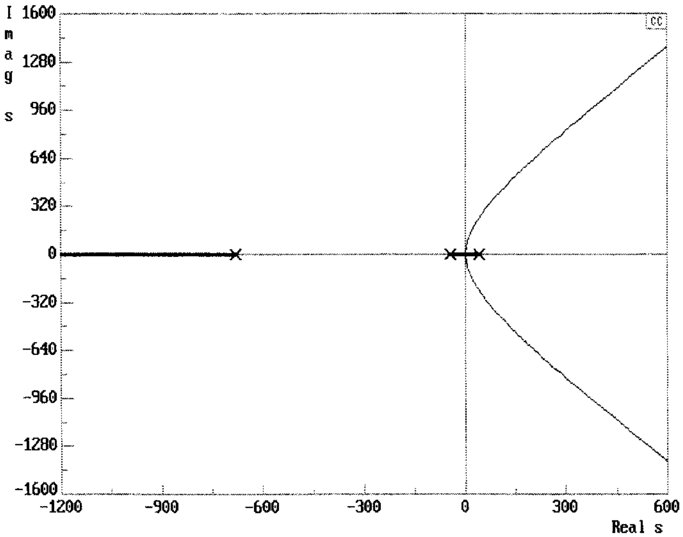
$$\frac{\Delta Z(s)}{\Delta I_c(s)} = -\frac{2KI_c/mZ}{s^2 - KI_c^2/mZ^2}. \tag{4}$$

And so we see that the incremental plant is unstable ($KI_c^2/mZ^2 > 0$). Given that we use a controller and a power electronic driver stage to the coil with transfer function $G_c(s)$ then the block diagram is as seen in Fig. 6 applies. Note the reverse action in calculating the error signal. This is in cognizance of the standard method of hardware implementation of a light detecting sensor arrangement (see implementation).

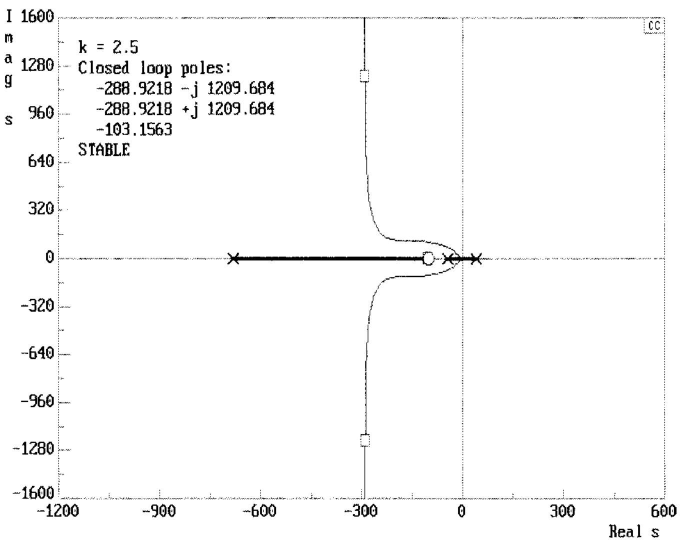
The steady-state effect of the desired position giving rise to a current I_c and a consequential levitation distance Z can be subtracted off when a proportional plus integral controller is used. The integral is required to wind out the steady-state error which a proportional controller would leave. The steady-state values (including the integral) are subtracted off the system giving the following closed-loop transfer function of the incremental plant, with disturbance voltage $V'(s)$ as the input shown in (5) at the bottom of the page.

It is quickly seen that if $G_c(s)$ is a proportional or a proportional plus derivative controller then the steady-state

$$\frac{\Delta Z(s)}{V'(s)} = \frac{2KI_c/(mZL'_c)}{(s^2 - KI_c^2/(mZ^2))\left(s + \dfrac{R'_c}{L'_c}\right) + G_c(s)2K_bKI_c/(mZL'_c)} \tag{5}$$



(a)



(b)

Fig. 8. (a) Root locus with proportional control and (b) root locus with proportional and derivative control.

error to an impulse disturbance in V' is zero. The characteristic equation for the system is given as

$$1+\frac{G_c(s)2K_bKI_c/(mZL_c')}{\left(s^2-\dfrac{KI_c^2}{mZ^2}\right)\left(s+\dfrac{R_c'}{L_c'}\right)}=0.$$

The last two expressions allow us to look at the root locus of the proposed controller and the closed-loop response of the system to various inputs [11].

V. Implementation and Results

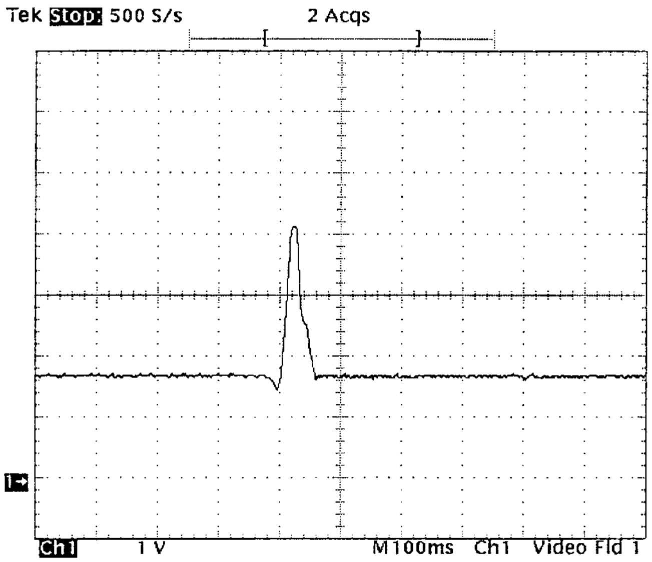
The laminated core was made from 263 soft iron ferrous rods of 1-mm diameter. The radius of the core $D/2$ was 0.9 cm, giving a fill factor of 81% for the core. The coil was made of copper SWG 24 wire consisting on 267 turns with a resistance of 1.8 Ω. The levitated ring was made of aluminum. The frequency of operation was picked as 2000 Hz. Table I shows the parameters given in the left-hand column and the calculated circuit values in the right-hand column.

For the geometry given the flux at the ring location is down to 0.6 that at the central core position. The ferrous rods were assumed to have a μ of at least 5000, giving from Fig. 4 a μ_{rod} of 35. Using that value in conjunction with Fig. 5 and taking a value of two with $l_c/l = 0.28$, the coil inductance was obtained, giving the calculated force of 8.36 mN. The actual force is 6.31 mN. If when calculating the coil inductance the top value in the band of values permissible was taken (see Fig. 5) a force of 5.267 mN results. Generally for different frequencies and heights it was found that the method of predicting the force agreed reasonably well with the actual force, once the correct value was picked from the band of values in Fig. 5.

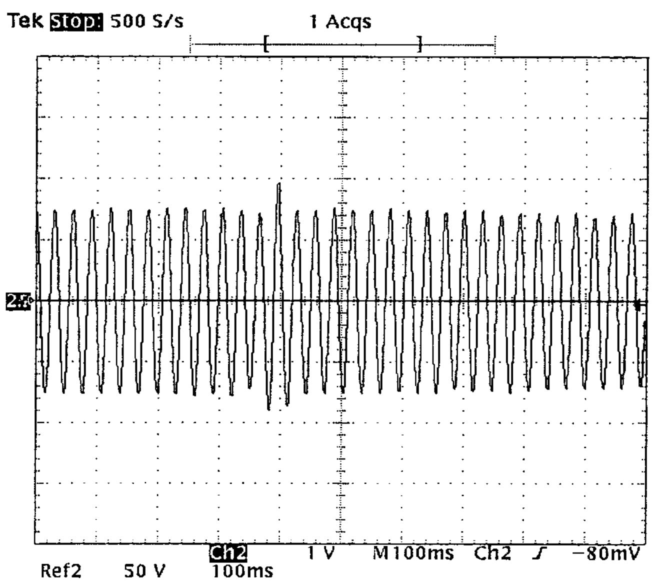
The circuit used to implement the levitation is as shown in Fig. 7, with dedicated set point hardware and integral controller. This avoids having a quiescent operating point, which effectively sets the set point and so the integral term. (Note though that in such arrangements the sensor signal is then the error signal [1].) The aluminum ring has a small paper like strut attached to it which affected the light to the detector, as the ring moved up and down, decreasing the sensor output as the ring moved up and so is in reverse operation to a normal control feedback signal. The output of the proportional plus integral controller, V_c, was feed to the bias input of an operational transconductance amplifier (OTA), CA3080, [12], [13], through an input series resistance R_b. The output current of the OTA was applied to a load resistance R_o. The input to the OTA is a sinusoidal voltage of constant amplitude V_i and frequency ω, respectively. The output voltage V_o of the OTA across R_o is given as $V_o = g_mV_iR_o$. However, g_m, the transconductance is directly proportional to the bias input current, V_c/R_b, and approximately equal to 20 times it, giving $V_o = 20(V_c/R_b)(V_iR_o)$ from which we see that the controller output voltage, V_c, amplitude modulates the ac voltage to the power amplifier in a linear manner. The gain of the sensor is $K_b = 646$ V/m. The gain of the OTA and power amplifier stage is 1.16. (An audio power amplifier was convenient to use because it could drive a low impedance load, as is the case with the coil/ring when ω is brought low to observe the frequency effect of the force.) The root locus equation of the system is

$$1+G_c(s)\,\frac{6.43\times 10^6}{(s+43.1)(s-43.1)(s+680)}=0.$$

When the controller is a proportional only controller the root locus is as shown in Fig. 8(a) and is seen to be unstable for all gain values. However if derivative action is added with $T_d = 0.01$ s, (see hardware circuit), then the root locus is as seen in Fig. 8(b) and if the poles are picked with the controller gain $K_c = 2.5$, the following closed-loop transfer function is obtained, as shown in (6) at the bottom of the next page, which is seen to be stable. The response of the sensor position to an impulse disturbance is shown in Fig. 9(a) and it is seen to be rejected. The calculated response is very similar. Fig. 9(b) shows the response of the output of the OTA to the impulse disturbance, changing the amplitude of the drive



(a)



(b)

Fig. 9. (a) Actual disturbance response of ring and (b) response of OTA to the disturbance.

voltage to overcome the disturbance. The response of the ring with only proportional control was seen to oscillate. Finally Fig. 10 shows a picture of the ring being levitated with the coil and core below it and the sensor mountings on the side.

VI. Discussion and Conclusion

Because the ring has a slightly larger inner diameter than the core and because of any out of balance masses in the ring or nonperfect symmetry in the field, the ring is inclined to rest against the core obliquely. This leads to slight distinction between the ring and core. A large radius in the ring was seen to diminish this problem. (However, the lateral stability of the ring is another problem.) Care has to be taken in the sensor area to avoid induced voltages due to the changing magnetic fields. The frequency of the ac signal to the coil has to be well above the natural frequencies of the system, namely $\omega_n = \sqrt{KI_c^2/mZ^2}$ (approximately 7 Hz for the implementation). Also as the frequency is brought low the instantaneous force effects, rather than averages over a cycle, would become prevalent. At high frequencies the skin effect and losses become a problem. (Skin depth in aluminum is 0.186 cm at 2000 Hz.)

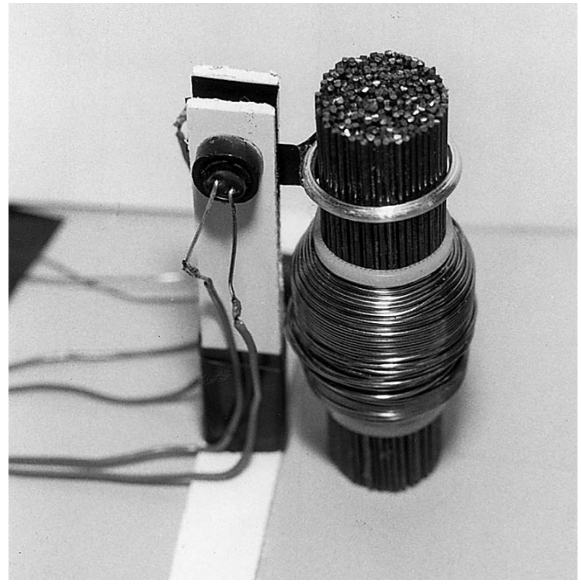


Fig. 10. Picture of the levitated ring, core with driving coil, and sensor mountings.

Various other ideas can easily be tried. For example frequency control using a voltage to frequency converter, or a sinusoidal pulse width modulation dispatch technique to the coil in an inverter arrangement, avoiding the losses associated with an audio power amplifier. Digital techniques with DSP devices can be used to implement various type of control strategies such as optimal or fuzzy logic controllers [14]. Simulation using magnetics packages should help in obtaining better values for inductances values [15].

The project proved a great success with the students who have undertaken it. In class both the jumping ring experiment (open loop) and levitation (closed loop) can be clearly shown, demonstrating the effect of electromagnetic induction (the ring can get quite warm) with an ac derived force and negative feedback control. It was found that the project motivated students to take up the area of machines, control, and power electronics, something which is necessary given the alluring nature of other areas in electrical and electronic engineering.

$$\frac{\Delta Z(s)}{V'(s)} = \frac{8607}{(s+43.1)(s-43.1)(s+680)+2.5(s+100)6.43\times 10^4} \tag{6}$$

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