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The jumping ring and Lenz's law—an analysis

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Abstract

Lenz's law is sometimes invoked to explain the behaviour of the jumping, or levitating, ring. This is shown to be incomplete, and an alternative explanation using Faraday's laws and circuit analysis is offered. This leads to the choice of optimum material and dimensions for the ring.

Introduction

A popular demonstration relating to induced currents is the jumping ring experiment. Here, a conducting non-magnetic ring is placed over the extended vertical core of a solenoid or demountable transformer. When ac power is applied to the solenoid the ring is thrown off or held in a state of levitation (from whichever limb it is placed on). A good practical description of this may be found in an article by Ford and Sullivan [1], and there is a more analytical article by Sumner and Thakkrar [2]. An explanation of this effect that is sometimes offered is that it happens because the induced current in the ring obeys Lenz's law, and as the fields are opposing the ring will be thrown off [3, 4].

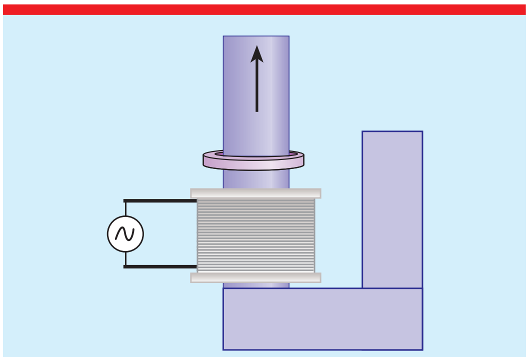

Figure 1. Demountable transformer and ring.

Qualitative analysis

The sole use of Lenz's law to explain this phenomenon is an incomplete and misleading explanation. In figure 1, the flux in the core is shown increasing in the vertical direction and coming out of the top of the core. Lenz's law states that the field produced by the current in the ring shall be such as to oppose the changing field generated in the core. In the case shown, the current in the ring will produce an opposing magnetic field and hence an upward force on the ring. However, as the cycle passes through its peak, the induced current (and hence the magnetic field) changes direction. The effect of the reversal of the induced magnetic field will be to pull the ring back downwards. This results in no net force on the ring when integrated over a whole cycle, so suggesting that the ring should remain resting on the coil former.

That this must be so can be seen if we complete the magnetic circuit as shown in figure 2. It is now obvious that as the ring forms a single turn secondary of a transformer it will have no tendency to move in any direction. So why does the ring fly off when the magnetic circuit is

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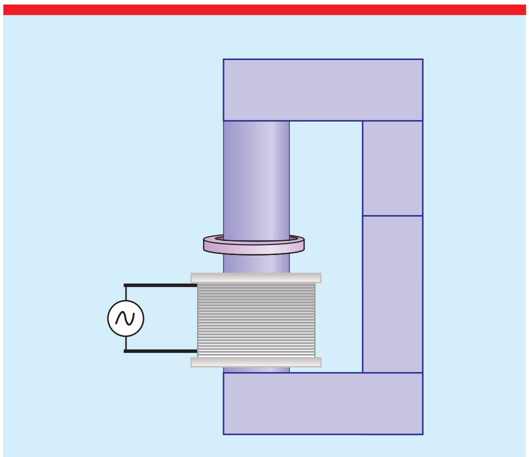

Figure 2. Magnetic circuit completed.

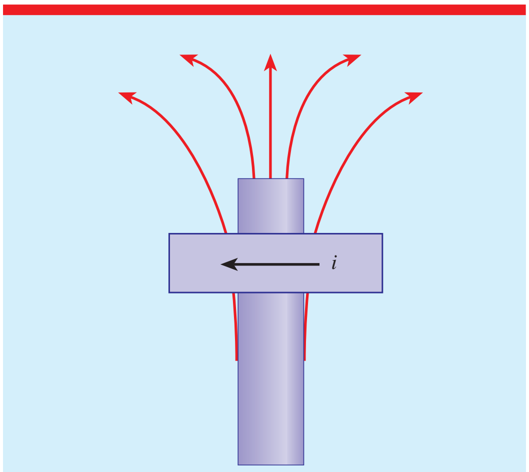

Figure 3. Field pattern.

incomplete? And why does it move upwards, not downwards, on whichever limb it is placed? Part of the reason is that when the magnetic circuit is broken there is a significant amount of flux leakage, giving rise to a horizontal component of flux (see figure 3 of [2]).

The vertical component of the changing flux gives rise to the circulating current in the ring by normal transformer action. This circulating current interacts with the vertical component of the field to give a force on the ring perpendicular to the current and the flux, as predicted by Faraday's left-hand rule. However, this force acts radially on the ring and has no vertical component.

Any vertical force must therefore be a result of the horizontal component of the leakage flux and the circulating current (i) in the ring (figure 3).

Assume a vertical sinusoidal varying field $\Phi_v \sin(\omega t)$. Then from Faraday's law the induced emf in the ring will be $-\Phi_v\omega\cos(\omega t)$, resulting in a current flow in the ring, the voltage induced in a single turn being equal to the rate of change of the flux linking the ring. If the ring is purely resistive then the current in the ring is $-\Phi_v\omega\cos(\omega t)/r$, where r is the resistance of a single turn. The vertical force F_v on the ring is then the product of the horizontal component of the flux, the current and the length of the conductor:

$$F_v = -B_h \sin(\omega t) \times \Phi_v\omega\cos(\omega t)/r \times \text{circumference of coil}$$

where $B_h \sin(\omega t)$ is the horizontal component of the field strength at the ring.

So

F_v is proportional to $B_h\Phi_v\omega\sin(2\omega t)/r$.

$$[\sin 2\theta \equiv 2\sin\theta\cos\theta]$$

Integrating this over a whole cycle gives a zero result, implying no net upward force and so no net movement of the ring! But it will vibrate at a frequency of twice the applied field.

However, this result assumes that the ring is purely resistive, which it is not. As it has some self-inductance, this results in the current in the ring lagging the applied field. The result of this is a net upward force sufficient to levitate the ring.

If the current in the ring lags the applied field by δ, then the upward force is proportional to

$$-\omega\sin(\omega t)\cos(\omega t + \delta),$$

i.e.

$$\tfrac{1}{2}[\omega\sin(2\omega t)\cos(\delta)] + \omega\sin^2(\omega t)\sin(\delta)$$

$$[\cos(A + B) \equiv \cos A\cos B - \sin A\sin B].$$

The first term integrated over a cycle is zero, but the second term is always positive, giving a net upward force on the ring and hence the lift that is observed.

This leads to the following conclusions:

(1) Lenz's law cannot be used to explain the motion of the ring.

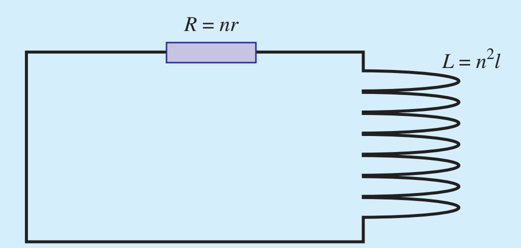

Figure 4. Equivalent circuit.

(2) Faraday's laws of induction, together with a recognition that the ring has some self-inductance (which leads to a phase shift of the induced current), is the correct explanation of the observed behaviour.

Quantitative analysis

What follows is a quantitative analysis leading to the best choice of material for the ring and its optimum dimensions.

Assume the ring has the following properties.

$$\sigma = \text{resistivity } (\Omega\text{ m})$$

$$\rho = \text{density (kg m}^{-3})$$

$$n = \text{number of turns}$$

$$D = \text{diameter of ring}$$

$$d = \text{diameter of wire forming ring}$$

$$l = \text{inductance of a single turn}$$

$$r = \text{resistance of a single turn}$$

From Faraday's law, the induced voltage in a single turn is given by

$$v = -\mathrm{d}(\Phi)/\mathrm{d}t.$$

If the exciting field is sinusoidal and the ring has n turns then the induced voltage v will be $-n\Phi_\mathrm{v}\omega\cos(\omega t)$, where $\Phi_\mathrm{v}\sin(\omega t)$ = vertical component of field.

The equivalent circuit of the coil is a resistor in series with an inductor (figure 4).

Standard circuit analysis gives the following result:

$$i = -n\Phi_\mathrm{v}\omega\cos(\omega t - \Psi)/Z,$$

where $Z = \sqrt{R^2 + \omega^2 L^2}$ and $\Psi = \tan^{-1}\frac{\omega L}{R}$, which in this case gives

$$Z = \sqrt{n^2r^2 + \omega^2 n^4 l^2} \text{ and } \Psi = \tan^{-1}\frac{n\omega l}{r}.$$

The upward force on the coil is equal to the product of current, horizontal component of the field strength and conductor length.

So

$$\begin{aligned} F\uparrow &= \frac{n\Phi_\mathrm{v}\omega\cos(\omega t - \Psi)}{Z} B_\mathrm{h}\sin(\omega t)\pi Dn \\ &= \frac{n^2 B_\mathrm{h}\Phi_\mathrm{v}\omega\sin(\omega t)}{Z}[\cos(\omega t)\cos\Psi \\ &\quad + \sin(\omega t)\sin\Psi]\pi D \\ &= \frac{n^2 B_\mathrm{h}\Phi_\mathrm{v}\omega}{Z}\left(\frac{\sin(2\omega t)}{2}\cos\Psi\right. \\ &\quad \left. + \sin^2(\omega t)\sin\Psi\right)\pi D. \end{aligned}$$

The term $\sin(2\omega t)$ integrates to zero over a cycle, so the net upward force is given by

$$F\uparrow = \frac{n^2 B_\mathrm{h}\Phi_\mathrm{v}\omega}{Z}\left(\sin^2(\omega t)\sin\Psi\right)\pi D.$$

So the peak upward force (in N) is equal to

$$B_\mathrm{h}\Phi_\mathrm{v}\pi D\frac{n^2\omega}{Z}(\sin\Psi).$$

From the identity $\sin\theta \equiv \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$ and using the expression for Ψ derived above i.e. $\Psi = \tan^{-1}\frac{n\omega l}{r}$, leads to

$$\sin\Psi = \frac{n\omega l}{\sqrt{r^2 + n^2\omega^2 l^2}}.$$

So $F_{\max}\uparrow = \frac{n^2\omega}{Z}(\frac{\omega nl}{\sqrt{r^2+n^2\omega^2l^2}}\pi D)B_\mathrm{h}\Phi_\mathrm{v}$, which reduces to $F_{\max}\uparrow = \frac{n^2\omega^2 l}{r^2+n^2\omega^2l^2}\pi D B_\mathrm{h}\Phi_\mathrm{v}$.

From Newton's second law, $F = ma$, it can be shown that the upward acceleration on the ring is given by $a = (F\uparrow/m) - g$, where

$$\begin{aligned} m &= \text{mass of ring} \\ &= \frac{n\pi^2 d^2 D\rho}{4}. \end{aligned}$$

Table 1. Metals listed in order of performance.

Material	Resistivity (σ) ($10^{-8}\Omega$ m)	Density (ρ) (kg m^{-1})	$\rho\sigma$
Sodium	4.70	970	4559
Calcium	3.4	1550	5270
Potassium	6.70	860	5762
Beryllium	3.30	1840	6072
Aluminium	2.82	2700	7614
Magnesium	5.00	1738	8690
Copper	1.72	8930	15 360
Silver	1.60	10500	16 800
Titanium	40.00	4500	180 000

So the maximum acceleration a of the ring occurs when $f(=F_{\max}\uparrow/m)$ is a maximum:

$$f=\frac{4\omega^2 nl}{\pi d^2\rho\left(r^2+n^2\omega^2 l^2\right)}B_{\mathrm{h}}\Phi_{\mathrm{v}}.$$

To determine the optimum number of turns, keeping all other variables constant, we differentiate f with respect to n and equate to zero. This leads to

$$n=\frac{r}{\omega l}.$$

So the optimum number of turns is directly related to the resistance of a single turn. The thicker the conductor the fewer turns required.

Assuming that the optimum number of turns is selected then the best material can be determined. Substituting this expression back into the expression for f leads to

$$f\propto\frac{\omega}{\rho r}$$

or

$$f\propto\frac{\omega}{\rho\sigma D},$$

so selecting a material with the smallest $\rho\sigma$ product will give the greatest upward acceleration to the ring.

Table 1 lists $\rho\sigma$ for some metals.

From table 1 we see that if the optimum number of turns is used then the best of the common metals to use is aluminium. This also explains the enhanced effect caused by cooling the ring in liquid nitrogen ([1] p 380, paragraph 3), as the resistivity of aluminium drops to 0.5×10^{-8} Ω m, giving a $\rho\sigma$ product of 1350.

Using the result for the optimum number of turns ($n=r/\omega l$), if we decide to use a single turn coil then for maximum acceleration we should ensure that $\omega l=r$.

We can now determine the ring's optimum dimensions. Consider a cylindrical aluminium ring of 48 mm diameter, 3 mm wall thickness and length x. From tables from various sources [5] the inductance of a single turn ring of 48 mm diameter is about 100×10^{-9} H. This value does not change very much with varying thickness, so for simplicity it will be assumed to be constant.

Then

$$2\pi\times50\times100\times10^{-9}=\frac{28.2\times10^{-9}\times\pi\times48\times10^{-3}}{3\times10^{-3}\times x\times10^{-3}},$$

which leads to $x=45$ mm, which is not too far from the 31 mm ring supplied in the Griffin demountable transformer kit, considering the approximations made.

Application of Lenz's law to other electromagnetic experiments

If Lenz's law cannot be used to explain the behaviour of the jumping ring it is reasonable to ask if it can be applied to other school demonstrations. The short answer is yes. Electromagnetic braking as exhibited by a non-magnetic conducting plate swinging in a magnetic field, or a disc spinning in a magnetic field, is readily explained by the application of Lenz's law (or Faraday's law of induction). In these cases a conductor is moving through a steady (non-uniform) magnetic field. Considering a closed path, it is seen that the flux varies with time, and so Lenz's law can be used to determine the resulting induced magnetic field, which will be seen to have the effect of opposing the motion of the pendulum or spinning disc.

Summary

Lenz's law alone cannot be used to explain the behaviour of a jumping or levitating ring. The correct explanation requires an application of Faraday's law together with circuit analysis of the system. This then leads to the reasons for selecting an aluminium ring of a particular size.

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[4] University of Glasgow website http://www.physics.gla.ac.uk/~kskeldon/PubSci/exhibits/E5/

[5] For example, go to http://emclab.umr.edu/index.html and select Inductance Calculator.

Mike Bostock-Smith has an electrical engineering degree from University College London and is a Chartered Engineer. His early career in the Post Office/BT involved the commissioning of microwave radio links for the distribution of 625 line colour TV signals and the planning of early data networks. At the time of retirement he was leading research into the detection, propagation and location of the cause of errors in the digital network. He now helps in the physics department of a local school. He is also a magistrate and a Parish Councillor.